Atmospheric Effects on the Meson Intensity Recorded by the International Standard Cube at a High Latitude Station

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Abstract

Cosmic ray data and upper air data recorded at Murchison Bay during the IGY have been used to estimate the atmospheric effects on the sea level meson intensity recorded by the standard cube. The pressure corrected nucleonic intensity has been used to eliminate variations due to fluctuations in the primary intensity. It is shown that the regression equations proposed by Duperier and Dorman lead to practically identical corrections for the atmospheric effects.

1. Introduction

The IGY plans for the cosmic ray investigations included a recommendation to use cubical telescopes with 10 cm lead as standard instruments for recording of the meson intensity at sea level. The plans have now been realized: meson data recorded by standard cubes are available for a net-work of stations covering all the world. To simplify comparisons between the results based on these data and presented by different authors, it is desirable that the recorded data should be corrected for atmospheric influences in the same way. Many theoretical and experimental investigations of the atmospheric effects on the sea level meson intensity have been published, especially during the last ten years. A general survey with detailed references is given e.g. by BACHELET and CONFORTO (1956).

The work carried out by the Swedish-Finnish-Swiss research expedition at Murchison Bay during the IGY (LILJEQUIST, 1959) has made it possible for us to present this contribution to the problem of adequate corrections. We are dealing exclusively with the meson component recorded by the standard cube at Murchison Bay (geographical coordinates: 80°03′ N, 18°18′ E). In a following paper we will treat cosmic ray recorders with other characteristics as well as data from the two cosmic ray stations in Sweden (Uppsala and Kiruna).

The cosmic ray data on which the present investigation is based, were recorded by a duplex cubical meson telescope with an iron shield equivalent to 10 cm lead (normal counting rate: 96,000 counts/hour) and a duplex neutron monitor of the Simpson type (normal counting rate: 25,000 counts/hour). The ground pressure was measured continually with a barograph and every third hour with a mercury barometer. The aerological measurements were made with the Väisälä sounding system.

2. Details of the Analysis

The period for which both cosmic ray data and aerological data are available, starts with September 1957 and covers eleven full months, as the radiosonde flights ceased in the middle of August 1958. Only the first seven months have been used to determine the atmospheric coefficients presented in the following sections. The reasons are that the ground pressure was unusually stable in the following two months, April and May 1958, and that some months had to be reserved for tests of the calculated coefficients.

Only the results from the ordinary radiosonde flights at 00 and 12 GMT have been used. In the tabulated results from 390 flights it has been necessary to interpolate fourteen 200 mb values and fifty-three 100 mb values. The interpolations were made on the assumption that the temperature gradient of an atmospheric layer between two fixed pressure surfaces, changes linearly with time.

The above-mentioned period, 1 September 1957 to 31 March 1958, has been analysed month by month. The number of days available for analysis in each month, varies between 23 and 29 (Table 11). Lacking aerological and cosmic ray data have caused the exclusion of 24 days from a total of 212 days.

Daily means of the following variables have been used:

Z = recorded meson intensity
N_1 = nucleonic intensity corrected for pressure changes with −0.737 per cent/mb
N_2 = nucleonic intensity corrected with −0.700 per cent/ mb
P = ground pressure
H_{200} = height of the 200 mb level
H_{100} = height of the 100 mb level
T_{200} = temperature at the 200 mb level
T_{150} = temperature at the 150 mb level
ΔH = thickness of the layer between 100 mb and 200 mb

Three more variables U, V, and W are introduced in Section 5. They are essentially linear combinations of simple height and temperature variables.

The quantities contained in the definitions of N_1 and N_2 require some remarks. The coefficient −0.737 per cent/mb is the pressure coefficient adopted for the nucleonic component by the Cosmic Ray Group at Uppsala. Unpublished investigations by one of the authors seem to indicate that this should be considered as an upper limit and that the true value is somewhat smaller (LINDGREN, 1961). The coefficient −0.700 per cent/mb has therefore been introduced as a plausible lower limit.

Conversions between height and temperature changes have been made according to the relation

$$\delta h = \frac{R_0}{g} \ln \frac{p_1}{p_2} \delta T \qquad (1)$$

where h is the height and T the mean temperature of a layer between the surfaces defined by the pressures p_1 and p_2, $p_1 > p_2$. R_0 is the specific gas constant of air and g the acceleration due to gravity. Equation (1) is easily deduced from the ideal gas law and the relation

$$-\delta p = \varrho g \delta h \qquad (2)$$

where $\varrho = \varrho(p, T)$ is the density of air.

The dependence of Z on different linear combinations of the variables described above has been studied by least squares regression analysis (WILLIAMS, 1959). We have thus calculated the coefficients α_i in a series of regression equations, all of the same type:

$$\delta Z = \sum_i \alpha_i \, \delta X_i \qquad (3)$$

Most of the calculations have been carried out on BESK, an electronic computer in Stockholm. The regression coefficients are presented in Tables 1—11 and Fig. 1. The errors listed together with the regression coefficients are standard errors calculated by conventional methods (WILLIAMS, 1959). The errors of the mean values, listed in the last row of each table, are the standard deviations of the monthly means. R denotes the multiple correlation coefficient.

3. Consideration of the Primary Variations

The period studied is characterized by strong solar activity and large fluctuations of the primary intensity. At the end of 1957 the solar activity as revealed by the relative sunspot numbers was in fact higher than ever since 1750. Under such conditions it is not advisable to estimate the atmospheric effects without considering the primary intensity variations, especially if the periods analysed are short as in this investigation. If no variable representing the primary variations is included in the regression equations, these variations are interpreted as atmospheric. The consequence is an incorrect estimation of especially the smaller atmospheric effects. This can be seen in Table 1, which is based on a regression relation of the Duperier type

$$\delta Z = \alpha_{11} \, \delta P + \alpha_{12} \, \delta H_{200} + \alpha_{13} \, \delta T_{200} \qquad (4)$$

Two of the height coefficients listed are positive and the mean value is as small as −1.13 per cent/km and not significantly different

Table 1. Regression coefficients calculated according to Equation (4)

Period	α_{11} partial pressure coefficient %/mb	α_{12} partial height coefficient %/km	α_{13} partial temperature coefficient %/°C	R
September 1957	-0.072 ± 0.030	-5.02 ± 5.86	-0.102 ± 0.105	0.563
October 1957	-0.151 ± 0.033	0.96 ± 1.96	0.006 ± 0.076	0.827
November 1957	-0.142 ± 0.044	-0.22 ± 2.67	0.202 ± 0.170	0.811
December 1957	-0.130 ± 0.015	-2.19 ± 2.00	-0.026 ± 0.034	0.887
January 1958	-0.135 ± 0.010	-3.66 ± 0.62	-0.083 ± 0.026	0.968
February 1958	-0.122 ± 0.011	-0.42 ± 0.74	-0.114 ± 0.040	0.955
March 1958	-0.155 ± 0.023	2.61 ± 2.58	0.147 ± 0.096	0.855
Mean values	-0.130 ± 0.028	-1.13 ± 2.66	0.030 ± 0.128	0.838

from zero. The pressure coefficient for September 1957 is abnormally low. The inclusion of the variable N_1 in the regression equation

$$\delta Z = \alpha_{21}\,\delta N_1 + \alpha_{22}\,\delta P + \alpha_{23}\,\delta H_{200} + \alpha_{24}\,\delta T_{200} \quad (5)$$

makes the height coefficients and their mean value, -4.81 per cent/km, agree better with expected values (Table 2).

Reasonable objections could be made against the use of the corrected nucleonic intensity to reflect the variations of the primary radiation associated with the meson component. An alternative in this case would be the assumption of a constant primary radiation but the consequences just demonstrated in Table 1 make this alternative unacceptable.

It is however important that the variable chosen to represent the primary variations, is as true as possible. Table 3 which is based on

Table 2. Regression coefficients calculated according to Equation (5).

Period	α_{22} partial pressure coefficient % /mb	α_{23} partial height coefficient % /km	α_{24} partial temperature coefficient % /°C	R
September 1957	-0.128 ± 0.005	-4.98 ± 0.99	0.026 ± 0.018	0.991
October 1957	-0.133 ± 0.011	-3.68 ± 0.69	-0.049 ± 0.024	0.985
November 1957	-0.141 ± 0.020	-6.07 ± 1.38	-0.043 ± 0.082	0.966
December 1957	-0.145 ± 0.007	-4.71 ± 0.92	-0.087 ± 0.016	0.980
January 1958	-0.123 ± 0.006	-4.19 ± 0.35	-0.077 ± 0.015	0.991
February 1958	-0.154 ± 0.007	-1.87 ± 0.44	-0.016 ± 0.025	0.988
March 1958	-0.134 ± 0.011	-8.20 ± 1.75	-0.072 ± 0.052	0.970
Mean values	-0.137 ± 0.011	-4.81 ± 1.98	-0.045 ± 0.040	0.982

Table 3. Regression coefficients calculated according to Equation (6)

Period	α_{32} partial pressure coefficient % /mb	α_{33} partial height coefficient % /km	α_{34} partial temperature coefficient % /°C	R
September 1957	-0.113 ± 0.005	-4.97 ± 0.99	0.026 ± 0.018	0.991
October 1957	-0.116 ± 0.011	-3.67 ± 0.69	-0.049 ± 0.024	0.985
November 1957	-0.123 ± 0.020	-6.09 ± 1.38	-0.045 ± 0.082	0.966
December 1957	-0.129 ± 0.006	-4.72 ± 0.92	-0.087 ± 0.016	0.980
January 1958	-0.106 ± 0.007	-4.18 ± 0.35	-0.078 ± 0.015	0.991
February 1958	-0.142 ± 0.006	-1.87 ± 0.44	-0.016 ± 0.025	0.988
March 1958	-0.118 ± 0.012	-8.17 ± 1.75	-0.071 ± 0.052	0.970
Mean values	-0.121 ± 0.012	-4.81 ± 1.97	-0.046 ± 0.040	0.982

the regression equation

$$\delta Z = \alpha_{31}\,\delta N_2 + \alpha_{32}\,\delta P + \alpha_{33}\,\delta H_{200} + \alpha_{34}\,\delta T_{200} \quad (6)$$

shows, compared with Table 2, that the pressure coefficient calculated for the meson component is very sensitive to changes in the pressure coefficient used to correct the recorded nucleonic intensity. The height and temperature coefficients are not affected. The question which pressure coefficient should be used to correct the nucleonic component will be treated by one of the authors in a following paper (LINDGREN, 1961).

4. A Comparison between Two Models of the Duperier Type

Two models of the simple Duperier type (DUPERIER, 1949) are compared in Tables 2 and 4. The first model, represented by Equation (5), takes no account of the atmosphere above the 200 mb level, the second corresponding to

$$\delta Z = \alpha_{41}\,\delta N_1 + \alpha_{42}\,\delta P + \alpha_{43}\,\delta H_{100} + \alpha_{44}\,\delta \Delta H \quad (7)$$

includes the 100 mb level.

The mean values obtained for the 100 mb model are compared with other experimental results and Trefall's theoretical coefficients (TREFALL, 1956) in Table 5. Our pressure coefficient is recalculated for the case of a nucleonic pressure coefficient = −0.720 per cent/mb, a generally adopted value. The agreement between the listed coefficients is good with one exception: the theoretical height coefficient is 50 per cent larger than the experimental values.

The 200 mb model gives the same pressure coefficient as the 100 mb model. The height coefficient is somewhat larger, which could be expected. A negative value of the temperature coefficient is not unreasonable as the reference level is lower than the mean level of meson production (TREFALL, 1955).

A critical examination of the errors and the multiple correlation coefficients seems to indicate that it is indifferent which of the two models is used. The 100 mb model has also been examined with another temperature variable. In Table 4 we have used the thickness of the layer between 100 mb and 200 mb as a

Table 4. Regression coefficients calculated according to Equation (7)

Period	α_{42} partial pressure coefficient % /mb	α_{43} partial height coefficient % /km	α_{44} partial temperature coefficient % /km	% /° C	R
September 1957	−0.127 ± 0.005	−5.19 ± 1.03	7.26 ± 2.02	0.145 ± 0.040	0.991
October 1957	−0.135 ± 0.010	−3.29 ± 0.69	−0.98 ± 1.99	−0.020 ± 0.040	0.986
November 1957	−0.136 ± 0.020	−5.78 ± 1.30	5.18 ± 4.74	0.104 ± 0.095	0.966
December 1957	−0.147 ± 0.006	−4.47 ± 0.85	−0.73 ± 0.97	−0.015 ± 0.019	0.982
January 1958	−0.124 ± 0.004	−3.69 ± 0.29	−0.97 ± 0.76	−0.019 ± 0.015	0.995
February 1958	−0.153 ± 0.007	−1.79 ± 0.43	0.39 ± 1.19	0.008 ± 0.024	0.989
March 1958	−0.133 ± 0.011	−7.90 ± 1.56	3.48 ± 2.13	0.070 ± 0.043	0.970
Mean values	−0.136 ± 0.010	−4.59 ± 1.96	1.95 ± 3.36	0.039 ± 0.067	0.983

Table 5. Regression coefficients for the 100 mb Duperier model. Meson telescopes with 10 cm lead.

Reference	Recording period	Opening angle	Partial pressure coefficient % /mb	Partial height coefficient % /km	Partial temperature coefficient % /° C
Experimental results:					
Trumpy-Trefall 1953	1950—1952	±59°x ±71°	−0.127 ± 0.008	−4.45 ± 0.53	0.036 ± 0.021
Dawton-Elliot 1953	1951—1952	±58°x ±58°	−0.125 ± 0.012	−4.00 ± 0.43	0.056 ± 0.018
Lindgren-Lindholm 1960	1957—1958	±45°x ±45°	−0.129 ± 0.010	−4.59 ± 1.96	0.039 ± 0.067
Theoretical values:					
Trefall 1956		zenith direction	−0.12	−6.7	0.06

measure of the mean temperature of that layer. In Table 6 the temperature at the 150 mb level is used instead according to

$$\delta Z = \alpha_{51}\,\delta N_1 + \alpha_{52}\,\delta P + \alpha_{53}\,\delta H_{100} + \alpha_{54}\,\delta T_{150} \quad (8)$$

The mean values of the atmospheric coefficients are not affected.

If we now take into account that the aerological data referring to the 200 mb model are more reliable from a statistical point of view than those of the 100 mb level (TREFALL and NORDÖ, 1959) and further that information about the lower level is obtained in many cases where data are lacking for higher levels, then we have to decide upon the 200 mb model as the more satisfactory.

As the temperature coefficient is of less importance than the pressure and height coefficients, as far as the magnitude of the corresponding corrections are concerned, we have tried a 200 mb model without temperature variable (Table 7) according to the relation

$$\delta Z = \alpha_{61}\,\delta N_1 + \alpha_{62}\,\delta P + \alpha_{63}\,\delta H_{200} \quad (9)$$

A comparison with Table 2 shows no remarkable change of the pressure coefficient but the height coefficient has been reduced by more than 10 per cent in qualitative agreement with theoretical expectance (TREFALL, 1956). The scattering of the monthly values is about the same in both cases.

5. An Examination of the Dorman Model

According to Dorman's theory the variations in the meson intensity at some level of recording x_0 may be represented by the relation

$$\delta Z = \alpha \delta P + \int_0^{x_0} W_T(x) \delta T(x) dx \quad (10)$$

where $\delta T(x)$ represents the temperature variation at the atmospheric depth x and $W_T(x)$ is a function describing the contribution from the various layers of air to the production of the temperature effect (DORMAN, 1957). The function $W_T(x)$, usually referred to as the density of the temperature coefficient, can

Table 6. Regression coefficients calculated according to Equation (8)

Period	α_{52} partial pressure coefficient % /mb	α_{53} partial height coefficient % /km	α_{54} partial temperature coefficient % /°C	R
September 1957	−0.126 ± 0.006	−4.55 ± 1.10	0.104 ± 0.040	0.989
October 1957	−0.130 ± 0.010	−3.57 ± 0.69	0.007 ± 0.039	0.986
November 1957	−0.135 ± 0.020	−5.74 ± 1.30	0.106 ± 0.096	0.966
December 1957	−0.147 ± 0.006	−4.53 ± 0.85	−0.012 ± 0.019	0.982
January 1958	−0.123 ± 0.004	−3.75 ± 0.25	−0.016 ± 0.011	0.995
February 1958	−0.155 ± 0.007	−1.90 ± 0.44	0.025 ± 0.036	0.989
March 1958	−0.133 ± 0.012	−7.98 ± 1.67	0.059 ± 0.047	0.969
Mean values	−0.136 ± 0.012	−4.57 ± 1.91	0.039 ± 0.052	0.982

Table 7. Regression coefficients calculated according to Equation (9)

Period	α_{62} partial pressure coefficient % /mb	α_{63} partial height coefficient % /km	R
September 1957	−0.126 ± 0.005	−4.73 ± 0.99	0.990
October 1957	−0.121 ± 0.009	−3.84 ± 0.72	0.982
November 1957	−0.134 ± 0.014	−5.73 ± 1.19	0.966
December 1957	−0.137 ± 0.009	−2.54 ± 1.21	0.954
January 1958	−0.125 ± 0.008	−4.85 ± 0.48	0.979
February 1958	−0.155 ± 0.007	−1.92 ± 0.42	0.988
March 1958	−0.141 ± 0.010	−6.15 ± 0.95	0.968
Mean values	−0.134 ± 0.012	−4.25 ± 1.58	0.975

be read from a series of curves. One of them applies to the international cube at sea level and is reproduced in Fig. 1 as curve A. It is however very difficult to make an experimental test of it by means of regression analysis, as it would be necessary to include a large number of temperature variables, which furthermore display a considerable intercorrelation.

In order to make a test we divided the atmosphere between ground and the 100 mb level into seven layers using the surfaces of 850 mb, 700 mb, 500 mb, 400 mb, 300 mb, and 200 mb as separating levels. In a first calculation we started from Dorman's curve of the density of the temperature coefficient and formed a variable W (not to be confused with W_T) which except for an arbitrary constant is defined by

$$\delta W = \Sigma c_i \delta T_i \, x_i \tag{11}$$

where c_i = the mean value for the ith layer of the density of the temperature coefficient, x_i = the difference in atmospheric depth between the lower and upper boundaries of the ith layer and δT_i = the deviation of the daily mean temperature of the ith layer from some standard temperature. A calculation of the coefficients in the equation

$$\delta Z = \alpha_{71} \, \delta N_1 + \alpha_{72} \, \delta P + \alpha_{73} \, \delta W \tag{12}$$

ought to give a regression coefficient $\overline{\alpha_{73}}$ not differing very much from 1. Table 8 shows that the resulting mean value, $\overline{\alpha_{73}} = 0.65 \pm 0.24$, is significantly smaller than 1. The corresponding pressure coefficient is about 25 per cent larger than the coefficients obtained from the Duperier models (Tables 2 and 4).

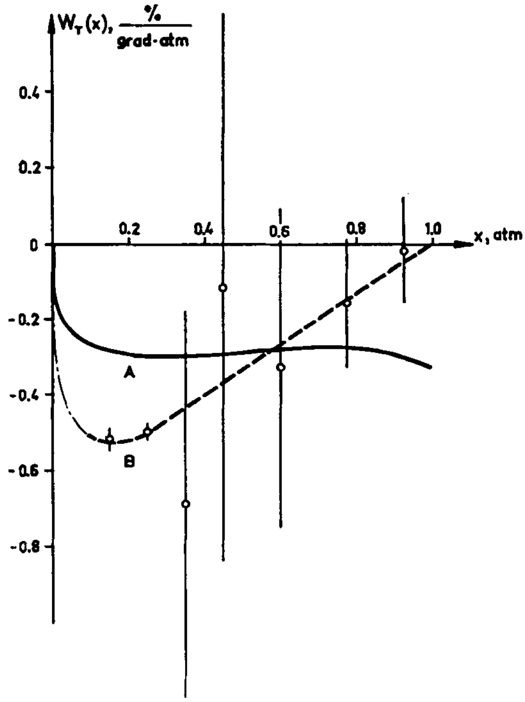

Fig. 1. Curve A gives the density of the temperature coefficient according to Fig. 41 in DORMAN 1957. The plotted points are experimentally determined in the present investigation (Section 5). Curve B is drawn to fit the experimental points in the region between ground and the 100 mb level.

For comparison the 100 mb Duperier model has been analysed in a similar way. The variables H_{100} and ΔH were transformed into a new variable U defined by

$$\delta U = b_1 \, \delta H_{100} + b_2 \, \delta \Delta H \tag{13}$$

The mean values of Table 4 ($\overline{\alpha_{43}} = -4.59$

Table 8. Regression coefficients calculated according to Equation (12)

Period	α_{72} partial pressure coefficient % /mb	α_{73}	R
October 1957	-0.155 ± 0.007	0.66 ± 0.14	0.980
November 1957	-0.179 ± 0.020	0.86 ± 0.21	0.959
December 1957	-0.159 ± 0.012	0.44 ± 0.20	0.955
January 1958	-0.168 ± 0.008	0.83 ± 0.09	0.977
February 1958	-0.171 ± 0.007	0.28 ± 0.06	0.988
March 1958	-0.185 ± 0.010	0.83 ± 0.12	0.971
Mean values	-0.170 ± 0.011	0.65 ± 0.24	0.972

per cent/km and $\overline{\alpha_{44}} = 1.95$ per cent/km) were used as weighting factors, b_1 and b_2. According to Table 9, thus based on

$$\delta Z = \alpha_{81}\,\delta N_1 + \alpha_{82}\,\delta P + \alpha_{83}\,\delta U \qquad (14)$$

the scattering of the monthly values is about the same as in the Dorman analysis above. R is somewhat larger.

The value $\overline{\alpha_{73}} = 0.65$ means that perfect agreement with the experimental results ($\overline{\alpha_{73}} = 1.00$) would be obtained in the case examined here if a new temperature curve were constructed from the original by multiplying each ordinate by 0.65. That would of course not affect the pressure coefficients or the multiple correlation coefficients. It is quite clear that a curve of arbitrary shape could be changed in a similar way to give an $\overline{\alpha_{73}}$ value $= 1.00$. In the selection of the most reasonable shape one should take into consideration the sample to sample scattering of the calculated coefficients and the agreement with existing physical theories.

To get an indication of another physically justified temperature curve we treated a regression equation with seven variables T_1, $T_2, \ldots, T_7$ to represent the mean temperatures of the layers mentioned above. As three more variables, Z, N_1 and P, were necessary we had to deal with ten variables in the regression equation:

$$\delta Z = \beta_1\,\delta N_1 + \beta_2\,\delta P + \beta_3\,\delta T_1 + \beta_4\,\delta T_2 + \ldots + \beta^9\,\delta T_7 \qquad (15)$$

The resulting mean value of the pressure coefficients was -0.170 ± 0.016 per cent/mb in good agreement with the first calculation (Table 8). The mean values of the temperature coefficients obtained are plotted in Fig. 1 together with Dorman's curve of the density of the temperature coefficient. As could be expected most of the points have very large errors, the points referring to 150 mb and 250 mb being exceptions.

Assuming curve B in Fig. 1 (essentially a straight line drawn to fit the experimental temperature coefficients) to be an acceptable representation of the temperature effects, we now formed a variable V in the same way as W but using curve B instead of curve A to obtain the weighting factors c_i in Equation (11). Table 10, based on

$$\delta Z = \alpha_{91}\,\delta N_1 + \alpha_{92}\,\delta P + \alpha_{93}\,\delta V \qquad (16)$$

reveals, compared with Table 8, a somewhat larger scattering of the month to month values of the pressure coefficient but also larger values of R. The mean value of α_{93} implies perfect agreement between curve B and the experimental results.

The deviation of the points for 150 mb and 250 mb from the Dorman curve might partly be explained by the exclusion of the air layer above 100 mb from the calculations.

The differential spectrum of π-meson generation can be represented by

$$f_\pi(E, x, \varphi) = AE^{-(2+\gamma)}g(x, \varphi) \qquad (17)$$

where $E =$ the energy of the π-meson, $x =$ the atmospheric depth, $\varphi =$ the zenith angle and $A =$ constant (DORMAN, 1957). γ is expected to lie between 0 and 1. Dorman's temperature curve seems to be drawn for $\gamma = 0.5$. A higher value of γ would make the curve agree better with our points for 150 mb and 250 mb (DORMAN, 1957). Also the pressure effect is strongly dependent on γ. A sensitivity curve for the cubical telescope (e.g. Fig 21 in DORMAN 1957) shows that the effective zenith angle is

Table 9. Regression coefficients calculated according to Equation (14)

Period	α_{82} partial pressure coefficient %/mb	α_{83}	R
October 1957	-0.127 ± 0.008	0.82 ± 0.13	0.985
November 1957	-0.143 ± 0.015	1.29 ± 0.27	0.965
December 1957	-0.140 ± 0.007	1.07 ± 0.21	0.974
January 1958	-0.123 ± 0.005	0.96 ± 0.06	0.991
February 1958	-0.153 ± 0.007	0.40 ± 0.08	0.989
March 1958	-0.133 ± 0.010	1.73 ± 0.25	0.971
Mean values	-0.137 ± 0.011	1.04 ± 0.45	0.979

about 20°. The pressure coefficient corresponding to that angle is -0.185 per cent/mb for $\gamma = 1$ and -0.095 per cent/mb for $\gamma = 0$ (Fig. 43 in DORMAN 1957). The dependence on γ is linear. The results presented in Tables 8 and 10 would then indicate a γ value >0.8. Now it is possible that the nucleonic intensity data represented by the variable N_1 are overcorrected (Section 2), which would mean that the pressure coefficients calculated for the meson component are too large (Section 3). A recalculation of the coefficients in Table 8 based on nucleonic intensity data corrected with -0.720 per cent/mb, does not affect the α_{73} values but gives for the pressure effect the mean value $\overline{\alpha_{72}} = -0.163$ per cent/mb. That corresponds to $\gamma = 0.7$.

6. Effects of Random Errors in the Aerological Data

TREFALL and NORDÖ have examined the influence of random errors of measurement on the coefficients obtained in correlation and regression analyses (TREFALL and NORDÖ, 1959). They propose that observed discrepancies between theoretical and experimental results in the estimation of atmospheric effects might be due to random errors in the upper air data.

We have examined the influence on the results obtained for the 200 mb Duperier model (Table 2) with the following assumptions: 1) the errors of H_{200} and T_{200} are random with respect to the true values of the variables; 2) the errors are not correlated with each other; 3) the error variances of H_{200} and T_{200} are 2,500 m² and 4.00 (°C)² respectively. According to TREFALL and NORDÖ the random errors of measurement would then disturb the calculations only by increased values of the square sums $\Sigma(\delta H_{200})^2$ and $\Sigma(\delta T_{200})^2$ if the samples considered were infinitely large. In spite of the small samples considered here, containing from 23 to 29 members, we assume that the influence of the random errors can be eliminated simply by a correction of the two square sums mentioned above. The corrections have been introduced in three steps: a) only $\Sigma(\delta H_{200})^2$ has been corrected, b) only $\Sigma(\delta T_{200})^2$ has been corrected, c) both square sums have been corrected. The results are given in Table 11

Table 11. Regression coefficients calculated according to Equation (5).

Corrections made for the influence of random errors of measurement (Section 6)
n = number of days
C_1 = ratio between total variance and error variance of H_{200}
C_2 = ratio between total variance and error variance of T_{200}
a) only $\Sigma(\delta H_{200})^2$ corrected
b) only $\Sigma(\delta T_{200})^2$ corrected
c) both sums corrected
d) no corrections introduced

Period	n	C_1	C_2	α_{22} partial pressure coefficient %/mb a)	b)	c)	d)	α_{23} partial height coefficient %/km a)	b)	c)	d)	α_{24} partial temperature coefficient %/°C a)	b)	c)	d)
September 1957	28	2.2	4.0	−0.119	−0.128	−0.120	−0.128	−10.26	−5.08	−10.61	−4.98	0.043	0.036	0.061	0.026
October 1957	29	27.0	9.8	−0.129	−0.136	−0.132	−0.133	−4.05	−3.65	−4.01	−3.68	−0.048	−0.060	−0.058	−0.049
November 1957	23	16.3	4.1	−0.145	−0.150	−0.159	−0.141	−6.76	−6.52	−7.51	−6.07	−0.062	−0.099	−0.149	−0.043
December 1957	28	10.9	23.5	−0.145	−0.146	−0.145	−0.145	−5.37	−4.85	−5.54	−4.71	−0.092	−0.093	−0.098	−0.087
January 1958	28	45.9	15.0	−0.122	−0.123	−0.122	−0.123	−4.31	−4.13	−4.25	−4.19	−0.075	−0.084	−0.082	−0.077
February 1958	26	25.7	3.5	−0.153	−0.153	−0.153	−0.154	−2.05	−1.83	−2.01	−1.87	−0.014	−0.028	−0.025	−0.016
March 1958	26	13.6	5.3	−0.104	−0.112	−0.248	−0.134	−17.04	−14.12	23.83	−8.20	−0.296	−0.282	0.937	−0.072
Mean values				−0.131	−0.135	−0.154	−0.137	−7.12	−5.74	−1.44	−4.81	−0.084	−0.087	0.084	−0.045

Table 10. Regression coefficients calculated according to Equation (16)

Period	α_{92} partial pressure coefficient %/mb	α_{93}	R
October 1957	-0.154 ± 0.006	0.78 ± 0.13	0.985
November 1957	-0.185 ± 0.019	1.22 ± 0.25	0.967
December 1957	-0.178 ± 0.011	1.04 ± 0.23	0.971
January 1958	-0.157 ± 0.005	0.94 ± 0.06	0.991
February 1958	-0.167 ± 0.007	0.42 ± 0.09	0.989
March 1958	-0.194 ± 0.012	1.53 ± 0.28	0.960
Mean values	-0.173 ± 0.016	0.99 ± 0.38	0.977

together with the uncorrected coefficients. The height coefficients are certainly larger in case a) than in the uncorrected case but the changes have also increased the scattering of the monthly values considerably. The small value of February 1958 has not been affected very much, whereas the large value of March 1958 has been doubled. The result when both variables have been corrected, is even more confusing. The coefficients of March 1958 have assumed quite abnormal values and the influence on the other six months has not been such as to reduce the month to month scattering.

The influence of the random errors is no doubt considerable but in the case just examined it can not explain the deviations of individual values from those expected.

7. Application of the Calculated Regression Coefficients

On the basis of the coefficients determined for the two Duperier models (Tables 2 and 4) and the Dorman model (Table 10, curve B in Fig. 1) we have calculated the corrections for the monthly mean values of the meson intensity from October 1957 to July 1958. The corrections are plotted on a percentage scale in Fig. 2. The partial corrections are seen to

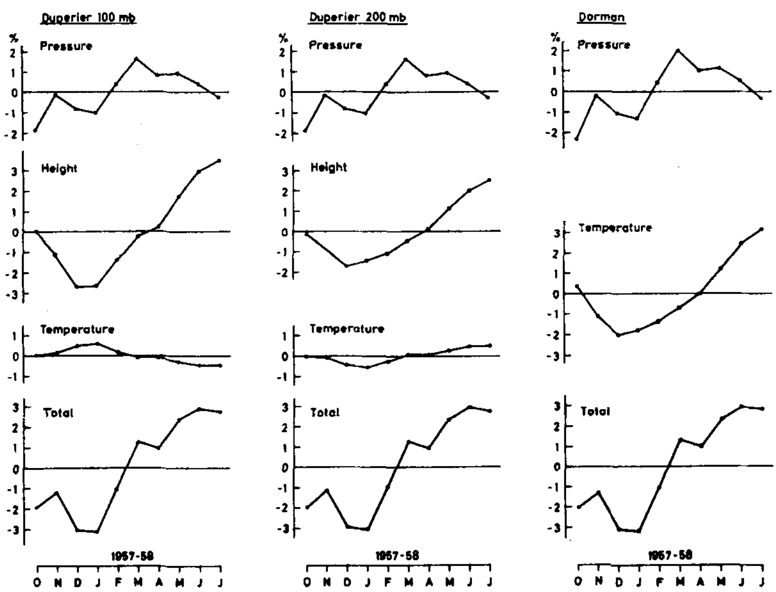

Fig. 2. Corrections for changes in the atmospheric conditions to be added to the recorded monthly mean intensities of the meson component for the period October 1957—July 1958. The corrections are calculated according to the mean values of the regression coefficients given in Tables 4, 2 and 10. Partial corrections (due to changes in pressure, height and temperature) are given as well as total corrections.

differ appreciably from model to model. Nevertheless the total corrections are practically identical. The differences between the Dorman model and the two Duperior models are of the order of 0.1 per cent (mean difference = 0.05 per cent) for the ten months treated, although the monthly values have not been used in the determination of the coefficients. These differences have to be compared with the standard error of the daily meson intensity which is 0.066 per cent for a counting rate of 96,000 counts per hour (Section 1).

Curves A1—A4 in Fig. 3 show the day to day corrections for June 1958, which month has been chosen quite arbitrarily among the months April—July 1958 not already used in the calculation of the regression coefficients. Curves A1—A3, corresponding to the two Duperier models and the Dorman model (curve B in Fig. 1), are essentially identical. The differences between them are of the order of 0.1 per cent. Curve A4 shows the corrections predicted by Dorman for $\gamma = 0.5$ (curve A in Fig. 1) with one exception: no account has been taken of the atmosphere above 100 mb. This curve deviates somewhat from the other three. The corrections plotted in curves A1 and A4 are now added to the recorded meson data: B1 and B2 are the resulting curves. For comparison the nucleonic intensity corrected with -0.737 per cent/mb (curve B3) is shown together with the corrected meson curves. The three curves agree quite well. If the degree of consistency with the nucleonic intensity curve could be used as a criterion on the reliability of the corrections used, it would nevertheless be very difficult to decide between the two alternatives represented by curves B1 and B2. The simple correlation coefficient calculated for curves B1 and B3 on the basis of the deviations from the means, is 0.870. About the same value, 0.865, is obtained for the correlation between curves B2 and B3.

8. Summary and Conclusions

It is possible to found reliable determinations of the atmospheric coefficients on data recorded under geomagnetically unfavourable conditions, if a variable representing the primary intensity variations is included in the calculations. Reasonable results are obtained for the two Duperier models examined. The month

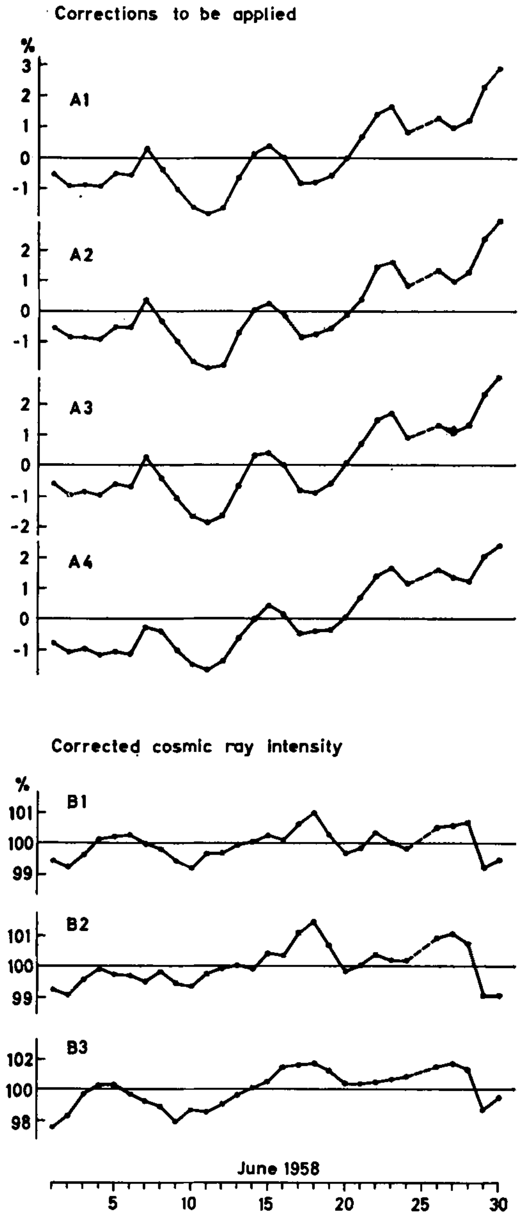

Fig. 3. A1—A4: corrections to be added to the recorded daily mean intensities of the meson component for June 1958 according to the mean values of the regression coefficients given in Table 4 (Duperier 100 mb), Table 2 (Duperier 200 mb), Table 10 (Dorman, curve B in Fig. 1) and Table 8 (Dorman, curve A in Fig. 1).

B1—B2: Daily mean intensities of the meson component corrected according to Tables 4 and 8.

B3: Daily mean intensities of the nucleonic component corrected for pressure changes with —0.737 per cent/mb.

to month scattering of the calculated regression coefficients and the multiple correlation coefficients are essentially the same for both models. For statistical reasons the 200 mb model should be preferred. If the 100 mb model is chosen, it seems justified to use the 200 mb model for interpolations, where data are lacking for the 100 mb level. The temperature effects predicted by Dorman for $\gamma = 0.5$ (Equation 17) do not fit the recorded data very well ($\overline{\alpha_{73}}$ significantly smaller than 1, Table 8). The pressure coefficients calculated according to Equation (12) indicate that γ should lie about half-way between 0.5 and 1.0. This is supported by the results obtained for the temperature effects (Section 5).

It has been shown that a Duperier model including three atmospheric variables leads to essentially the same atmospheric corrections of the sea level meson intensity as a more complicated Dorman model with eight variables, if the relative weights of the variables are first assigned by the same procedure for both models. The close agreement between the models examined (as regards the resulting corrections) has been demonstrated in the correction of daily as well as monthly mean intensities.

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