On the Angular Momentum Balance in the Equatorial Trough Zone of the Eastern Hemisphere

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Abstract

A study has been made of the conditions for seasonal balance of zonal angular momentum, during Southern summer, in the troposphere overlying the equatorial (monsoonal) westerlies. The observational data are not adequate to determine the finer details of balance. It is shown that by and large it can be accounted for by zonal pressure gradients and meridional transport of Earth's angular or Ω-momentum (a Coriolis effect). Zonal and meridional stresses due to advective and eddy transports of relative zonal or $\bar{u}$-momentum as well as the surface frictional stresses are of considerably smaller magnitude. This confirms for a monsoonal region the conclusions of Palmer (1958), from observations in the North Pacific in June, that quasi-geostrophic equilibrium occurs down to very low latitudes above the planetary friction layer.

The high-level easterly jet stream is shown to play an important part in two aspects of balance.

1. Introduction

The summer monsoonal wind regimes of Southern Asia and Indonesia-Northern Australia are distinguished from wind regimes in comparable latitudes of the eastern hemisphere mainly by the presence of a westerly stream which in the region of maximum development extends upward to about three, in places five or more kilometers. They are overlain by jet-stream like easterly winds in the upper troposphere exceeding in strength the easterlies during non-monsoonal (intermediate) seasons, and those of the western hemisphere at all times (cf. KOTESWARAM 1958). It is perhaps appropriate, if somewhat trivial, to regard both the monsoonal westerlies and upper easterlies as displacements and developments of the quasi-permanent equatorial flow regime in the eastern hemisphere.[1]

[1] FLOHN (1955) and others emphasized the role of the trade winds in the winter hemisphere, in holding up the concept of the monsoonal westerlies as 'deflected planetary air streams'.

Connections between the tropical summer monsoons and the general circulation of the atmosphere have been explored in many ways since the study of world wide correlations by Walker. Year-to-year variations, in particular of the trade wind regimes and subtropical high pressure belts, would cause variations in monsoonal activity as seen, for example, in the occurrence of widespread droughts in some years and of excessive rains in others.

Another although probably related problem of summer monsoon is the within-season processes such as abrupt onset ('burst'), breaks and pulsations all of which are clearly evident in rainfall. These have been studied mainly in relation to geographically bound factors such as heating of land masses — and also of cooling in the winter hemisphere as in the case of 'surges' of the Australian monsoon; to synoptic-scale disturbances in the monsoonal trough; and to orographic and topographic effects. In current studies of the Australian monsoon we have found some evidence that monsoonal rainfall varies as a result of an interplay between continental heating and the travelling disturbances of the middle latitude westerlies (BERSON and TROUP, 1960). Different methods have been applied in establishing these connections but they led to results similar to those obtained in studies of the south-west monsoon in India (cf. RAHMATULLAH 1952).

A revision of the problem concerning the vagaries of monsoonal regimes in the tropics was suggested by a recognition of the importance of the high-tropospheric easterlies as a link with the general zonal circulation. This followed upon the discovery of a close connection between time of monsoon onset in India and a preceding abrupt change of 200 mb flow from light westerly to strong easterly of remarkably high constancy (SUTCLIFFE and BANNON, 1956). Subsequently similar changes were found from soundings over a wide region of Southern Asia in relation to the onset of the May-Yu rainy season in China (DAO SHIH-YEN, 1958). For the monsoonal regime of the Indonesia-Australia sector, analysis of 200 mb winds at Singapore, Cocos Island (in the Indian Ocean), Darwin and other places and of rainfall in the extreme North of the continent has revealed, among other things, relationships of a similar kind (TROUP, 1960).

A first necessary step in a wider investigation of the behaviour of monsoons is the dynamic aspect of interactions, day by day and week by week. It can be facilitated if one has gained some insight into the average seasonal balance of zonal angular momentum in the part of the atmosphere in which the monsoonal circulations develop, in which one would expect the upper easterly jet to play an important part. Although data are sparse, we can attempt the assessment of the magnitudes of the various terms in the equation of zonal momentum balance.

2. The equation for the generation of zonal angular momentum

We consider a portion of an annular strip of the atmosphere bounded by vertical surfaces in latitude circles φ_0, φ_1 and in longitude circles λ_0, λ_1, and at bottom and top by isobaric surfaces $\bar{p}_0$, $\bar{p}_1$ (where $\bar{p}_0$ approximates the mean surface pressure). The change of relative zonal angular momentum in this volume is given with sufficient approximation by the equation

$$\frac{a^3}{g}\frac{\partial}{\partial t}\int^{\delta p}\!\!\int^{\delta\lambda}\!\!\int^{\delta\varphi}\bar{u}\cos^2\varphi\,d\varphi\,d\lambda\,d\bar{p} = -\frac{a^2}{g}\Bigg[\underbrace{\left|\cos^2\varphi\int^{\delta p}\!\!\int^{\delta\lambda}\overline{uv}\,d\lambda\,d\bar{p}\right|_{\varphi_0}^{\varphi_1}}_{(A)} + \underbrace{\left|\int^{\delta p}\!\!\int^{\delta\varphi}\overline{u^2}\cos^2\varphi\,d\varphi\,d\bar{p}\right|_{\lambda_0}^{\lambda_1}}_{(B)}\Bigg]$$

$$+\frac{a^3}{g}\Bigg[-\underbrace{\int^{\delta p}\!\!\int^{\delta\lambda}\!\!\int^{\delta\varphi} g\gamma_\lambda\cos^2\varphi\,d\varphi\,d\lambda\,d\bar{p}}_{(C)} + \underbrace{2\Omega\int^{\delta p}\!\!\int^{\delta\lambda}\!\!\int^{\delta\varphi}\bar{v}\cos^2\varphi\sin\varphi\,d\varphi\,d\lambda\,d\bar{p}}_{(D)}$$

$$+\underbrace{2\int^{\delta p}\!\!\int^{\delta\lambda}\!\!\int^{\delta\varphi}\frac{\overline{uv}}{a}\sin\varphi\cos\varphi\,d\varphi\,d\lambda\,d\bar{p}}_{(E)}\Bigg] + \underbrace{a^2\int^{\delta\lambda}\!\!\int^{\delta\varphi}\cos^2\varphi\,\bar{\tau}_{0\lambda}\,d\varphi\,d\lambda}_{(F)} \qquad (1)$$

Here $\delta p = \bar{p}_0 - \bar{p}_1$, $\delta\varphi = \varphi_1 - \varphi_0$, etc.; u and v are zonal and meridional components of wind, Ω the angular velocity of rotation and a the mean radius of the earth, g acceleration of gravity and $\gamma_\lambda = \frac{1}{a\cos\varphi}\frac{\partial\bar{p}}{\partial\lambda}\Big/\frac{\partial\bar{p}}{\partial z}$ the zonal slope of an isobaric surface. $\bar{\tau}_{0\lambda}$ is the zonal component of surface frictional stress.

A horizontal bar denotes averages over a time interval of the order of a month, but we have ignored the time variation of pressure at the surface as a very small effect in connection with the present case. The upper pressure (p_1) will be taken as 100 mb throughout.

Eq. (1) is correct under the following, further assumptions:

1. Eddy stresses along the isobaric surface $p_1 = \text{const.}$ are negligibly small in comparison with surface stresses. This proviso imposes certain limitations on the interpretation of results obtained below, but the data position makes this unavoidable.

2. A mountain torque effect has not been allowed for in eq. (1). This is not considered a very restrictive assumption in the present case.

Eddy stresses at the vertical boundaries will be considered having regard to the inequalities $\overline{uv} \neq \bar{u}\,\bar{v}$, $\overline{u^2} \neq (\bar{u})^2$. Thus terms A and B can be broken up into terms with $\bar{u}\,\bar{v}$ and $\overline{u'v'}$, $(\bar{u})^2$ and $\overline{(u')^2}$, the primed terms being Reynolds stresses on the synoptic scale, as u', v' are taken as daily departures from the monthly or seasonal averages $\bar{u}$, $\bar{v}$. In this we follow with PALMÉN (1955) the procedure adopted earlier in studies of the extratropical cells of the general circulation.

Terms A and B together represent the vertically integrated horizontal divergence of transport of $\bar{u}$-momentum, and term F is the frictional torque exerted by the earth on the atmosphere. For a complete annular strip term B vanishes; so also does term C, the term containing the zonal pressure gradient. (This may be written as

$$-a^2\left[\left|\int^{\delta\varphi}\!\!\int^{\delta p}\cos\varphi\, Z_p\, dp\, d\varphi\right|_{\lambda_0}^{\lambda_1} + \int^{\delta\lambda}\!\!\int^{\delta\varphi} Z_{p_1}\frac{\partial p_0}{\partial\lambda}\cos\varphi\, d\varphi\, d\lambda\right]$$

where Z_p is the height above sea-level of an isobaric surface. As is readily seen, it vanishes for $\lambda_1 = \lambda_0 + 2\pi$.)

As we deal here with portion of an annular strip only, those terms do not necessarily vanish. The portion of B involving $\bar{u}^2$ is the divergence of mean zonal transport of zonal momentum, whereas that part involving $\overline{(u')^2}$ and the pressure gradient term allow for meridional stresses on the volumes exerted by motions on scales below the mean motion (synoptic-eddy and molecular scales). While the $\overline{uv}$ term introduces interaction across latitudes, i.e. with neighbouring zonal strips, the meridional stress terms account for interaction with the hemispheric remainder of the zonal strip of which we consider a portion.

Terms D and E are a Coriolis effect and a metric term in the meridional momentum transport, arising from the transformation of the relevant basic momentum equation in Cartesian framework into a system of spherical polar co-ordinates. These will be discussed further below.

3. The region considered and data employed

We shall apply the above equation to the seasonal (December—February) flow between the equator and 15° S, and between longitudes 60° E and 180°. (Some computations will also be made on the monsoon area in northern hemisphere summer for comparison.) These limits enclose almost all of the surface tropical westerlies except for a restricted region near West Africa. These westerlies occur in a belt whose width varies from five to fifteen degrees latitude (twenty degrees north west of Australia). In a more comprehensive study one could assess from observations at six to eight stations near the boundaries of the region, the divergence of meridional and zonal transports. Here we shall treat a smaller sample of stations in addition to climatological surface and upper air data, mainly from published sources. For meridional transfer of zonal momentum, we have used once daily rawin soundings at Singapore (01°21′ N, 103°54′ E) at the northern boundary, and Cocos Island (12°05′ S, 96°53′ E) and Darwin (12°26′ S, 130°52′ E), just south and north respectively of the poleward boundary of the westerlies.

The surface stresses have been evaluated from data used by PRIESTLEY (1951) in computing the zonal stress between ocean and atmosphere; these data were kindly made available by the author. We employed the same values of drag coefficient, density, and of the correction factor for the combination of wind speeds, as were used in that paper.

Use was made of charts of vector mean wind, and of vector standard deviation (BROOKS *et al.* 1950; British Meteorological Office 1954; JENKINSON 1956) and of surface winds over the oceans, in computing the zonal advective and eddy stresses $(\bar{u})^2$ and $\overline{(u')^2}$. The latter is equal to σ_u^2 where σ_u is the standard deviation of zonal wind component: this was assessed by consideration of the standard vector deviations and of the ratio σ_v/σ_u evaluated for a number of stations in comparable locations, at selected levels, from the data of CRUTCHER (1958).

4. Horizontal momentum transfer

RAO (1958) calculated for July and August 1955 from soundings at three stations in India, at Singapore and at Nairobi (East Africa) meridional transfer of mean momentum associated with the southwest monsoon. His data indicate that the convergence of meridional transport which would be required to balance the frictional torque of the extensive zone of surface west winds, is brought about largely by strong northward transport across the equator into the eastern half of the zone. Between 70 and 80 percent of the zonal stress (meridional transport of ZM integrated from 1,000 to 100 mb) at Singapore was due to advection by mean flow, most of it above the 300 mb level in the vicinity of the easterly jet. The remainder, also concentrated at these levels, was due to time-eddy transports of the species $\overline{u'v'}$. The Indian stations, between five and fifteen degrees latitude north, showed mainly southward transport in that month, but of considerably smaller magnitude. At Nairobi the zonal stress integrated out almost to zero.

No attempt was made by Rao to assess angular momentum balance from a consideration of surface friction torque and the other effects discussed above. Only a comparatively small fraction of the region is covered by land, but the zonal stresses computed from the soundings in India and at Nairobi would not necessarily be representative of stresses over the open ocean. On the other hand, surface winds over the Indian subcontinent would contribute some substantial proportion of the torque of the surface westwind regime on the eastern hemisphere, during Northern summer mainly north of the equator.

In Table 1 are given seasonal (December—February) means of wind components and meridional transport of $\bar{u}$-momentum per unit mass at various pressure levels. The last column shows pressure means of wind components and zonal stresses. Meridional mass transport at Singapore is southward, the drift (pressure mean of $\bar{v}$) being 0.44 m/sec. This is comparable with northward mass transport in Northern summer as e.g. shown by drifts of 0.36 m/sec in July 1955 and 0.23 m/sec in July 1957 (RAO 1960). Thus in Southern and Northern summer alike, tropospheric drift is in the same direction as the 'spillover' of wintermonsoon (or of tradewinds) from the opposite hemisphere: the countercurrent in the upper troposphere does not appear to compensate fully for the outflow beneath.

During Southern summer, at Singapore total zonal stress, i.e. the sum of integrated advective and eddy transports, agrees qualitatively with Rao's findings for July, the average December—February mean zonal stress is 7.6 southward compared with 14.8 northward in July and $26.6 \cdot 10^7$ dyne cm^{-1} northward in August 1955. It may not be justified to compare our seasonal means with Rao's means for two months in one individual season. However, in selecting that year, the author was presumably not taking a very abnormal season. One might therefore compare the proportions of eddy stress in total zonal stress. As will be seen from Table 1, during Southern summer eddy transport represents less than 10 percent of the total. The corresponding proportions for July and August 1955 are 40 and 20 percent approximately. It will also be seen from the table that advective flux is concentrated in the upper troposphere owing to the high zonal velocities associated with the easterly jet stream. This is so during Northern summer too, but in contrast to this eddy transports northward in the lower and southward in the upper troposphere, while of considerable magnitude, tend to integrate out vertically. There is of course a large scatter about this average distribution, the consequences of which are treated in the Appendix. Vertical distribution of fluxes is illustrated in Figure 1, which gives values for two individual seasons for Singapore and Cocos Island. We notice in this figure, as in Table 1, the inactive layer in the middle troposphere which is a typical feature of the general tropical circulation (PALMÉN, RIEHL and VUORELA 1958).

Cocos Island during Southern summer is just poleward of the zone of surface westerlies and shows therefore considerable mean northward flow up to 850 mb associated with the southeasterly trades, but here the vertically integrated mass transport almost vanishes. This would conform with the concept of a Hadley type circulation between the southern trade wind regime and a (poleward shifted) equatorial trough. Similarly, as in the winter Hadley cell, net transport of ZM is directed southward.

Table 1. Meridional and zonal components (m/sec) and meridional transports (m^2/sec^2).

Positive value eastward and southward.

The last column shows vertical means, or integrated transport of $\bar{u}$ momentum (dyne cm^{-1}). Averages for Dec.—Feb., mainly over years as stated, except for eddy flux (2 seasons, 1957, 1958 for all stations).

Height (thousands of feet)		1.5	3	5	7	10	15	20	25	30	40	45	50	55	60	Mean or integral
Approximate pressure (mb)		950	900	850	800	700	600	500	400	325	200	150	125	100	80	
SINGAPORE	$\overline{V}$	+4.8	+ 4.5	+3.8		+ 1.4	+0.8	+ 0.2	+0.7	-0.6	- 4.3		- 2.9		+0.1	+0.44
(1951—5)	$\overline{U}$	-0.1	+ 0.7	+0.1		+ 0.6	-0.8	- 2.2	-4.0	-4.9	-11.2		-16.3		+2.5	-4.0
01°21′ N	$\overline{V}\,\overline{U}$	-0.5	+ 3.2	+0.4		+ 0.8	-0.6	- 0.4	-2.8	+2.9	+48.5		+47.0		+0.2	+ 7.2·10^7
103°54′ E	$\overline{V'U'}$		- 7.5		-2.2	- 1.2		- 2.1		+0.4	+ 6.5		+24.8		-4.9	+0.4·10^7
COCOS ISLAND	$\overline{V}$	-3.0[1]	- 2.2		-0.2	- 0.4		+ 0.1		+0.2	+ 2.1	+ 3.3	+ 3.7	(+ 1.1)		-0.03
(1956 - 9)	$\overline{U}$	-3.1[1]	- 4.9		-1.3	- 0.8		- 0.7		-1.4	- 2.5	- 4.4	- 7.5	(-10.9)		-3.1
12°05′ S	$\overline{V}\,\overline{U}$	+9.3[1]	+ 6.2		+0.3	+ 0.3		- 0.1		-0.3	- 5.3	-14.7	-27.8	-12.3		-0.8·10^7
96°53′ E	$\overline{V'U'}$		+14.2		+6.2	+ 1.6		- 1.4		0	+ 6.0	+ 9.9		- 3.6		+3.5·10^7
DARWIN	$\overline{V}$	+0.5	- 0.1	-1.3		- 1.8		- 0.7		-0.1	+ 1.4	+ 1.1	+ 0.6	+ 0.6		-0.41
(1953—9)	$\overline{U}$	+2.9	+ 2.4	0		- 3.2		- 0.1		-0.3	- 1.4	- 4.7	- 7.7	- 9.7		-1.3
12°26′ S	$\overline{V}\,\overline{U}$	+1.4	- 0.2	0		+ 5.6		+ 0.1		0	- 2.0	- 5.3	- 4.4	- 5.5		+0.8·10^7
130°52′ E	$\overline{V'U'}$		- 4.5		+3.9	+16.5		+28.7		-1.9	- 8.2	-11.4		- 1.0		+1.6·10^7

[1] 1000 ft.

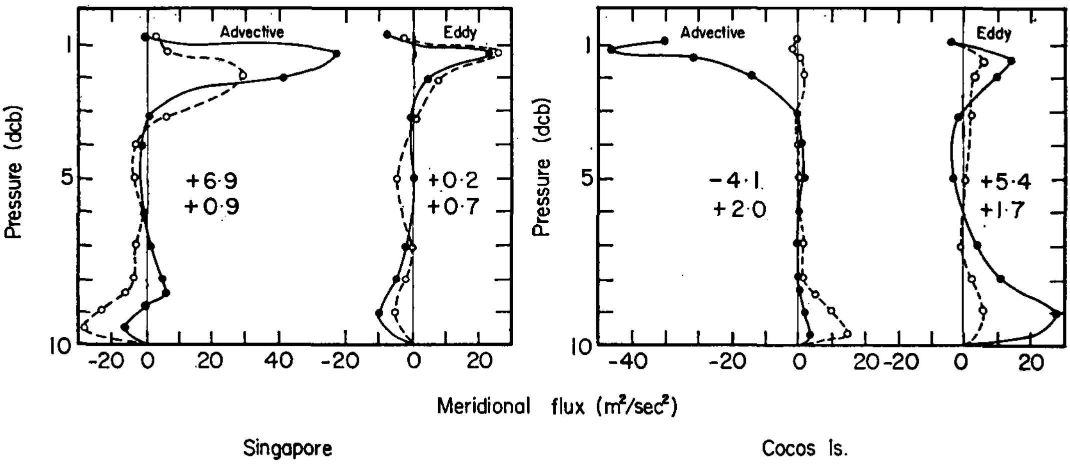

Figure 1. Vertical distribution of meridional fluxes of zonal momentum; solid lines 1956/7 season, dashed lines 1957/8 season. Inset values give vertical integrals of $\varrho \bar{U}\,\bar{V}$ and $\varrho \overline{U'V'}$ (upper value 1956/7 in units of 10^7 dyne/cm.

Comparing with PALMÉN's (1956) assessment of transport across latitude circles in the winter Hadley cell of the northern hemisphere, we find (after converting to flux per unit width) the values given in Table 2.

Table 2. Integrated meridional transport of $\bar{u}$-momentum (in 10^7 dyne cm^{-1}) at Cocos Island and at a comparable latitude in the winter Hadley cell of the general circulation. (Positive values poleward.)

	Cocos Island (summer)	Hadley Cell (winter)
Advective Flux	− 0.8	3.7
Eddy Flux....	3.5	4.1
Total.........	2.7	7.8

As is well known, during summer the poleward export of zonal angular momentum from the trade wind belt is generally much less than during winter. Although possibly the surface momentum sink of the middle latitude westerlies is larger in the southern hemisphere, the integrated zonal stress at Cocos could be a fair estimate of the average stress at about latitude 12° S at the southern boundary of the equatorial westwind regime. There is still a large contribution of eddy flux to the total meridional transport. However, it should be noted that in the eastern parts of the Southern Indian Ocean this eddy flux occurs predominantly in the lowest kilometers below, and just above, the tradewind inversion whereas in the winter Hadley circulation, according to Palmén's calculations, the contribution by the layer 700 to 100 mb amounts to 90 percent of the total zonal eddy stress. That large low-level eddy flux is presumably due to disturbances on and variations in position of the equatorial trough.

At Darwin meridional velocity components are similarly distributed although there is considerably less poleward drift at upper levels. Darwin's position on the coast of the mainland and in a synoptically disturbed region does not permit us to take its meridional net drift as anywhere near representative of a wider region. The vertical distribution of zonal wind indicates two eastwind maxima, one immediately above the monsoonal westerlies and the other at 100 mb. Whereas the latter must be regarded as the edge of the dynamically important high-level easterly jet which judging from the values for Cocos and Darwin appears to slope slightly upwards south of the equator, the lower eastwind maximum would indicate the existence of a strongly sloping belt of easterlies connecting the tropical upper easterlies with the low-level easterlies (trade winds) farther poleward. This feature is also characteristic of the Northern summer circulation in the eastern hemisphere (STAFF MEMBERS, ACADEMIA SINICA 1957).

As would be expected the products $\bar{u}\,\bar{v}$ show a rather confused vertical pattern at Darwin, but the integrals bear out qualitatively the position at Cocos.

The aforementioned and a more detailed comparison with Rao's calculations during July at tropical stations in the eastern hemisphere suggests that meridional drift and momentum transports are at lower level of intensity during Southern summer. This might be expected on account of smaller frictional torque exerted on the atmosphere owing to a less intense monsoonal regime in the Southern tropics. An assessment of that torque for both seasons will be made in the following section.

5. Frictional torque and divergence of meridional momentum transport

In July the westwinds cover land areas such as India and Burma. Approximating the surface zonal stress over these regions to the mean value in the corresponding latitude zone over neighbouring ocean areas, the land contribution to the torque south of 20° N—the contribution north of that latitude being negligible—was less than 10 percent. Thus different but comparable values of $\bar{\tau}_{0\lambda}$ over land would have made little difference in the assessment of total torque.

The land area covered by westwinds south of the equator in January is not negligible either, but most of it is in what remains at this time of year of the broad doldrums zone. The land area has been thus neglected in January.

Results of the computation are shown in Table 3. It is of interest to compare the mean stresses with those obtaining over all oceans. Priestley's study gave in a ten-degree broad belt of minimum stress south and north of the equator, in January and July respectively, values of 0.25 and − 0.06 dyne cm^{-2}. The negative mean stress in the latter case emphasizes the importance of the Asian southwest monsoon as a disturbance on the scale of the general circulation. In January, the strength of the Indonesia-Australia west monsoon is insufficient to reverse the sign of the average ocean stress in the equatorial belt.

An estimate of *divergence* of meridional transport of angular momentum (term A in eq. (1) transposed to the left) requires an estimate of the mean value of advective and eddy flux on the northern and southern boundaries of the zone. We have one pair of stations in approximately the same longitude, viz. Singapore and Cocos Island, both in the regime of upper easterlies; and we shall boldly assume the divergence of flux between them is representative for the zone, using the seasonal averages in Table 1 for computation. (Incidentally, the total integrated fluxes at Darwin and Cocos Island are approximately equal.) In northern summer we shall use Rao's values for July and August 1955. Singapore and Nairobi are both near the equator and serve as southern while Madras, Bombay and Nagpur serve as northern boundary stations. Resulting divergences of meridional transport of zonal angular ($\bar{u}$) momentum are given in the fourth column of Table 3.

Table 3. Surface torque and integrated meridional flux divergence.

Equatorial Zone of westerlies	Area (10^6 km^2)	Mean Surface Zonal stress (dyne cm^{-2})	Friction Torque (10^{25} g cm^2 sec^{-2})	Flux Divergence (10^{25} g cm^2 sec^{-2})
Dec.—Feb., S. Hem. 60°—180° E	16.8	− 0.21	− 2.1	− 4.3
July 1957, N. Hem. 50°—125°E	22.7	− 0.73[1]	− 9.5	− 4.8

[1] Indian Ocean only, − 0.93

It will be seen that in the two cases they are of the correct sign and order of magnitude but that in the northern hemisphere balances only a fraction of the frictional torque. In view of the uncertainty of the computations on inadequate observational data, it would be futile at this stage to speculate on the significance of this unbalance for the magnitude of a 'mountain torque'. Moreover we have so far left out the effect of meridional stresses at the eastern and western ends of the zone, of zonal pressure gradient and of Ω-momentum transfer.

6. Interaction with the western hemisphere

Interaction in longitude, and especially between the monsoon dominated portion of the zone and the hemispheric remainder, must be ultimately due to unequal distribution of land and sea on a broad scale. For this causes differences in radiation balance, in convection and vertical eddy transfer of momentum and will lead to circulations in vertical zonal planes.

To obtain some insight into this kind of interaction vertical profiles of $(\bar{u})^2$, $\overline{(u')^2}$ and of zonal pressure gradient were studied. Here we shall only give results of computations on the relevant integrals in equation (1). These are proportional to

$$\delta_\lambda \int_{100}^{p_0} \{\bar{u}^2\}\, dp; \quad \delta_\lambda \int_{100}^{p_0} \{\overline{(u')^2}\}\, dp;$$

$$\delta_\lambda \int_{100}^{p_0} \{\bar{Z}_p\}\, dp + \int_{\lambda_0}^{\lambda_1} \left\{\bar{Z}_{100} \frac{\partial p_0}{\partial \lambda}\right\} \partial\lambda$$

where { } brackets denote integrals over the lengths of the boundaries at the eastern and western ends, and δ_λ differences of quantities between them.

In the last term the second part is the vertically integrated pressure ascendant of an atmosphere with no zonal temperature gradient. It has been assessed from seven-year (1954—1960) January averages of sea-level pressures at a number of stations in the region.

The first part of the third term is that contribution to the total integral which allows for zonal thermal inhomogêneity. The thickness term has been evaluated from soundings, pairing where necessary sea-level pressure with thickness at a sounding station.

The assessment of the $\bar{p}_0$ term is considerably more accurate than the thickness term, for several reasons. One is the better network of sea-level pressure recording stations; another is the uncertainty introduced into the values for thickness differences between stations owing to differences in instrumentation and radiation corrections. Semi-diurnal variations of pressure would cause fictitious gradients in both terms, but owing to differences in G.M.T. observation times between stations at the eastern and western end, systematic errors of this kind are thought to be small.

Orders of magnitude. — By and large, the eddy flux term in the western hemisphere exceeds that of the eastern hemisphere. $\bar{u}$-momentum is therefore transported from the Eastern Central Pacific and Atlantic regions into the Western Pacific and Indian Ocean sectors of the equatorial trough zone, at this time of year. In terms of means over height and latitude, typical values of $\delta_\lambda \overline{(u')^2}$ are of the order 3 m^2/sec^2, the largest value occurring between a maximum in the Eastern Atlantic (49 m^2/sec^2) and a minimum in the Eastern Indian Ocean (37 m^2/sec^2).

The differences in advective flux are of the order 3 times this amount, due mainly to differences in speeds in the upper troposphere. These differences will be discussed further below.

Typical differences in sea-level pressure between the eastern and western ends are of the order of a few millibars, the average over the width of the latitude zone being $\delta_\lambda \bar{p}_0 = -2.3$ mb. The eastward fall of pressure is mainly between the northern periphery of the Indian Ocean anticyclone and the Indonesia-Australia-Southwest Pacific monsoonal trough. However, the troposphere over the equatorial trough zone in the western hemisphere tends to be warmer than over the monsoon dominated zone of the Eastern hemisphere. The relevant differences $\delta_\lambda \bar{T}$ are of the order 1° C. The pressure-and latitude-averaged slope of the isobaric surfaces remains however negative (10 metres/10,000 km), the thermal ascendent failing to compensate sea-level pressure gradient. (An inspection of stratospheric temperatures from the IGY period suggests that stratospheric compensation of tropospheric zonal temperature differences does exist, but is probably small.)

An evaluation of the isobaric slope from the charts of Heastie (1957) prepared for a different period, gives a value of −20 metres per 10,000 km, averaged from 700 to 100 mb. The values at individual levels suggest that the thermal (tropospheric) compensation does not extend above 300 mb; we believe this discrepancy to be due to such factors as radiation errors at the highest levels.

In Table 4 are given the equivalent torques

Table 4. Integrated zonal flux divergence and pressure gradient.

Southern hemisphere equatorial trough, Dec.—Feb. (Units 10^{25} g cm^2 sec^{-2}.)

Zonal Flux Divergence		Zonal Pressure Ascendant		
Advective	*Eddy*	*Homogeneous*	*Thermal*	*Total*
1.0	0.5	−17.8	8.6	−9.2

due to meridional stresses calculated from the aforementioned terms.

It is evident that the zonal barocline exerts a far greater torque than advective and eddy meridional stresses due to zonal transport of $\bar{u}$-momentum and, by referring to Table 3, than the torques due to surface frictional stresses and meridional transports of $\bar{u}$-momentum. This would still be the case if allowance were made of an error of fifty percent in the thermal component of the pressure term. Even an *underestimate* of this component by $5 \cdot 10^{25}$ would leave a value of $-4 \cdot 10^{25}$ g cm^2 sec^{-2} for the zonal pressure ascendant.

The budget of angular momentum in the space considered, without regard to the two remaining volume integrals in (1), would be as follows:

	+	−
Meridional flux convergence	4	
Friction torque		2
Zonal flux convergence		1
Pressure *gradient*	9	
Unbalance	10	$\times 10^{25}$ g cm^2 sec^{-2}

The unbalance would be equivalent to an increase of *eastward* zonal velocity component by 2 m/sec in a month, taken as an average value in the entire space considered. A generation (or "storage") of momentum of this amount is far in excess of that observed in the middle of the summer season. It is true that an increase of low-tropospheric westerlies of this magnitude does occur from December to February, as we have ascertained from the data, but it is all but compensated by a similar increase in the high-tropospheric easterlies during this time. Again, while the position might be rather different in individual years, a gain of momentum by that amount is unlikely.

7. Quasi-geostrophic balance in the sub-equatorial trough

We shall now consider the last two of the volume integrals on the right of equation (1). Since sin φ occurs in the integrand — $\cos^2\varphi$ being close to unity — it appeared at first sight that these integrals would be negligibly small in comparison with other terms in the equation. This is almost certain to be the case for the metric $\overline{uv}$ term which for low latitudes is merely a small correction to the first term. However, the Ω-momentum transport (Coriolis) term can be large for small mass transports determined by the integral $\int \bar{v}\, dp$. This term may be written as

(3) Integrated meridional transport of Earth's angular momentum

$$= -\frac{2a^3\Omega}{3g}\int^{\delta p}\int^{\delta\varphi}\int^{\delta\lambda} \bar{v}\, d(\cos^3\varphi)\, dp\, d\lambda$$

Let $\bar{v}^*$ denote meridional wind averaged over $\bar{p}$, $\cos^3\varphi$ and λ. The value of $\bar{v}^*$ which would remove an unbalance of $10 \cdot 10^{25}$ in our preliminary budget, is readily computed and turns out to be -1.5 m/sec (implying an equatorward net mass transfer across the zone).

In Figure 2 are plotted pressure means of $\bar{v}$

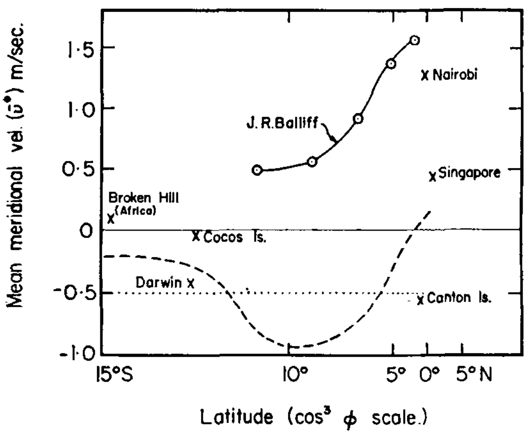

Figure 2. Meridional drifts in portion of the equatorial trough zone in December—February. For explanation of curves see text.

in *m/sec* versus $\cos^3\varphi$. Station plots are seasonal averages obtained from mainly four years' data. It is seen that variations with longitude are as large as those with latitude. This deprives us from estimating $\bar{v}^*$ directly from these data.

It is possible to obtain some information on latitude-profile of meridional mass-transport in equatorial regions during Northern summer. BALIFF (1958) evaluated from soundings in a comparatively dense network set up during June 1956 in the Nuclear Test regions around the Marshall Islands, among other things monthly mean meridional wind at various levels up to 60,000 ft. He published the average values between longitudes 155° and 175° approximately, separately for latitudes 3,5 ..... 11° N. From these values we computed the equivalent pressure means and these are shown in Figure 2 joined by a smooth curve. They are actually south wind components, but are interpreted here as equivalent poleward winds and therefore taken as positive.

Another source of information is our general knowledge of average mass transport across the whole equator, required by the seasonally progressing accumulations and depletions of mass in either hemisphere. RAO (1960) has shown that transport values at individual stations close to the equator differ from the required average transfer (of order of cm/sec) by factors of up to 100 and higher, southward and northward flow alternating with longitude. In view of this circumstance, mean mass transport across the equator along our northern boundary — the length of which is one third of the circumference — will not be nearly as large as observed at any stations.

The broken curve in Fig. 2 is constructed on the basis of the available data permitting a variation with latitude near the equator as large as that resulting from Baliff's analysis. It is drawn in such a manner as to accommodate a value of $\bar{v}^*$ equal to -0.5 m/sec which would restore momentum balance if we accept the minimum estimate of unbalance in the previous section as a best estimate.

The hypothetical curve implies very large gradient of meridional mass transport just south of the equator. Uncompensated by lateral (zonal) divergence it would be equivalent to a tremendous mass accumulation and corresponding surface pressure rise of approx. 70 mb per day between 3° and 8° S. Our data indicate that the integrated zonal mass transport diverges in fact rapidly, mainly owing to the transition from light westerlies to strong easterlies in the upper troposphere, between the Western Pacific and Singapore and continuing so westward. Mass continuity would be preserved if the ratio of difference in $\int \bar{v}dp$ between southern and northern boundaries, to the difference in $\int \bar{u}dp$ between eastern and western boundaries be equal to the inverse ratio of the respective lengths. The latter ratio is 8, while from seasonal means of zonal flow at Canton Is. (171° E) and Nairobi the former ratio turns out to be about 24. The seemingly large discrepancy might be removed by a consideration of the latitude position of the high-level easterly jet. Judging from the available literature, the southern summer easterly jet is located at 125 mb and about 2° S (RAMSEY 1955) in the East Asia region, but at about 200 mb and somewhat further south over Africa. This could indicate that the maximum divergence of zonal transport occurs some five degrees south of the equator. In this case, with the value of $\bar{v}^*$ required for seasonal momentum balance, total horizontal divergence of mass transport over the 3°—8° S latitude zone would reduce to a vanishingly small residual which could readily be accounted for by the small surface pressure changes observed.

These results are at first sight surprising since it is widely held that for such low latitudes, and particularly in monsoonal regions, accelerations and friction are as important as pressure gradients in consideration of air movement.

Conclusions

The foregoing analysis of wind, pressure and temperature data in the tropical atmosphere, from sea level to near tropopause level, has furnished evidence for interactions between the (monsoonal) wind system overlying the equatorial trough zone south of the equator during Southern summer and the atmosphere surrounding it. Interaction across the southern boundary of this zone is an obvious consequence of the fact that the wind system is generally within the northern limb of a Hadley-type circulation, modified chiefly by seasonal displacement of the equatorial trough zone. Interaction across the northern boundary, near the equator, brings out the importance of the coupling between the northern hemisphere tropical cell (modified by the Asian winter monsoon) and the summer monsoon in the Indonesia-Australia region. The associated meridional momentum transfers are, in order of magnitude, sufficient to maintain the surface westerlies against friction.

A surprising result is the magnitude of two terms in the equation of angular momentum balance determining interaction across the eastern and western boundaries of the zone. Whereas divergence of zonal momentum transfer, by mean motions and eddies on the synoptic scale is relatively small, zonal pressure gradient and meridional transport of Ω-momentum are both of a higher order of magnitude, or nearly so. This suggests agreement with results obtained by PALMER (BALIFF, PALMER ET AL. 1958) for the troposphere over the Western Pacific equatorial trough during Northern summer, that quasi-geostrophic balance obtains in low latitudes close to the equator, i.e. for the troposphere above a planetary friction layer.

In the angular momentum balance the existence of an upper easterly jet stream close to the equator is of importance in two respects. It is a prerequisite for convergence of the meridional transfer of momentum to balance (approximately) with frictional destruction of west momentum at the surface. The increase of easterly winds, from the jet's entrance region in the Western Pacific to its exit region near Africa, is consistent with strong meridional gradient of integrated northward mass flow, and thereby transport of Ω-momentum close to the equator. Such transport appears to be required to balance zonal pressure gradients in the monsoon dominated trough of the eastern hemisphere.

Data sources

The sources of data used in this study, and not otherwise mentioned in the text are as follows:

Wind data have been taken from SCOTT (1957) for Singapore mean values and from CRUTCHER (1958), LAMOND (1959), the New Zealand Daily Weather Bulletin, Pacific Islands Section, Malayan Meteorological Service, Pilot Balloon and Radar Wind Data, from manuscript data kindly provided by the Bureau of Meteorology, Australia; and also from Monthly Climatic Data for the World, which publication also provided most of the radiosonde data used in Section 6. This was supplemented by the publication Upper Air Data Australia (Bureau of Meteorology 1958), the Pacific Islands Bulletin, and manuscript data. We are indebted to the Director, Malayan Meteorological Service, for radiosonde data for Singapore.

For some stations other published values from various sources have been used to obtain values below 850 mb.

Sea level pressure data have been taken from Monthly Climatic Data for the World, and from routine publications of the Australia, Malayan and New Zealand Meteorological services.

Appendix

In the present study, largely concerned as it is with orders of magnitude, we have had resource mainly to published material, or to data already processed for other investigations, and comparatively little processing of daily data has been performed, because of the very large demands on time and labour required. There may therefore be some inadequacies and inaccuracies in the results due to missing data and selective effects, differing lengths of periods used for various stations in parts of the study, accuracy of charts and interpolations from them, and so on. We feel that in a work such as this such errors are of minor importance; the major factor which we feel should be considered, is the high variability from year to year of the mean flow in the region, which occurs even in regions of strong mean flow (TROUP 1960). Consider the flow at 40,000 ft, which is close to or at the level of maximum advective and eddy transports. In January, for instance, the mean easterly component at Singapore over 9 years was 18 knots, varying from 6 to 24 knots in individual Januaries and the variability is even greater at 50,000 ft. If such variations and the corresponding variations in the v component which are also large, occur due to small shifts of wind systems in regions of strong gradient, then the investigation should be conducted over a considerable number of years, or over a few years with many more stations.

Examination of the data for two seasons, contemporaneous at Cocos Island and Singapore, suggests that the main variation is in the advective flux and to a lesser extent in the eddy flux at the southern boundary. Curves of advective and eddy flux for the two stations are shown in Figure 1. It will be noted that at Singapore the eddy flux is consistently small and varies comparatively little from year to year, in fact the values level for level up to 200 mb are close to those which can be obtained from the values of r_{uv} σ_u and σ_v given by CRUTCHER (1958), for a different period and for somewhat selective data. There are large variations in advective flux, although the general form of the curve is the same. The variation is greater at Cocos, but the eddy flux shows the same type of *variation* in both years.

If the behaviour of different stations is compared, we find for instance, taking the standard deviations of 40,000 ft monthly means at Singapore, Cocos Island and Darwin, for four seasons, and of differences between these components, after eliminating trend, that the following results:

	Singapore	Difference Cocos—Sing. (and correlation)	Cocos	Difference Cocos—Darwin (and correlation)	Darwin
S.D. (knots^2)	4.4	3.1	6.9	7.0	6.0
		0.95		0.45	

The standard deviation of the differences depends only on the individual standard deviations and the correlations between the values at the two stations. The standard deviations given above give correlation coefficients of 0.95 between Cocos and Singapore monthly means, and 0.45 between Cocos and Darwin monthly means. Taking these as representative of latitudinal and longitudinal differences we infer:

(a) Differences between latitudes reflect major year-to-year changes in the flow, and not just small-scale shifts, since on the available evidence (RAMSEY 1955 for instance) the jet axis lies close to or just to the south of Singapore.

(b) Large-scale simultaneous and similar changes of the flow are to a considerably less extent contributors to the variations in longitude; the variance of monthly means at any one station would be made up to 25 percent of large-scale variations which affect all longitudes simultaneously, and 75 percent of variations which represent differences between longitudes due to either time or to space variations.

Hence taking a few years at any station-pair such as Singapore and Cocos does give some indication of the pattern of the mean in the region, in spite of the large individual variations from year to year. Some smoothing is of course effected by taking seasonal instead of monthly means.

The above statement has only been demonstrated on one component of the flow at one level; the full investigation of year to year variations, is of course a separate and most complex problem. However, we feel that the statement will apply to other quantities as well, and in particular to values of fluxes computed at individual stations.

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