The Exchange of Kinetic Energy between Larger Scales of Atmospheric Motion

By BARRY SALTZMAN and AARON FLEISHER,[1] Massachusetts Institute of Technology

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Abstract

The rate of transfer of kinetic energy between the different scales of eddies of the 500 mb geostrophic wind has been measured for each day of 1951. In the mean, there is a net transfer from intermediate (cyclone) scales to both longer and shorter scales.

The rate, per unit area and per unit pressure difference, at which kinetic energy is transferred from eddies of wave number n to eddies of all other wave numbers is (SALTZMAN 1957)

$$L^*(n) = L^1(n) + L''(n) \tag{1}$$

$$\begin{aligned}
L'(n) = -\frac{1}{2gp_0}\int_0^{p_0}\int_{-\pi/2}^{\pi/2}\sum_{\substack{m=-\infty\\ \neq 0}}^{\infty}\Bigg\{ & \frac{in}{a\cos\phi}\{U(m)\,[U(-n)\,U(n-m) - U(n)\,U(-n-m)] \\
& + V(m)\,[V(-n)\,U(n-m) - V(n)U(-n-m)]\} \\
& + \frac{1}{a\cos\phi}\{U(-n)\,[U(m)\,V(n-m)\cos\phi]_\phi + U(n)\,[U(m)\,V(-n-m)\cos\phi]_\phi \\
& + V(-n)\,[V(m)V(n-m)\cos\phi]_\phi + V(n)\,[V(m)V(-n-m)\cos\phi]_\phi\} \\
& - \frac{\tan\phi}{a}\{V(m)\,[U(-n)\,U(n-m) + U(n)\,U(-n-m)] \\
& - U(m)\,[V(-n)\,U(n-m) + V(n)\,U(-n-m)]\}\Bigg\}\cos\phi\, d\phi\, dp
\end{aligned}$$

$$\begin{aligned}
L''(n) = -\frac{1}{2gp_0}\int_0^{p_0}\int_{-\pi/2}^{\pi/2}\sum_{\substack{m=-\infty\\ \neq 0}}^{\infty}\Big\{ & U(-n)\,[U(m)\,\Omega(n-m)]_p + U(n)\,[U(m)\,\Omega(-n-m)]_p \\
& + V(-n)\,[V(m)\,\Omega(n-m)]_p + V(n)\,[V(m)\,\Omega(-n-m)]_p\Big\} \\
& \cos\phi\, d\phi\, dp
\end{aligned}$$

[1] Now with the Department of City and Regional Planning, Mass. Inst. of Technology.

λ = longitude

ϕ = latitude

p = pressure

t = time

m, n = wave number around a latitude circle

$u\ (\lambda, \phi, p, t) = (a \cos \phi)\, d\lambda/dt$ = zonal wind component

$v\ (\lambda, \phi, p, t) = a\, d\phi/dt$ = meridional wind component

$\omega\ (\lambda, \phi, p, t) = dp/dt$ = individual pressure change

$F(x) = (2\pi)^{-1} \int_0^{2\pi} x(\lambda, \phi, p, t)\, e^{-in\lambda} d\lambda$ = Fourier transform of x

$U(n, \phi, p, t) = F(u)$

$V(n, \phi, p, t) = F(v)$

$\Omega\ (n, \phi, p, t) = F(\omega)$

p_0 = pressure at the earth's surface

a = radius of the earth

g = acceleration of gravity,

$(x)\xi = \partial x/\partial \xi$.

Since $\sum_{n=1}^{\infty} L^*(n) = 0$ we can think of $L^*(n)$ as the rate at which the existing kinetic energy is being redistributed among all the scales of atmospheric eddies, this process being independent of other processes which can alter the total eddy kinetic energy—i.e., transfers to or from the zonal motion ($n = 0$), viscous dissipation, and conversions to or from potential and internal energy.

We are concerned now with estimating $L'(n)$, the contribution to $L^*(n)$ from interactions of the horizontal winds, for the scales represented on hemispheric charts. For this purpose we apply the geostrophic approximation. We do not think these charts justify a resolution in more than 15 wave numbers and therefore take all scales of motion smaller than wave number 15 to be zero. We label the result of this approximation $L(n)$. Then,

$$\sum_{n=1}^{15} L(n) = 0 \qquad (2)$$

which is to say that $L(n)$ measures the geostrophic redistribution of kinetic energy among only the fifteen wave numbers. Transfers due to non-geostrophic components and transfers between the wave group $n = 1$ to 15 and shorter waves are considered to be separate processes and are not treated here.

Our data are the 500 mb northern hemisphere charts for every day of 1951. Contour heights are given at every ten degrees of longitude and every five degrees of latitude from 15° N to 80° N. These are the data from which we obtained the spectrum of the rate of transfer of kinetic energy between the zonal current and the eddies (SALTZMAN and FLEISHER, 1960).

We assume that events at 500 mb are representative of the vertical average, at least in regard to sign.

Our finite difference scheme limits the integration to the region between 22.5° N and 72.5° N. For these new boundaries,

$$\sum_{n=1}^{15} L(n) = \frac{2\pi a \cos \phi}{g} \left[v' \left(\frac{u'^2 + v'^2}{2} \right) \right] \Bigg|_{\phi=72.5}^{\phi=22.5}$$

(brackets denote a zonal average and primes the deviation therefrom). However, if our results are to be representative of the globe it is necessary that (2) be satisfied, which means that the net eddy transport of eddy kinetic energy into this region must be zero. We have measured this and found it quite small. In place of a sufficient condition we assume that the region we treat is large enough to represent the interactions over the globe.

The integral involves a summation of triple products of Fourier components—effectively a triple correlation. The variability of the atmosphere and the quality of the data hardly encourages such an enterprise. It should be expected, therefore, that only a long-time mean will have a variance sufficiently small to be worth quoting. We present the annual mean and, in computing its error, take half of the 365 days to be independent. To obtain additional smoothing, we consider three groups of wave numbers: 'long' waves $1 \lessapprox n \lessapprox 5$, 'cyclone' waves $6 \lessapprox n \lessapprox 10$, 'short' waves $11 \lessapprox n \lessapprox 15$, and we denote the transfer function for these groups by L_i, $i = 1, 2, 3$, respectively; i.e.,

$$L_1 = \sum_{n=1}^{5} L(n)$$

$$L_2 = \sum_{n=6}^{10} L(n)$$

$$L_3 = \sum_{n=11}^{15} L(n)$$

The loss in resolution is appreciable, but physically less than might be supposed since the same wave number at different latitudes corresponds to different physical scales.

Results

We define

$\bar{x}$ = annual average of x

$\sigma(x) = (\overline{x^2} - \bar{x}^2)^{\frac{1}{2}}$ = standard deviation of x

$\frac{2\sigma(x)}{\sqrt{182}}$ = error of $\bar{x}$

The mean values obtained and their errors, in 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}, are

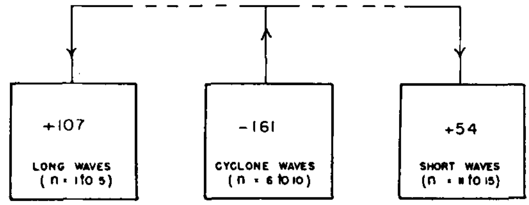

Fig. 1. Estimate of the mean rate of kinetic energy exchange between long waves ($n=1$ to 5), cyclone waves ($n=6$ to 10) and shorter waves ($n=11$ to 15), based on daily 500 mb geostrophic measurements for the year 1951. Units are 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}.

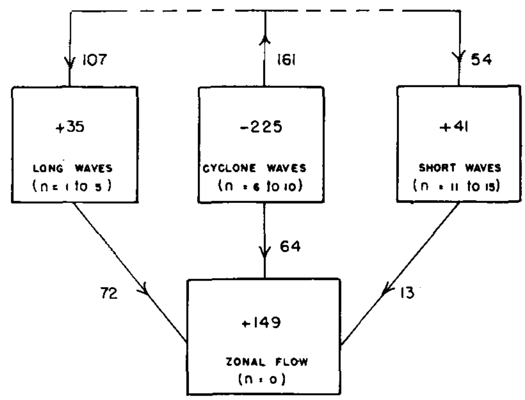

Fig. 2. Estimate of the mean rate of kinetic energy exchange between the zonal current ($n=0$), long waves ($n=1$ to 5), cyclone waves ($n=6$ to 10) and shorter waves ($n=11$ to 15), based on daily 500 mb geostrophic measurements for the year 1951. Units are 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}.

$$\bar{L}_1 = +124 \pm 81$$
$$\bar{L}_2 = -145 \pm 96$$
$$\bar{L}_3 = +\ 71 \pm 54$$

and

$$\sum_{i=1,2,3} \bar{L}_i = +\ 50 \pm 56$$

from which we conclude that $\bar{L}_1$ and $\bar{L}_3$ are significantly positive, $\bar{L}_2$ is significantly negative, and as required, $\sum_{i=1,2,3} \bar{L}_i$ is not significantly different from zero. In physical terms, there is, in the mean, a net transfer of kinetic energy from the 'cyclone' band ($n=6$ to 10) to the long-wave band ($n=1$ to 5) and to the short-wave band ($n=11$ to 15). FJØRTOFT (1953) has demonstrated the possibility of transfers of this kind, and SALTZMAN (1959) has constructed a model which displays these energy flow characteristics.

To estimate the numerical magnitude of this effect we assume that $\sum_i \overline{L}_i$ is entirely in the nature of a computational noise which affects each of the three wave-groups equally. Thus, we subtract $1/3 \sum_i \overline{L}_i$ from $\overline{L}_1$, $\overline{L}_2$ and $\overline{L}_3$, to achieve the balance required by (2). The results are pictured in Figure 1. An arrow indicates a net loss or gain to or from the other two wave groups.

In Figure 2 we display the results shown in Figure 1 combined with those presented by SALTZMAN and FLEISHER (1960) for the interactions between the mean zonal current and the eddies. This is our estimate of the mean geostrophic energy flow in the domain $n = 0$ to 15. Estimates of the release of potential and internal energy in this domain are given by SALTZMAN and FLEISHER (1959) and WIIN-NIELSEN (1959). There are no direct measurements of the transfer of kinetic energy between these scales of disturbances and scales of wave number larger than 15.

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