The Dynamics of Flow on a Submarine Ridge

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Abstract

The equations of motion are developed in a form appropriate to flow parallel to a uniformly sloping bottom. Some cases of two-layer flow are considered in which the flow is concentrated in the lower layer, as in the overflow over a submarine ridge. Whereas in the absence of friction such currents would tend to follow the contours of the bottom, the effect of bottom friction is to deflect the current to the downslope side of the contours. Numerical solutions show that in some cases the angle of deflection might be as large as 30°, but in most circumstances it is probably less, of the order of 5° or 10°. An indication is given of how the theory may be applied to the case of density varying continuously with depth, as in a real ocean. Some general conclusions, which are relevant to the interpretation of observational data, are drawn.

1. Introduction

The flow of water over a submarine ridge into an oceanic basin is frequently the source of supply of an important water mass. The overflowing currents differ from other oceanic currents in that they appear to reach their maximum velocity at, or near to the bottom, the overlying water being almost at rest. The flow of such currents is likely to be influenced to an unusual extent by the contours of the bottom and the effect of bottom friction. Observations indicating the presence of such flow into the North Atlantic over the Iceland-Faeroe and Greenland-Iceland Ridges have been described by DIETRICH (1956, 1957). STEELE (1959) has also described observations of deep overflow across the Iceland-Faeroe ridge. The process of cascading (COOPER and VAUX, 1949) would also seem to imply the existence of currents of this type.

The equations of motion will be developed in a form appropriate to flow parallel to a uniformly sloping bottom. Solutions will be given for some cases of two-layer flow, the water in each layer being of uniform density and the thickness of the lower layer small compared with the total depth of water. The net flow in the upper layer will be assumed to be zero. An indication will be given of how the method may be applied to the case of density varying continuously with depth, as in a real ocean.

2. Equation of motions

Consider, in the first instance, flow on the side of a ridge with a straight horizontal crest and a downslope at a constant inclination α to the horizontal. Let rectangular axes XYZ be taken, with OY along the crest of the ridge, OX down the slope, at an angle α to the horizontal, OZ perpendicular to OX and OY and directed upwards, as in Fig. 1. Let OY be orientated at an angle Θ to the west of north.

Let u, v, w be the velocity components, p pressure, ϱ density, ϕ latitude, ω angular velocity of the earth's rotation, X, Y and Z the components of external force, other than gravity, per unit mass.

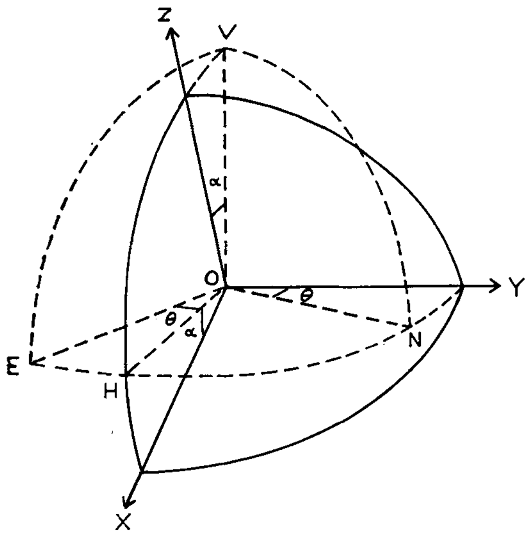

Fig. 1. System of co-ordinates: OY is parallel to crest of ridge; OX is parallel to line of greatest slope.

Let $\frac{D}{Dt}$ denote $\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$

Then the equations of motion may be written in the form

$$\left.\begin{aligned}
&\frac{Du}{Dt} + 2\omega\left\{ -v\,(\cos\alpha\sin\phi + \sin\alpha\sin\Theta\cos\phi) + w\cos\Theta\cos\phi \right\} = -\frac{1}{\varrho}\frac{\partial p}{\partial x} + g\sin\alpha + X \\
&\frac{Dv}{Dt} + 2\omega\left\{ u\,(\cos\alpha\sin\phi + \sin\alpha\sin\Theta\cos\phi) + w\,(\sin\alpha\sin\phi - \cos\alpha\sin\Theta\cos\phi) \right\} = -\frac{1}{\varrho}\frac{\partial p}{\partial y} + Y \\
&\frac{Dw}{Dt} - 2\omega\left\{ u\cos\Theta\cos\phi + v\,(\sin\alpha\sin\phi - \cos\alpha\sin\Theta\cos\phi) \right\} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} - g\cos\alpha + Z
\end{aligned}\right\} \quad (1)$$

with the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{dw}{dz} = 0 \qquad (2)$$

In considering flow near the bottom, w will be small and terms in w may be neglected.

If, also, the inclination α is small enough (of order 10^{-2}, say) for terms in $\sin\alpha$ on the left hand side of (1) to be neglected compared with those in $\cos\alpha$, and if $\cos\alpha$ is taken as 1, equations (1) reduce to

$$\left.\begin{aligned}
&\frac{Du}{Dt} - fv = -\frac{1}{\varrho}\frac{\partial p}{\partial x} + g\sin\alpha + X \\
&\frac{Dv}{Dt} + fu = -\frac{1}{\varrho}\frac{\partial p}{\partial y} + Y \\
&-2\omega\cos\phi\,(u\cos\Theta - v\sin\Theta) = -\frac{1}{\varrho}\frac{\partial p}{\partial z} - g\cos\alpha + Z
\end{aligned}\right\} \quad (3)$$

where $f = 2\omega\sin\phi$.

If the terms on the left of the third equation of (3) and the external force Z are negligible compared with the main gravity component $g\cos\alpha$, this equation reduces to

$$\frac{\partial p}{\partial z} = -g\varrho\cos\alpha \qquad (4)$$

corresponding to the normal hydrostatic equation.

Allowing for shearing stresses tangential to planes perpendicular to OZ but neglecting the lateral shearing stresses, the force components may be written

$$X = \frac{1}{\varrho}\frac{\partial F_{zx}}{\partial z},\quad Y = \frac{1}{\varrho}\frac{\partial F_{zy}}{\partial z},\quad Z = 0 \qquad (5)$$

where F_{zx} and F_{zy} are the components of shearing stress by which water above a plane $z =$ const. acts on the water below that plane.

3. Equations for two-layer flow

Consider an upper and a lower layer of density ϱ_1 and ϱ_2 respectively, the thickness of the lower layer being $\eta\,(x, y)$. Let the free surface be given by

$$z = h_0 + x\tan\alpha + \zeta$$

as in Fig. 2. Let $p_a =$ atmospheric pressure.

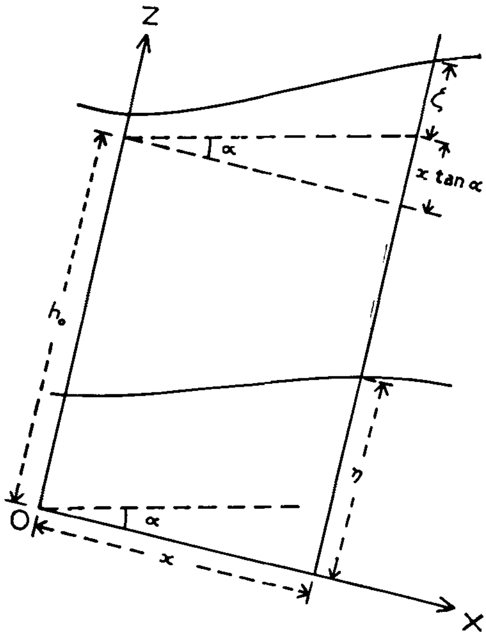

Fig. 2. Co-ordinates for two-layer flow: η = thickness of bottom layer.

Then, from (4), the pressure is given, in the upper layer ($z > \eta$), by

$$p_1 = p_a + g\varrho_1 \cos\alpha\,(h_0 + x\tan\alpha + \zeta - z) \quad (6)$$

and in the lower layer ($z < \eta$) by

$$p = p_a + g\cos\alpha\,\{\varrho_1(h_0 + x\tan\alpha + \zeta - \eta) + \varrho_2(\eta - z)\} \quad (7)$$

From (6) and (5) in (3), and integrating throughout the depth of the upper layer, it is found that

$$\left.\begin{aligned} \overline{\frac{Du_1}{Dt}} - f\bar{v}_1 &= -\frac{1}{\varrho_1}\frac{\partial p_a}{\partial x} - g\cos\alpha\frac{\partial\zeta}{\partial x} + \\ &\quad + \frac{F_{sx} - F_{\eta x}}{\varrho_1 h_1} \\ \overline{\frac{Dv_1}{Dt}} + f\bar{u}_1 &= -\frac{1}{\varrho_1}\frac{\partial p_a}{\partial y} - g\cos\alpha\frac{\partial\zeta}{\partial y} + \\ &\quad + \frac{F_{sy} - F_{\eta y}}{\varrho_1 h_1} \end{aligned}\right\} \quad (8)$$

where $h_1 = h_0 + x\tan\alpha - \eta$

$$\bar{u}_1 = \frac{1}{h_1}\int_{\eta}^{\eta+h_1} u_1\,dz \text{ etc.}$$

F_{sx} and F_{sy} are the components of surface stress and $F_{\eta x}$, $F_{\eta y}$ those of stress at the interface.

It will be assumed now that p_a is uniform, the surface stress zero and that the mean motion in the upper layer vanishes. Then (8) reduces to

$$\left.\begin{aligned} g\cos\alpha\frac{\partial\zeta}{\partial x} &= -\frac{F_{\eta x}}{\varrho_1 h_1} \\ g\cos\alpha\frac{\partial\zeta}{\delta y} &= -\frac{F_{\eta y}}{\varrho_1 h_1} \end{aligned}\right\} \quad (9)$$

In the lower layer, from (7) and (5) in (3) and then integrating throughout the depth of the layer and using condition (9),

$$\left.\begin{aligned} \overline{\frac{Du}{Dt}} - f\bar{v} &= g'\left(\sin\alpha - \frac{\partial\eta}{\partial x}\cos\alpha\right) + \frac{F_{\eta x}}{\varrho_2}\left(\frac{1}{\eta} + \right. \\ &\quad \left. + \frac{1}{h_1}\right) - \frac{F_{0x}}{\varrho_2\eta} \\ \overline{\frac{Dv}{Dt}} + f\bar{u} &= -g'\frac{\partial\eta}{\partial y}\cos\alpha + \frac{F_{\eta y}}{\varrho_2}\left(\frac{1}{\eta} + \frac{1}{h_1}\right) - \\ &\quad - \frac{F_{0y}}{\varrho_2\eta} \end{aligned}\right\} \quad (10)$$

where $g' = \left(\dfrac{\varrho_2 - \varrho_1}{\varrho_2}\right) g$, $\bar{u} = \dfrac{1}{\eta}\displaystyle\int_0^{\eta} u\,dz$, etc., and F_{0x}, F_{0y}, are the components of stress at the bottom.

If $h_1 \gg \eta$, the frictional terms in (10) reduce to

$$(F_{\eta x} - F_{0x})/\varrho_2\eta \text{ and } (F_{\eta y} - F_{0y})/\varrho_2\eta.$$

Since $w = 0$ at $z = 0$ and $w = \dfrac{\partial\eta}{\partial t}$ at $z = \eta$, the equation of continuity for the lower layer gives

$$\frac{\partial}{\partial x}(\eta\bar{u}) + \frac{\partial}{\partial y}(\eta\bar{v}) + \frac{\partial\eta}{\partial t} = 0 \quad (11)$$

4. Form of the frictional terms

Let the resultant mean velocity in the lower layer be V, i.e.

$$V^2 = \bar{u}^2 + \bar{v}^2$$

and let the combined shearing stress at the interface and at the bottom be related to V by a quadratic law, so that if F is the resultant frictional stress,

$$F = k\varrho_2 V^2$$

If it is assumed also that the stress F is in a direction directly opposed to the mean velocity V, then the stress components on the lower layer are given by

$$F_{0x} - F_{\eta x} = k\varrho_2 V\bar{u}$$
$$F_{0y} - F_{\eta y} = k\varrho_2 V\bar{v}$$

Thus in (10), with $h_1 \gg \eta$,

$$\left.\begin{aligned} \frac{\overline{Du}}{Dt} - f\bar{v} &= g'\left(\sin\alpha - \frac{\partial\eta}{\partial x}\cos\alpha\right) - \frac{kV\bar{u}}{\eta} \\ \frac{\overline{Dv}}{Dt} + f\bar{u} &= -g'\frac{\partial\eta}{\partial y}\cos\alpha - \frac{kV\bar{v}}{\eta} \end{aligned}\right\} \quad (12)$$

There is little experimental evidence to indicate the magnitude of k in these conditions, i.e. a bottom current flowing below water sensibly at rest. From a number of observations of tidal currents in comparatively shallow water, values of k in the range 1.5 to 4×10^{-3} and averaging 2.5×10^{-3} have been derived (summarised data have been given by PROUDMAN, 1953, and BOWDEN, 1956). From wind-induced currents in the Straits of Dover, superimposed on tidal currents, BOWDEN (1956) found $k = 3.8 \times 10^{-3}$ and ROSSITER (1958) found $k = 3.5$ to 4.5×10^{-3}. WYRTKI (1956) has summarised a number of observations in which the friction has been estimated from wind drift in the surface layer in the open ocean. The corresponding values of k are mostly in the range 1.0 to 2.7×10^{-3}, apart from some rather doubtful values of 10 to 12×10^{-3} for the Atlantic.

In dealing with the flow through straits, DEFANT (1955) proposed the value $k = 3 \times 10^{-2}$ and found that this led to estimates of velocity of the correct order of magnitude for flow through passages such as the Dardanelles, Bosporus and Straits of Gibraltar. The flow in these cases differs from the bottom currents envisaged above in that the current in the bottom layer is bounded above by a current flowing with comparable velocity in the opposite direction. The resistance to the bottom current is likely to be greater, therefore, than if the overlying water were at rest or if the current extended to the free surface. The irregular bottom topography in the straits probably increases the resistance further.

DIETRICH (1956) has used the value $k = 3 \times 10^{-2}$ in estimating the rate of flow down the southwestern slope of the Iceland-Faeroe Ridge, from density data obtained in the "Anton Dohrn" cruises. The flow on the Iceland-Faeroe Ridge has also been considered by Steele (1960), who has suggested that the high apparent value of k may be due mainly to the loss of energy from the bottom layer, associated with turbulent mixing of the bottom water with that in the layer above.

At the other extreme, STONELEY (1957), in dealing with turbidity currents, has suggested a value as low as $k = 2 \times 10^{-4}$ for a turbidity current with velocity of the order of 28 m/s. This value is based on an extrapolation of data obtained by Keulegan, in model-scale experiments on the flow of saline wedges, in which he found that k decreased with increasing Reynolds number.

5. Solutions for steady motion

If all accelerations relative to earth are negligibly small compared with the other terms in (12), one may put

$$\frac{\overline{Du}}{Dt} = \frac{\overline{Dv}}{Dt} = 0, \text{ and then}$$

$$\left.\begin{aligned} -f\bar{v} &= g'\left(\sin\alpha - \frac{\partial\eta}{\partial x}\cos\alpha\right) - \frac{kV\bar{u}}{\eta} \\ f\bar{u} &= -g'\frac{\partial\eta}{\partial y}\cos\alpha - \frac{kV\bar{v}}{\eta} \end{aligned}\right\} \quad (13)$$

Putting $\frac{\partial\eta}{\partial t} = 0$ in (11)

$$\bar{u}\frac{\partial\eta}{\partial x} + \bar{v}\frac{\partial\eta}{\partial y} + \eta\left(\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{v}}{\partial y}\right) = 0 \quad (14)$$

(a) *Layer of uniform thickness*

If the thickness of the bottom layer is uniform, i.e. $\frac{\partial \eta}{\partial x} = \frac{\partial \eta}{\partial y} = 0$ (13) becomes

$$\left.\begin{aligned} -f\bar{v} &= g' \sin\alpha - \frac{kV\bar{u}}{\eta} \\ f\bar{u} &= -\frac{kV\bar{v}}{\eta} \end{aligned}\right\} \qquad (15)$$

Let $\tan\gamma = -\bar{u}/\bar{v}$, so that $\bar{u} = V\sin\gamma$, $\bar{v} = -V\cos\gamma$.

Then, from (15),

$$\tan\gamma = \frac{kV}{f\eta} \qquad (16)$$

and

$$V\left(1 + \frac{k^2V^2}{f^2\eta^2}\right)^{\frac{1}{2}} = \frac{g' \sin\alpha}{f} \qquad (17)$$

Thus V may be calculated, given g', $\sin\alpha$, f, η and k and hence, also, $\tan\gamma$, $\bar{u}$ and $\bar{v}$.

From (16) and (17),

$$\left.\begin{aligned} \bar{u} &= \frac{g' \sin\alpha}{f} \frac{kV}{f\eta} \left(1 + \frac{k^2V^2}{f^2\eta^2}\right)^{-1} \\ \bar{v} &= -\frac{g' \sin\alpha}{f} \left(1 + \frac{k^2V^2}{f^2\eta^2}\right)^{-1} \end{aligned}\right\} \qquad (18)$$

Since $\bar{u}$ and $\bar{v}$, as given by (18), are independent of x and y, equation (14) is satisfied. Thus flow in a layer of uniform thickness, with uniform velocity given by (18), is a possible form of motion.

If conditions are uniform parallel to the crest of the ridge, i.e. $\partial\eta/\partial y = 0$, $\partial\bar{v}/\partial y = 0$, and if $\partial\bar{u}/\partial x$ is negligibly small, it follows from (14) that $\partial\eta/\partial x = 0$ also, so that the problem reduces to that just considered.

The case of flow in a bottom layer of uniform thickness has been treated by Steele (1960), who obtained a solution essentially the same as that given above.

In the frictionless case, $k = 0$, the above solution becomes

$$\bar{u} = 0, \quad \bar{v} = -V = -\frac{g' \sin\alpha}{f}$$

The flow is then purely geostrophic and parallel to the crest of the ridge.

If, on the other hand, the friction is very great, so that

$$\frac{kV}{f\eta} \gg 1, \text{ then } \bar{u} \to V, \ \bar{v} \to 0$$

and

$$\bar{u} = V = \left(\frac{g'\eta \sin\alpha}{k}\right)^{\frac{1}{2}}$$

so that the flow is directed down the slope and its velocity is limited by the friction coefficient k.

(b) *No transverse component of flow*

If there is no flow parallel to the crest of the ridge, i.e. $\bar{v} = 0$, $\bar{u} = V$, and if $\partial\bar{u}/\partial x$ is negligibly small, then from (14), $\partial\eta/\partial x = 0$.

Hence in (13)

$$\left.\begin{aligned} 0 &= g' \sin\alpha - \frac{k\bar{u}^2}{\eta} \\ f\bar{u} &= -g' \frac{\partial\eta}{\partial y} \end{aligned}\right\} \qquad (19)$$

Thus

$$\bar{u} = \left(\frac{g'\eta \sin\alpha}{k}\right)^{\frac{1}{2}} \qquad (20)$$

and

$$\frac{\partial\eta}{\partial y} = -\frac{f\bar{u}}{g'} \qquad (21)$$

Thus, in order to maintain a flow directly down the slope, a transverse gradient, given by (21), is required.

(c) *More general case*

If the slope is small enough for the approximations discussed in § 2 to be valid, the axes OX and OY may be taken in any two perpendicular directions in the plane of the bottom. In this case, let OX and OY be inclined downwards at angles α and β respectively to the horizontal. Then equation (13) would be replaced by

$$\left.\begin{aligned} -f\bar{v} &= g' i_x - \frac{kV\bar{u}}{\eta} \\ f\bar{u} &= g' i_y - \frac{kV\bar{v}}{\eta} \end{aligned}\right\} \qquad (22)$$

where $i_x = \sin\alpha - \frac{\partial\eta}{\partial x}\cos\alpha$, $i_y = \sin\beta - \frac{\partial\eta}{\partial y}\cos\beta$. i_x and i_y represent the components of slope of the upper surface of the bottom layer in the OX and OY directions.

If the OX axis is taken in the direction of the resultant current in the bottom layer, then $\bar{u} = V$, $\bar{v} = 0$ and (22) becomes

$$g' i_x = \frac{kV^2}{\eta} \tag{23A}$$

$$fV = g' i_y \tag{23B}$$

(14) becomes

$$\frac{\partial}{\partial x}(\eta V) = 0 \tag{23C}$$

Thus V can be computed from g' and the slope i_y of the interface in the OY direction only. Then, if i_x and η are known, (23A) enables an estimate of k to be made.

From (23A) and (23B)

$$\tan\gamma \equiv \frac{i_x}{i_y} = \frac{kV}{f\eta} \tag{24}$$

This equation, corresponding to (16) for the case of a layer of uniform thickness, gives the angle γ between the current vector and the contours of the upper surface of the bottom layer.

6. Numerical solutions

To obtain an order of magnitude for the quantities involved, consider the values given by Dietrich (1956) for section IIIb across the centre of the Iceland-Faeroe Ridge. In the above notation, for a position at a depth of 700 m on the south-west slope, $\varrho_1 = 1.0277$, $\varrho_2 = 1.0280$, $\sin\alpha = 1/125$, $\eta = 50$ m. Then $g' = 0.29$ cm/s², and taking $\phi = 63°$, $f = 1.30 \times 10^{-4}$ s^{-1}.

Considering solution 5(a), i.e. the bottom layer of uniform thickness, and with $k = 3 \times 10^{-2}$, equations (16) and (17) give

$$V = 14.5 \text{ cm/s}, \ \tan\gamma = 0.67, \ \gamma = 33.8°$$

so that

$$\bar{u} = 8.1 \text{ cm/s}, \ \bar{v} = 12.0 \text{ cm/s}.$$

If $\qquad k = 3 \times 10^{-3}$,

then

$$V = 17.5 \text{ cm/s}, \ \tan\gamma = 0.081, \ \gamma = 4.6°,$$
$$\bar{u} = 1.4 \text{ cm/s}, \quad \bar{v} = 17.5 \text{ cm/s}.$$

In the purely geostrophic case, i.e. $k = 0$,

$$V = 17.6 \text{ cm/s}, \ \tan\gamma = 0.$$

Even with the higher value of k, 3×10^{-2}, the component of flow parallel to the ridge is greater than the component down the slope.

Considering solution 5 (b), i.e. with no transverse component of flow, and taking $k = 3 \times 10^{-2}$, equation (20) gives $u = 19.6$ cm/s and then (21) gives $\frac{\partial\eta}{\partial y} = -0.89 \times 10^{-2}$. With $k = 3 \times 10^{-3}$, (20) gives $\bar{u} = 62$ cm/s, and from (21), $\frac{\partial\eta}{\partial y} = -2.82 \times 10^{-2}$.

Thus the flow can be directed down the slope only if the thickness of the layer increases to the right of the flow with a gradient of order 10^{-2}, i.e. 10 m per km. This could be maintained in a narrow channel or canyon but not on an open slope. A comparison of Dietrich's sections IIIb and IIIa does indicate an increase in the thickness of the bottom layer from southeast to northwest, but only of the order of 100 m in 100 km, i.e. 10^{-3}, which is too small by a factor of 10 to maintain a purely downslope flow. It appears, therefore, that the interpretation of a section such as IIIb should be based on solution (a) rather than solution (b), with the major component of flow parallel to the contours of the ridge.

The conditions postulated in solution (c), equations (23 A) and (23 B), appear to resemble those reported by Dietrich (1957) in the bottom current flowing southwards at the foot of the continental slope off East Greenland. From the velocities computed by Dietrich across sections off Cape Dan and Cape Farewell, the mean velocities in the bottom layer may be taken as 10 cm/s and 5 cm/s respectively. The increase in depth of the isosteric surface, $\sigma_t = 27.9$, from the C. Dan to the C. Farewell section is about 500 m in 500 km, i.e. 10^{-3}. Taking $\Delta\sigma_t = 0.1$ gives $g' = 0.095$ cm/s², and putting $i_x = 10^{-3}$, $\bar{u} = 7.5$ cm/s in (23 A), leads to

$$\frac{k}{\eta} = 1.7 \times 10^{-6} \text{ cm}^{-1}.$$

η may be estimated at 200 m, corresponding to $k = 3.4 \times 10^{-2}$.

Thus if the flow in this current may be assumed to be in a steady state, it is consistent with a value of k of the order 3×10^{-2}. Then, in the C. Dan section, taking $\bar{u} = 10$ cm/s, $\eta = 250$ m gives $\tan \gamma = 0.105$, $\gamma = 6°$. In the C. Farewell section, with $\bar{u} = 5$ cm/s, $\eta = 150$ m the values $\tan \gamma = 0.087$, $\gamma = 5°$ are obtained.

It is of some interest to compare the features of the bottom currents considered here with those of a turbidity current. Taking the values

$\varrho_1 = 1.0$, $\varrho_2 = 1.4$, corresponding to $g' = 280$ cm/s^2, $k = 3 \times 10^{-3}$, $\eta = 100$ m, $V = 28$ m/s, $f = 1.1 \times 10^{-4}$ s^{-1}, then $\tan \gamma = 7.6$, $\gamma = 82{,}5°$.

If the current is constrained to flow down the line of greatest slope, the transverse gradient of its upper boundary, from (21), is $\partial\eta/\partial y = 1.1 \times 10^{-3}$, which is about 1/10 of that associated with a bottom current of the type considered above.

7. Density varying continuously with depth

Let it be assumed that there is some surface, at a distance z' from the bottom, on which the velocity and the shearing stress may be taken to be zero, as in Fig. 3. Then from equation (3), at $z = z'$,

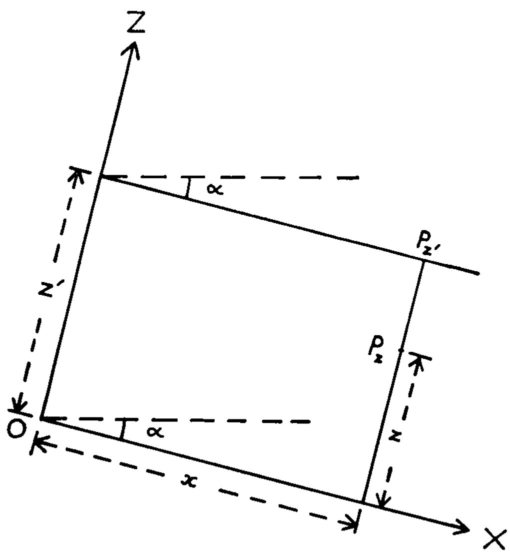

Fig. 3. Co-ordinates for density varying with depth: velocity and shearing stress zero on the surface $z = z'$.

$$\frac{1}{\varrho}\frac{\partial p}{\partial x} = g \sin \alpha, \quad \frac{\partial p}{\partial y} = 0 \tag{25}$$

If $p_{z'}$ = pressure at the point (x, z'), then the pressure at (x, z) is given by

$$p = p_{z'} + g \cos \alpha \bar{\varrho}_z (z' - z)$$

$$\text{where } \bar{\varrho}_z = \frac{1}{z' - z}\int_z^{z'} \varrho \, dz$$

Hence, using (25),

$$\frac{1}{\varrho}\frac{\partial p}{\partial x} = g \sin \alpha + g \cos \alpha (z' - z) \frac{1}{\varrho}\frac{\partial \bar{\varrho}_z}{\partial x}$$

Then in (3),

$$\left.\begin{aligned} \frac{Du}{Dt} - fv &= -g \cos \alpha (z' - z) \frac{1}{\varrho}\frac{\partial \bar{\varrho}_z}{\partial x} + \\ &\quad + \frac{1}{\varrho}\frac{\partial F_{zx}}{\partial z} \\ \frac{Dv}{Dt} + fu &= -g \cos \alpha (z' - z) \frac{1}{\varrho}\frac{\partial \bar{\varrho}_z}{\partial y} + \\ &\quad + \frac{1}{\varrho}\frac{\partial F_{zy}}{\partial z} \end{aligned}\right\} \tag{26}$$

If the accelerations relative to earth are zero, integrating from the bottom to $z = z'$ gives,

$$\left.\begin{aligned} -\int_0^{z'} v\,dz &= -\frac{g \cos \alpha}{f}\int_0^{z'} \frac{1}{\varrho}\frac{\partial \bar{\varrho}_z}{\partial x}(z' - z)\,dz - \\ &\quad - \frac{F_{0x}}{\varrho f} \\ \int_0^{z'} u\,dz &= -\frac{g \cos \alpha}{f}\int_0^{z'} \frac{1}{\varrho}\frac{\partial \bar{\varrho}_z}{\partial y}(z' - z)\,dz - \\ &\quad - \frac{F_{0y}}{\varrho f} \end{aligned}\right\} \tag{27}$$

since $F_{z'x} = F_{z'y} = 0$.

The terms on the left hand side of (27) represent the components of volume transport by the bottom current. The first terms on the right hand side represent the components of transport in the absence of friction, while the second terms may be regarded as correction terms due to friction. Then (27) may be written

$$\left.\begin{aligned} T_y &= T_{0y} + \Delta T_y \\ T_x &= T_{0x} + \Delta T_x \end{aligned}\right\} \qquad (28)$$

where $T_x = \int_0^{z'} u\,dz$, $T_y = \int_0^{z'} v\,dz$,

$$T_{0x} = -\frac{g\cos\alpha}{f}\int_0^{z'} \frac{1}{\varrho}\frac{\partial\bar{\varrho}_z}{\partial y}(z'-z)\,dz,$$

$$T_{0y} = \frac{g\cos\alpha}{f}\int_0^{z'} \frac{1}{\varrho}\frac{\partial\bar{\varrho}_z}{\partial x}(z'-z)\,dz,$$

$$\left.\begin{aligned} \Delta T_x &= -\frac{F_{0y}}{\varrho f} \\ \Delta T_y &= \frac{F_{0x}}{\varrho f} \end{aligned}\right\} \qquad (29)$$

Thus the transport correction terms depend only on the components of bottom friction and the Coriolis parameter. The numerical examples discussed in § 6 suggest that in many cases the correction terms will not exceed about 10 % of the transports computed neglecting friction.

A procedure for the analysis of observational data may now be outlined: —

(i) Assuming that the position of a surface of no motion parallel to the bottom may be estimated, the velocity components across two mutually perpendicular sections, each perpendicular to the bottom, may be computed, neglecting friction. This may be done, for example, by Helland-Hansen's method and the terms T_{0x}, T_{0y} determined.

(ii) From the computed velocity at the bottom, the frictional terms F_{0x}, F_{0y} may be estimated, assuming a value of the coefficient k.

(iii) The transport correction terms ΔT_x, ΔT_y may then be computed from (29).

8. Conclusions

The following conclusions, relevant to the interpretation of observational data, may be drawn.

(i) In bottom currents, flowing on a submarine slope, the effects of bottom friction are likely to be appreciable and should not be neglected. At the same time, the geostrophic effects must also be taken into account.

(ii) In the absence of friction, the currents would tend to follow the contours of the bottom. The effect of friction is to deflect the current to the down-slope side of the contours. The angle between the current vector and the contours might be as great as 30° in some cases, but it seems probable that in most circumstances it will be less, of the order of 5° or 10°.

(iii) It is unreliable, under these conditions, to attempt to estimate the velocity of flow from data in a single vertical section. Observations in two sections at right angles should be obtained.

(iv) There is at present considerable uncertainty in the value of the friction coefficient k. It is desirable, therefore, that in as many cases as possible, direct measurements of bottom currents should be made at the same time as the density data are being obtained. If reliable estimates of k could be established, it might then be possible to compute bottom currents satisfactorily from density data alone.

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