On the Problem of Initial Data for the Primitive Equations

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(Manuscript received September 18, 1959)

Abstract

It is first demonstrated that in a forecast with the primitive equations, pure geostrophic or purely non-divergent initial wind fields will result in noticeable meteorological "noise", especially in a baroclinic atmosphere. It is then shown in a linear example that this noise is greatly suppressed if the divergence of the initial wind field is set equal to the divergence implied by the usual geostrophic theory of numerical weather prediction. The absence of noise in Charney's 1955 test computation with non-divergent "balanced" initial data is explained.

Finally, the quasi-geostrophic expansion is used to define initial data for a non-linear forecast with the primitive equations. Two cases are considered, that where only the geopotential is known from observations, and that where only the non-divergent part of the wind is known from observation. It is shown that the initial tendencies computed from initial data of this kind preserve the assumed geostrophic character of the flow, and thereby make it possible to get more accurate forecasts with the primitive equations.

1. Introduction

It is well known that the "primitive equations" (the Eulerian hydrodynamical equations modified by the assumption of hydrostatic balance) are very sensitive to the initial ($t=0$) fields of wind and pressure; unless these fields are suitably adjusted to one another, the forecast may contain primarily *noise* in the form of high-frequency gravity-inertia waves, and the more important motions of lower frequency will thereby be hidden. HINKELMANN (1951) has shown that the amplitude of this unwanted noise can be made smaller than that of the desired low frequency motion by simply using the geostrophic values for the initial wind field. His theoretical treatment was for a linear system, however. It was later shown in a non-linear example by CHARNEY (1955) that better results could be obtained by relating the initial wind and pressure (geopotential) fields by the "balance equation":

$$\nabla \cdot l \nabla \psi + 2\left[\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial y^2} - \left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2\right] = \nabla^2 \varphi \qquad (1)$$

Here the initial wind $\mathbf{v}$ is given by the stream-function ψ, so that $\mathbf{v}$ is initially non-divergent. The symbols l and φ are the Coriolis parameter and the geopotential, respectively, and ∇ is the horizontal gradient operator. (1) is derived from the equations of motion by simply requiring not only that the horizontal divergence $\nabla \cdot \mathbf{v}$ vanish at $t=0$, but also that $\partial(\nabla \cdot \mathbf{v})/\partial t$ vanish at $t=0$.

It is easy to demonstrate that, in a linear system, the relation (1) is not sufficient to eliminate gravity-inertia waves. In a linear system (1) reduces to

$$\nabla \cdot l \nabla \psi = \nabla^2 \varphi, \qquad (2)$$

which, if l is considered constant, is simply the geostrophic relation. We will now consider a particular simple linear system and show that geostrophic initial conditions, although setting an upper limit to the amplitude of the meteorological noise, are not precise enough, and that the initial wind field must contain some divergence if the gravity-inertia waves are to be satisfactorily eliminated from the forecast.

2. Noise from geostrophic initial data

As a simple linear system we take that studied by HINKELMANN (1951). A constant basic current U (from west to east along the x-axis), in a homogeneous incompressible atmosphere of average height H, has small perturbations superimposed on it. The perturbations are assumed to have infinite extent from north to south (y), and the Coriolis parameter l is considered a constant. Three types of wave motions are found to be possible; a "meteorological" wave moving approximately with the speed $c_1 \approx U$, and two gravity-inertia waves moving approximately with the speed $c_{2,3} \approx U \pm \sqrt{gH + l^2/\nu^2}$. ($\nu$ is the wave-number $= 2\pi/L$, where L is the wave length in the west-east direction.)

In general all three waves will be present. Following Hinkelmann, we let the total amplitude in the geopotential be 1, and let A_1, A_2, A_3 be the amplitude in the geopotential associated with each of the three different waves (i. e. $A_1 + A_2 + A_3 = 1$). We now wish to know the values of A_1, A_2, and A_3 when the initial wind field is geostrophic. This is easily determined by setting Hinkelmann's parameters δ and ε [in his equations (16) and (14b)] equal to zero. Substitution of the resulting values of F and G in his equation (15) gives the desired result:

$$\left.\begin{aligned} A_1 &= 1 + O(\mu^2), \\ A_2 \sim -A_3 &= \frac{1}{2}\mu\left[1 + \frac{l^2}{\nu^2 gH}\right]^{-1} + O(\mu^2) \end{aligned}\right\}, \quad (3)$$

$$\mu = U[gH + l^2/\nu^2]^{-\frac{1}{2}} \quad (4)$$

The non-dimensional term $l^2/\nu^2 gH$ in (3) is less than 1, even for very long waves.

The use of strict geostrophic initial conditions for the primitive equations in this simple system will therefore result in some noise. This noise will have an amplitude (in φ) equal approximately to μ times the amplitude of the low frequency wave. In the barotropic system we have used, μ is indeed small ($\mu \sim 1/10$). However, in a baroclinic system, with internal gravity waves, the value of g is effectively reduced, and μ will not be so small. Geostrophic initial conditions will therefore no longer be even partially satisfactory for predictions with the primitive equations.

3. Elimination of the noise in a linear system

We now ask for a better definition of the initial u- and v-fields in Hinkelmann's linear barotropic model—better in the sense that A_2 and A_3 will be smaller than the values given by (3). From Hinkelmann's analysis we readily find that the u- and v-fields (u_1 and v_1) associated with the meteorological wave (subscript 1) are related to the geopotential field (φ_1) of that wave by the following equations:

$$u_1 = \frac{[\mu + O(\mu^3)]}{\sqrt{gH + l^2/\nu^2}}\,\phi_1 \sim \frac{\mu\phi_1}{\sqrt{gH + l^2/\nu^2}} \quad (5)$$

$$v_1 = [1 + O(\mu^2)]\,\frac{i\nu}{l}\,\phi_1 \sim \frac{1}{l}\,\frac{\partial\phi_1}{\partial x} \quad (6)$$

If the (total) initial u- and v-fields are related to the (total) initial φ-distribution by the exact forms of (5) and (6), only the meteorological wave will be present, and there will be *no* noise motion. If the approximate forms of (5) and (6) are used, an analysis similar to that in section 2 will show that A_2 and A_3 are at most of order μ^2 instead of μ.

In the preceding analysis in section 2 *both* the initial u- and v-fields were assumed to be given geostrophically by the initial φ-field. As we saw, this did not eliminate the noise completely, but only reduced it to something like 10 % of the geostrophic wave motion. (5) and (6) on the other hand show that only v in this linear system should be geostrophic if noise is to be eliminated to order μ^2. The initial u-field should not be geostrophic ($u_{geo} = 0$ since $\partial\varphi/\partial y = 0$ by assumption), but should be given instead by (5).

Since the entire analysis has been made assuming that $\partial u/\partial y, \partial v/\partial y$ and $\partial\varphi/\partial y$ are zero, the horizontal divergence is given by

$$\nabla\cdot\mathbf{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = i\nu u.$$

The approximate form of the relation (5) between u and φ is therefore equivalent to requiring that

$$(\nabla\cdot\mathbf{v})_{t=0} = \frac{i\nu\mu}{\sqrt{gH + l^2/\nu^2}}\,\phi_{t=0} = \frac{\mu}{\sqrt{gH + l^2/\nu^2}}\left(\frac{\partial\varphi}{\partial x}\right)_{t=0} \quad (7)$$

Thus, the elimination of the noise motion requires a certain amount of *divergence* in the initial velocity field. Although this has been demonstrated here for a linear system only, it is obvious that the presence of non-linearities will not make this needed divergence zero, but on the contrary will only make the initial-value problem still more complicated. We may therefore conclude that the use of "balanced" initial data, as suggested by Charney, is not sufficient to eliminate noise motions in general.

We will now show that the initial field of divergence given by (7), whose introduction is necessary to eliminate the unwanted gravity-inertial waves in this linearized system, is exactly that field of divergence implied by the usual quasi-geostrophic vorticity equation. The quasi-geostrophic equations for this simple model are (we still neglect $\beta = dl/dy$):

$$\begin{aligned} &\frac{\partial\zeta}{\partial t} + U\frac{\partial\zeta}{\partial x} + l\nabla\cdot\mathbf{v} = 0, \\ &\frac{\partial\varphi}{\partial t} + gH\nabla\cdot\mathbf{v} = 0, \\ &\zeta = \partial v/\partial x = l^{-1}\partial^2\varphi/\partial x^2. \end{aligned} \tag{8}$$

These may be solved for $\nabla\cdot\mathbf{v}$ by eliminating $\partial\varphi/\partial t$.

We get

$$\left[l^2 - gH\frac{\partial^2}{\partial x^2}\right](\nabla\cdot\mathbf{v}) = -U\frac{\partial^3\varphi}{\partial x^3} = U\nu^2\frac{\partial\varphi}{\partial x},$$

or

$$(\nabla\cdot\mathbf{v})_{geo} = \frac{U}{gH + l^2/\nu^2}\frac{\partial\varphi}{\partial x} = \frac{\mu}{\sqrt{gH + l^2/\nu^2}}\frac{\partial\varphi}{\partial x} \tag{9}$$

This relation, by its agreement with (7), shows that the initial field of divergence, which is necessary to eliminate the gravity-inertia waves in this linear initial-value problem, is just that field of divergence which is implied by the usual quasi-geostrophic system of forecasting. Although the analysis has been confined to a linear system, we may infer that the same procedure should be followed for a flow pattern in which non-linear terms are important.[1]

4. The absence of noise in Charney's test computation

The analysis above has shown that the more-or-less complete elimination of noise for the primitive equations is possible only if the initial wind field contains a divergence field equal to the field of divergence implied by the quasi-geostrophic theory. This seems to be in disagreement with CHARNEY's result (1955) that the use of the balance equation (1) with a non-divergent initial wind field is necessary to eliminate the noise. We will see that there is no disagreement because the idealized flow pattern used by him to test his hypothesis was a special case.

Charney's initial flow pattern was of the simple form

$$\psi = A\sin\frac{2\pi x}{L}\sin\frac{\pi y}{D}, \tag{10}$$

where ψ is the stream function, and A, L and D are constants. This flow pattern existed in the upper layer of a two layer model atmosphere, with the lower layer being so deep that its motions could be ignored.

The (non-linear) quasi-geostrophic equations for this model may be written:

$$\frac{\partial\zeta}{\partial t} + J\left(\frac{\psi,\zeta}{x,y}\right) + l\nabla\cdot\mathbf{v} = 0, \tag{11}$$

$$l\frac{\partial\psi}{\partial t} + g'H\nabla\cdot\mathbf{v} = 0, \tag{12}$$

$$\zeta = \nabla^2\psi \tag{13}$$

(The Coriolis parameter l was also taken as constant by Charney.) Here g' is a reduced value of gravity equal to $(1-\varrho'/\varrho)\,g$, where ϱ' and ϱ are the densities of the upper and lower layers. These equations are readily combined to give an equation for $\nabla\cdot\mathbf{v}$:

$$\left[\nabla^2 - \frac{l^2}{g'H}\right](\nabla\cdot\mathbf{v}) = \frac{l}{g'H}J\left(\frac{\psi,\nabla^2\psi}{x,y}\right). \tag{14}$$

This equation gives the quasi-geostrophically implied value of $\nabla\cdot\mathbf{v}$ which, according to the results of section 2 and 3, should be included in the initial wind field if no noise is to appear.

It now becomes clear why Charney's neglect of this initial divergence field did not give rise

[1] HINKELMANN (1959) has recently performed a numerical experiment verifying this result.

to gravity-interia waves in his computation, for it is apparent from the simple form of (10) that the right side of (14) vanishes. This being the case $\nabla \cdot \mathbf{v}$ also vanishes, and it was therefore unnecessary to include divergence in the initial wind field in his special case. On the other hand, the results of Charney's test do show conclusively that, in a non-linear flow pattern, the relation (1) is better than the geostrophic relation as a constraint on the initial data. Evidently (1) gives a better definition of the initial *vorticity* than does the geostrophic formulae.

5. Initial data for non-linear problems

When the equations of hydrodynamics are not linearized, it is no longer possible to separate the different types of motion, and by a clever choice of initial conditions, to exclude forever any unwanted "noise" from the forecast. As a compromise, however, we can require this of the initial data—that when at $t=0$ we compute the local rates of change of $\partial \mathbf{v}/\partial t$, $\partial \varphi/\partial t$, etc., the latter quantities should all have values very close to what they would have (initially) in a quasi-geostrophic forecast. This is a reasonable reqirement first of all because we know that the geostrophically computed values are at least approximately correct. Secondly, as is apparent from section 3, this requirement does eliminate the unwanted noise almost completely in a linear system.

A technique for imposing this requirement comes very neatly out of the familiar theory of the quasi-geostrophic expansion (PHILIPPS, 1939). So far the most useful application of this theory has been to derive the well-known geostrophic system of prognostic equations (ELIASSEN, 1948; IUDIN, 1957; KIBEL, 1957; MONIN, 1958). Here we will emphasize its usefulness in deriving satisfactory initial data for the primitive equations. In the process we will see that the appropriate initial values of $\nabla \cdot \mathbf{v}$ and $\mathbf{k} \cdot \nabla \times \mathbf{v}$ — the importance of the former being apparent from section 3 and the importance of the latter from Charney's 1955 computation—will both appear as improvements on the use of geostrophic initial data.

We first set the horizontal wind $\mathbf{v}$ equal to geostrophic wind $\mathbf{v}_g$ plus the geostrophic deviation $\mathbf{v}'$,

$$\mathbf{v} = \mathbf{v}_g + \mathbf{v}', \tag{15}$$

whereupon the horizontal equation of motion can be written

$$\mathbf{v}' = \frac{1}{l}\mathbf{k} \times \frac{d\mathbf{v}}{dt} \cdot \tag{16}$$

$\mathbf{k}$ is the vertical unit vector. The acceleration $d\mathbf{v}/dt$ is now approximated geostrophically,

$$\mathbf{v}' \approx \frac{\mathbf{k}}{l} \times \left[\frac{\partial \mathbf{v}_g}{\partial t} + (\mathbf{v}_g \cdot \nabla)\mathbf{v}_g\right], \tag{17}$$

and this equation is used to evaluate $\nabla \cdot \mathbf{v}'$ and $\zeta' = \mathbf{k} \cdot \nabla \times \mathbf{v}'$:

$$\nabla \cdot \mathbf{v}' = -\frac{1}{l}\left[\frac{\partial \zeta_g}{\partial t} + \mathbf{v}_g \cdot \nabla \zeta_g\right], \tag{18}$$

$$l\zeta' = -\frac{2}{l^2}\left[\frac{\partial^2 \varphi}{\partial x^2}\frac{\partial^2 \varphi}{\partial y^2} - \left(\frac{\partial^2 \varphi}{\partial x \partial y}\right)^2\right]. \tag{19}$$

The Coriolis parameter l has so far been considered as a constant. As the final step we compute also $\nabla \cdot \mathbf{v}_g$ and $l\zeta_g$, now letting l vary,

$$\nabla \cdot \mathbf{v}_g = -l^{-1}\mathbf{v}_g \cdot \nabla l \tag{20}$$

$$l\zeta_g = \nabla^2 \varphi - l^{-1}\nabla l \cdot \nabla \varphi, \tag{21}$$

and combine these expressions with (18) and (19) to get:

$$l\nabla \cdot \mathbf{v} = -\left[\frac{\partial \zeta_g}{\partial t} + \mathbf{v}_g \cdot \nabla (l + \zeta_g)\right] \tag{22}$$

$$l\zeta = \nabla^2 \varphi - \frac{1}{l}\nabla \varphi \cdot \nabla l - \frac{2}{l^2}\left[\frac{\partial^2 \varphi}{\partial x^2}\frac{\partial^2 \varphi}{\partial y^2} - \left(\frac{\partial^2 \varphi}{\partial x \partial y}\right)^2\right]. \tag{23}$$

If the right side of these equations can be evaluated, the field of $\mathbf{v}$ itself can be computed by integration of the $\nabla \cdot \mathbf{v}$- and ζ-fields.

Let us for the moment take the simple view that the initial distribution of φ is known, while that of $\mathbf{v}$ is only poorly specified by the wind observations. In this case we consider (22) and (23) as determining more accurate values of $\mathbf{v}$ than are given by the hypothetically poor wind observations. Evaluation of the right side of (23) gives no difficulty. However, (22), which is the usual geostrophic form of the vorticity equation, contains the term $\partial \zeta_g/\partial t = l^{-1}\nabla^2 \partial \varphi/\partial t$. This term we treat by introducing the geostrophic and hydrostatic form of the thermal energy equation for adiabatic motion,

$$\frac{\partial}{\partial p}\left(\frac{\partial \varphi}{\partial t}\right) = -\mathbf{v}_g \cdot \nabla \left(\frac{\partial \varphi}{\partial p}\right) - \bar{\sigma}\omega, \quad (24)$$

and the continuity equation,

$$\nabla \cdot \mathbf{v} + \partial \omega / \partial p = 0. \quad (25)$$

[$\bar{\sigma} = \bar{\sigma}(p)$ is normally taken as a standard value of $(\partial\varphi/\partial p) \times (\partial \ln \Theta/\partial p)$ where Θ is the potential temperature.] (22), (24), and (25), together with appropriate boundary conditions are, in fact, the quasi-geostrophic prognostic equations and can be solved for both $\partial\varphi/\partial t$ and $\nabla \cdot \mathbf{v}$. We are therefore able to determine $\mathbf{v}$, since both ζ and $\nabla \cdot \mathbf{v}$ are now known.

Let us now assume that the initial data for a forecast with the primitive equations is the field of $\mathbf{v}$ we have computed in this way together with the original field of φ. We examine first the initial value of $\partial \mathbf{v}/\partial t$ computed from the *original* equation of motion (16), realizing that $\mathbf{v}'$ in (16) is now equal to the value given it by (17).

$$\begin{aligned}\frac{\partial \mathbf{v}}{\partial t} &= -(\mathbf{v}\cdot\nabla)\mathbf{v} - \omega\frac{\partial \mathbf{v}}{\partial p} - l\mathbf{k}\times\mathbf{v}' \\ &= \frac{\partial \mathbf{v}_g}{\partial t} + [(\mathbf{v}_g\cdot\nabla)\mathbf{v}_g - (\mathbf{v}\cdot\nabla)\mathbf{v}] - \omega\frac{\partial \mathbf{v}}{\partial p} \quad (26) \\ &\approx \frac{\partial \mathbf{v}_g}{\partial t}.\end{aligned}$$

The last step here is possible because, as shown by CHARNEY's scale theory (1948), $\omega\partial\mathbf{v}/\partial p$ is smaller than any of the other terms in (26) for geostrophic types of motion.[1] The difference between the two horizontal advection terms is also smaller (presumably) than the individual terms. Assuming finally that $\partial/\partial t$ is of the same order of magnitude as $\mathbf{v}\cdot\nabla$, we end up with only $\partial\mathbf{v}_g/\partial t = l^{-1}\mathbf{k}\times\nabla(\partial\varphi/\partial t)$ on the right side. [$l=$ constant here, as it was in (17).] We may then conclude that the initial values of $\partial\mathbf{v}/\partial t$ are such that

$$\frac{\partial \zeta}{\partial t} \approx \frac{\partial \zeta_g}{\partial t} = l^{-1}\nabla^2\frac{\partial \varphi}{\partial t} \quad (27)$$

$$\frac{\partial}{\partial t}(\nabla\cdot\mathbf{v}) \approx 0 \quad (28)$$

[1] This is not true, though, for extremely long waves (BURGER 1958). For such waves, however, $\partial\mathbf{v}/\partial t \approx 0$.

We now investigate the value of $\partial(\partial\varphi/\partial p)/\partial t$ computed with the complete (i.e. non-geostrophic) form of the thermal energy equation:

$$\frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial p}\right) = -\mathbf{v}\cdot\nabla\left(\frac{\partial \varphi}{\partial p}\right) - \sigma\omega \quad (29)$$

On the right side of this equation $\mathbf{v}$ is again the $\mathbf{v}$ determined from (22) and (23). ω is identical with the ω appearing in (24) and (25), as is also the case with $\nabla(\partial\varphi/\partial p)$. σ, however, is presumably the actual value of $(\partial\varphi/\partial p)$ $(\partial \ln \Theta/\partial p)$ instead of a standard value as in (24). Applying a subscript g to the value of $\partial(\partial\varphi/\partial p)/\partial t$ computed geostrophically on the left side of (24), and eliminating the common value of ω between (29) and (24), we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial p}\right) &= \left(\frac{\sigma}{\bar{\sigma}}\right)\left[\frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial p}\right)\right]_g + \\ &+ \left[\left(\frac{\sigma}{\bar{\sigma}}\right)\mathbf{v}_g - \mathbf{v}\right]\cdot\nabla\frac{\partial \varphi}{\partial p} \approx \left[\frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial p}\right)\right]_g \quad (30)\end{aligned}$$

Here the final approximate equality is true only if $(\sigma/\bar{\sigma})$ is not very different from 1.

The importance of establishing the final approximate equalities of (26) and (30) is that they show that, although the fields of $\mathbf{v}$ and φ are changing with time, *they will remain in approximate geostrophic balance.*[1] This, of course, is the implication of the approximation used between (16) and (17).

Equation (23), which gives an improved value for the initial vorticity, is the same as the balance equation (1), except that the nonlinear terms in ψ and the term $\nabla\psi\cdot\nabla l$ have been "approximated" by their geostrophic equivalents. As MONIN has shown (1958), (23) and (1) are both of the same accuracy as far as the quasi-geostrophic expansion is concerned.

Let us now consider the opposite situation, namely that situation where the wind observations are "better" than the observations of φ. As CHARNEY has demonstrated in his scale analysis (1948) ζ is larger than $\nabla\cdot\mathbf{v}$ (again, except for extremely large scale motions) so that we may assume that ζ is given satisfactorily by the wind observations, but that $\nabla\cdot\mathbf{v}$ is not. In this

[1] A more complete demonstration would require also an investigation of the boundary condition on the pressure at the surface of the earth. To save space it is not reproduced here, especially since there are several alternate ways of formulating this condition.

case we take the balance equation (1) to determine the unknown φ,

$$\nabla^2\varphi = \nabla \cdot l\nabla\psi + 2\left[\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2} - \left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right], \quad (1)$$

after ψ has first been determined by solving the Poisson equation

$$\nabla^2\psi = \zeta = \mathbf{k}\cdot\nabla\times\mathbf{v}_{\text{observed}}\,. \quad (31)$$

$\nabla\cdot\mathbf{v}$ has still to be determined. We determine it again from the quasi-geostrophic system except that we use the non-divergent wind $\mathbf{v}_\psi = \mathbf{k}\times\nabla\psi$ in place of the $\mathbf{v}_g$ in (22):

$$l\nabla\cdot\mathbf{v} = -\left[\frac{\partial\nabla^2\psi}{\partial t} + \mathbf{v}_\psi\cdot\nabla(l+\nabla^2\psi)\right]. \quad (32)$$

The counterpart of (24) is written

$$l\frac{\partial}{\partial p}\left(\frac{\partial\psi}{\partial t}\right) = -l\mathbf{v}_\psi\cdot\nabla\left(\frac{\partial\psi}{\partial p}\right) - \bar{\sigma}\omega, \quad (33)$$

by making the substitution $\psi \sim l^{-1}\varphi$. The continuity equation (25) is still retained. Both $\partial\psi/\partial t$ and $\nabla\cdot\mathbf{v}$ can now be determined from (32), (33) and (25), just as $\partial\varphi/\partial t$ and $\nabla\cdot\mathbf{v}$ could be determined from (22), (24) and (25). Finally, it is again possible to show that the approximate relations (26) and (30) are valid in this case also.

In actual practice, both wind and geopotential observations are available, and it would seem advisable to somehow combine the two analysis approaches described by (22), (23) and (24) on the one hand and by (1), (32) and (33) on the other hand. An extension of the type of procedure suggested by SASAKI (1958) may be very useful in this connection.

The above methods of determining "good" initial data for the primitive equations are clearly based on the assumption that the wind is nearly geostrophic. The extent to which the method will prove useful in low latitudes, or in high latitudes when $\nabla\mathbf{v}$ approaches l in magnitude, can perhaps be answered only by experience.

In closing, it should be emphasized that the approximate equalities of (26) and (30) are not goals to be attained for their own sake. The whole idea of using the primitive equations instead of the usual geostrophic system is to obtain an *increased* accuracy, not merely to duplicate the results of the latter method. This increased accuracy in $\partial\mathbf{v}/\partial t$, for example, is to be obtained by including a term like $\omega\partial\mathbf{v}/\partial p$ in (26), a procedure which is not possible with the geostrophic system. However, since $\omega\partial\mathbf{v}/\partial p$ is, by the scale theory smaller than $(\mathbf{v}\cdot\nabla)\mathbf{v}$ or $l\mathbf{k}\times\mathbf{v}'$, it does no good to include $\omega\partial\mathbf{v}/\partial p$ unless the larger terms $(\mathbf{v}\cdot\nabla)\mathbf{v}$ and $l\mathbf{k}\times\mathbf{v}'$ are at the same time evaluated with sufficient accuracy themselves. This may be considered as the fundamental reason for being so careful with the initial data for the primitive equations.

Acknowledgments

The research described in this paper was sponsored by the Office of Naval Research and the Geophysical Research Directorate under contract Nonr 1841(18).

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