The Atmospheric Budget of Angular Momentum

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Abstract

The mean meridional circulation derived from wind observations is combined with surface torque and mountain torque measurements to obtain a complete angular momentum budget for the northern Hemisphere between 15°N and 70°N for summer and winter. A previously proposed scheme of vertical flux of angular momentum involving downward flux everywhere except in the lower layers of the tropical easterlies is confirmed. The layer of zero vertical flux in the tropics is approximately at the level of maximum easterly wind.

1. Introduction

The mean meridional velocities at the surface over the oceans, observed by RIEHL and YEH (1950), have been used by PALMÉN and ALAKA (1952) to obtain an estimate of the various terms in the angular momentum budget of the atmosphere over a limited latitude range. The variation of mean meridional velocity with height had not been observed and therefore a hypothetical decrease to zero at 700 mb. with a compensating flow above was assumed. Recently however, the mean meridional circulation in summer and winter has been derived as a function of latitude and height from an analysis of observed winds in the area 160°W—O—40°E, 75°N—15°N (TUCKER, 1959). These observations have been combined with observations of surface torque (PRIESTLEY 1951a) and mountain torque (WHITE, 1949) to arrive at a fairly detailed and complete angular momentum budget for the Nothern hemisphere in winter and summer.

The mean meridional (and zonal) velocities were previously derived (TUCKER, 1959) in an analysis based upon charts of the field of mean meridional motion in the Northern hemisphere drawn from as complete a network of surface and upper air radar and radio-wind stations as possible. These charts were constructed at various standard pressure levels from the surface

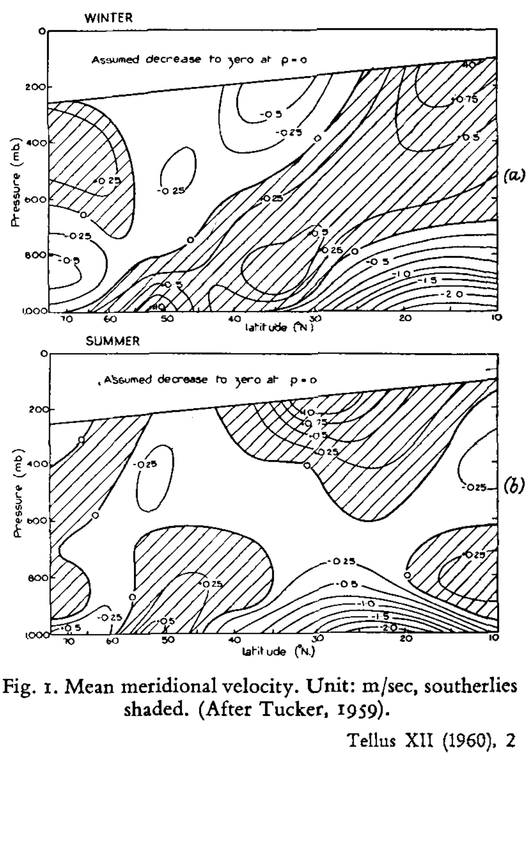

Fig. 1. Mean meridional velocity. Unit: m/sec, southerlies shaded. (After Tucker, 1959).

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to 300 mb. Complete longitude cover was possible for the surface but not for the upper levels and the meridional velocities were corrected for the limited longitude field. The results are represented in Figure 1 which depicts the mean meridional velocity for summer (June, July and August) and winter (December, January and February) as a function of latitude and height. A two-year period of observation was chosen for the computations, from October 1949 to September 1951 inclusive.

2. The budget of angular momentum

(a) *Theory*

Consider an element of volume dV and an element of the surface of this volume dS with $d\sigma$ the projection of dS on the meridional plane. If Ω is the absolute angular momentum per unit mass, then following Widger (1949) the equation of balance of angular momentum can be written as

$$\frac{\partial}{\partial t}\int_V \varrho\Omega\, dV = \int_S \varrho\Omega v_n\, dS + \int_\sigma pr\, d\sigma + \int_S r\tau_x\, dS \quad (1)$$

where τ_x is the total zonal (eastward) frictional stress at the boundary, r the distance from the earth's axis, v_n the *inward* component of velocity across the surface dS, and the integral is taken over a complete latitude belt extending from the surface to the top of the atmosphere.

The term $\int_\sigma pr\, d\sigma$ represents the torque due to pressure differences across mountain ranges where $pd\sigma$ does not vanish (White, 1949). The term $\int_S r\tau_x\, dS$ represents an exchange of angular momentum due to frictional interaction with the earth's surface. Both these terms are only effective at the earth's surface.

$\int_S \varrho\Omega v_n\, dS$ represents the convergence of anglar momentum advected across the boundary surface. If we consider a ring of the atmosphere with horizontal surfaces, H, bounded by latitudes Φ_1, Φ_2 and vertical surfaces, A, bounded by pressures p_1, p_2 then writing $\Omega = ur + \omega r^2$,

$$\int_S \varrho\Omega v_n\, dS = \Big[\int_A \varrho ruv\, dA + \omega \int_A \varrho r^2 v\, dA\Big]_{\Phi_1}^{\Phi_2} + \Big[\int_H \varrho ruw\, dH + \omega \int_H \varrho r^2 w\, dH\Big]_{p_1}^{p_2} \quad (2)$$

where u, v, w are components of motion in the x (east), y (north) and z (vertical) directions respectively. Squared brackets imply the difference between the value of the expression inside the brackets at the two boundary surfaces indicated outside.

We will consider the long term average of total angular momentum around a latitude belt to be constant, that is

$$\frac{\partial}{\partial t}\int_V \varrho\omega\, dV = 0 \quad (3)$$

Neglecting the correlation between ϱ and u, v or w the equation of balance of angular momentum becomes

$$\begin{aligned}
&\int_\sigma pr\, d\sigma + \int_H r\tau_x\, dH_{\text{surface}} \;\ldots\ldots\ldots\ldots \quad \text{(i)}\\
&\left.\begin{aligned}&+\Big[\int_A \varrho r\bar{u}\bar{v}\, dA + \omega\int_A \varrho r^2\bar{v}\, dA\Big]_{\Phi_1}^{\Phi_2}\ldots\\ &+\Big[\int_H \varrho r\bar{u}\bar{w}\, dH + \omega\int_H \varrho r^2\bar{w}\, dH\Big]_{p_1}^{p_2}\ldots\end{aligned}\right\}\ldots \quad \text{(ii)}\\
&+\Big[\int_A \varrho r\overline{u'v'}\, dA\Big]_{\Phi_1}^{\Phi_2} \;\ldots\ldots\ldots\ldots \quad \text{(iii)}\\
&+\Big[\int_H \varrho r\overline{u'w'}\, dH\Big]_{p_1}^{p_2} \;\ldots\ldots\ldots\ldots \quad \text{(iv)}\\
&= 0 \quad (4)
\end{aligned}$$

The bars in terms (*ii*) imply a mean value with regard to time and around a circle of latitude; these terms represent the effect of a mean meridional circulation. Terms involving ω represent the effect of the earth's rotation (ω-angular momentum); terms involving $\bar{u}$ represent the effect of the motion of the atmosphere relative to the earth (relative angular momentum). The prime implies a departure from the mean, and primed terms represent the effects of turbulent motion on all scales less than the mean meridional motion, this is called the *eddy flux*.

The following sections will be devoted to the evaluation of the terms in equation 9 throughout the atmosphere using latitude intervals (Φ_2—Φ_1) of five degrees, and pressure intervals (p_1—p_2) of 200 mb.

(b) *Surface torques* (terms (i), Equ. 4)

In order to determine the sources and sinks of angular momentum at the bottom of the atmosphere the surface frictional torque must be combined with the torque due to pressure differences across mountains.

Table 1. Surface friction torque (S) and torque due to pressure difference across mountains (M). Also conversion table for 1 dyne cm^{-2} throughout a five degree latitude belt. Unit 10^{25} g cm^2 sec^{-2}. Brackets indicate extrapolated values.

Latitude (°N)		70—65	65—60	60—55	55—50	50—45	45—40	40—35	35—30	30—25	25—20	20—15
WINTER	S	(0)	(− 1.0)	− 3.0	− 6.0	− 9.0	−10.5	− 10.5	− 10.5	+ 1.5	+ 6.0	+ 12.0
	M	(+ 1.0)	+ 0.5	0	− 1.0	− 2.0	− 2.0	0	+ 1.5	+ 2.0	+ 2.0	(+ 1.0)
SUMMER	S	(— 0.5)	— 1.5	— 1.5	— 3.0	— 3.0	— 1.5	— 1.5	0	+ 4.5	+ 7.5	+ 4.5
	M	(+ 1.0)	+ 0.5	— 0.5	— 1.5	— 2.0	— 2.0	— 1.5	— 0.5	+ 0.5	+ 2.0	(+ 1.0)
Conversion table 1 dyne	=	2.1	3.0	4.1	5.3	6.5	7.7	8.9	10.3	11.6	12.3	12.9

(i) *Surface frictional torque.* The contribution of the surface friction term $\int_H r\tau_x\, dH_{\text{Surface}}$ within a latitude belt is

$$\int_0^{2\pi}\int_{\phi_1}^{\phi_2} c\varrho u(u^2+v^2)^{\frac{1}{2}} r^2 R\, d\lambda\, d\phi$$

where c is the surface drag coefficient, R the earth's radius and longitude. This term is not accurately known mainly because of uncertainties about the value of c. PRIESTLEY (1950) estimated a generation of 4.1×10^{26} g cm^2 sec^{-2} between 0 and 30°S using $c=0.0013$, while WIDGER (1949) using $c=0.003$ computed the larger value of 7.3×10^{26} g cm^2 sec^{-2} between 10°N and 30°N for January 1946. The most reliable estimate is probably that of PRIESTLEY (1951 a) using ocean surface wind data and assuming a value of 0.0013 for the drag coefficient. Priestley recognised that for various reasons (notably because the inclusion of land surfaces would require a higher value of drag coefficient) his value may be an underestimate by as much as 50 per cent. *Priestley's values are used in this work and are multiplied by the arbitrary factor of 1.5.* This is probably the largest source of error in the following analysis. The surface friction torque (S) for each five degrees latitude belt is given in Table 1; values have been rounded off to 0.5×10^{25} g cm^2 sec^{-2}. A conversion table is included in Table 1 relating angular momentum flux for the five degree latitude belt to linear momentum flux per unit area.

(ii) *Mountain torque.* The contribution of the term $\int_\sigma pr\, d\sigma$ in equation 3 has been calculated by WHITE (1949) who used normal surface pressure fields for each month.

Using White's data the torque due to mountains (M) has been determined for Winter and Summer and is included in Table 1; values have been rounded off to 0.5×10^{25} g cm^2 sec^{-2}.

A comparison of surface friction torque (S) and mountain torque reveals several interesting points:

(a) S and M are generally of the same sign (i.e. in the mean, pressure is highest on the windward side of mountains).

(b) The seasonal variation of S is larger than that of M; the two quantities are of the same order of magnitude in mid-latitudes in summer but elsewhere M is only about 1/5 of S.

(c) North of 65°N there appears to be an input of westerly angular momentum into the atmosphere due to the mountain effect (M positive). Widger (1949) has located a weak source of westerly angular momentum due to surface easterlies as far south as 60°N in January 1946, but information is still scanty as to the magnitude, extent and persistence of polar easterlies.

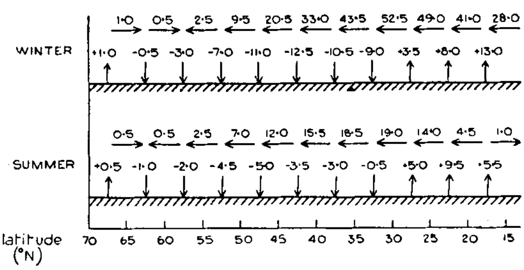


Fig. 2. Sources and sinks of angular momentum at the earth's surface, and total horizontal transport across lines of latitude necessary for balance. No transport is assumed to occur across 70°N. Arrows indicate the direction of transport of westerly angular momentum. Unit: 10^{25} g cm^2 sec^{-2}.

Table 2. Northward mass transport between 1 000 mb and 700 mb. (unit 10^6 tons sec^{-1}).

Latitude °N	60	55	50	45	40	35	30	25	20	15	10	5	0
Palmén and Alaka..							−58	−84	−126	−183	−190	−134	−67
Winter	−2	+12	+18	+21	+40	+46	−8	−49	−92	−110			
Summer	−7	+8	+21	+15	+2	−30	−72	−77	−46	−1			

Table 3. Upward mass transport through the 700 mb. surface (unit 10^6 tons sec^{-1}).

Latitude °N	65—60	60—55	55—50	50—45	45—40	40—35	35—30	30—25	25—20	20—15	15—10	10—5	5—0
Palmén and Alaka								−26	−42	−57	−7	+56	+67
Winter	+12	+9	0	0	+13	0	+34	−51	−26	−9			
Summer	−1	+13	+9	−8	−16	−27	−33	0	+26	+4			

(iii) *Total surface torque*. Values of S and M (Table 1) are combined in Figure 2 to give the total surface torque together with the horizontal transfer of angular momentum necessary for balance. In order to derive this horizontal transfer it was assumed that the source of angular momentum north of 70°N is negligible and therefore there is no horizontal transfer across this latitude.

(c) *The contribution of the mean meridional circulation* (terms [ii], Equ 4)

The mass transport across a given vertical surface on a complete latitude circle is given by

$$M_\Phi = \frac{2\pi r}{g} \int_{p_1}^{p_2} v \, dp$$

$$= \frac{2\pi r}{g} \tilde{v} (p_1 - p_1) \qquad (5)$$

where $\tilde{v}$ is the mean with regard to pressure of $\bar{v}$. $M\phi$ has been calculated between 1000 mb. and 700 mb. for each 5 degree latitude circle, these values given in Table 2 are compared with mass transports used by PALMÉN and ALAKA (1952). These authors used the January surface meridional components computed by RIEHL and YEH (1950) and assumed them constant to 900 mb. decreasing to zero at 700 mb. PALMÉN and ALAKA used a 1010 mb. surface whereas this analysis is based on a 1000 mb. surface.

The mass transport through a given horizontal pressure surface between two complete circles of latitude is given by

$$M_p = 2\pi R^2 (\sin \phi_2 - \sin \phi_1) \varrho \tilde{w} \qquad (6)$$

where $\tilde{w}$ is the mean with regard to latitude of $\bar{w}$. $M_{700\,mb}$ has been calculated for 5 degree latitude belts, these values are given in Table 3 and compared with the values of Palmén and Alaka.

There is reasonable agreement between the values of Palmén and Alaka for January 1946 and the winter values except that their maximum descent occurs between 15°N and 20°N whereas this analysis shows it to occur some 10° further north.

The flux of angular momentum due to the mean meridional circulation involves the computation of the four terms in (ii) of equation 4. Consider the block A (p_2, Φ_1), B (p_1, Φ_1), C (p_1, Φ_2), D (p_2, Φ_2) in Figure 3. Suffixes $\alpha, \beta, \gamma, \delta$

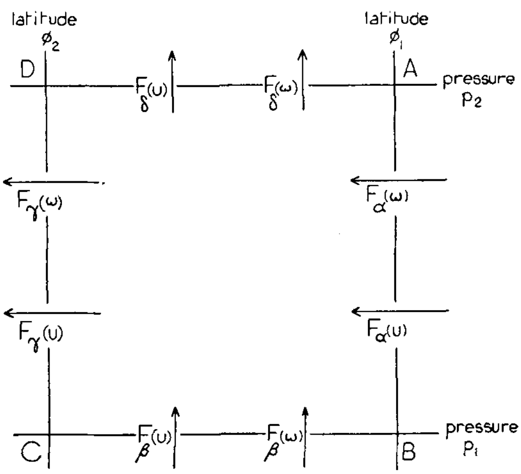
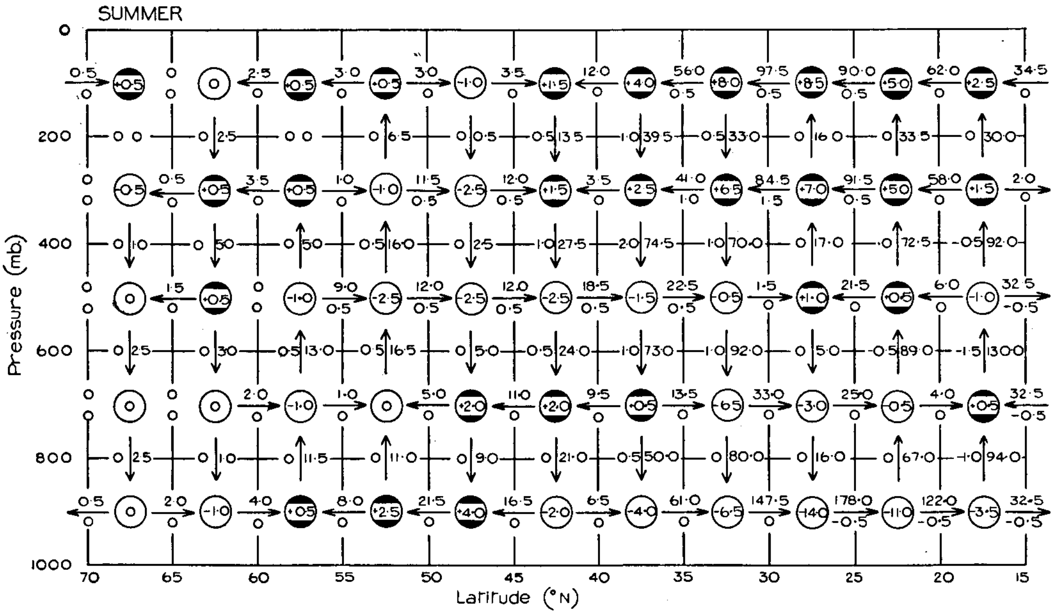


Fig. 3. Angular momentum flux across the boundaries of the block.



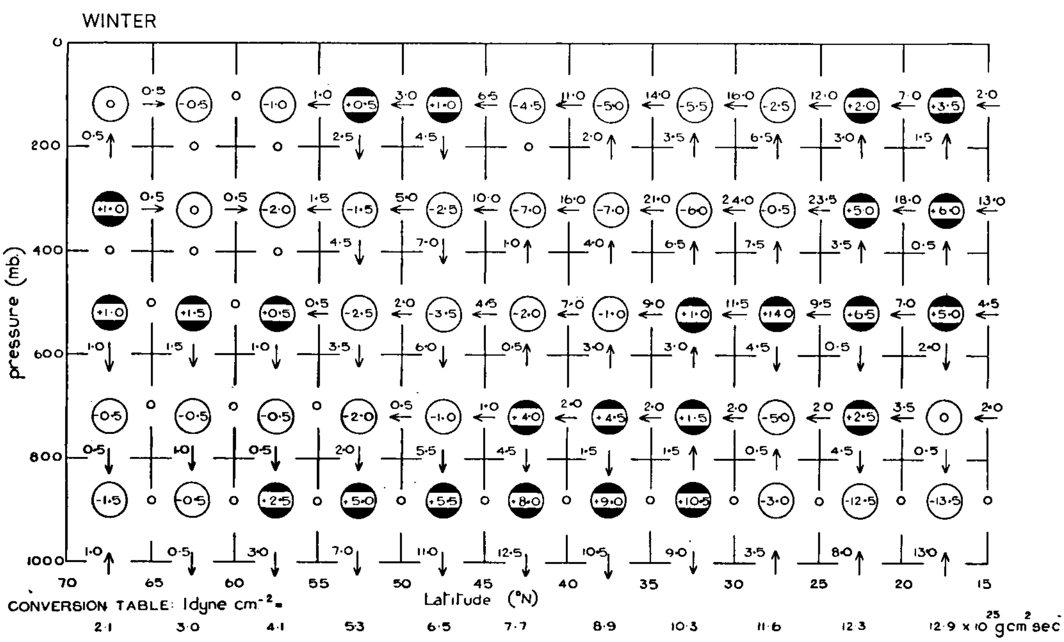


Fig. 4 and Fig. 5. Flux of relative angular momentum and ω-angular momentum due to the mean meridional circulation. Arrows indicate the direction of flux. Relative angular momentum is shown above horizontal arrows and on the right of vertical arrows. ω-angular momentum is shown below horizontal arrows and on the left of vertical arrows. Figures inside circles denote net convergence (shaded) or divergence within each block. Unit: 10^{25} g cm^2 sec^{-2}.

refer to the surfaces AB, BC, CD and DA respectively, the bar $^{-}$ denotes a mean value over the surface considered.

The northward fluxes of relative angular momentum across AB, CD are $F_\alpha(u)$ and $F_\gamma(u)$ respectively, where

$$F_\alpha(u) = M_{\phi_1} \bar{u}_\alpha R \cos\phi_1$$
$$F_\gamma(u) = M_{\phi_2} \bar{u}_\gamma R \cos\phi_2$$

and

$$F_\alpha(u) - F_\gamma(u) = \Big[\int_A \varrho r \bar{u} \bar{v}\, dA\Big]_{\phi_1}^{\phi_2} \text{ in equation 4}$$

The northward fluxes of ω-angular momentum across AB, CD are $F_\alpha(\omega)$ and $F_\gamma(\omega)$ respectively, where

$$F_\alpha(\omega) = M_{\phi_1} \omega R^2 \cos^2\phi_1$$
$$F_\gamma(\omega) = M_{\phi_2} \omega R^2 \cos^2\phi_2$$
$$F_\alpha(\omega) - F_\gamma(\omega) = \Big[\omega \int_A \varrho r^2 \bar{v}\, dA\Big]_{\phi_1}^{\phi_2} \text{ in equation 4}$$

The upward fluxes of relative angular momentum across BC, DA are $F_\beta(u)$ and $F_\delta(u)$ respectively, where

$$F_\beta(u) \simeq M_{p_1} \bar{u}_\beta R \cos\left(\frac{\phi_1+\phi_2}{2}\right)$$
$$F_\delta(u) \simeq M_{p_2} \bar{u}_\delta R \cos\left(\frac{\phi_1+\phi_2}{2}\right)$$

and

$$F_\beta(u) - F_\delta(u) \simeq \Big[\int_H \varrho r \bar{u} \bar{w}\, dH\Big]_{p_1}^{p_2} \text{ in equation 4}$$

The upward fluxes of ω -angular momentum across BC, DA are $F_\beta(\omega)$ and $F_\delta(\omega)$ respectively, where

$$F_\beta(\omega) \simeq M_{p_1} \omega R^2 \cos^2\left(\frac{\phi_1+\phi_2}{2}\right)$$
$$F_\delta(\omega) \simeq M_{p_2} \omega R^2 \cos^2\left(\frac{\phi_1+\phi_2}{2}\right)$$

and

$$F_\beta(\omega) - F_\delta(\omega) \simeq \Big[\omega \int_H \varrho r^2 \bar{w}\, dH\Big]_{p_1}^{p_2} \text{ in equation 4}$$

These terms have been computed for Winter and Summer and are presented in diagramatic form in Figures 4 and 5. The encircled figures give the convergence (shaded) or divergence of angular momentum in each block due to the mean meridional circulation. Since the total mass transfer across a latitude is zero the ω-angular momentum terms cancel out when summed through the depth of the atmosphere. This is not true of relative angular momentum transfer which contributes to the total horizontal flux, the significance of this "circulation transport" has been a source of disagreement amongst meteorologists. The contribution of this relative angular momentum to the total horizontal transport has been calculated from Figures 4 and 5 and is given in Table 4 to a nominal accuracy of 0.1×10^{25} g cm^2 sec^{-2}, the ratio (in per cent) with the total horizontal transfer necessary for balance is also given. Included in this table are similar values for January computed by Palmén and Alaka.

TUCKER (1959) described a method of adjusting values of $\bar{v}$ to correct for taking a 200 degree latitude arc instead of the complete circle. No similar correction can be applied to the values of $\bar{u}$. In winter therefore the very strong zonal velocities near Japan are not sampled. To attempt to appreciate this the geostrophic zonal

Table 4. Contribution of the mean meridional circulation (F_M) to the total horizontal flux of angular momentum (F_T) across a latitude circle.

Latitude °N		65	60	55	50	45	40	35	30	25	20	15
Winter	F_M	+ 0.2	— 0.1	— 0.5	— 0.8	— 1.1	— 2.4	— 3.1	— 1.0	+ 2.3	+ 5.5	+ 6.5
	$100\, F_M/F_T$	— 20	— 20	— 20	— 8	— 5	— 10	— 9	— 2	+ 6	+ 18	+ 36
	F_M	Using Petterssen's values of $\bar{u}$				— 2.3	— 2.9	— 4.8	+ 2.9	+ 5.5	+ 13.1	+ 16.4
	$100\, F_M/F_T$					— 11	— 13	— 14	+ 7	+ 14	+ 42	+ 91
	F_M	Palmén and Alaka							+ 6	+ 11	+ 14	
	$100\, F_M/F_T$								+ 10	+ 19	+ 30	
Summer ..	F_M	0	+ 0.1	— 0.1	— 0.8	— 0.8	0	+ 1.3	+ 2.0	+ 1.7	+ 0.8	+ 0.4
	$100\, F_M/F_T$	0	+ 20	— 4	— 11	— 7	0	+ 7	+ 11	+ 12	+ 18	+ 40

velocities for Winter computed for complete latitude circles by PETTERSSEN (1950) were combined with the values of $\bar{v}$ to assess the contribution of the mean meridional circulation to the horizontal flux of angular momentum. This composite model was used only for this calculation and the results are encorporated in Table 4.

The results agree qualitively with those of Palmén and Alaka — the contribution of the mean meridional circulation is generally small *but significant.* In low latitudes the contribution is quite large and the composite model shows it to be predominantly important. In middle latitudes the indirect circulation and the increase of westerlies with height cause the contribution to be southward so that the northward horizontal eddy flux must be *greater* than required by surface stress measurements.

(d) *The horizontal eddy flux*

To complete the angular momentum budget the horizontal and vertical eddy fluxes must be included (terms (iii) and (iv) in equation 4). Not withstanding much work by PRIESTLEY (1949), STARR and WHITE (1952 a, b; 1954 a, b) and WIDGER (1949), the horizontal eddy fluxes are known only very approximately. Therefore the meridional circulation transport, F_M, was substracted from the total transport, F_T, and the residue was attributed to the horizontal eddy flux. A percentage of this cross-latitude eddy transport F_T—F_M, was then allocated to each layer on the basis of the computations of STARR (1951) and STARR and WHITE (1952 a, b). At the time the computations for the present study were carried out Starr and White's values were available only for 13°N and 31°N, consequently it was assumed that the relative importance of each level at 31°N applied to all higher latitudes. Subsequent work (STARR and WHITE 1954 b) shows this to be a reasonable assumption. The percentage of total horizontal eddy transfer allocated to each layer are given in Table 5, and the values are entered in the complete angular momentum budgets (Figures 6 and 7).

(e) *The complete angnlar momentum budget*

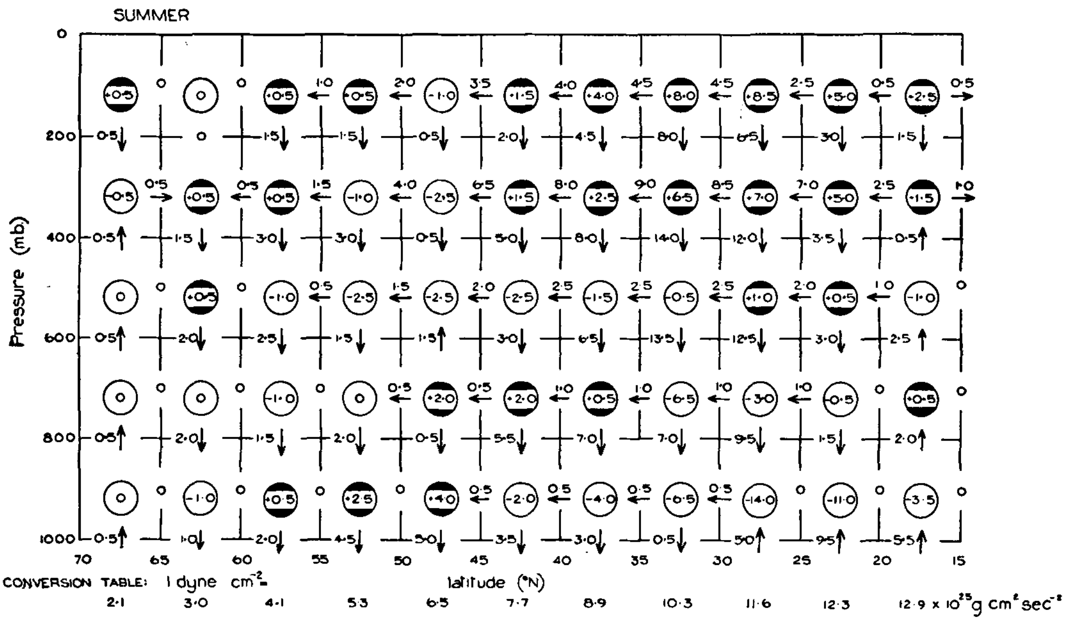
All the terms in equation 4 have now been computed except the vertical eddy flux (iv), this can be determined from the necessity for balance. The complete budgets for winter and summer are given in Figures 6 and 7, the encircled figures in each block are the convergence or divergence due to the existence of a mean meridional circulation (the encircled values in Figures 4 and 5) and the vertical eddy fluxes have been computed to give balance. The arrows across the boundaries of each block and the associated figures indicate the direction and magnitude of the eddy fluxes. The fluxes through the earth's surface were discussed in section 2 (b).

Table 5. Percentage of total horizontal eddy transfer allocated to each layer.

Latitude °N	Layer (mb.)	15	20	25	30	35
Winter	0— 200	10	20	25	30	30
	200— 400	60	50	50	45	45
	400— 600	20	20	20	20	20
	600— 800	10	10	5	5	5
	800—1 000	0	0	0	0	0
Summer	0— 200	0	15	20	25	25
	200— 400	70	55	55	50	50
	400— 600	15	15	15	15	15
	600— 800	10	10	10	5	5
	800—1 000	5	5	0	5	5

3. The vertical eddy flux of angular momentum

Since it was realised that the hypothetical upward flux of angular momentum in the tropics presented by WIDGER (1949) must be in error (PALMÉN 1951), much attention has been focussed on the vertical transfer in the general circulation of the atmosphere. Recently SHEPPARD (1953, 1954) has proposed a scheme whereby the upward transfer of westerly momentum near the surface in the Trade Wind region decreases rapidly with height and reverses to become a downward flux over the remainder of the troposphere — at least up to the level of maximum westerlies. He has incorporated a direct low-latitude circulation in his scheme to balance the divergence of vertical and meridional eddy flux at the upper (low latitude) levels and convergence at lower levels. In the region of surface westerlies Sheppard suggests that the downward flux is maintained with height up to the level of maximum westerlies where there is a convergence of horizontal eddy flux. He further suggests that no substantial meridional circulation is required for balance,



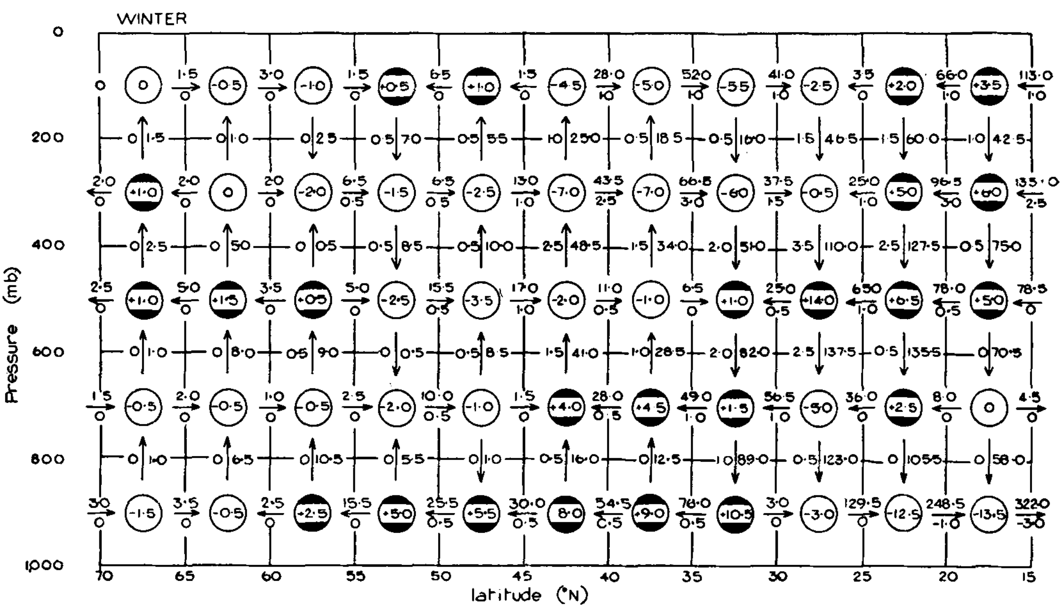


Fig. 6 and Fig. 7. Eddy flux of angular momentum. The surface (1 000 mb.) stress is taken from Fig. 2. Figures inside circles are the convergence and divergence of angular momentum due to the mean meridional circulation (Figures 4 and 5), vertical eddy fluxes have been computed to give balance. Unit: 10^{25} g cm^2 sec^{-2}. A conversion table relating angular momentum to linear momentum is given beneath each figure.

the convergence of horizontal eddy flux being approximately balanced by a divergence of vertical eddy flux.

We are now in a position to compare the derived scheme of vertical momentum transfer with the empirical pattern proposed by Sheppard for a uniform earth with no monsoon or topographic effects.

(a) *Winter* (Figure 6)

North of 45°N the direction of the eddy flux is downwards throughout the atmosphere; this is strongest between 45°N and 50°N, exceeding 4×10^{25} g cm^2 sec^{-2} (0.6 dynes/cm^2) throughout the column. The extension of the indirect circulation southwards to latitude 30°N (Figure 4A) counterbalances and exceeds the convergence of horizontal eddy flux causing an upward eddy flux in the upper levels. Also south of 25°N in the upper levels the convergence due to the direct circulation is insufficient to offset the divergence of horizontal eddy flux, again requiring an upward eddy flux. In the lower levels of low latitudes where the direct circulation is quite strong the resulting divergence of angular momentum far outweighs any convergence of horizontal eddy flux, consistent with a rapid changeover at about 800 mb. from an upward flux at the surface (in the easterlies) to a downward eddy flux above.

Although there may be no *a priori* reason for demanding a downward eddy flux of westerly momentum in areas where westerlies increase or easterlies decrease with height in the mean (i. e. a positive coefficient of eddy viscosity), from physical considerations this seems more reasonable. The validity of Figure 5 can therefore be queried in the upper levels south of 45°N. Since the upward eddy fluxes inferred here arise mainly from the existence of a strong indirect circulation it appears that the part of the Northern hemisphere not included in the observational analysis, in particular the strong winter jet in Southern Asia and near Japan, is strongly linked with the global meridional circulation. Thus it would seem that in winter a strong direct circulation may exist to the South of the Asian sub-tropical jet and this in conjunction with a strong horizontal temperature gradient plays a major role in the maintenance of global westerlies.

(b) *Summer* (Figure 7)

Apart from the upward flux at 600 mb., 45—50°N, which may be attributed to the accumulation of errors, the scheme of vertical angular momentum transfer is in good agreement with that proposed by Sheppard. The strongest downward transfer occurs at about 30°N, reaching 14×10^{25} g cm^2 sec^{-2} (1.3 dynes/cm^2) at about 500 mb. An estimate of the coefficient of eddy viscosity has been attempted, because of the crudity of the data this indicates only little more than an order of magnitude. A mean coefficient of eddy viscosity, $\bar{K}$, was defined as

$$\bar{K} = \bar{\tau}_{xz} \Big/ \varrho \frac{\partial \bar{u}}{\partial z}$$

where $\bar{\tau}_{xz}$ is the mean shearing stress in the eastward direction, i.e. the vertical eddy flux of westerly momentum. The values computed for Summer are given in Table 6 and considering the crudity of the data they show a reasonable uniformity. Various estimates of the values of K for crosswind and downwind momentum transfer in the lowest few hundred metres of the atmosphere (Riehl et al 1951, Sheppard and Omar 1952, Sheppard 1954) place it at about 1×10^5 cm^2/sec with indications of an increase with height. An increase of K with height would, in the case of the Trades, provide downward fluxes aloft of the same order as upward fluxes near the surface where the velocity gradient with height (of opposite sign) is greater. Table 6 shows an increase of $\bar{K}$ from 800 mb. to 600 mb. and agrees fairly well with Riehl et al (1951) who found an eddy viscosity of 6×10^5 cm^2/sec at about 700 mb. in the E. Pacific above the trade inversion.

(c) *Level of zero vertical eddy flux in low latitudes*

Several studies of vertical eddy fluxes in the lowest layers of the Trades assume that the

Table 6. The 'mean' eddy viscocity in Summer. Unit 10^5 cm^2/sec.

Level	Latitude °N									
	65—60	60—55	55—50	50—45	45—40	40—35	35—30	30—25	25—20	20—15
400 mb.	15	13	10	3	7	15	23	27	13	0
600 mb.	17	8	5	—2	5	10	32	30	33	17
800 mb.	10	5	4	1	4	6	9	20	12	12

turning point in the wind profile is the level at which τ_{sz} (the shearing stress along the motion) is zero. This assumption may be tested on a climatological basis using the profiles of $\bar{v} = \bar{v}(p)$ from Figure 1.

The equation of mean zonal motion is approximately

$$\frac{d\bar{u}}{dt} = l\bar{v} + \frac{1}{\varrho}\frac{\partial\bar{\tau}_{xz}}{\partial z} + \frac{1}{\varrho}\frac{\partial\bar{\tau}_{xy}}{\partial y} \qquad (7)$$

The bar implies a mean in time and around a latitude circle, whence if we assume $\frac{d\bar{u}}{dt} = 0$

$$\frac{\partial\bar{\tau}_{xz}}{\partial z} + \frac{\partial\bar{\tau}_{xy}}{\partial y} = -\varrho l\bar{v} \qquad (8)$$

Figures 6 and 7 show that in low latitudes a high value of $\partial\bar{\tau}_{xy}/\partial y$ below 800 mb. is 0.5×10^{25} g cm^2 sec^{-2} (5° latitude)$^{-1}$, or about 1×10^{-5} dynes cm^{-2} (100 m)$^{-1}$, while a low value of $\partial\bar{\tau}_{xz}/\partial z$ is 3.0×10^{25} g cm^2 sec^{-2} (200 mb.)$^{-1}$, or about 6×10^{-3} dynes cm^{-2} (100 m)$^{-1}$. $\partial\bar{\tau}_{xy}/\partial y$ can therefore be neglected in low latitudes at low levels and equation 7 becomes

$$\frac{\partial\bar{\tau}_{xz}}{\partial z} = -\varrho l\bar{v} \qquad (9)$$

therefore

$$\bar{\tau}_{xz\,(p)} = \bar{\tau}_{xz\,(p_0)} + \frac{l}{g}\int_{p_0}^{p}\bar{v}\,dp \qquad (10)$$

$\bar{\tau}_{xz}(p_0)$. is the surface torque. Values of the surface friction torque were taken from Table 1 and curves of $\bar{v} = \bar{v}(p)$ from Figure 1 were used to calculate the levels at which $\bar{\tau}_{xz}(p) = 0$. Results are represented in Figure 8. Since the first level of observation above the surface was 850 mb. and the next 700 mb., the level of maximum easterlies cannot be accurately placed. However the line of zero zonal velocity has been drawn in Figure 8. RIEHL et al (1951) in a quasi-climatological study found the level of maximum north easterlies in the E. Pacific to be about 900 mb. at 20°N decreasing northwards; SHEPPARD and OMAR (1952) in an analysis of winds over four tropical islands at about 20°N found a mean maximum of easterlies at about 0.5 km. for all wind speeds. These values agree fairly well with the level of zero $\bar{\tau}_{xz}$ in Figure 8.

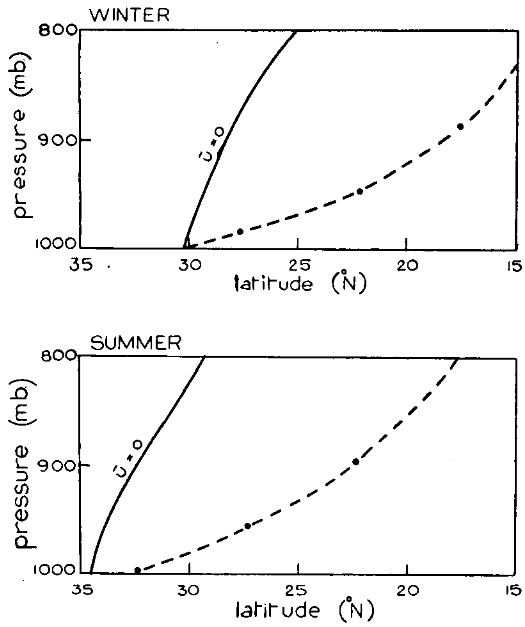


Fig. 8. The level (dotted line) above which the upward eddy flux of westerly angulat momentum decreases to zero. The full line denotes the boundary between mean westerly and mean easterly flow.

4. Conclusion

The investigation of the angular momentum budget confirms the small contribution of the mean meridional circulation to the net horizontal flux of relative angular momentum except in low latitudes where it becomes comparable with the horizontal eddy flux. Perhaps the most important part played by the mean meridional circulation is in the horizontal and vertical flux of ω-angular momentum. These large fluxes do not contribute to the net horizontal transfer but result in areas of convergence and divergence of advected angular momentum which balance corresponding areas of divergence and convergence of eddy flux in such a way that the direction of vertical eddy flux is downgradient. This confirms a scheme proposed by SHEPPARD and OMAR (1952) and SHEPPARD (1954) namely that in low latitudes the divergence of angular momentum in the lower layers due to the mean meridional circulation is consistent with an upward flux in the surface easterlies which decreases rapidly to zero at the level of maximum easterlies and becomes downwards at higher levels.

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