A Preliminary Study of the Potential to Kinetic Energy Conversion Process in the Stratosphere

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Abstract

The potential to kinetic energy conversion process in the lower stratosphere associated with the vertical exchange of warm and cold air is evaluated using adiabatically derived vertical velocities for the North American region for a five-day period. Preliminary results suggest the possibility that on the average the kinetic energy of stratospheric motions may not result from a conversion of potential energy within the stratosphere by this process. The further implication is that stratospheric motions are maintained by the motions in the adjacent layers of the atmosphere.

1. Introduction

A fundamental question about stratospheric motions refers to the manner of the maintenance of their kinetic energy. It seems clear that the kinetic energy of the motions of the troposphere is maintained by the conversion from available potential energy as described and verified by many investigators, most recently by LORENZ (1955) and WHITE and SALTZMAN (1956), and that this process is associated with the rising of warm and sinking of cold air. On the other hand, it has not been established that a similar process operates in the stratosphere. This preliminary study represents an attempt to evaluate from observations the nature of this potential to kinetic conversion process in the stratosphere.

2. Procedure

Following WHITE and SALTZMAN (1956) we may write the equations expressing the time rate of change of the kinetic energy of the horizontal wind and the total potential and internal energy of the entire mass of the atmosphere as follows:

$$\frac{\partial}{\partial t}\int_M k\,dm = -\int_M \omega\alpha\,dm - \int_M D\,dm \qquad (1)$$

$$\frac{\partial}{\partial t}\int_M (\Phi + I)\,dm = \int_M \omega\alpha\,dm + \int_M \frac{\partial Q}{\partial t} \qquad (2)$$

where dm is the element of mass and the integration is carried out over the entire mass of the atmosphere, k is the kinetic energy of the horizontal wind, $\omega = \frac{dp}{dt}$ is the individual time rate of change of pressure, α is the specific volume, D is the rate of frictional dissipation of kinetic energy, Φ is the geopotential, I is the internal energy, and $\frac{dQ}{dt}$ is the net rate of heat addition. The appearance of the integral

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$\int_M \omega\alpha\, dm$ with opposite signs in both equations represents the process of potential to kinetic energy conversion in the atmosphere which is familiarly associated with the vertical exchange of warm and cold air.

The critical problem in evaluating this integral lies in the determination of ω which is closely related to the field of vertical velocity. In this investigation ω was evaluated by the adiabatic relation

$$\omega = -\frac{\dfrac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T}{\dfrac{\partial T}{\partial p} - \dfrac{\alpha}{c_p}} \tag{3}$$

where T is the temperature, $\mathbf{v}$ is the horizontal wind vector on an isobaric surface, and C_p is the specific heat at constant pressure.

The finite difference form of equation (3) was evaluated from constant pressure charts at 200, 100, 50 and 25 mb over the region of North America extending from 30° to 60°N and from 70° to 120°W. These charts had been carefully analyzed for other purposes by Mr. S. Muench of the Atmospheric Circulations Laboratory, Air Force Cambridge Research Center, and kindly provided us by Mr. W. Hering and Mr. S. Muench for this investigation. The available data consisted of seven 200, 100, and 50 mb and six 25 mb charts covering the period 28 January to 3 February, 1957, a period of intense stratospheric warming as descibed by Craig and Hering (1959). The wind velocities were evaluated geostrophically and the temperatures hydrostatically. Contour height values were abstracted at a grid system of 273 points spaced 2.5 degrees apart and the ω's were obtained at a grid system of 60 points spaced 5 degrees apart. The time derivatives were approximated by finite differences over 24 hour intervals. The computation yielded fields of the 24-hour average values of ω and α for the layers 200—50 mb and 100—25 mb which were taken to be representative of these fields at the 100 and 50 mb levels.

Using the values of ω and α thus derived it was possible to sample the integrand for this region of the hemisphere for the short period of time as indicated, and for the layers of the atmosphere at approximately the height of the 100 and 50 mb levels. Average values of the integrand $\omega\alpha$ over space and time were computed and an analysis of the covariance of $\omega\alpha$ was performed in a manner entirely analogous to that of White and Saltzman (1956). Following their procedure we may write:

$$\overline{\{[\omega\alpha]\}} - \overline{\{[\omega]\}}\;\overline{\{[\alpha]\}} = \overline{\{[\omega]''[\alpha]''\}} + \overline{\{[\omega'\alpha']\}} + \overline{\{[\omega]\}'''\{[\alpha]\}'''} \tag{4}$$

where the single, double and triple primes denote deviations from east-west, north-south, and time averages respectively, the brackets indicate an average in the east-west direction, the braces an average in the north-south direction and the bars an average in time.

The left side of equation (4) is a more representative measure of the integrand average than the term $\{[\omega\alpha]\}$ alone, since the second term on the left side must vanish when the average is taken over the entire hemisphere and will give rise to spurious non-zero values if not subtracted out because of the limited area treated. The first term on the right is a measure of the conversion process due to overturnings in the north-south direction, the second is a measure of the process due to overturnings in the east-west direction. The last term is associated with temporal pulsations of the space average values of ω and α.

3. Results

The values of ω obtained are reasonable. When transformed by means of the hydrostatic equation to equivalent vertical velocities, the standard deviation is 0.86 cm sec^{-1} at 100 mb and 1.35 cm sec^{-1} at 50 mb. A typical example of the distribution of the vertical velocity in relation to the temperature and contour height fields at 50 mb is shown in Fig. 1 for 29 January, 1957.

The numerical values of the potential to kinetic energy conversion processes are shown in Table 1.

Table 1. The rate of conversion between potential and kinetic energy. A minus sign indicates a conversion from potential to kinetic energy. Units in ergs gm^{-1} sec^{-1}.

	1	2	3	4
	$\overline{\{[\omega\alpha]\}} - \overline{\{[\omega]\}}\;\overline{\{[\alpha]\}}$	$\overline{\{[\omega]''[\alpha]''\}}$	$\overline{\{[\omega'\alpha']\}}$	$\overline{\{[\omega]\}'''\{[\alpha]\}'''}$
50 mb	7.5	—0.1	7.5	0.1
100 mb	—0.5	—0.3	—0.4	0.2

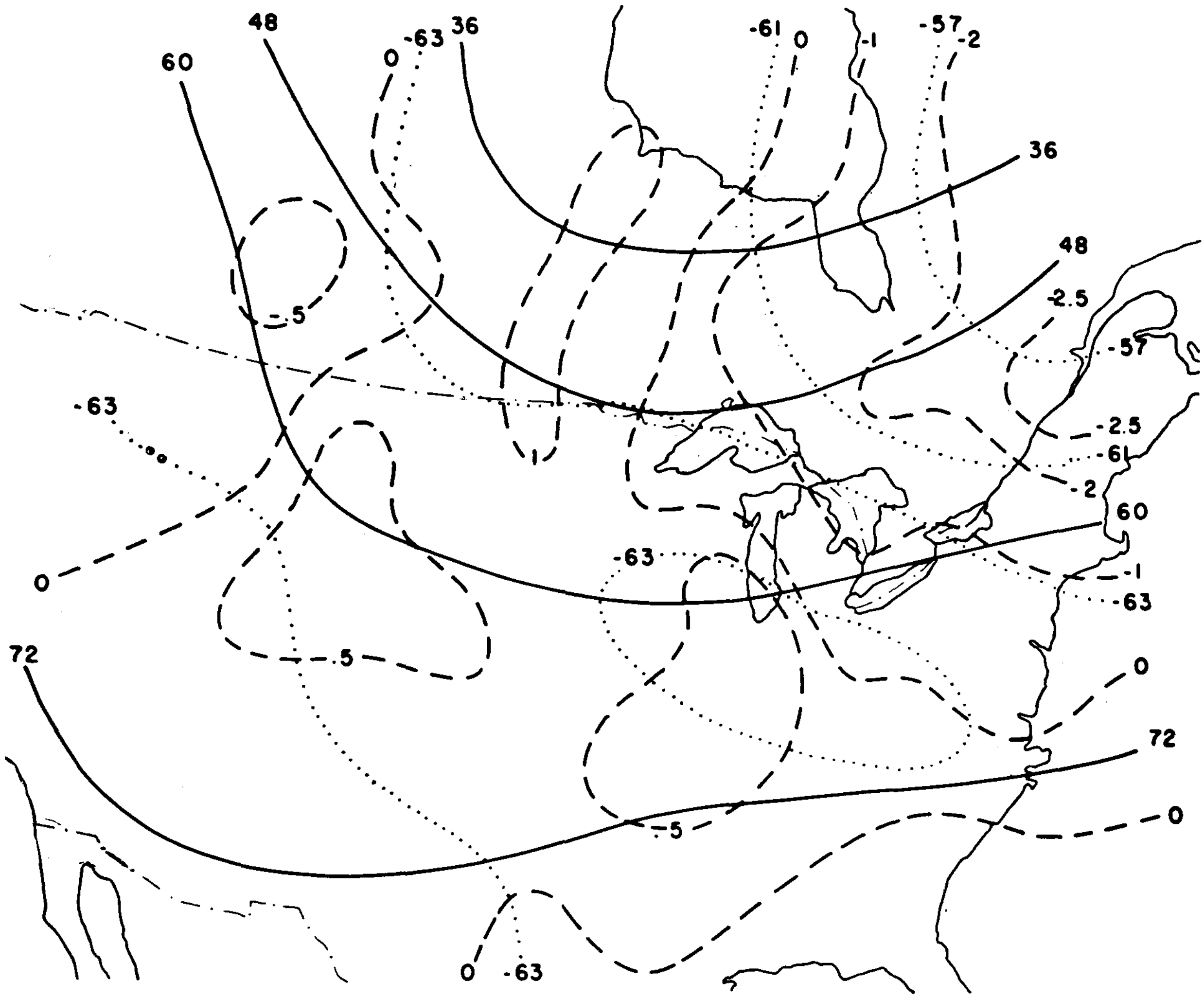

Fig. 1. The vertical velocity, temperature, and contour height distribution at 50 mb for January 29, 1957, 03 GMT. Vertical velocity units in cm sec^{-1}.

The first column of Table 1 indicates that at 50 mb the conversion process is opposite to that found in the troposphere being from kinetic to potential energy in this region of the stratosphere. The sense of this conversion process was the same on each of the four individual days examined. At 100 mb the magnitude of the conversion term is not essentially different from zero and the six individual daily values on which the mean is based vary in sign.

A more detailed analysis of this conversion process is given in columns 2, 3, and 4 of table 1. At 50 mb the principal contribution to the conversion process is associated with overturnings in the east-west direction as indicated in column 3. The contributions of the other two terms in columns 2 and 4 are negligible. At 100 mb the conversion rates associated with each of the individual terms are small.

Since the 100 mb data are representative of the layer between 200 and 50 mb, this layer will be wholly contained in the stratosphere only at high latitudes and will be partially within the troposphere at low latitudes. It should be possible upon examination of the latitudinal variation of the term $[\overline{\omega'\alpha'}]$ to detect whether

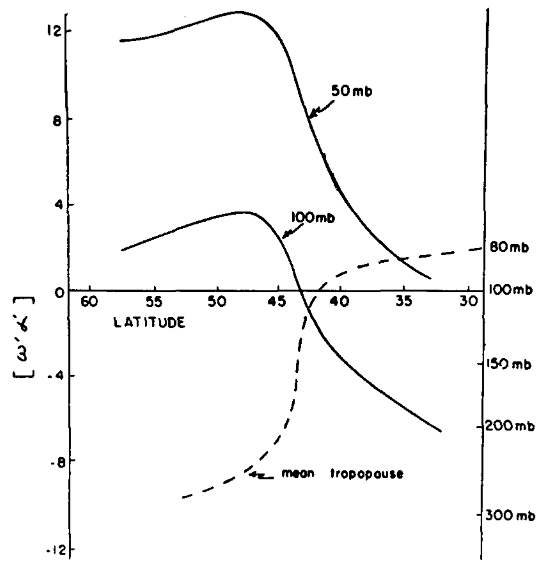

Fig. 2. The latitudinal variation of the potential to kinetic energy conversion process due to east-west overturnings in units of ergs gm^{-1} sec^{-1}. Also shown is the latitudinal variation of the mean tropopause height as a function of pressure.

there is any systematic difference in the sense of the conversion process as one proceeds from troposphere to stratosphere. The latitudinal variation of this term at both 100 and 50 mb together with a plot of the mean tropopause height is shown in Fig. 2.

It can be seen that the sign of the conversion process changes at approximately that latitude (43°N) where the mean tropopause crosses the 100 mb level, being from kinetic to potential to the north and from potential to kinetic to the south of this latitude. This observational feature agrees with the concept of a reversal of the potential to kinetic energy conversion process from troposphere to stratosphere.

If these observational findings are further substantiated by more extensive investigations on a hemispherical basis and over longer periods of time then the implications seem far-reaching. If the kinetic energy of the stratospheric motions is not maintained by conversion from potential energy within the stratosphere by the processes associated with the vertical exchange of warm and cold air then it must be maintained by boundary interactions with the adjacent layers above and below. Such processes involve the vertical transport of existing kinetic energy through the top and bottom boundaries of the layer as pointed out by STARR (1959) and other boundary processes associated with variations in the height of the bounding pressure surfaces. Should general verification of these concepts be obtained, one possible implication would be that stratospheric motions must to a large extent vary in response to tropospheric changes, and that the explanation for many observed characteristics of the stratosphere, such as the size and motion of circulation systems and the seasonal and latitudinal distribution of ozone, may very well lie in a better understanding of the linkages between the stratosphere and troposphere.

4. Critical Remarks

As in all such limited investigations care must be exercised in generalization before confirmation on the basis of more extensive data. This particular study suffers from the following deficiencies:

a. The results are based on a sample of data from a small area of the hemisphere and for a very short period of time.
b. The period of time studied was one of abnormal high level temperature changes, and the results may not be typical of more normal conditions.
c. The vertical velocity computations on which the results fundamentally depend are based upon the adiabatic assumption.

REFERENCES

LORENZ, E. N., 1955: Available potential energy and the general circulation. *Tellus*, **7**, pp. 157—167.

WHITE, R. M. and SALTZMAN, B., 1956: On conversions between potential and kinetic energy in the atmosphere. *Tellus*, **8**, pp. 357—363.

CRAIG, R. A. and HERING, W. S., 1959: The stratospheric warming of January and February, 1957. *J. Meteor.* **16**, pp. 91—107.

STARR, V. P., 1959: Questions concerning the energy of stratospheric motions. (In press).