Spectrum of Kinetic Energy Transfer due to Large-Scale Horizontal Reynolds Stresses

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All whirls do trade their velocity—but,
The biggest whirl exacts its cut.

We have made estimates of the integral,

$$M(n) = \frac{2\pi a}{g} \iint \Big[U(n,\phi,p)V(-n,\phi,p) + U(-n,\phi,p)V(n,\phi,p) \Big] \cos^2\phi \frac{\partial}{\partial\phi} \left[\frac{\bar{u}(\phi,p)}{\cos\phi} \right] d\phi\, dp,$$

which measures the rate of transfer of kinetic energy between the mean zonal flow and the harmonic components of the eddy flow. Only the action of the horizontal Reynolds stresses on the zonal current is considered. n is the wave number around latitude circles; ϕ is the latitude; p is the pressure; U and V are the complex Fourier coefficients respectively of the zonal and meridional components of the wind; $\bar{u}$ is the mean zonal wind; a is the radius of the earth; g is the acceleration of gravity; and the integration spans the entire atmosphere. Further details are contained in two papers by B. SALTZMAN (1957, 1958).

The estimates were made for each day of the year 1951 using geostrophic winds at 500 mb and were based on a fifteen wave-number Fourier resolution along every five degrees of latitude from 15 deg N to 80 deg N of contour heights spaced every ten degrees of longitude. To estimate the contribution from the unsampled portions of the hemisphere we assumed that the integrand fades to zero linearly from 22.5 deg N to 10 deg N, and from 72.5 deg N to the pole. We hope this extrapolation is conservative.

The results are shown in the figure and

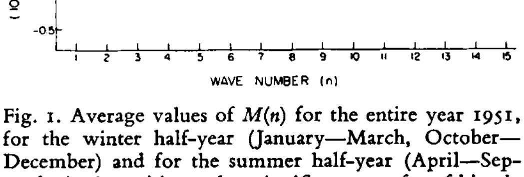

Fig. 1. Average values of $M(n)$ for the entire year 1951, for the winter half-year (January—March, October—December) and for the summer half-year (April—September). A positive value signifies a transfer of kinetic energy from disturbances of wave, number n to the mean zonal current.

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Table 1. Standard deviations of the daily values of *M(n)* from the averages shown in fig. 1, in units of (±) 10^{-17} ergs sec^{-1} mb^{-1}. The last column gives the standard deviation of the net daily transfer.

Wave number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15} M(n)$
Winter ..	1.54	2.59	2.76	1.90	2.20	2.41	1.63	1.47	.91	.79	.48	.47	.34	.33	.34	9.10
Summer .	.73	1.22	1.52	1.08	1.02	.99	1.03	.92	.62	.45	.41	.34	.24	.30	.23	3.66
Annual ..	1.26	2.03	2.33	1.55	1.72	1.87	1.37	1.23	.78	.65	.45	.41	.29	.32	.29	6.98

the accompanying table. These place in perspective the spectrum obtained for the single month, January 1949, previously reported (SALTZMAN, 1958).

The net transfer, i.e., the sum over the fifteen wave numbers, is directed from the eddies to the zonal flow. If we take this 500-mb estimate to be representative of the entire depth of the atmosphere then the annual average is 3.8×10^{20} ergs sec^{-1}, the winter average is 5.8×10^{20} ergs sec^{-1} and the summer average is 1.8×10^{20} ergs sec^{-1}, which are smaller but still of the same order as obtained by V. P. STARR (1959) for the same year using observed winds (see, also, STARR 1953).

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