On the Maintenance of the Large-Scale Quasi-Permanent Disturbances in the Atmosphere

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Abstract

It is proposed that large-scale quasi-permanent flow systems are maintained against dissipative effects by a transfer of kinetic energy from smaller, cyclone-scale, disturbances which have baroclinic energy sources, the processes involved being non-linear and barotropic. To illustrate how such processes can operate, an example is given based on an idealized flow consisting of a limited number of double-Fourier components.

1. Introduction

In Fig. 1 a 500 mb synoptic map for a region covering the North American Continent and the North Atlantic Ocean is presented. The most prominent features of this map—namely, the large cyclonic area to the north sloping from northwest to southeast and the large, more diffuse, anticyclonic area to the south sloping from northeast to southwest—are typical of the quasi-steady winter conditions observed in this region. The largest scale systems shown on this map correspond roughly to disturbances of wave number three around the hemisphere and are so oriented that they transfer their kinetic energy to the zonally-averaged mean current which has its maximum velocity somewhere between the two main pressure centers. As such, these large systems constitute a representative example of those disturbances which, in the winter average, are most effective in maintaining the mean zonal current (see SALTZMAN 1957, 1958; SALTZMAN and FLEISHER 1960).

In viewing the persistence of flow patterns of this type over long time intervals (see, e.g., monthly mean maps presented by LAHEY, et al, 1958) one is led to pose the following question: How is the kinetic energy of these large-scale systems maintained in the face of a steady drain of their energy through viscous effects and through transfer to the zonal current? There are two possible answers to this question: Either the kinetic energy of the large disturbances is supplied by a direct *conversion* of potential and internal energy (i.e., by risings of warm air and sinkings of cold air on this large scale) or the kinetic energy is supplied by *transfer* from disturbances of other scales which themselves grow at the expense of potential and internal energy (e.g., from the smaller transient cyclone waves which tend to develop in the baroclinic zone south of the large quasi-stationary cyclonic vortex, such as the shorter wave length disturbances clearly evident in Fig. 1).

If, as presently indicated in studies by REED and TANK (1956), and WHITE and SALTZMAN (1956), for example, the indirect circulations in mid-latitudes, whose upward branches are

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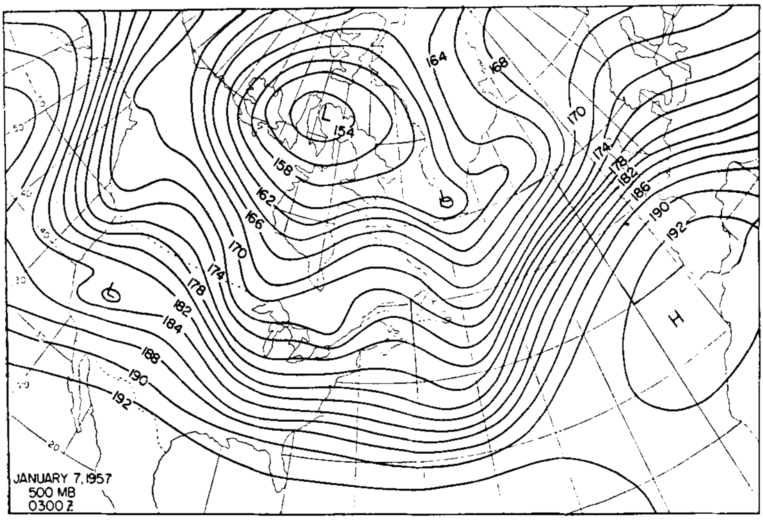

Fig. 1. Contour map of the 500 mb pressure surface for 7 Jan. 1957.

represented by rising motion in the large cold vortices to the north and whose downward branches are represented by sinking motion in the large warm highs to the south, are predominant over direct circulations on this scale, then one is forced to conclude that the large-scale features such as those shown in Fig. 1 are, in fact, steadily damped rather than amplified by direct conversion processes. In this event the maintenance of the kinetic energy of these features must rest entirely on the transfer processes mentioned above as the second alternative. In any event, however, it will be of interest to illustrate the operation of this transfer process,—by nature, a *non-linear* process—with a simple theoretical case. It is to this end that the present discussion is directed.

2. Governing equations

We assume that the transfer processes important for the maintenance of the large-scale disturbances are, as in the case of the maintenance of the zonal westerlies, of an essentially horizontal character and are governed by the barotropic vorticity equation. For simplicity, we shall apply this equation to a plane rather than a spherical surface, in which case the equation takes the form

$$\frac{\partial}{\partial t}\nabla^2\psi = -\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\nabla^2\psi + \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\nabla^2\psi - \beta\frac{\partial\psi}{\partial x} \quad (1)$$

where x and y are Cartesian coordinates pointing eastward and northward respectively, t is time, ψ is a stream function proportional to the contour height of a pressure surface, $\nabla^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$, and β is the derivative with respect to y of the Coriolis parameter. Following procedures similar to those employed by KAMPÉ DE FÉRIET (1948), GAMBO, et al (1955), WIPPERMANN (1956) and LORENZ (1957), we express ψ over a fundamental region whose dimensions in x and y are K and L, respectively, in terms of a double Fourier expansion of the form

$$\psi(x, y, t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \Psi(m, n, t)\, e^{i(kmx+lny)}, \quad (2)$$

where $k = 2\pi/K$, $l = 2\pi/L$, m and n denote the wave number in the x and y directions respectively, and the complex Fourier coefficients, $\Psi(m, n, t)$, are given by the relation

$$\Psi(m, n, t) = = \frac{1}{KL}\int_0^K\int_0^L \psi(x, y, t)\, e^{-i(kmx+lny)}dxdy \quad (3)$$

To simplify the boundary conditions we take ψ to be periodic of wave lengths K and L in x and y, respectively.

By multiplying both sides of (1) by $(KL)^{-1}$ $exp\,[-i(kmx+lny)]$, integrating over the fundamental region, and applying (3) we obtain the transformed vorticity equation governing the Fourier coefficients of the stream function. This equation has the form

$$\frac{d}{dt}\Psi(m,n) =$$

$$= \frac{kl}{(k^2m^2+l^2n^2)}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}$$

$$[(mq-np)(p^2k^2+q^2l^2)\Psi(p,q)\Psi(m-p,n-q)]$$

$$-\frac{\beta ikm}{(m^2k^2+n^2l^2)}\Psi(m,n) \qquad (4)$$

The total kinetic energy integrated over the fundamental region

$$\overline{E} = \frac{1}{KL}\int_0^K\int_0^L\frac{1}{2}\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right]dx\,dy \quad (5)$$

may be expanded in terms of a Fourier spectrum of the form

$$\overline{E} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{1}{2}K(m,n) \qquad (6)$$

$$= \sum_{n=1}^{\infty}K(o,n) +$$

$$+\sum_{m=1}^{\infty}\Big\{K(m,o)+$$

$$+\sum_{n=1}^{\infty}[K(m,n)+K(m,-n)]\Big\} \qquad (7)$$

where $K(m,n) = (k^2m^2+l^2n^2)\,|\Psi(m,n,)|^2$. The first term on the right of (7) gives the kinetic energy of the mean zonal wind, while the term in curled brackets represents the spectral function for the mean disturbance kinetic energy superimposed on the mean zonal field. For the purpose of this study we are interested in obtaining an equation for the rate of change of kinetic energy of a given double Fourier component as a function of the transfer of kinetic energy to or from the other double Fourier components which comprise the complete stream field. Such an equation is obtained by multiplying both sides of (4) by $\Psi(-m,-n)$ and using the fact that $\Psi(-a,-b)$ is the complex conjugate of $\Psi(a,b)$. Thus we find,

$$\frac{d}{dt}K(m,n) = kl\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}(p^2k^2+q^2l^2)(mq-np)$$

$$[\Psi(-m,-n)\Psi(p,q)\Psi(m-p,n-q)-$$

$$-\Psi(m,n)\Psi(p,q)\Psi(-m-p,-n-q)] \quad (8)$$

By means of this last relation we are now able to calculate the instantaneous barotropic energy transfers due to the non-linear interactions among the Fourier components which comprise a field of flow in the fundamental region. It may be verified from (6) and (8) that the total kinetic energy is conserved (i.e., $d\overline{E}/dt = o$), as is implicit in the barotropic system treated.

3. An example

In this section we shall discuss the energetical properties of a simple flow system having the general features of the actual case shown in Fig. 1. This flow system will be composed of a few selected double Fourier components, grouped to represent, respectively, the mean zonal current, the long-wave disturbance, and the smaller-scale disturbance. We shall see that in such a system kinetic energy is transferred from the small-scale disturbance (which may be viewed as baroclinically-generated) to the long-wave component and, simultaneously, from the long-wave component to the mean zonal current.

For this example we take the fundamental region to be a square so that $l = k$. The stream function, ψ, is taken to be of the form

$$\psi(x,y,t) = Z(y,t) + L(x,y,t) + S(x,y,t) \quad (9)$$

where

$$Z(y,t) = A(t)\sin ky \qquad (10)$$

represents the zonal current (wave number zero in x),

$$L(x,y,t) = B(t)[\cos k(x+y)+$$

$$+\cos k(x-y)] + C(t)\sin kx \qquad (11)$$

represents the long-wave component (wave number one in x), and

$$S(x,y,t) = D(t)\cos k(2x-y) +$$

$$+E(t)\cos 3kx \qquad (12)$$

represents the short-wave component (a combination of wave numbers two and three in x). Here $A = 2Im\,\Psi(0,1)$, $B = 2Re\,\Psi(1,1) = 2Re\,\Psi \cdot (1,-1)$, $C = 2Im\,\Psi(1,0)$, $D = 2Re\,\Psi(2,-1)$ and $E = 2Re\,\Psi(3,0)$. The dynamical properties of a system consisting of Z and L alone has already been studied in some detail by LORENZ (1957).

The components Z, L, and S are shown in igures 2, 3, and 4 with the following values of the Fourier coefficients:

$$\begin{aligned} A &= +3.00 \\ B &= -0.75 \\ C &= -1.50 \\ D &= +1.00 \\ E &= +1.00 \end{aligned} \tag{13}$$

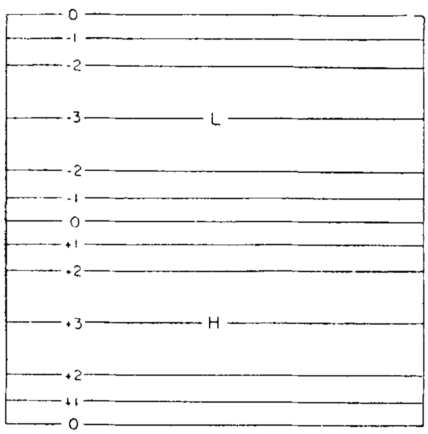

Fig 2. Stream function pattern, Z, representing the zonal motion: $Z(y) = 3.00 \sin ky$.

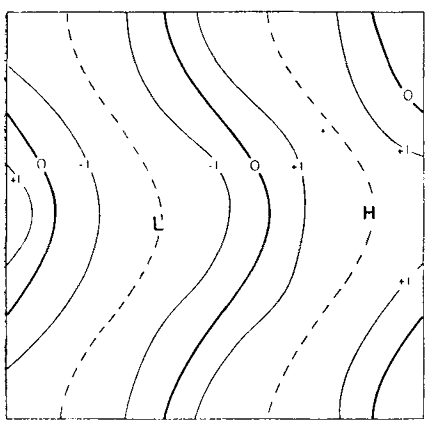

Fig. 3. Stream function pattern, L, representing the large-scale disturbances: $L(x, y) = -.75\,[\cos k\,(x + y) + \cos k\,(x - y)] - 1.50 \sin kx$.

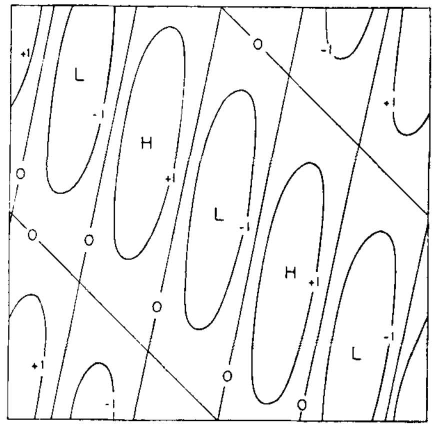

Fig. 4. Stream function pattern, S, representing the small-scale disturbances: $S(x, y) = \cos k\,(2x - y) + \cos 3\,kx$.

In Fig. 5 the sum of Z and L, representing a composite of the largest scales of motion, is

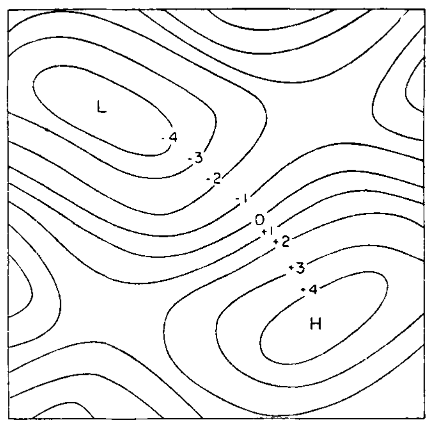

Fig. 5. Composite stream function pattern, $(Z + L)$, representing the largest scales of flow.

shown. In Fig. 6 the complete stream field, $(Z + L + S)$, obtained by superimposing the small-scale disturbance field, S, on $(Z + L)$, is shown. This stream field has many of the characteristics of the actual case shown in Fig. 1, the most obvious differences being the intensity of the large high pressure region and the strength of the easterlies.

The kinetic energy in each of these components is

$$\overline{E}_Z = .25\,k^2 A^2 \tag{14}$$

$$\overline{E}_L = .25\,k^2 (4B^2 + C^2) \tag{15}$$

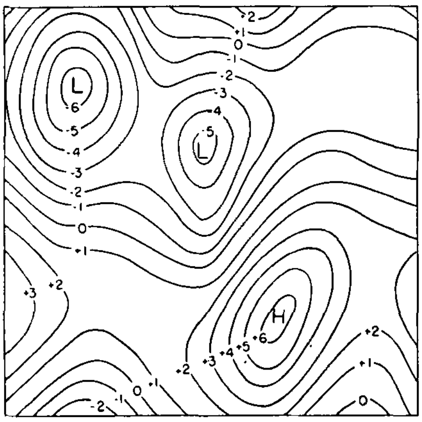

Fig. 6. Composite stream function pattern, $(Z + L + S)$, representing the complete field of flow.

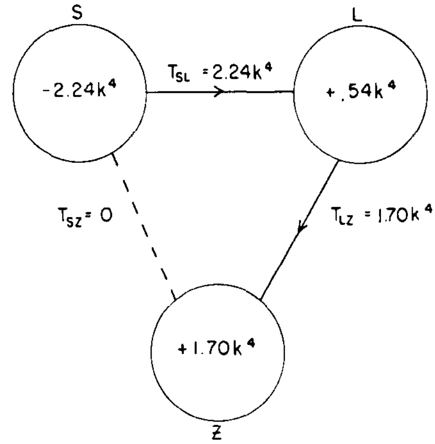

Fig. 7. Kinetic energy flow diagram for the example treated. Numbers within circles indicate the instantaneous rate of energy loss or gain due to the barotropic transfer.

$$\overline{E}_S = .25\, k^2(5D^2 + 9E^2) \qquad (16)$$

and, given the values listed in (13), $\overline{E}_Z = 2.25\, k^2$, $\overline{E}_L = 1.14\, k^2$, and $\overline{E}_S = 3.50\, k^2$.

By applying (8) we obtain the following expressions for the rate of change of $\overline{E}_Z$, $\overline{E}_L$ and $\overline{E}_S$:

$$\frac{d}{dt}\overline{E}_Z = T_{LZ} \qquad (17)$$

$$\frac{d}{dt}\overline{E}_L = -T_{LZ} + T_{SL} \qquad (18)$$

$$\frac{d}{dt}\overline{E}_S = -T_{SL} \qquad (19)$$

where

$$T_{LZ} = .50\, k^4 ABC \qquad (20)$$

measures the transfer of kinetic energy between the long wave and the mean zonal motion (i.e., between L and Z), and

$$T_{SL} = -3.00\, k^4 BDE \qquad (21)$$

measures the transfer between the short and long waves (i.e., between S and L). For this system there can be no kinetic energy exchange between the short waves and the mean flow, (i.e., $T_{SZ} = 0$). With the numbers given in (13) these energy transfer functions have the values $T_{LZ} = +1.70\, k^4$ and $T_{SL} = +2.24\, k^4$, implying an instantaneous kinetic energy flow simultaneously from the smallest scale component, S, to the larger scale component L, and to a lesser degree, from the component L to the mean zonal component Z (see Fig. 7). If we take $K = 10^9$ cm (corresponding to the dimensions of a comparable region of the atmosphere) and let the units of ψ be 10^{11} cm^2/sec (corresponding to a maximum mean zonal westerly velocity of 19 m/sec at $y = K/2$), the energy transfers are of sufficient strength to regenerate the existing kinetic energy of L in six days and that of Z in four days. These figures are of the order required by estimates of atmospheric frictional dissipation. Thus we have constructed one example in which the energy of the larger synoptic features is being maintained against dissipative effects by a transfer of energy from the smaller scale features. The latter may be presumed to grow baroclinically and hence have an independent energy source.

It will be recalled (see eq. 12) that $S(x, y)$ consists of two double-Fourier components, one of wave numbers $(2, -1)$ and the other of wave numbers $(3, 0)$. In agreement with the results of FJØRTOFT (1953), the loss of energy by the component, $(2, -1)$, is balanced by a small gain of energy by the shortest wavelength component, $(3, 0)$, and a larger gain of energy by the long-wave component, $L(x, y)$, the net effect being the transfer from S to L described above. It may be remarked, also, that kinetic energy of disturbances of higher wave numbers than are included initially in our example would tend to be generated as *second-order* effects. Since we have been concerned here with the problem of the maintenance of quasi-steady features we have restricted attention only to the first order transfers. If potential energy conversions are not adequate to maintain the large-scale disturbances directly, these first-order transfers must, in the long-time average, be predominantly of the sign illustrated here. On specific days, however, first-order energy transfers of opposite sign (i.e., transfers from the larger to smaller scale components) may also occur and perhaps are of great actual importance in the sudden deepening of individual intermediate-scale waves observed on weather charts. An example of this reverse transfer may be obtained simply by changing the sign of either B, D or E. Clearly, many other examples illustrating transfers in either direction may be constructed using different Fourier components. It has been our purpose here to demonstrate at least one case which has the energetical properties mentioned in the introductory discussion, and, at the same time, is somewhat realistic in its correspondence with observed phenomena.

In connection with the above remarks it is pertinent to note that for our case the sign of the kinetic energy transfer between S and L, or between S and $(L + Z)$, is independent of the phase of the small-scale disturbances along the diagonal from northwest to southeast in Fig. 4. Thus the small-scale disturbances will continue to feed energy into the larger features at the same rate even if the highs and lows in Fig. 4 were interchanged, as might result from the motions of the small systems relative to the larger ones.

4. Further remarks

a) The view concerning the role of the smaller scale disturbances expressed here undoubtedly has certain connections with the experience of synoptic forecasters who regard the large features, particularly the large cold vortex to the north, as a sort of 'graveyard' for the smaller scale disturbances. The process of energy transfer from the smaller to larger waves is probably related to the familiar 'occlusion' process.

b) According to this discussion, the large scale features of the mean pressure map (i.e., of the average of the ensemble of many maps of the type shown in Fig. 1) are decisively influenced by transient smaller scale phenomena which are entirely absent from mean maps. If this view is correct, it is suggested that a linear, steady-state, theory will not be completely adequate in explaining the large-scale mean features. It would appear that a primary influence of land-sea thermal differences is to create favored areas for the baroclinic development of small-scale disturbances which, in turn, damp by non-linear barotropic processes, to maintain the larger systems. The discussion by SUTCLIFFE (1951) is of pertinence in this connection.

c) Computations from actual atmospheric data of the energy transfers occurring in the wave number domain can be performed feasibly with the use of the high speed computer. Such computations, based on equations presented by the writer (1957), are now under way at M.I.T.

Acknowledgement

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