On a Graphical Method for an Approximate Determination of the Vertical Velocity in the Mid-Troposphere

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Abstract

A simple graphical method for determining the vertical velocity from a simplified, but consistent form of a diagnostic equation for this quantity is developed. It is shown that the main term in the equation is the advection of temperature in the vorticity field. Section 2 contains simple rules from which the sign of the vertical velocity may be determined.

An approximate solution to the equation is found in section 3, while section 4 contains an investigation of the accuracy of this solution as function of the scale of the motion. Different types of approximate solutions are investigated.

Section 5 contains an example of the use of the method.

1. Introduction

The vertical velocity is one of the most important quantities in the atmosphere and one of the most difficult to determine. It is hardly necessary to stress the desirability of methods for its computation. The role of the vertical velocity in large-scale conversion of potential energy into kinetic energy is well-known. In predictions of large-scale precipitation patterns (SMEBYE, 1958) we need methods for a determination of this quantity.

By a numerical integration of the equations for two or multiparameter models designed for physical weather prediction we obtain the vertical velocity as a by-product at one or several levels in the troposphere. There seems, however, to be some time to go before these maps are available almost immediately to every meteorologist in the weather services. It seems therefore worth while to consider methods which allow us to compute an instantaneous value of the vertical velocity by simple graphical methods.

The methods which so far have been available for research work have been based upon an evaluation of all terms except those containing the vertical velocity in the vorticity equation or the adiabatic equation. The other possibility, to compute the ascent or descent directly from the continuity equation, has usually been avoided because of the great sensitivity of this method to error in the observed winds. The former methods can at best give a mean value for the time-period between observations and give therefore not instantaneous values. A method which can give a good first approximation is therefore needed. The method reported here is believed to fulfil this requirement.

In the solution of equations for the vertical velocity in two or multilevel models we need usually a good first guess in order to decrease

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the number of relaxations to solve the equation accurately. The approximate solution given here can be used with advantages in such problems.

2. The method for determination of the vertical velocity

The following procedure for determination of the vertical velocity, $\omega = dp/dt$, consists in finding an approximate solution to the so-called ω-equation, which is obtained by an elimination of the local time derivative between the quasi-geostrophic vorticity equation and the adiabatic equation. The ω-equation has earlier been considered, first by FJØRTOFT (1955) and later by ELIASSEN (1956). Although it is possible to derive this equation from a very general form of the vorticity equation we shall here restrict ourselves to the most simple consistent form of the equation, which is

$$\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla (\zeta + f) = f_0 \frac{\partial \omega}{\partial p} \tag{2.1}$$

where f_0 is a standard value of the Corioli's parameter. The different consistent forms of the vorticity equation has earlier been considered by the author (1959).

The adiabatic equation will be written:

$$\left.\begin{aligned} \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial p}\right) + \mathbf{v} \cdot \nabla \left(\frac{\partial \phi}{\partial p}\right) + \sigma\omega = 0, \\ \sigma = -\alpha \frac{\partial \ln \Theta}{\partial p} \end{aligned}\right\} \tag{2.2}$$

We shall in the following assume that the static stability, σ, is constant in any isobaric surface, i.e., $\sigma = \sigma(p)$ and shall further compute the relative vorticity by the formula:

$$\zeta = \frac{1}{f_0} \nabla^2 \phi \tag{2.3}$$

Using (2.1), (2.2) and (2.3) we obtain the ω-equation in the form:

$$\sigma \nabla^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} = f_0 \frac{\partial}{\partial p} (\mathbf{v} \cdot \nabla \eta) - \\ - \nabla^2 \left(\mathbf{v} \cdot \nabla \frac{\partial \phi}{\partial p}\right) = F(x, y, p) \tag{2.4}$$

The forcing function $F(x, y, p)$ may be expressed in a number of different ways. We shall here choose to express $F(x, y, p)$ with the aid of the height and temperature in an isobaric surface. We will further neglect variations in the Corioli's parameter in the evaluation of F, because this simplifies the procedure to a great extent. By doing so we naturally restrict the latitudional extent of the field. Using the approximations stated above we can write the forcing function in the following form

$$F(x, y, p) = \frac{2Rg}{f_0 p} \{J(\nabla^2 z, T) + A(z, T)\} \tag{2.5}$$

$J(\nabla^2 z, T)$ represents the usual Jacobian, while $A(\alpha, \beta)$ is a notation for the operator

$$A(\alpha, \beta) = -\left(\frac{\partial^2 \alpha}{\partial x \partial y}\right)\left(\frac{\partial^2 \beta}{\partial x^2} - \frac{\partial^2 \beta}{\partial y^2}\right) + \\ + \left(\frac{\partial^2 \alpha}{\partial x^2} - \frac{\partial^2 \alpha}{\partial y^2}\right)\left(\frac{\partial^2 \beta}{\partial x \partial y}\right) \tag{2.6}$$

It is seen that $A(\alpha, \beta)$ is related to the deformation properties of the two fields α and β. It will consequently be called the deformation function. The deformation function obviously vanishes, if α is proportional to β. There is a general tendency for the isoterms in the midtroposphere to be parallel to the contour lines. At most places $A(z, T)$ will therefore be small compared to $J(\nabla^2 z, T)$

A direct computation of $A(z, T)$, using finite differences at 500 mb, shows the following characteristics:

1. The deformation function is locally of a smaller magnitude than the Jacobian in (2.5). A mean value of the absolute values of A turned out to be less than half of the corresponding mean value of $J(\nabla^2 z, T)$.

2. The deformation function is generally on a smaller scale than the Jacobian.

In view of the two statements above it is suggested that the deformation function can be neglected in comparison with the Jacobian, if we are interested in a fast method to obtain a first estimate of the vertical velocity. The statements 1. and 2. do not hold in general for levels above or below the 500 mb level, where we find a greater baroclinicity in the atmosphere.

We may derive a "geostrophic" expression for $A(z, T)$ which clearly show how it is depending upon the deformation properties of the fluid. Introducing the expressions for the geostrophic wind and the geostrophic thermal wind in the form:

$$\frac{\partial u}{\partial p}=\frac{R}{f_0 p}\frac{\partial T}{\partial y}, \ \frac{\partial v}{\partial p}=-\frac{R}{f_0 p}\frac{\partial T}{\partial x} \qquad (2.7)$$

in the expression for Λ we can, after some manipulations, arrive at the expression

$$\Lambda(z,T)=\frac{pf_0^2}{2gR}\left\{A\frac{\partial B}{\partial p}-B\frac{\partial A}{\partial p}\right\} \qquad (2.8)$$

where

$$A=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}, \ B=\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y} \qquad (2.9)$$

are the two deformation expressions for the wind. (2.8) shows first of all how the geostrophic estimate of the deformation function depends on the deformation expressions for the wind. It shows further that the deformation function will be small if A and B do not vary greatly with pressure.

In the following we shall approximate the forcing function by

$$F(x, y, p)=\frac{2Rg}{f_0 p}J(\nabla^2 z, T) \qquad (2.10)$$

Let us next in order to get a first crude idea about the solution to the ω-equation consider a single component of the ω-field, say

$$\omega=A\sin kx\sin ly\sin mp \qquad (2.11)$$

where k, l and m are wave numbers in the x, y and p direction, respectively. We have then

$$L(\omega)=\nabla^2\omega+\frac{f_0^2}{\sigma}\frac{\partial^2\omega}{\partial p^2}=-s^2\omega=$$
$$=\frac{2Rg}{f_0 p\sigma}J(\nabla^2 z, T) \qquad (2.12)$$

where $s^2=k^2+l^2+f_0^2 m^2/\sigma$ is a measure of scale. Approximating the Laplacian of Z in the usual way:

$$\nabla^2 z=-\frac{4}{d^2}(z-\bar{z}) \qquad (2.13)$$

we get the following solution:

$$\omega=\frac{2Rg}{s^2 f_0 p\sigma}\cdot\frac{4}{d^2}J(z-\bar{z}), T) \qquad (2.14)$$

which also may be written in the form:

$$\omega=a\mathbf{v}_\zeta\cdot\nabla T \qquad (2.15)$$

where a is a positive quantity and

$$\mathbf{v}_\zeta=-\frac{g}{f_0}\nabla(z-\bar{z})x\mathbf{k} \qquad (2.16)$$

The solution (2.15), which of course only may be considered as a first approximation, shows that $\omega<0$ *(upward motion) if we have warm air advection by the "wind" in the vorticity field, while* $\omega>0$ *(downward motion), if cold air is advected by* $\mathbf{v}_\zeta$.

The above rule may be used to locate the regions of upward and downward motion directly on a weather map, where an isotherm analysis is made, and when the vorticity field is represented by isolines for $(z-\bar{z})$. It should be mentioned that the reversed rule applies if the vorticity field is represented by $\nabla^2 z=4/d^2(\bar{z}-z)$, which is the usual finite difference approximation.

Although the above rule may be good enough to determine the sign of the vertical velocity, we need something better if we want to go beyond this. We are here mainly interested in the horizontal distribution of ω, and shall therefore represent the vertical distribution as simple is possible. The representation.

$$\omega(x, y, p, t)=(p-p_s)(p-p_*)^{M(x,p)} \qquad (2.17)$$

where $p_s=100$ cb represents the surface of the earth and $p_*=20$ cb a level in the lower stratosphere, where $\omega=0$, is the simplest representation which gives a reasonable distribution of convergence and divergence in the vertical direction. From (2.17) it follows that

$$\frac{\partial^2\omega}{\partial p^2}=\frac{\omega}{\frac{1}{2}(p-p_s)(p-p_*)} \qquad (2.18)$$

and (2.4) can be written

$$\nabla^2\omega—b\omega=h(x, y) \qquad (2.19)$$

where

$$b=\frac{2f_0^2}{\sigma(p_s-p)(p-p_*)}>0 \qquad (2.20)$$

and

$$h(x, y) = \frac{2Rg}{f_0 \sigma p} J(\nabla^2 z, T) \qquad (2.21)$$

The equation (2.19) is of the Helmholz type and is similar to equations used in numerical prediction of two parameter models. It is further of the same type as the equation used by SMEBYE (1958) in his investigation of large scale precipitation forecasting. In his solution of the equation he approximated

$$\nabla^2 \omega = -\frac{4}{d^2}(\omega - \overline{\omega}) \qquad (2.22)$$

and got the equation in the form

$$\omega - B\overline{\omega} = hd^2 \qquad (2.23)$$

where however the term containing $\overline{\omega}$ was neglected. This procedure corresponds to the solution given in (2.15) and may in many cases lead to an underestimate of ω as shown in the next section. It actually only gives an accurate solution for one particular scale, while the accuracy rapidly falls off, when we remove ourselves from this scale. We shall return to the form (2.19) and in the next section discuss a procedure for an approximate solution and its accuracy.

3. Solution of the Helmholz equation

FJØRTOFT (1952) has discussed approximate solutions to Poisson and Helmholz equations. It is shown by Fjørtoft that the speed of convergence of the series solution increases rapidly in the Poisson case, if an increasing gridsize is used in the smoothing operators used for evaluation of the terms in the formal solution. Further, it is pointed out that the convergence of the solution in the Helmholz case may be speeded up in the same way, but that this is not necessary, because already applying the same gridsize in all terms leads to a fairly accurate solution including only a few terms in the series. Although this is true it may still be desirable to investigate whether a varying gridsize will increase the accuracy to such a degree that still fewer terms give the same accuracy.

The method depends upon the use of a varying gridsize. Let $d_1, d_2 \ldots d_p \ldots$ denote a steadily increasing series of gridsizes. The finite difference form of the Helmholz equation can now be written in any of the forms

$$\nabla_p \omega - bd_p^2 \omega = d_p^2 h \qquad (3.1)$$

where

$$\nabla^2 \omega \simeq \frac{1}{d_p^2} \nabla_p^2 \omega$$

It is convenient for the following computations to define a smoothing operator:

$$\omega^{[p]} = \omega + \frac{\nabla_p^2 \omega - bd_p^2 \omega}{\alpha_p + bd_p^2} \qquad (3.2)$$

where α_p so far is an arbitrary constant. We shall later relate this smoothing to the well-known operator.

$$\omega^{(p)} = \omega + \frac{\nabla_p^2 \omega}{\alpha_p} \qquad (3.3)$$

which for $\alpha_p = 4$ is identical with the Fjørtoft-operator. By formal computations we get by repeated use of (3.2) for different values of the gridsizes:

$$\omega = -\frac{\nabla_1^2 \omega - bd_1^2 \omega}{\alpha_1 + bd_1^2} - \frac{(\nabla_2^2 \omega - bd_2^2 \omega)^{[1]}}{\alpha_2 + bd_2^2} - \ldots - \frac{(\nabla_p^2 \omega - bd_p^2 \omega)^{[1] \ldots [p-1]}}{\alpha_p + bd_p^2} + \omega^{[1] \ldots [p]} \qquad (3.4)$$

where the last term is the rest term.

Substituting from (3.1) we obtain:

$$\omega = -\frac{d_1^2 h}{\alpha_1 + bd_1^2} - \frac{d_2^2 h^{[1]}}{\alpha^2 + bd_2^2} - \ldots - \frac{d_p^2 h^{[1] \ldots [p-1]}}{\alpha_p + bd_p^2} + \omega^{[1] \ldots [p]} \qquad (3.5)$$

The next problem is to bring the solution (3.5) into a form which is easier to handle in practice. This is done by relating the two smoothing operators (3.2) and (3.3) to each other. Eliminating $\nabla_p^2 \omega$ between (3.2) and (3.3) we obtain

$$\omega^{[p]} = \frac{\alpha_p}{\alpha_p + bd_p^2} \omega^{(p)} \qquad (3.6)$$

(3.6) shows that $\omega^{[p]}$ act in principally the same way as $\omega^{(p)}$. The only difference is a further reduction of the amplitude of the different Fourier-components by the proportionality factor. When finally (3.6) is inserted in (3.5) we obtain:

$$\omega = -\frac{d_1^2}{\alpha_1 + bd_1^2} h - \frac{d_2^2}{\alpha_2 + bd_2^2} \cdot \frac{\alpha_1}{\alpha_1 + bd_1^2} h^{(1)} - \dots$$
$$\dots - \frac{d_p^2}{\alpha_p + bd_p^2} \left\{ \prod_{\nu=1}^{\nu=p-1} \frac{\alpha_\nu}{\alpha_\nu + bd_\nu^2} \right\} h^{(1)\dots(p-1)} +$$
$$+ \left\{ \prod_{\nu=1}^{\nu=p} \frac{\alpha_\nu}{d_\nu + bd_\nu^2} \right\} \omega^{(1)\dots(p)} \qquad (3.7)$$

A truncated form of (3.7) may be used as an approximative solution, when we have decided which values we will ascribe to the series d_ν and α_ν. Many possibilities exist in this respect depending on what we want to obtain. If we for instance want to use the approximate solution as a first guess for a relaxation procedure it may be convenient to choose the values α_ν in such a way that a fairly accurate solution is obtained over a large interval of wave-numbers. It is not the purpose here to give any complete discussion of the solution (3.7). We shall be satisfied by showing that for the specific choice

$$\alpha_1 = \alpha_2 = \dots = \alpha_p = 4$$

and

$$d_1^2 = \frac{1}{2} d_2^2 = \dots = \frac{1}{2^{p-1}} d_p^2$$

we can obtain a rather accurate solution to the special form of the ω-equation discussed here.

The accuracy of the solution is of course also depending on the particular value of the constant b, which appears in the Helmholz equation. Generally speaking we obtain a more accurate solution with the same number of terms the larger b is.

4. Accuracy of the approximate solution

This section will contain an evaluation of the accuracy of the approximate solution. In order to be able to compute numerical values we shall decide upon the units to be used. A convenient unit for ω is mb per 12 hours. We shall further use a value for $d_1 = 6 \times 10^5$ m. Les us finally assume that the right hand side $h(x, y)$ has been computed with a usual finite difference procedure, i.e.

$$h = \frac{2Rg}{f_0 \sigma p} J(\nabla^2 z, T) =$$
$$= \frac{2Rg}{f_0 \sigma p} \frac{1}{4 d_1^4} \mathrm{J}(\nabla^2 z, T) = \frac{c}{4 d_1^4} H \qquad (4.1)$$

where

$$c = \frac{2Rg}{f_0 \sigma p}, \quad H = \mathrm{J}(\nabla^2 z, T) \qquad (4.2)$$

Using MTS-units and the above unit for ω we obtain for the 500 mb surface

$$bd_1^2 \simeq \frac{5}{2}, \quad c \simeq \frac{1}{5} \qquad (4.3)$$

The finite difference equation to which we want to find an approximate solution is

$$\nabla_1^2 \omega - bd_1^2 \omega = \frac{c}{4 d_1^2} H(x, y) \qquad (4.4)$$

We shall investigate the accuracy of the solution by assuming that ω can be written in the form

$$\omega = \text{const.} + \Sigma A_{mn} e^{2\pi i \left(\frac{x}{\lambda_m} + \frac{y}{\nu_n}\right)} \qquad (4.5)$$

In this problem we shall assume that $\omega \to 0$ for $(x, y) \to \infty$. This means that we are allowed to put the constant in (4.5) equal to zero. From the expression (4.5) we may easily find the Fourier expansion for H by inserting in (4.4).

$$H = -\frac{4 d_1^2}{c} \Sigma B_{mn} e^{2\pi i \left(\frac{x}{\lambda_m} + \frac{y}{\nu_n}\right)} \qquad (4.6)$$

with

$$B_{mn} = \left(4 - 2 \cos \frac{2\pi}{\lambda_m} d_1 - 2 \cos \frac{2\pi}{\nu_n} d_1 + bd_1^2 \right) A_{mn} \qquad (4.7)$$

When we now insert the series (4.6) in the approximate solution (3.7) we can investigate with which accuracy it is possible to reproduce the series (4.5) with a truncated form of (3.7)

We note then first that for any quantity

$$\delta = \Sigma \delta_{mn} e^{2\pi i \left(\frac{x}{\lambda_n} + \frac{y}{\nu_n}\right)} \qquad (4.8)$$

we have

$$\delta^{(p)} = \Sigma R_{mn}^{(p)} \delta_{mn} e^{2\pi i \left(\frac{x}{\lambda_m} + \frac{y}{\nu_n}\right)} \qquad (4.9)$$

with

$$R_{mn}^{(p)} = \left(1 - \frac{4}{\alpha_p}\right) + \frac{2}{\alpha_p}\left(\cos\frac{2\pi}{\lambda_m}d_p + \cos\frac{2\pi}{\nu_n}d_p\right) \tag{4.10}$$

With these notations we can derive an expression for the ratio between ω_{mn}^*, which is the amplitude of the general component in the series, and ω_{mn}, which is the corresponding amplitude in the complete development of ω. The ratio $\omega_{mn}^*/\omega_{mn}$, will be used as a measure of the accuracy of the approximate solution. In order to have the possibility to express the accuracy in numbers for a rather general case, we shall select the case, where $\lambda_m = \nu_n$ and where α_p and d_p have the values stated above. We arrive at the following expression for the accuracy:

$$\frac{\omega_{mn}^*}{\omega_{mn}} = \left[4\left(1 - \cos 2\pi\frac{d_1}{\lambda_m}\right) + bd_1^2\right]\cdot$$
$$\cdot\left\{\frac{1}{\alpha_1 + bd_1^2} + \left(\frac{d_2}{d_1}\right)^2 \frac{\alpha_1}{\alpha_1 + bd_1^2}\,\frac{R_{mn}^{(1)}}{\alpha_2 + bd_2^2} + \ldots\right.$$
$$\left.\ldots + \left(\frac{d_p}{d_1}\right)^2\cdot\left[\prod_{\nu=1}^{p-1}\frac{\alpha_\nu}{\alpha_\nu + bd_\nu^2}\right]\cdot\frac{R_{mn}^{(1)}\ldots R_{mn}^{(p)}}{\alpha_p + bd_p^2}\right\} \tag{4.11}$$

With our special selection of the parameters $(\lambda_m = \nu_n,\ d_p,\ \alpha_p)$ we have

$$R_{mn}^{(p)} = \cos 2\pi\frac{d_p}{\lambda_m} \tag{4.12}$$

If $\lambda_m = 4d_1$, we see immediately that $R_m = 0$. Consequently we will have complete accuracy, i.e. $\{\omega_m^*/\omega_m\}_{\lambda=4d_1} = 1$ for this wave length, independent of the number of terms. Next we want to investigate the accuracy of the solution for other wave lengths, when we incorporate a certain number of terms in the approximate solution. The number of terms is characterized by the index p. $p = 1$ means that we include two terms etc. The accuracy of the solution for $p = 1$ and $p = 2$ is given in table 1, which shows that $p = 2$ will give an accuracy which is sufficient for most diagnostic purposes, especially when we consider the other approximations in the formulation of the problem. Also, it is worth while mentioning that $p = 1$ gives a solution with an error less than 10 % up to a wavelength close to 6,000 km.

Table 1.

λ_m	4 d	5 d	6 d	8 d	10 d	12 d	14 d	16 d
ω_m^*/ω_m, $p=1$	1.00	1.03	1.00	0.93	0.86	0.83	0.80	0.79
ω_m^*/ω_m, $p=2$	1.00	1.00	1.02	1.03	1.01	1.00	0.99	0.98

We shall next compare the accuracy of the solution given above with the one obtained without varying the gridsize in the Helmholz case. The derivation is made in a completely similar fashion as before. The approximate solution takes now the form:

$$\omega^* = -\frac{c}{4d_1^2}\cdot\frac{1}{\alpha_1 + bd_1^2}\left[H + \frac{\alpha_1}{\alpha_1 + bd_1^2}H^{(1)} + \ldots\right.$$
$$\left.\ldots + \left(\frac{\alpha_1}{\alpha_1 + bd_1^2}\right)^{p-1}\cdot H^{\overbrace{(1)(1),\ldots(1)}^{(p-1)}}\right] \tag{4.13}$$

The accuracy, measured in the same way as before, is now given by the formula:

$$\frac{\omega_m^*}{\omega_m} = \frac{4\left(1 - \cos 2\pi\frac{d_1}{z_m}\right) + bd_1^2}{\alpha_1 + bd_1^2}\cdot$$
$$\cdot\left\{1 + \frac{\alpha_1}{\alpha_1 + bd_1^2}R_m^{(1)} + \ldots + \left(\frac{\alpha_1}{\alpha_1 + bd_1^2}R_m^{(1)}\right)^{p-1}\right\} \tag{4.14}$$

Table 2 gives the accuracy for different values of the wave length for $p = 1$ and $p = 2$.

A comparison between table 1 and 2 shows clearly that the former method is an advantage. This is particularly so, because it is not followed by any extra work, if the computations are made graphically. It should further be pointed out that the two formulas are different for $p = 1$. Only for $p = 0$ are they identical.

The reason that we obtain such a high accuracy already for $p = 1$ and 2 is due to

Table 2.

λ_m	4 d	5 d	6 d	8 d	10 d	12 d	14 d	16 d
ω_m^*/ω_m, $p=1$	1.00	0.96	0.91	0.83	0.76	0.72	0.69	0.68
ω_m^*/ω_m, $p=2$	1.00	0.99	0.97	0.94	0.87	0.85	0.83	0.82

the fact that the coefficient to ω in the Helmholz equation in this particular case is as great as 2.5.

It is at this point worth while to mention that Helmholz equations which with advantage may be solved graphically appear in other problems. The prognostic equation for the thickness field, derived by FJØRTOFT (1955) and tested by SIGGTRYGGSSON and the author (1957), also leads to a Helmholz-equation. Denoting the thickness field between 1,000 mb and 500 mb by ϕ_T we can write the prognostic equation in the form

$$\nabla_1^2 \phi_T - e d_1^2 \phi_T = d_1^2 h \qquad (4.15)$$

The value of ed_1^2, to be used in (4.15) turns out to be about 4. The accuracy of the solution with varying gridsize is shown in table 3 for this case. Again the value for $p=1$ and 2 are given. p = 1 gives a good accuracy, while $p=2$ is remarkably good due to the large value of the coefficient.

Table 3.

λ_m	4 d	5 d	6 d	8 d	10 d	12 d	14 d	16 d
ω_m^*/ω_m, $p=1$	1.00	1.02	1.00	0.95	0.92	0.90	0.88	0.87
ω_m^*/ω_m, $p=2$	1.00	1.01	1.01	1.00	1.00	1.00	0.99	0.98

We shall finally comment a little on the numerical values used in the practical application of the ω-equation. When we introduce the numerical values of bd_1^2, and $c/4d_1^2$ and measure ω in mb/12h as before we obtain:

$$\omega = q + 0.88\, q^{(1)} + 0.50\, q^{(1)(2)} + 0.21\, q^{(1)(2)(3)} + \ldots \qquad (4.16)$$

where $q = -2/65\, H$.

In graphical work it is extremely convenient if the coefficients are simple numbers. It will do no harm, if we replace (4.16) by

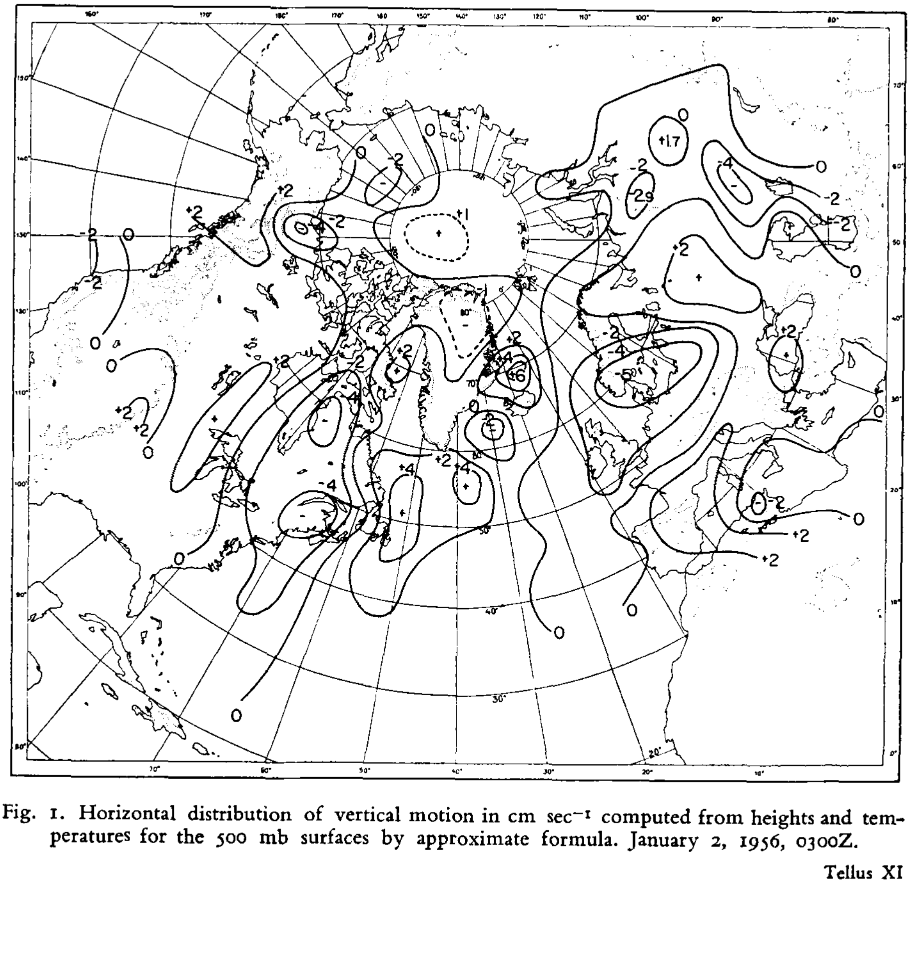

Fig. 1. Horizontal distribution of vertical motion in cm sec⁻¹ computed from heights and temperatures for the 500 mb surfaces by approximate formula. January 2, 1956, 0300Z.

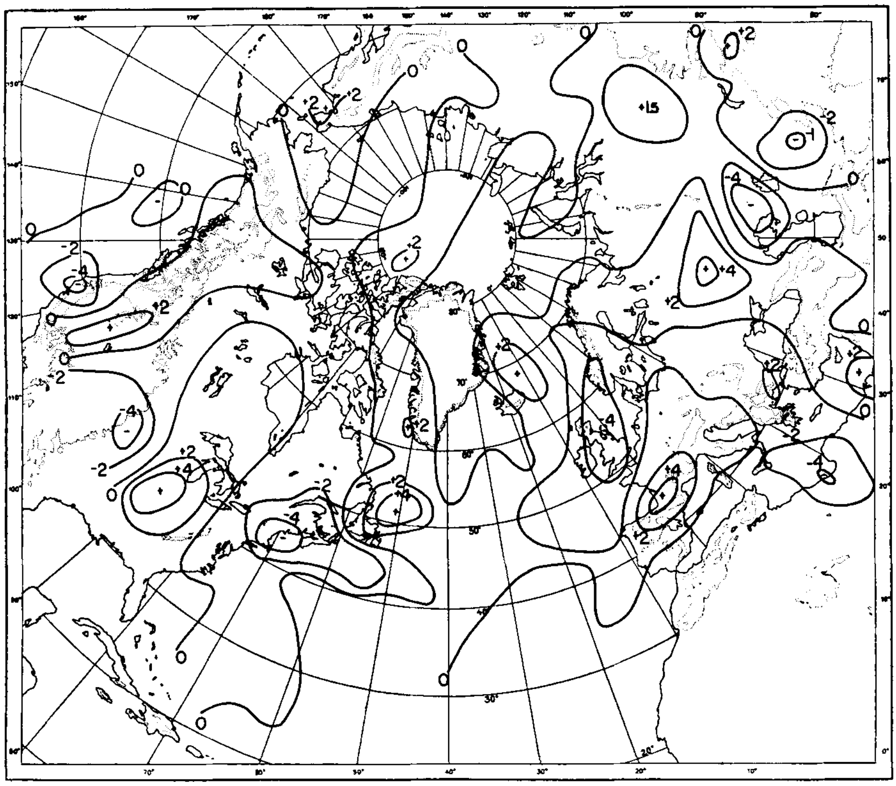

Fig. 2. Horizontal distribution of vertical motion in cm sec^{-1} computed 850, 500, and 300 mb heights using a two-parameter baroclinic model. Time as in figure 1.

$$\omega = q + q^{(1)} + \frac{1}{2} q^{(1)(2)} + \frac{1}{5} q^{(1)(2)(3)} \quad (4.17)$$

and compute q by the approximate relation

$$q = -\frac{2}{65} J(\nabla^2 z, T) \simeq \frac{5}{4} J(z - \bar{z}, T) \quad (4.18)$$

where it is understood that $(z - \bar{z})$ in the last expression is measured in decameters.

5. An example of horizontal distribution of ω

A test of the formula developed in the preceding section was made by computing ω using the method and compare the obtained distribution with the corresponding distribution obtained from a baroclinic model.

The two data fields used for the approximate solution were the height and temperature fields for the 500 mb surface for January 2, 1956, 0300Z.

The baroclinic vertical velocity was obtained from a model using the height fields for 850, 500 and 300 mb as initial data.

Fig. 1 and 2 give the two distributions, where fig. 1 is the approximate solution. It is seen that all maxima and minima which appear on one of the figures can be found on the other. There are some differences in the configurations, but the agreement is in general good, when it is realized that the two fields are computed from different, although not independent, data.

It should finally be mentioned that the graphical method naturally also can be formulated in such a way that the two fields used in the computation are a height field and a thickness fields.

6. Summary and conclusions

A simple, but consistent form of an equation for the vertical velocity is analyzed in section 2.

It is found that the dominating term in the forcing function is the advection of temperature in the field of isolines for relative vorticity. A simple rule from which the sign of the vertical velocity may be determined is formulated.

Section 3 contains a method for solving a Helmholz-equation similar to the one developed by Fjørtoft for a Poisson-equation and using a variable gridsize.

The accuracy of the solution is computed in section 4, which also contains a comparison between the solution and one obtained without varying the gridsize.

An example is shown in section 5.

The method developed in this paper to obtain the horizontal distribution of the vertical velocity will be of use in cases, where the distribution is not available from integration of a baroclinic model of the atmosphere. The main advantage of the method is that it can give instantaneous values of the vertical velocity. Local values of the vertical velocity can easily be obtained without laborious graphical manipulations.

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