The Horizontal and Vertical Wind Profiles of the Subtropical and Polar Jet for January 1—7, 1956 and the Variation of the Equivalent Barotropic Level

By HESSAM TABA

International Meteorological Institute, University of Stockholm[1]

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Abstract

All the available wind information over central, west and north Europe, the Atlantic Ocean, the United States and Canada has been studied for a period of seven days, January 1—7, 1956 0300 Z. The axis of the subtropical and the polar jets have been chosen as reference coordinates. Mean horizontal and vertical wind profiles have been computed at 100 km intervals from the axis of both jets and are discussed. Based on these profiles, the average wind and the equivalent barotropic level is computed. It is shown that the height of the equivalent barotropic level changes as the distance from the axis of the jet varies. The equivalent barotropic level is shown to have a maximum height of 432 mb at the axis of the subtropical jet and a minimum height of 560 at a distance of 1,000 km north of the polar jet. Finally, an average E.B.L. is computed and is shown to have a height of 505 mb.

Part I

Introduction

In a series of recent articles DEFANT and TABA (1957—1958) have studied changes in the type of general circulation by means of tropopause maps. It was shown that hemispheric tropopause maps were most suitable for depicting changes in the three dimensional structure of the atmosphere. The authors had also emphasized the close relation between the breaks of the tropopause height and the main wind belts. It has also been demonstrated that the vertical structure of the atmosphere is quite different on either sides of the two jets and that the subtropical and polar jet divided the atmosphere into three distinct parts with different characteristics. The conservative properties of the atmosphere on either side of the jets suggest that in statistical studies of the large scale atmospheric motion the most suitable reference coordinate would be the axis of the jets. In the first part of this paper the horizontal and vertical wind profiles north and south of the two jets are presented. It should be stressed since all the wind information used in this study is what was available for the part of the hemisphere mentioned, one can say for sure that the average values obtained are at least a good representation of the mean motion of the atmosphere for this period of the year.

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Preparation of the material

All available wind information for a period ofs even days, January 1—7, 1956 0300 GMT has been plotted. This covers the area of the United States, Canada, the Atlantic Ocean, North, West, and Central Europe and North Africa. A total number of 950 stations have been studied which means about 132 stations per day. The following procedure has been followed:

1) With the help of pressure maps of 850, 500, 300, 200 and 100 mb and tropopause maps, the position of the axis of both the polar and the subtropical jet has been located.

2) These jets are used as coordinates of reference. The horizontal distance of every station from the axis of the jet has been measured. Here the map factor is $m = \frac{1+\sin 50}{1+\sin \varphi_m}$ where φ_m is the latitude of the half of the distance from the station to the jet axis.

3) For each isobaric level the wind speed of each station has been plotted on a diagram with the distance from the jet axis as one coordinate and the speed as the other coordinate. In this way a point cloud distribution has been obtained. For the sake of presentation one of these levels, namely the 250 mb (subtropical jet), is presented in Fig. *1*. As one can see, the number of points are quite different within each 100 km horizontal distance. This means that the value of the mean speed computed is based on different number of observations.

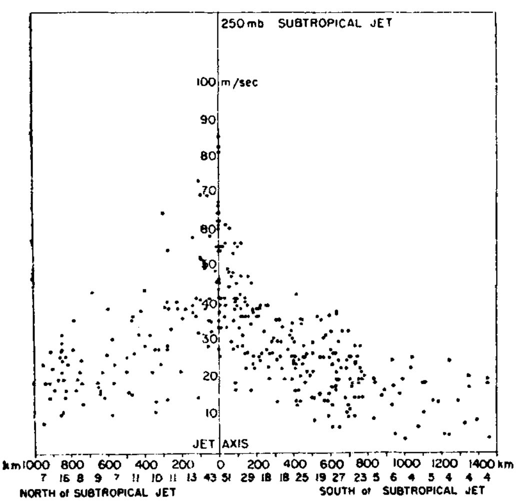

Fig. 1.

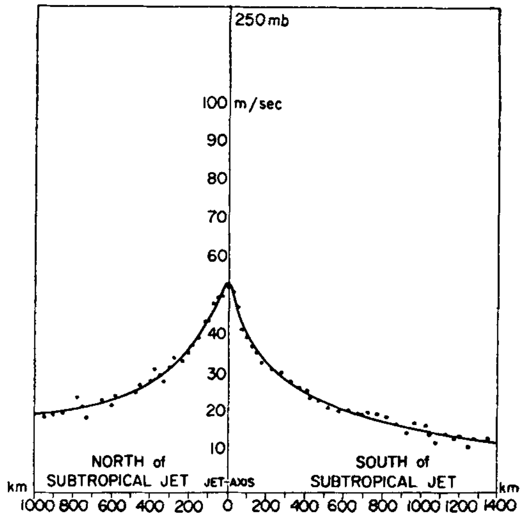

Fig. 2. The mean wind at 250 mb as a function of distance from the axis of the subtropical jet. The horizontal axis shows distances in km and the vertical axis the speed in m/sec.

It is quite clear from the figure that for example within a certain horizontal distance, say 100 km, on either side of the jet, one finds quite a different range of speeds. This is an indication of the fact that in this case the subtropical jet has been more intense in some regions of the northern hemisphere and relatively weaker in other regions and at other times.

Fig. *2* represents the mean wind as a function of distance from the jet. Within every 50 km horizontal distance the average wind speed has been computed. To obtain a more smooth profile, a further average is computed covering a horizontal distance of 100 km. It should be mentioned that as there were not enough wind observations at heights above 50 mb, it was decided not to continue above this level.

Horizontal wind profiles

1. *The subtropical jet*

Figs. *3* and *4* show the horizontal wind profiles for the subtropical jet for different isobaric levels. The following are important points which deserve considerations.

a) The strongest wind to be observed in these two figures is about 55 meter per second

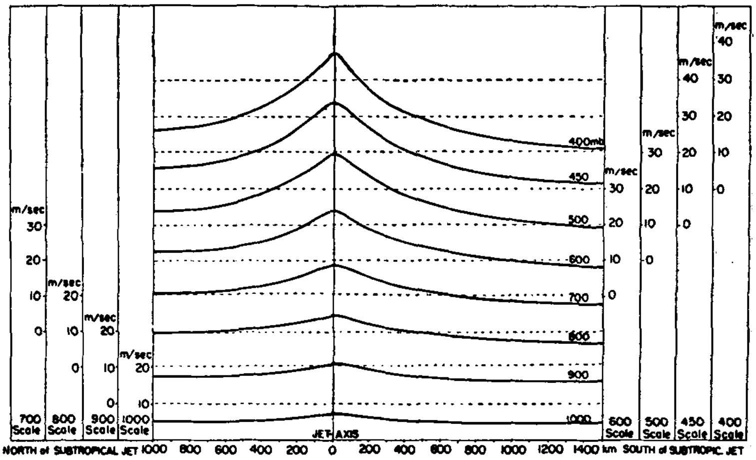

Fig. 3. The horizontal wind profile of the subtropical jet 1000—400 mb.

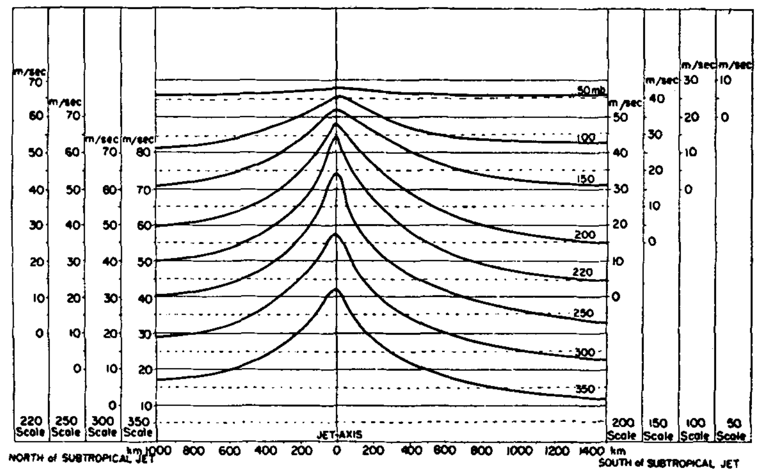

Fig. 4. The horizontal wind profile of the subtropical jet 350—50 mb.

and at the level of 220 mb. However, this is not the level of maximum wind. The level of maximum wind is at about 228 mb with a speed of 58—60 m/sec.

b) At each level the shear is strongest near the axis of the jet and decreases away from it. The strongest shear is found on the southern side of the subtropical jet.

c) As seen from the figures, towards south the wind profiles reach a horizontal distance of 1,500 km while towards north it does not exceed 1,000 km. The reason for this difference is that the space north of the subtropical jet was occupied partly by the polar jet as well. This means that the intermediate area between the two jets was covered by stations some of which belonged to north of the subtropical jet and the rest to south of the polar jet.

2. *The polar jet*

Figs. 5 and 6 show the horizontal wind profiles for the polar jet for different isobaric levels. Here are some noteworthy points:

a) The strongest wind to be observed in these two figures is about 50 m/sec at the 300 mb level. This is not really the level of maximum wind. The maximum wind is about

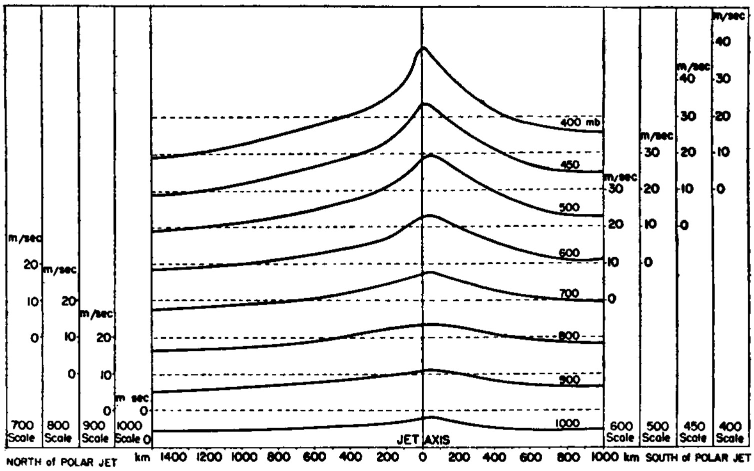

Fig. 5. The horizontal wind profile of the polar jet 1000—400 mb.

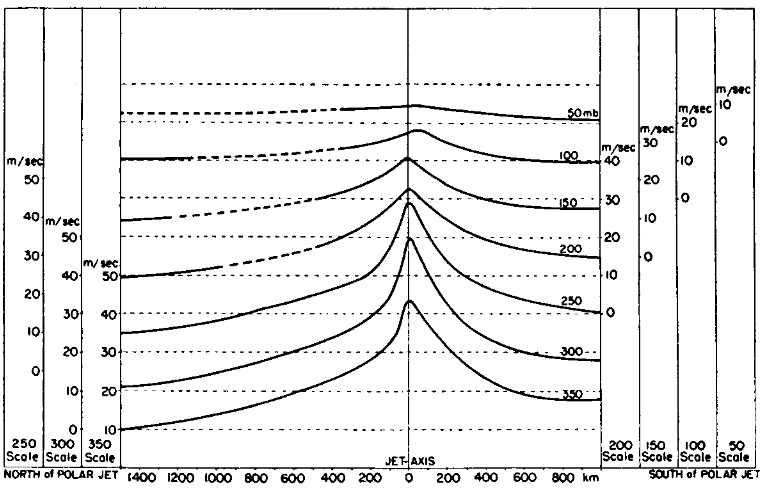

Fig. 6. The horizontal wind profile of the polar jet 350—50 mb.

53 m/sec and the height at which it is observed is at about 295 mb.

b) Near the axis of the polar jet the horizontal shear is much more pronounced than at distances further away from the jet. Observe the relatively much stronger shear on the *northern* side of the jet.

c) In the case of the polar jet, the wind profile to the north extends to a horizontal distance of 1,500 km while towards the south it reaches only to 1,000 km. The reason for this is exactly what was mentioned for the subtropical jet.

d) The profiles of 200, 150, 100, 50 mb have some dashed parts towards the north. This is to show that at these levels the data studied was not quite enough and reliable.

e) Notice the displacement of the strongest wind towards the south at the levels 400, 500, 600 and 700 mb. Fig. 5.

Vertical wind profiles

1. *South of the subtropical jet*

Fig. 7 shows the vertical wind profiles of south of the subtropical jet. The numbers

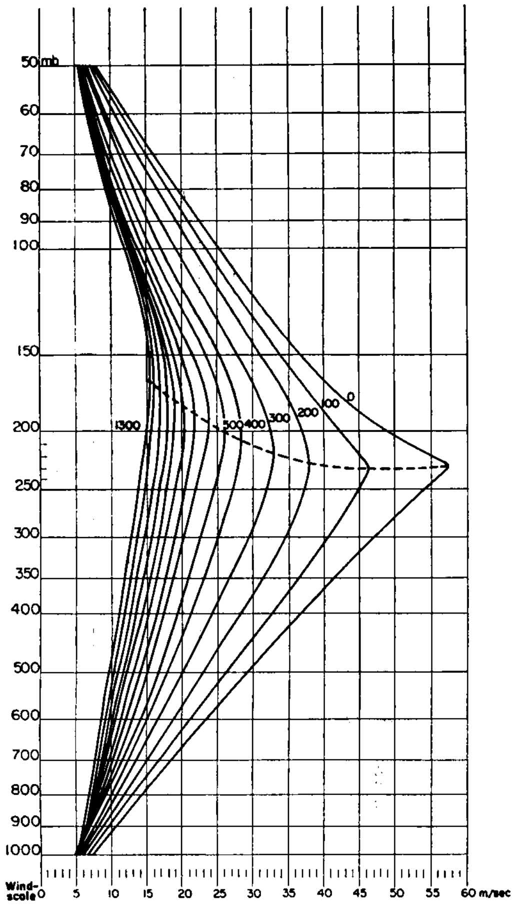

Fig. 7. The vertical wind profiles south of the subtropical jet. The number on each profile indicates the distance from the jet axis. Observe the increase of the height of maximum wind towards south.

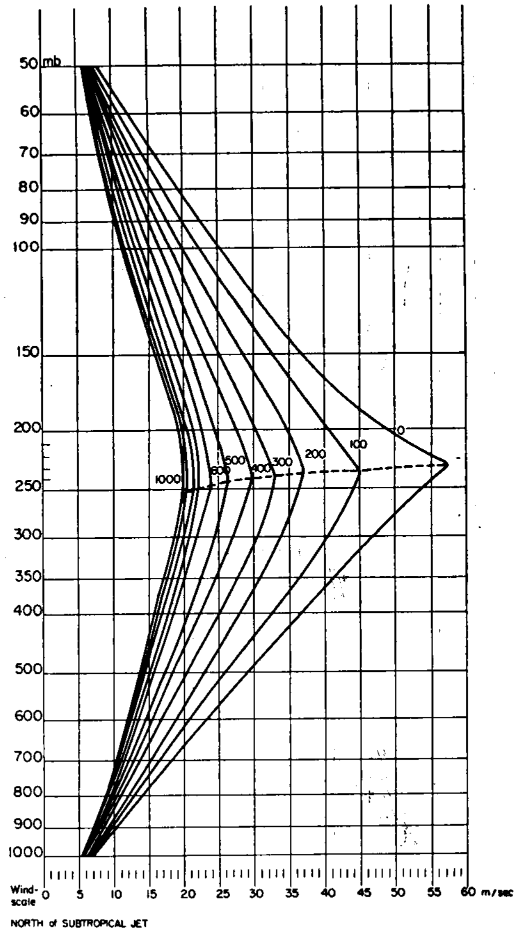

Fig. 8. The vertical wind profile north of the subtropical jet. Observe the decrease of the height of maximum wind towards north.

on each profile indicate the distance from the axis of the jet. Here are some interesting features:

a) As mentioned before, the strongest wind observed is about 58 m/sec at the level of 228 mb and at the axis of the jet.

b) The decrease of the wind with the distance from the axis of the jet is much more pronounced in the middle troposphere than at lower and higher levels.

c) Notice the increase of the height of maximum wind as one goes further towards the south. This is indicated by the dashed line.

2. *North of the subtropical jet*

Fig. *8* shows the vertical wind profiles north of the subtropical jet. There is a difference between this profile and the one to the south. This is the decrease of the height of maximum wind as one goes from the center of the jet towards the north as indicated by dashed lines.

3. *South and north of the polar jet*

Figs. *9* and *10* show the vertical wind profiles south and north of the polar jet.

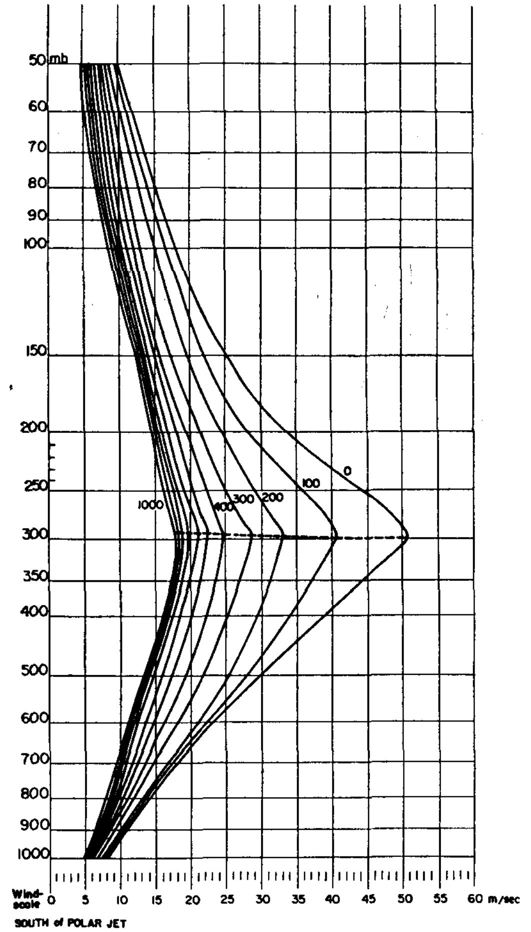

Fig. 9. The vertical wind profile south of the polar jet. The height of the maximum wind increases towards south.

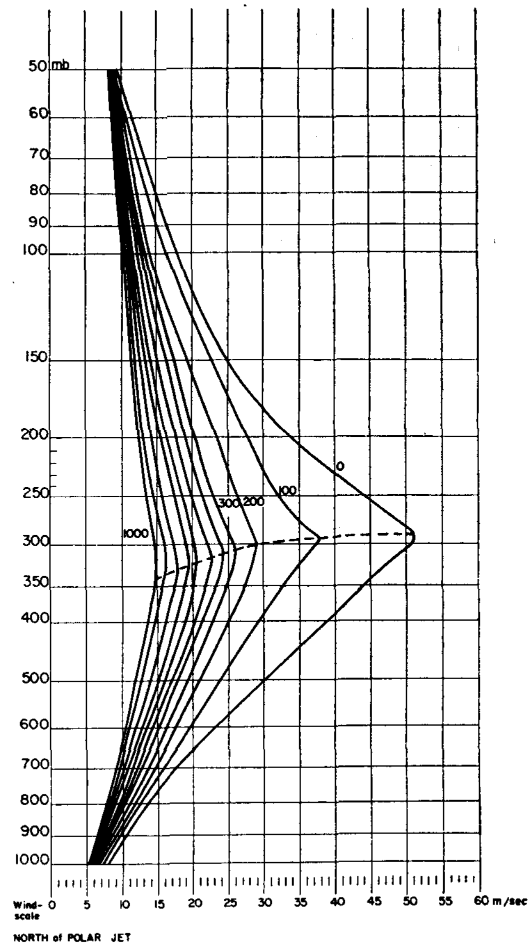

Fig. 10. The vertical wind profile north of the polar jet. The height of the maximum wind drops with the distance from the axis of the jet.

a) The strongest wind is about 52 m/sec and at a height of 295 mb.

b) The horizontal shear is strongest near the jet axis, the difference becomes less the further the distance increases. This is even noticed in the middle of the troposphere.

c) The height of the maximum wind increases towards south and decreases northward as indicated by the dashed line.

Conclusions and some discussion of the results

Before starting to draw any conclusions, it should be clearly mentioned that what follows is based on the average values obtained from a statistical treatment of seven days of wind data. As we know, the wind speed is far from being uniform along the axis of the two jets. This is seen when plotting the wind speed of different stations having the same distance from the jet at the same level. However, much more regular variations are observed along an axis perpendicular to the jet. This is the reason why in this study the axis of the jets has been chosen as reference coordinates. As the fluctuations of the jet axis means the advection of different air masses it seems in order to obtain any coherent picture of the large scale motions one simply has to follow the jets.

From this study of the assemblage of upper air data and as far as this statistical result can support the conclusions, one can say that the average maximum speed of *polar* jet is about

50 m/sec for *winter* and this maximum is located at an average height of 300 mb. For the *subtropical* jet the value obtained is around 60 m/sec at a height of 220—230 mb.

DEFANT has computed (1959 to be published) the average wind maximum at the axis of the *subtropical* jet for a period of 6 days in the summer (July 15—20, 1957). He has obtained a value of about 70 m/sec. As one can see, this value is about 10 m/sec more than the winter average obtained in this study.

This may be somewhat surprising and the major question is whether it may be due to the fact that no data from Asia and particularly Japan have been included in this study (as pointed out by Professor E. PALMÉN). In view of the quite large difference obtained it seems, however doubtful, whether this is sufficient to explain this reversed seasonal variation.

If for a moment we accept the idea that the maximum average subtropical jet is stronger in summer than in winter and then search for the mechanism which is responsible for this difference, we see that:

1) In the mean meridional cross sections constructed by DEFANT (DEFANT 1959 to be published) for the same period as the wind was computed, he finds that the subtropical front separating the subtropical air from middle air is much stronger in summer than winter. Now the question is whether the solenoidal field which goes together with this front can be an indication of the strong wind field. As the value of the average wind obtained from this study is not very different from what is needed for the geostrophic balance, it seems that the increase of the wind maximum of the jet could be related to the existing horizontal temperature field of the subtropical front. However, it is difficult to identify and isolate the cause and effect.

2) The second essential difference between Defant's summer profile and this winter profile is that in summer the level of zero wind is at a height of 80 mb and from there upward the winds are from east whereas in the winter the speed at 50 mb still is not less than 10 m/sec. Evidently this decrease of the height of the easterlies in summer relative to winter is due to the fact that in summer the subtropical jet is on the average in much higher latitude than in winter. Consequently, the reversed horizontal temperature field in the stratosphere is more intense. It seems possible that this decrease of the thickness of the layer of westerlies in summer relative to winter might be the cause of redistribution of existing momentum and as a result the sharpness of the peak.

Part II

Introduction

It was ROSSBY's suggestion (1939), that the large scale atmospheric motions could be considered as two dimensional and non-divergent. Following this hypothesis CHARNEY (1949) formulated the problem more precisely. In his study Charney uses the simplified vorticity equation

$$\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla \eta = f \frac{\partial \omega}{\partial p} \qquad (1)$$

with the boundary conditions

$$\omega = 0 \text{ at } P = 0 \text{ and } P = P_0$$

Here $\eta = f + \zeta$ is the absolute vorticity and ∇ indicates $\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}$ where $\mathbf{i}$, $\mathbf{j}$ are unit vectors.

Furthermore he assumes that in studying large scale atmospheric motions, the wind increases in magnitude with height while the pattern of the streamlines remains almost the same from level to level. In other words the wind at any level is given by the relation

$$\mathbf{v} = A(p) \cdot \bar{\mathbf{v}} \qquad (2)$$

where $\bar{\mathbf{v}}$ is defined as

$$\bar{\mathbf{v}} = \frac{l}{p_0} \int_0^{p_0} \mathbf{v} \cdot dp \qquad (3)$$

we also have $\zeta = A(P)\bar{\zeta}$. Introducing this into the vorticity equation (1) and integrating over the whole atmosphere, we can eliminate ω and obtain the prognostic equation

$$\frac{\partial \bar{\zeta}}{\partial t} + \overline{A^2}\, \bar{\mathbf{v}} \cdot \nabla \bar{\zeta} + \bar{\mathbf{v}} \cdot \nabla f = 0$$

Using (2) and (3) one gets

$$\frac{\partial \zeta}{\partial t} + \frac{\overline{A^2}}{A} \mathbf{v} \cdot \nabla \zeta + \mathbf{v} \cdot \nabla f = 0$$

This equation indicates that there must be a certain intermediate level (or levels) where the ratio $\frac{\overline{A^2}}{A} = 1$ and all vorticity changes are the result of advection. If such a level exists, the motion at this level can be said to be non-divergent. This intermediate level is very often mentioned to be in the vicinity of 500 mb. It is the purpose of this work to study the accuracy of this assumption and to find out the horizontal variation of the so-called equivalent barotropic level.

The equivalent barotropic level

From each vertical wind profile the average wind is computed. In evaluating the integral

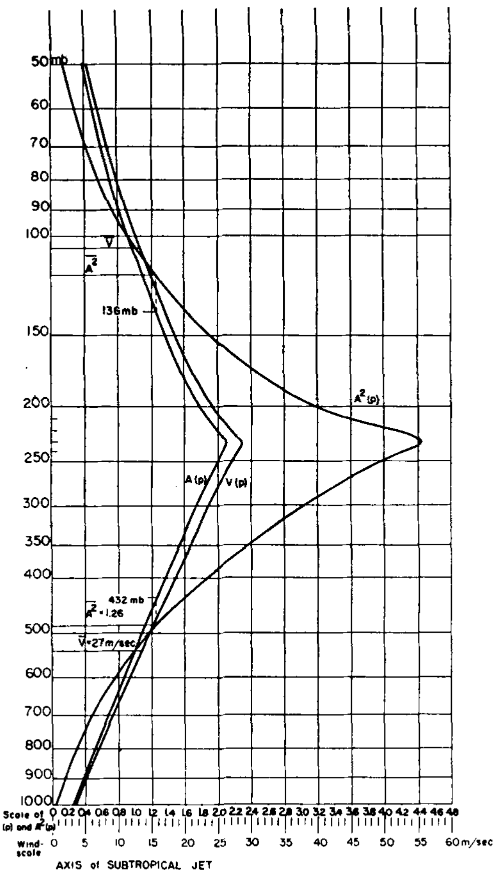

Fig. 11. The vertical wind profile at the axis of the subtropical jet together with the profiles of $A(p)$ and $A^2(p)$. The mean wind is indicated by $\overline{v} = 27$ m/sec. The equivalent barotropic levels are at 432 and 136 mb respectively. $\overline{A^2} = 1.26$. The wind speed is given in m/sec along the horizontal axis. The scale of $A(p)$ and $A^2(p)$ is also indicated along the horizontal axis.

(3) use has been made of the so-called trapezoidal rule. Due to the vertical wind structure, there are naturally two levels at which the wind and the average wind have the same value. By dividing the wind at any level by the mean wind the profile of $A(p)$ and by multiplying the A value by itself $A^2(p)$ is obtained. Finally using a similar integral as (3) the value of $\overline{A^2}$ is computed.

Fig. *11* shows the vertical wind profile at the axis of subtropical jet together with the graphs of the function $A(p)$ and $A^2(p)$. For this profile the mean wind is 27 m/sec and the corresponding heights are at 540 and 106 mb. The value of $\overline{A^2} = 1.25$ and the levels at which $\overline{A^2} = A$, that means the equivalent barotropic levels, are at 432 and 136 mb respectively.

The computation has been repeated for all wind profiles to the south and north of the subtropical jet and distances of 100 km. As one would expect, away from the axis of the jet, $\overline{v}$, $A(p)$, $A^2(p)$, $\overline{A^2}$ and the height of the equivalent barotropic level all vary. The details of this variation is tabulated in the following table.

Fig. *12* shows the wind profile at the axis of the polar jet together with the functions $A(p)$ and $A^2(p)$. As the graph of $A(p)$ and the wind profile coincide in all the levels, both are represented by the same curve. The mean wind for this profile is 25 m/sec. The levels at which the wind is equal to the mean wind is at 570 and 144 mb. In this case $\overline{A^2} = 1.26$ and the corresponding equivalent barotropic levels are at 485 and 188 mb respectively. In the case of polar jet as well, the height of the equivalent barotropic level changes as the distance from the axis of the jet increases. The details of these variations are also presented in table 1.

The variation of the height of equivalent barotropic level

Table 1 contains four different parts:

a) South of the subtropical jet
b) North of the subtropical jet
c) South of the polar jet
d) North of the polar jet

The details of variations of $\overline{v}$, $\overline{A^2}$ and the heights of E.B.L. can be studied from table 1.

Discussion of the results

To obtain a general picture of how the lower equivalent barotropic level varies in the space Fig. *13* is presented. The important features of this figure are:

1) In the average the equivalent barotropic level has a higher altitude for the wind structure which represents subtropical jet than the polar jet.

2) At the axis of both jets, the equivalent barotropic level has its maximum height.

3) Away from the axis of the subtropical jet on either side, the height of the E.B.L. decreases rapidly at first and slowly later on.

4) Towards the north of the polar jet, the height of the E.B.L. decreases rapidly first and less rapid later on.

5) To the south of the polar jet the level drops rather sharply at first, and slower later on until 600 km horizontal distance is reached. From there on the height of E.B.L. increases again. It seems that this increase should continue so much that the E.B.L. to the south of the polar jet joins the part to the north of the subtropical jet in such a way that there is no discontinuity in between.

General Conclusions

From this study one can easily see to what extent the assumption that 500 mb is the equivalent barotropic level is correct. The

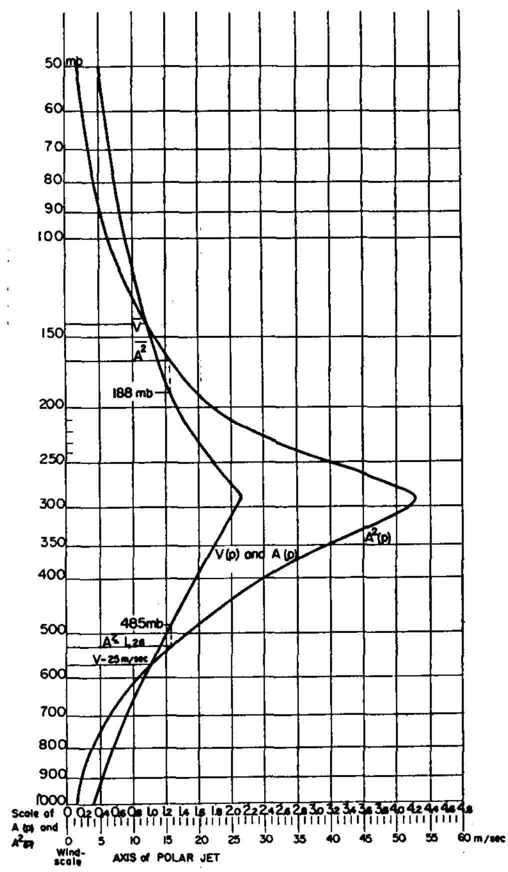

Fig. 12. The vertical wind profile at the axis of the polar jet together with the function $A^2(p)$. Notice that as the profile of $v(p)$ and $A(p)$ coincide in all levels, they are both given by the same curve. Here $\overline{v} = 25$ m/sec. $\overline{A^2} = 1.26$ and the equivalent barotropic levels are at 485 and 188 mb respectively.

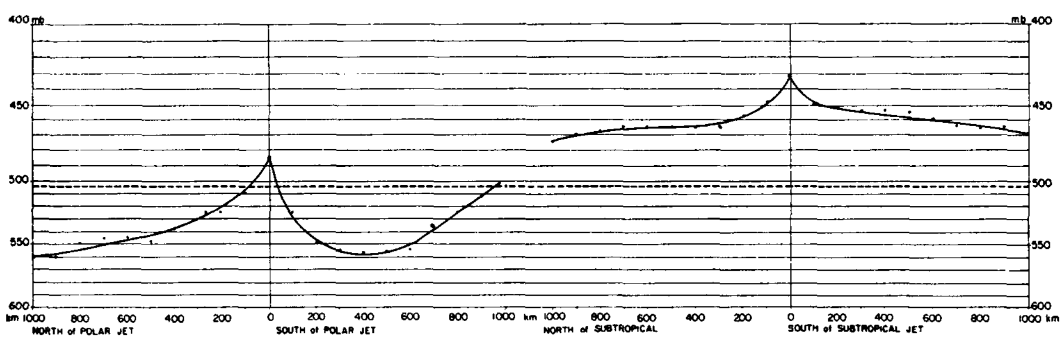

Fig. 13. The variation of the height of the equivalent barotropic level. Axis of the two jets are indicated by the two vertical lines. The dashed line indicates the height of the mean equivalent barotropic level.

Table 1

SOUTH OF SUBTROPICAL JET

distance from jet km	$\overline{A^2}$	lower E.B.L.	upper E.B.L.	$\bar{v}$ m/sec
0	1.26	432 mb	136 mb	27.0
100	1.22	450 "	113 "	23.9
200	1.20	452 "	111 "	20.4
300	1.19	454 "	111 "	18.2
400	1.17	454 "	111 "	16.5
500	1.17	455 "	111 "	15.1
600	1.43	460 "	108 "	14.0
700	1.13	464 "	106 "	13.0
800	1.11	465 "	101 "	12.3
900	1.11	465 "	100 "	11.6
1000	1.10	470 "	89 "	11.0

NORTH OF SUBTROPICAL JET

distance from jet km	$\overline{A^2}$	lower E.B.L.	upper E.B.L.	$\bar{v}$ m/sec
0	1.26	432 mb	136 mb	27.0
100	1.24	449 "	131 "	23.6
200	1.21	459 "	127 "	20.5
300	1.19	465 "	127 "	18.5
400	1.18	465 "	123 "	16.8
500	1.17	465 "	121 "	15.4
600	1.14	465 "	120 "	14.5
700	1.13	465 "	120 "	13.6
800	1.13	468 "	120 "	13.0
900	1.12	470 "	120 "	12.8
1000	1.11	474 "	120 "	12.7

SOUTH OF POLAR JET

distance from jet km	$\overline{A^2}$	lower E.B.L.	upper E.B.L.	$\bar{v}$ m/sec
0	1.26	485 mb	188 mb	25.0
100	1.12	525 "	191 "	22.6
200	1.11	550 "	185 "	20.1
300	1.14	555 "	183 "	17.4
400	1.13	556 "	180 "	15.7
500	1.12	556 "	168 "	14.3
600	1.11	555 "	165 "	13.4
700	1.10	535 "	157 "	12.6
800	1.11	525 "	158 "	12.2
900	1.12	512 "	160 "	11.8
1000	1.13	505 "	161 "	11.7

NORTH OF POLAR JET

distance from jet km	$\overline{A^2}$	lower E.B.L.	upper E.B.L.	$\bar{v}$ m/sec
0	1.26	485 mb	188 mb	25.0
100	1.16	510 "	168 "	20.6
200	1.14	515 "	167 "	17.7
300	1.13	515 "	165 "	16.3
400	1.12	528 "	164 "	15.2
500	1.10	538 "	165 "	14.4
600	1.10	545 "	160 "	13.4
700	1.09	545 "	147 "	12.6
800	1.08	550 "	145 "	11.7
900	1.07	560 "	135 "	11.0
1000	1.06	560 "	118 "	10.4

E.B.L. computed from this seven days mean has a maximum height of 432 mb and a minimum height of 560. This amounts to a difference of 128 mb or approximately 1,800 meters. It should be remembered that 130 mb is based on the average winds of seven days. This might be considerably less than what one may encounter in the study of a single day. From this discussion one can also say that the gradient of vorticity in the prognostic

Table 2

Pressure mb	1000	950	900	850	800	750	700	650	600	550
$\frac{\overline{A^2}}{\bar{A}}$	3.03	2.50	2.14	1.89	1.70	1.52	1.41	1.30	1.19	1.08

Pressure mb	500	450	400	350	300	250	200	150	100	50
$\frac{\overline{A^2}}{\bar{A}}$	1.00	0.91	0.84	0.77	0.72	0.73	0.77	0.93	1.21	2.38

equation should be multiplied by a different wind than what the equation suggests in different regions of the map. At least part of the inaccuracy of the results of barotropic forecasts can be due to incorrectness of the assumption that 500 mb is the E.B.L.

As one should expect, the greatest error in forecasts can be made over the regions of the map where the subtropical jet approaches the polar jet, as in these regions the height of the E.B.L. changes very much within a rather short horizontal distance.

To get an idea where the *mean* equivalent barotropic level is, a mean profile of $A(p)$ and $A^2(p)$ from all the individual values has been constructed. Fig. *14* shows the functions of $A(p)$ and $A^2(p)$. From the figure one can see that the level at which $A = \overline{A^2}$ is equal to 505 mb and 135 mb respectively.

One can conclude by saying that as long as we use such a simple model and so far as we cannot include the horizontal variation of the function A into the equation, the 500 mb is the closest approximation to the non-divergent level.

Finally, the ratio $\dfrac{\overline{A^2}}{A}$ computed from the profile of mean $A(p)$ and $A^2(p)$ is tabulated and given in Table 2. It is possible to apply the equivalent barotropic model at any level provided the advection of the gradient of the vorticity is multiplied by the factor $\mathbf{v} \cdot \dfrac{\overline{A^2}}{A}$ corresponding to that level.

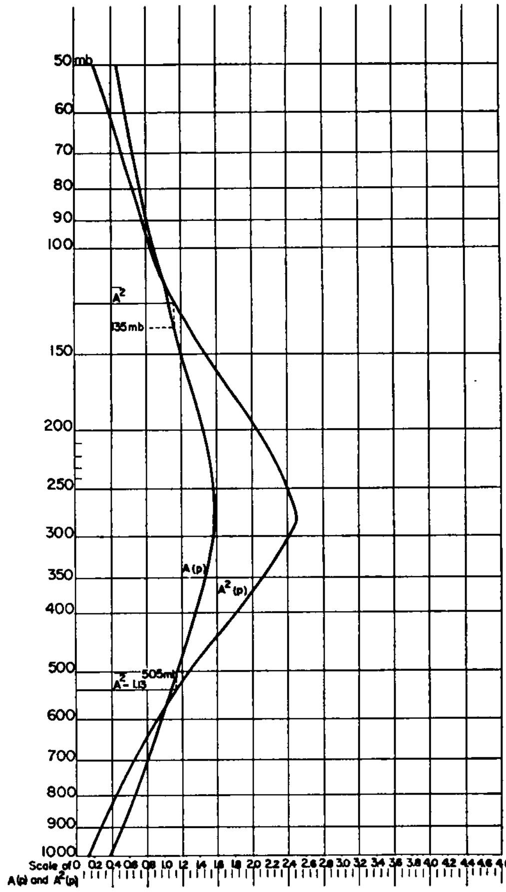

Fig. 14. The profiles of mean $A(p)$ and $A^2(p)$. The level at which $\overline{A^2} = A$ is 505 mb and 135 mb respectively. The mean $\overline{A^2}$ is equal to 1.13.

Acknowledgement

I would like to express my thanks to Dr. B. Bolin for suggesting the study of this problem and to Professor Friedrich Defant for the stimulating discussions throughout the work.

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