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The Evaporation-Precipitation Cycle of the Trades

By E. B. KRAUS, Snowy Mountains Hydro-electric Authority, Cooma North, N.S.W., Australia.

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Abstract

The area distribution of surface wind strengths between 30° S and 30° N is expressed as a function of the median velocity, on the basis of climatological data. This is used to obtain estimates of evaporation and energy dissipation over the same area.

The main energy supply of the trade circulation is due to condensation in the equatorial trough. It depends partly on evaporation into the trade winds at some earlier time. A study of the energy balance shows that this may lead to oscillations with a period of about 3 weeks; corresponding to the mode of the time which elapses between the evaporation and the precipitation of the same vapour molecules.

Disturbances or fluctuations in the direct meridional circulation of the sub-tropics may be initiated by changes in the infra-red cooling rate and by changes in the height of the tropopause. It is suggested tentatively that this could be caused by changes in the ozone distribution above the tropical tropopause.

1. Introduction

The trade winds are associated with a direct meridional circulation which produces kinetic energy. Heat is supplied to this circulation mainly by the condensation of water vapour in the equatorial trough. From figures tabulated by JACOBS (1951) it can be inferred that between 0° and 30° latitude in the North Atlantic and the North Pacific the heat supplied on the average to the atmosphere by condensation is twenty times as large as the sensible heat supplied directly by conduction from the sea surface. About 60% of this heat is released by precipitation in the belt between 0° and 10°N, though in fact the main release occurs in an even narrower belt which shifts with the season.

Rainfall in the tropics accounts for more than nine-tenths of the water which is evaporated from the tropical land and sea surfaces. Less than one-tenth is exported to extra-tropical latitudes. The evaporation from the oceans is roughly proportional to the wind velocity. If the mean surface wind velocity increases for some reason—a higher rate of evaporation results—this causes an increased moisture transport by the trades towards the equatorial trough—the resultant higher rate at which latent heat is liberated there, increases the energy of the trade wind circulation—this increases evaporation still further—and so on. The process would tend to amplify initially small disturbances. The amplification would proceed in time steps which must be of the same order as the time lag between evaporation and precipitation of the same molecules. Below it will be shown that this may lead to oscillations with a period of about 21—28 days. It will also be shown that stability cannot be maintained under these circumstances by frictional dissipation or by changes in the energy flux across the lateral boundaries. The trade wind circulation can only remain stable between its comparatively narrow observed limits if any increase in the heat supply is largely balanced by a corresponding increase in the radiational cooling rate. Small changes in the mean cooling rate will cause a change in the mean thermodynamic efficiency of the circulation and this may have a considerable effect upon the climatic regime.

For simplicity sake we shall deal in the first instance with a hypothetical earth without land surfaces between 30° north and south. The effect of tropical land masses will be considered at least qualitatively in a later chapter.

Most of the symbols used below, as well as the dimension of quantities which they represent are listed in the following table:

List of Symbols

A	Mechanical equivalent of heat (g cm^2 sec^{-2} cal^{-1})
c_p	Specific heat at constant pressure (g^{-1} cal K^{-1})
c_v	Specific heat at constant volume (g^{-1} cal K^{-1})
C	generalised stress factor (g cm^{-3})
e	vapour pressure (g cm^{-1} sec^{-2})
E	evaporation (g cm^{-2} sec^{-1})
d	loss of kinetic energy by turbulence friction per unit horizontal area (g sec^{-3})
h	latent heat production per unit area (erg cm^{-2} sec^{-1})
K	generalised evaporation factor (cm^{-2} sec^2)
l_k	components of horizontal poleward directed unit vector
L	latent heat of vaporization (cal g^{-1})
p	air pressure (g cm^{-1} sec^{-2})
P	rainfall intensity (g cm^{-2} sec^{-1})
q	specific humidity
dQ^C/dt	heat change due to condensation (g^{-1} cal sec^{-1})
dQ^R/dt	heat change due to radiation (g^{-1} cal sec^{-1})
S	global surface between 30° S and 30° N
ds	element of sea surface
T	temperature (degree K)
u	surface wind speed
u^*	critical surface wind speed
$\hat{u}$	median surface wind speed
v	wind speed
$v_k\ v_\lambda\ v_\varphi\ v_z$	Wind components
x_k	co-ordinates in a Cartesian system
z	height above mean sea surface
β	Bowen's ratio of sensible to latent heat production
j	resistance coefficient
Γ_{ik}	components of Reynold's stress (g cm^{-1} sec^{-2})
Γ_0	surface stress (g cm^{-1} sec^{-2})
δ	lag time between evaporation and precipitation
ε	evaporation coefficient
λ	longitude
ϱ	specific density (g cm^{-3})
$d\sigma$	element of lateral boundary area
$d\tau$	element of volume
φ	latitude

2. The area distribution of surface wind strength over the tropical oceans

As a preliminary to the following study, it was necessary to evaluate the area distribution of surface velocities and to see whether the changes of distribution with time followed an assessable pattern. Fortunately, frequency distributions of surface wind strength, at sampling points given by a 10° grid of meridians and parallel circles over the southern hemisphere oceans, had already been compiled from the Climatic Atlas of the Oceans by P. SQUIRES (unpublished) for a different purpose. These were used to compute the fractional frequency of velocities smaller than u at the various sampling points. The velocity u is supposed to represent conditions at a level $z = 600$ cm over the sea surface.

The fractional frequency $f(u, m)$ at the m'th sampling point equals zero for $u = 0$, and approaches unity as u approaches its maximum. If there are M representatively spaced sampling points in a sufficiently large area S, the part $s(u)$ of this area which is covered on the average by velocities smaller than u will be given by:

$$\frac{s(u)}{S} = \frac{1}{M} \sum_{m=1}^{M} f(u, m)$$

In figure 1, the ratio on the left hand side was plotted separately for different ocean areas S against the ratio $u/\hat{u}$ where $\hat{u}$ denotes the median velocity which is just exceeded over half of the area. There are no points for small values of $u/\hat{u}$ because winds of less than Beaufort strength 4 had been sampled together.

The general shape of the plot in figure 1 is determined by the value of $s(u)/S$ being necessarily zero for $u = 0$, equal to 0.5 for $u = \hat{u}$, and equal to unity for large values of u. All the same, the alignment of all the points, representing quite different seasons and oceans, along a single well-defined line is rather remarkable. It suggests a physical relationship, between the generation of kinetic energy over the sea and its frictional dissipation or export to extra-tropical latitudes. This relation appears to remain unchanged over a wide range of conditions.

It may be significant that the only point in figure 1, which deviates to any marked extent from the general alignment, represents conditions in July for the Indian Ocean. It is due to a limited area with persistently high velocities. This is associated with the Indian Summer Monsoon, which means it derives its energy not only from the ocean, but probably also from the heating of the adjacent land masses.

Squires' frequency distribution of surface wind strength covered only the oceans of the southern hemisphere. However, isotachs of "resultant" winds have been published by

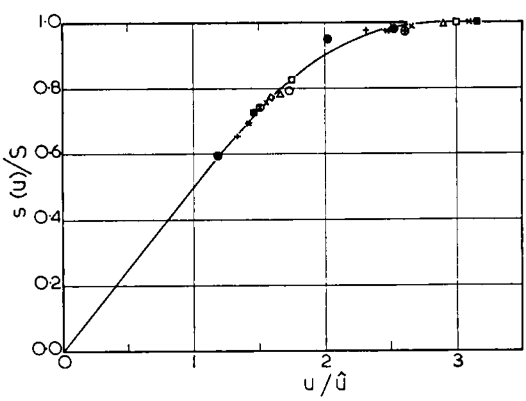

Fig. 1. The ordinate gives the fraction of ocean surface area between 0° and 30° S covered on the average by velocities smaller than u. The abscissa gives the ratio of u to the corresponding median velicity $\hat{u}$. Symbols and values of u in m sec^{-1} for different months and seasons are as follows:

	S. Atlantic	S. Pacific	S. Indian
January	□ 4.6	△ 4.8	○ 4.6
April	.. 4.6	× 5.1	⊗ 5.0
July	■ 5.4	✱ 5.6	● 6.8
October	◇ 5.1	+ 6.0	⊕ 5.1

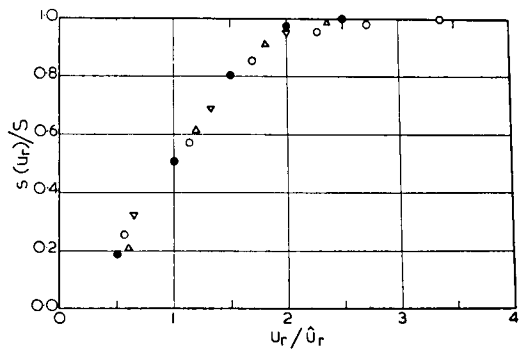

Fig. 2. The ordinate gives the fraction of ocean surface area between 0° and 30° covered by "resultant" winds smaller than u_r. The abscissa gives the ratio of u_r to the corresponding median resultant wind $\hat{u}_r$. Symbols and values of u_r in m sec^{-1} for different months and seasons are as follows:

	N. Hemisphere	S. Hemisphere
January	△ 3.4	▽ 3.1
July	○ 3.5	● 4.1

RIEHL (1954) for all of the tropical oceans. These isotachs were planimetered and then dealt with as above. The result is shown in figure 2 which is remarkably similar in shape to figure 1. This confirms the probable existence of an underlying physical relationship. Deviations from the general alignment are due again to July conditions in the Indian Ocean.

For the following derivation it is necessary to express the velocity distribution for the whole sea surface between 30°N and 30°S as an analytical function. Points, representing the entire surface, infact do fall in line with the other points, though this could not be shown in figure 1 because of crowding. A functional expression is obtained by dividing the alignment into three sections.

$$\text{For} \quad \frac{u}{\hat{u}} < 1 \quad \frac{s}{S} = 0.5 \cdot \frac{u}{\hat{u}}$$

$$\text{For } 1 < \frac{u}{\hat{u}} < 3 \; \frac{s}{S} = -0.2 + 0.85 \cdot \frac{u}{\hat{u}} - 0.15 \cdot \left(\frac{u}{\hat{u}}\right)^2$$

$$\text{For } 3 < \frac{u}{\hat{u}} \qquad \frac{s}{S} = 1 \tag{1.1}$$

These values are shown as a fully drawn line in figure 1. Equations revealing the physical basis of the observed area/velocity distribution would have almost certainly a different structure, but the necessary knowledge is not available at present, and the formalistic, descriptive expressions in (1.1) are sufficient for the present purpose.

3. The energy balance

The space to be considered in the following equation extends from 30°S to 30°N, and from the surface to the top of the atmosphere. The effect of motion on the molecular scale will be neglected. Following MILLER (1951) the energy balance equation may then be derived in the form:

$$A \iiint \varrho \frac{dQ^C}{dt} d\tau + A \iiint \varrho \frac{dQ^R}{dt} d\tau =$$
$$= \iiint \varrho \frac{d}{dt}(Ac_v T + gz + \frac{\bar{v}^2}{2} + \frac{v'^2}{2}) d\tau +$$
$$+ \iiint \frac{\partial (v_k p)}{\partial x_k} d\tau + \iiint \frac{\partial (v_k \Gamma_{ik})}{\partial x_k} d\tau \qquad (3.1)$$

Conventional tensor notation, which implies that all terms in which a suffix occurs twice represent a sum of three Cartesian components, has been used in the last two integrals. Equation (3.1) indicates that the supply of radiant heat and heat of condensation equals the changes in the internal energy, the potential energy and the kinetic energy of the mean ($\bar{v}^2/2$) and turbulent ($v'^2/2$) motion, plus the work carried out against pressure and frictional forces.

The integral of the kinetic energy changes is at least one order of magnitude smaller than any of the other terms in (3.1) and can be neglected. The remaining terms on the right-hand side can be transformed with aid of the equation of continuity and Gauss' theorem as follows:

$$\iiint \varrho \frac{d}{dt}(Ac_v T + gz) d\tau =$$
$$= \frac{\partial}{\partial t}\iiint \varrho (Ac_v T + gz) d\tau$$
$$+ \iint \varrho (Ac_v T + gz) v_k l_k d\sigma -$$
$$- \iint \varrho Ac_v T v_z ds$$

$$\iiint \frac{\partial (v_k p)}{\partial x_k} d\tau =$$
$$\iint p v_k l_k d\sigma - \iint p v_z ds$$
$$\iiint \frac{\partial (v_k \Gamma_{ik})}{\partial x_k} d\tau =$$
$$= \iint (v_\lambda \Gamma_{k\lambda} + v_z \Gamma_{kz}) l_k d\sigma -$$
$$- \iint (u_\lambda \Gamma_{\lambda z_0} + u_\varphi \Gamma_{\varphi z_0}) ds \qquad (3.2)$$

The velocity v_z is directed upwards into the volume, the horizontal surface integrals have, therefore, a negative sign. The upward flux of potential energy from the surface can be considered zero.

Some of the terms on the right-hand side of (3.2) can be combined by introducing the concept of sensible heat or enthalpy through the relations:

$$A \varrho c_v T + p = A \varrho c_v T + A\varrho (c_p - c_v) T = A \varrho c_p T$$
$$\int_0^\infty \varrho Ac_v Tdz + \int_0^\infty \varrho gz\, dz = \int_0^\infty \varrho Ac_v Tdz -$$
$$- \int_0^\infty z dp = \int_0^\infty (\varrho Ac_v T + p) dz =$$
$$= \int_0^\infty A \varrho c_p Tdz \qquad (3.3)$$

As regards the last two terms in (3.2) it may be observed that:

$$\Gamma_{kz} l_k = |\Gamma_{\varphi z}|;\ \Gamma_{k\lambda} l_k = |\Gamma_{\varphi\lambda}| \ll |\Gamma_{\varphi z}|;\ v_z \ll u_\varphi$$

Furthermore, the horizontal surface integral covers a far greater area than the integral over the lateral boundary. It follows that the effect of the stresses across the lateral boundary is very much smaller than that of the stresses at the sea surface.

$$\iint (v_\lambda \Gamma_{k\lambda} + v_z \Gamma_{kz}) l_k d\sigma - \iint (u_\lambda \Gamma_{\lambda z_0} +$$
$$+ u_\varphi \Gamma_{\varphi z_0}) ds \approx - \iint (u_\lambda \Gamma_{\lambda z_0} + u_\varphi \Gamma_{\varphi z_0}) ds =$$
$$= \iint u \Gamma_0 ds \qquad (3.4)$$

The sign of the last term is opposite to that of the preceding term because the direction of the stress at the surface is opposite to that of u.

The energy balance equation (3.1) becomes then after re-arrangement:

$$A\iiint \varrho \frac{dQ^C}{dt}\,d\tau + A\iiint \varrho \frac{dQ^R}{dt}\,d\tau + $$
$$+ A\iint \varrho\, c_p\, T v_z\, ds =$$
$$= \iint (Ac_p T + gz)\,\varrho\, v_k\, l_k\, d\sigma + \iint u\,\Gamma_0 ds +$$
$$+ A\frac{\partial}{\partial t}\iiint \varrho\, c_p\, T d\tau \qquad (3.5)$$

The first integral in this equation represents the rate at which heat of condensation is supplied to the tropical atmosphere. Its magnitude is about $22 \cdot 10^{22}$ erg sec^{-1}.

The atmosphere is cooled by radiation and the second integral is, therefore, always negative. From balance consideration, its magnitude is found to be about $-16 \cdot 10^{22}$ erg sec^{-1}.

The third integral represent the rate at which sensible heat is conducted to the air from the sea surface. Its magnitude is about 10^{22}.

On the right-hand side, the first integral representing the flux of sensible heat and potential energy across both meridional boundaries amounts to about $7 \cdot 10^{22}$ erg sec^{-2}.

The decrease of kinetic energy through turbulent friction at the surface as given by the last integral will be estimated below to be about $0.2 \cdot 10^{22}$ erg sec^{-1}.

The last term in equation (3.5) is always very small compared with any of the other terms. It will be zero if the circulation is steady.

4. The heat supplied by condensation and turbulent conduction from the sea.

The net condensation heat gain per unit time must be proportional to the precipitation intensity P. The heat gain by conduction on the other hand can be expressed by definition as the product of Bowen's dimensionless ratio β with the evaporation E.
In mathematical terms:

$$\iiint \varrho \frac{dQ^C}{dt}\,d\tau + \iint \varrho c_p\, T v_z\, ds =$$
$$= \iint L\,(P + \beta E)\, ds \approx L\iint P ds +$$
$$+ L\bar{\beta}\iint E\, ds \qquad (4.1)$$

In the last term, temperature variations of the latent heat of vaporization have been disregarded. It is also assumed that a mean value $\bar{\beta} = 0.05$ can be assigned to Bowen's ratio.

From continuity considerations for the specific humidity, q, it follows that

$$\iiint \frac{\partial(\varrho q)}{\partial t}\,d\tau + \iiint \frac{\partial\,(\varrho q v_k)}{\partial x_k}\,d\tau +$$
$$+ \iint P ds = 0 \qquad (4.2)$$

Using Gauss' theorem:

$$\iiint \frac{\partial\,(\varrho q v_k)}{\partial x_k}\,d\tau = -\iint \varrho q v_z\, ds +$$
$$+ \iint \varrho q v_k\, l_k\, d\sigma = -\iint E ds + \iint \varrho q v_k\, l_k\, d\sigma$$
$$(4.3)$$

To evaluate the local changes in moisture content given by the first term of 4.2, consider that a finite time δ elapses between evaporation and precipitation of the same water molecules. A change in the evaporation regime can therefore not affect the precipitation immediately. The resulting short-term change in the atmospheric moisture content may be expressed by the difference between the evaporation at time t and the evaporation at an earlier time $t - \delta$.

$$\iiint \frac{\partial\,(\varrho q)}{\partial t}\,d\tau \approx \iint E_t\, ds - \iint E_{t-\delta}\, ds$$
$$(4.4)$$

Combination of equations (4.1) to (4.4) gives:

$$\iiint \varrho \frac{dQ^C}{dt}\,d\tau + \iint \varrho\, c_p\, T v_z\, ds =$$
$$= L\Big(\bar{\beta}\iint E_t\, ds + \iint E_{t-\delta}\, ds -$$
$$- \iint \varrho\, q v_k\, l_k\, d\sigma\Big) \qquad (4.5)$$

which indicates that the instantaneous heat supply by condensation and turbulent conduction depends on the instantaneous evaporation and on the evaporation at some earlier time $t - \delta$, minus the moisture export to extratropical latitudes.

5. Estimate of evaporation and surface friction

The evaporation from the oceans has been computed by MONTGOMERY (1940) and SVERDRUP (1951) by a formula which may be written in the form:

$$\iint E ds = \iint \varrho\, k_0 j \varepsilon\, (q_w - q_0)\, u ds =$$
$$= \iint 0{,}623\, \varrho\, k_0 j \varepsilon \frac{e_w - e_0}{p_0}\, u ds =$$
$$= \iint K (e_w - e_0)\, u ds \tag{5.1}$$

Where $k_0 = 0.4$ is the Karman constant; j the resistance coefficient is equal to the square root of the surface drag; ε is the evaporation coefficient; q_w, q_0, e_w and e_0 denote the specific humidity and the vapour pressure at the sea surface and in the air at a height assumed to be 600 cm.

The coefficients j and ε in equation (5.1) and hence the generalised "evaporation factor" K depend upon wind velocity. MUNK (1947) considers the sea surface to be hydrodynamically smooth at wind speed less than $u^* = 650$ cm sec^{-1} and hydrodynamically rough at higher wind speeds. At the critical wind speed u^*, a discontinuous rise is assumed to occur in the value of the surface drag j^2 from about 0.0009 at low wind speeds to 0.0035 at $u > u^*$. The value of ε jumps at the same time from 0.085 to 0.148. These were the values used by JACOBS (1951) to study the large scale aspects of energy transformation over the oceans.

Evidence for a critical wind velocity for air sea boundary processes has also been published recently by MANDELBAUM (1956). However, the existence of a discontinuity in these processes is by no means generally accepted and the magnitude of the variation is also controversial. Deacon (in personal communication) considers that j^2 increases from about 0.0009 at low wind speed to around 0.0025 at winds of 10 m/sec. or more, with probably a tendency for a relatively rapid but continuous increase at a speed of about 7 m/sec. This is supported by the material presented by DEACON, SHEPPARD and WEBB (1956).

Table I. Adopted Values of Surface Variables.

	ϱ_0 (g cm^{-3})	p_0 (g cm^{-1} sec^{-2})
$u \leqslant 650$ cm sec^{-1}	1.2×10^{-3}	1.015×10^{6}
j	ε	K (cm^{-2} sec^{2})
0.0009	0.085	0.75×10^{-9}
	ϱ_0 (g cm^{-3})	p_0 (g cm^{-1} sec^{-2})
$u > 650$ cm sec^{-1}	1.2×10^{-3}	1.015×10^{6}
j^2	ε	K (cm^{-2} sec^{2})
0.0022	0.148	2.06×10^{-9}

As regards ε, BROCKS (1955) has analysed a large number of humidity profile data without finding a sudden rise, though this cannot be considered conclusive because of observational errors and the grouping of data, which would have tended to blur a discontinuity. The numerical values of ε given by Brocks for $u = 3$ m sec^{-1} and $u = 10$ m sec^{-1} agree well with those used by Jacobs.

Whether the surface drag changes discontinuously at some critical velocity, or whether there is a rapid, but continuous transition, has little effect on the following argument. In fact it will be shown below that the end results would be practically the same if a single mean value for K had been adopted. This was actually done by Jacobs. However as an electronic computer was available, there was no difficulty in dealing with different values of j and ε. The values chosen are shown in Table I.

As a first and rather rough approximation it will be assumed that there is a mean value of $\Delta e = e_w - e_0$ such that

$$\iint K (e_w - e_0)\, u ds \approx \Delta e \iint K u ds \tag{5.2}$$

where K assumes one of the two values given in Table I depending on the wind velocity. The procedure finds some justification in the fact that the difference in vapour pressure between the surface and the air over the open tropical seas will rarely vary by more than thirty per cent around its areal mean. By contrast the wind velocity can easily vary by more than one hundred per cent from place to place. The effect of variations in u upon the evaporation is further enhanced by their influence upon K.

Equation (5.2) can be integrated with the aid of (1.1). If K_1 and K_2 denote the values of K under smooth and rough conditions:

$$L\int\int E ds = \Delta e\, L \int\int K u ds =$$

$$= \Delta e\, L S \hat{u} \left\{ 1.125\, K_2 - 0.075\, K_1 + \right.$$

$$\left. + (K_2 - K_1)\left[0.1 \left(\frac{u^*}{\hat{u}}\right)^3 - 0.425 \left(\frac{u^*}{\hat{u}}\right)^2 \right] \right\} =$$

$$= S \cdot h(\hat{u}) \qquad (5.3)$$

The expression $h(\hat{u})$ denotes the supply of latent heat to the atmosphere per unit surface area of the tropics, as a function of the single variable $\hat{u}$.

The mean value of $\hat{u}$ can be seen from the legend of figure 1 to be equal to 5.1 m sec^{-1}. It will be assumed that Δe = 4.75 mbs. which is between Δe = 4 quoted by Jacobs for a selected strip (20°N — 25°N; 25°W — 65°W) in the North Atlantic; and Δe = 5.3 quoted for a similar strip in the North Pacific (20°N—25°N; 130°W—140°E). With this value for Δe and with $\hat{u}$ = 5.1 m sec^{-1}, the mean production of latent heat is computed from (5.3) to be equal to $9.65 \cdot 10^4$ erg cm^{-2} sec^{-1}.

From Jacobs' tables it can be inferred that heat in latent form is supplied to the atmosphere at a rate which—on the average over all seaons and oceans between 30° and the equator —amounts to 200 cal cm^{-2} day^{-1} or $9.7 \cdot 10^4$ erg cm^{-2} sec^{-1}. Jacobs based his evaluation directly on climatological wind data. The coincidende of his estimate with the present one suggests that the actual velocity distribution is, in fact, reasonably well represented by the relations (1.1).

The function $h(\hat{u})$ is plotted in figure 3. With a single mean value for K, the term in the curly brackets becomes a constant and $h(\hat{u})$ is then approximated by a straight line. This is shown in figure 3 for $K_1 = K_2 = 1.52 \times 10^{-9}$. The closeness of the two graphs proves that the actual relation of K or j to u has little effect upon the total evaporation.

The effect of friction at the surface can be evaluated in a similar way. The surface stress is given by:

$$\Gamma_0 = \varrho_0 j^2 u^2 \qquad (5.4)$$

Hence:

$$\int\int \Gamma_0 u ds = \int\int \varrho_0 j^2 u^3 ds = \int\int C u^3 ds$$

From the values of j and ϱ_0 in Table I, it can be readily found that for sub-critical velocities $u<u^*$ the generalized stress factor becomes $C_1 = 1.08 \times 10^{-6}$ g cm^{-3}. For higher velocities it is equal to $C_2 = 2.64 \times 10^{-6}$ g cm^{-3}.

Combination with (1.1) gives:

$$\int\int \Gamma_0 u ds = \int\int C u^3 ds = \hat{u}^3 S \left\{ 2.6325\, C_2 - \right.$$

$$- 0.0275\, C_1 + (C_2 - C_1)\left[0.06 \left(\frac{u^*}{\hat{u}}\right)^5 - \right.$$

$$\left.\left. - 0.2125 \left(\frac{u^*}{\hat{u}}\right)^4 \right] \right\} = \qquad (5.5)$$

$$= S \cdot d(\hat{u})$$

The function $d(\hat{u})$ is plotted in figure 4. With a single mean value for C, the term in curly brackets becomes again a constant. The resulting cubic approximation to $d(\hat{u})$ is shown as a dashed line in figure 4 for $C_1 = C_2 = 2.3 \times 10^{-6}$. The figure shows that the variations of C with u—though larger than those of K—could have again been safely neglected

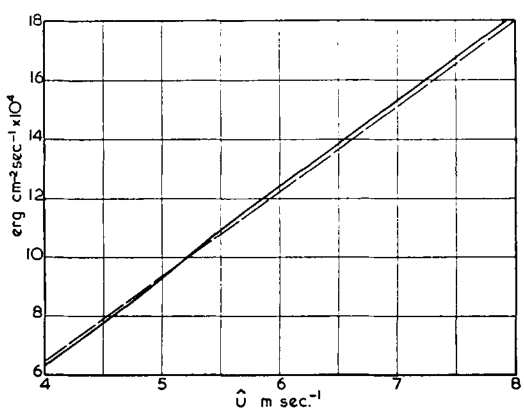

Fig. 3. Mean supply of latent heat energy per unit time and unit surface area. The full line gives values obtained from equation (5.3). The broken line gives values obtained from (5.3) by setting $K_1 = K_2 = 1.52 \times 10^{-9}$.

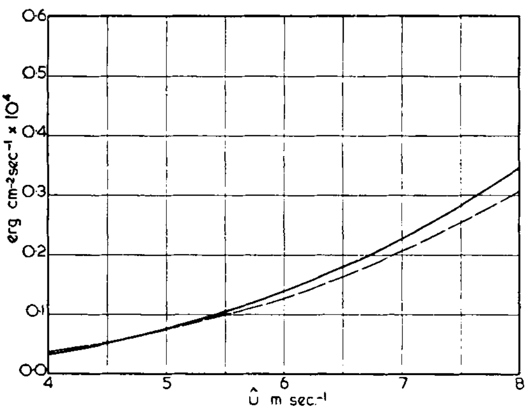

Fig. 4. Mean work of viscous forces per unit time in a column of unit area. The full line gives values obtained from equation (6.10). The broken line gives values obtained from (6.10) by setting $C_1 = C_2 = 2.3 \times 10^{-6}$.

for values of $\hat{u}$ between 4 m sec^{-1} and 6 m sec^{-1}, which is the range likely to be encountered in reality.

6. Oscillation of the trade wind circulation

Introduction of (4.5) into (3.1) gives with (5.3) and (5.5) after re-arrangement and division by S:

$$Ah\,(\hat{u})_{t-\delta} = d\,(\hat{u})_t - \bar{\beta} Ah\,(\hat{u})_t - \frac{A}{S}\int\!\!\int\!\!\int \varrho \frac{dQ^R}{dt}\,d\tau + \frac{I}{S}\int\!\!\int\!\!\int (Ac_p T + gz + {} + ALq)\,\varrho\, v_k\, l_k\, d\sigma + \frac{A}{S}\frac{\partial}{\partial t}\int\!\!\int\!\!\int \varrho c_p\, T\, d\tau \tag{6.1}$$

Equation (6.1) is a first order difference equation. The indices t and $t-\delta$ refer to events occurring at different times. From the theory of these equations it follows that the general solutions contains a periodic function $\hat{u}(t)$ with period δ.

The lag time δ which elapses between evaporation and precipitation, may vary in nature from a duration of minutes to one of months. In practice, however, most of the precipitation will fall in the equatorial trough near latitude φ_0. The most frequent value of δ should therefore be of the same order as the mean lag time which elapsed between equatorial precipitation and the preceding evaporation; that means it will be of the order:

$$\bar{\delta} \approx \frac{r}{\varphi_{30}-\varphi_0}\int_{\varphi_0}^{\varphi_{30}} \frac{\varphi}{|\bar{u}_\varphi|}\,d\varphi \approx \frac{r\,(\varphi_{30}-\varphi_0)}{2\,|\bar{u}_\varphi|} \approx \frac{900}{|\bar{u}_\varphi|}$$

(naut. miles/knots)

From data given by RIEHL (1954) it may be assumed that the mean meridional surface velocity towards the equator $|\bar{u}_\varphi|$ will be of the order of 1.4 to 1.8 knots. From that it follows that δ should be about 21 to 27 days. As more moisture will probably be precipitated before the moisture stream reaches the equatorial trough, than afterwards in return currents towards higher latitudes on the western side of the sub-tropical anticyclones, the true mode of δ may be rather less than that value.

If the time lag between evaporation and precipitation was the same everywhere, periodic disturbances with a period δ would be possible. RIEHL, YEH and LA SEUR (1950) have in fact reported the existence of perturbations in low latitudes with a period of the same magnitude as that mentioned above. They were observed as travelling bands of maximum and minimum velocity and they may well be related to the periodic solution of equation (6.1).

In actual fact, the value of δ is, of course, not uniform. However, even a continuous distribution of δ over a range of values would still allow damped periodic oscillations, provided there was a sufficiently pronounced mode. If that was not the case the perturbations would become aperiodic.

7. Stability and secular changes of the trade wind circulation

The amount of heat supplied to the trade wind circulation per unit time and surface may be written by (4.5) and (5.3) as:—

$$Q_1^* = \frac{I}{S}\int\!\!\int\!\!\int \varrho \frac{dQ^C}{dt}\,d\tau + \frac{I}{S}\int\!\!\int c_p\,\varrho\, T v_z\, ds = h\,(\hat{u})_{t-\delta} + \bar{\beta}\, h\,(\hat{u})_t - \frac{I}{S}\int\!\!\int \varrho\, L\, q v_k\, l_k\, d\sigma \tag{7.1}$$

The efficiency of the circulation may be defined as the fraction of Q_1^* which is used to overcome turbulent friction in the trades:—

$$\eta\, A\, Q_1^* = d\,(\hat{u}) \tag{7.2}$$

From the quoted figures it may be inferred that the average values of η is at present

0.009—or perhaps slightly larger if lateral friction had not been neglected. This means that the trade circulation may be said to operate as a heat engine with an overall efficiency of about 0.9 or at most, perhaps one per cent.

Combination of (7.1) with (7.2) gives after re-arrangement and division by η:—

$$A\,h\,(\hat{u})_{t-\delta} = \frac{I}{\eta}d\,(\hat{u})_t - A\,\bar{\beta}\,h\,(\hat{u})_t + \\ + \frac{A}{S}\int\!\!\int \varrho\,L\,qv_k\,l_k\,d\sigma \qquad (7.3)$$

The last term in (7.3) represents the mean contribution of the unit surface to the lateral export of latent heat from the tropics. On the basis of STARR's and WHITE's (1954) data, this term may be estimated equal to 0.98×10^4 erg cm^{-2} sec^{-1}. This is only about one-tenth of the total supply of latent heat from the tropical oceans to the atmosphere. Variation in this flux would be numerically too small to affect the logic of the following argument and this term will, therefore, be considered constant. With this assumption, and with the aid of (5.3) and (5.5), it is possible to solve equation (7.3) numerically or graphically, to give $\hat{u}_t$ as a function of $\hat{u}_{t-\delta}$ and η.

Consider first, the cast of a steady circulation, in which $\hat{u}_t = \hat{u}_{t-\delta}$ would be a function of η only. This relation is shown in Table II.

Figure 5 shows the evaporation term on the left side of (7.3) as a solid curve. The terms on the right hand side are shown as thin curves for various values of η. The steady state values of $\hat{u}$, which were listed in Table II, are determined by the intersection of the two sets of curves. The scheme of events in a changing circulation, when $\hat{u}_t \neq \hat{u}_{t-\delta}$ may be discussed, on the basis of a highly simplified disturbance, as follows:—

It will be assumed that the circulation was steady initially with $\eta = 0.008$; $\hat{u} = 4.8$ and a latent heat production of 8.7×10^4 erg cm^{-2} sec^{-1}. This equilibrium may be disturbed by a sudden hypothetical increase of the efficiency to a value $\eta = 0.010$. As a result the circulation will be speeded up until it is characterised

Table II. Median velocity $\hat{u}$ in a steady circulation as a function of the efficiency η.

$\eta \times 10^2$	0.6	0.7	0.8	0.9	1.0	1.1	1.2
$\hat{u}$ (m sec^{-1})	4.2	4.6	4.9	5.2	5.5	5.8	6.0

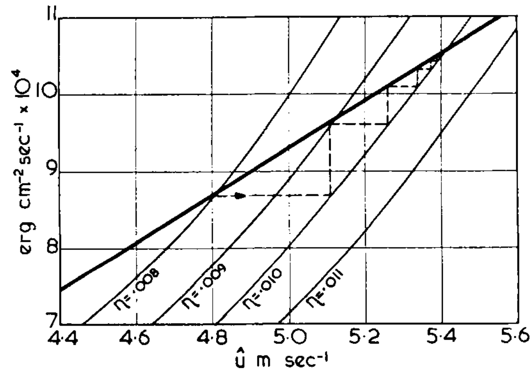

Fig. 5. The solid line shows latent heat production. The thin lines represent right hand terms of equation (8.5) for various values of η.

by a value of $\hat{u} = 5.1$ m sec^{-1}. At this velocity evaporation supplies latent heat at a rate of 9.6×10^4 erg cm^{-2} sec^{-1}, while the dissipation and other terms on the right hand side of (7.2) amount to only 8.7×10^4 erg cm^{-2} sec^{-1}. After a time δ the excess energy is made available to the circulation through condensation in the equatorial belt. When that happens, $\hat{u}$ will increase further to 5.26 m sec^{-1}. The whole process, as shown by the dotted line in figure 5 will continue, with intermittent steps of decreasing amplitude, until the circulation is again steady with a value of $\hat{u} = 5.4$ m sec^{-1}.

The same would have happened if η had decreased initially, or if it had changed temporarily only. In each case, there would have been a subsequent oscillatory approach to the relevant equilibrium and the associated steady state circulation. Changes in the model or a more realistic form of disturbance would not affect the character of this deducation.

It is obvious that the present discussion is based on a grossly simplified case. The value of δ will not be uniform in general and neither η nor the lateral export of latent heat are likely to remain exactly constant whilst the circulation changes. However, these changes are relatively small. Even if they were taken into account, they would not prevent the thin curves in figure 5 from sloping more steeply than the thick evaporation curve. As long as this is the case, the sequence of events would not differ in principle from that described above. Physically, this is due to the evaporation varying with the first power of $\hat{u}$, while the dissipation varies with a higher power. This creates essentially stable conditions. Random influences causing the circulation to speed up will be counteracted by rapidly increasing friction. If the circulation slows down for any reason, the input of energy exceeds the frictional losses and it will be accelerated again.

For a given operating efficiency the state of the trade circulation is, therefore, stable; it may oscillate about its equilibrium but will always tend to return towards it. This probably explains the stability of the subtropical climate and perhaps also the velocity distribution shown in figures 1 and 2.

The stability is not affected by changes in δ. If δ has no pronounced mode, the return to equilibrium after a disturbance may be aperiodic instead of being oscillatory. The distribution of δ is probably not constant. As a result, disturbances of the equilibrium cause perhaps subsequent oscillatory changes some times, while at other times they may cause a more or less monotonic return to equilibrium.

8. Variations of the atmospheric heat sinks

The efficiency of the circulation depends on the intensity and on the distribution in space of the heat sources and sinks. Heating occurs at the earth's surface and in regions where clouds form. The height where this occurs does not appear to vary significantly in the tropics. An increase of moisture transport towards the equator tends to produce a widening of the rainfall belt rather than a lowering of the condensation level.

The mean heat loss is given by:

$$Q_1^* = \frac{I}{S}\int\int\int \varrho \frac{dQ^R}{dt} d\tau - \frac{I}{S}\int\int\left(c_p T + \frac{gz}{A}\right)\varrho v_k l_k \, d\partial \tag{8.1}$$

The second term in this equation is, on the average, at least two and a half times larger than the last term. This estimate is derived from radiation balance consideration as quoted by PRIESTLEY (1951) which require a mean flux of about 4.5×10^{22} erg sec^{-1} across the 35th parallel. This flux is made up mainly of sensible heat, potential energy and latent heat. The most thorough estimate of the latent head flux appears to have been obtained by STARR and WHITE (1954), from whose figures it is inferred that it amounts to about 1.3×10^{22} erg sec^{-1}. The difference between these figures gives the flux of sensible heat and potential energy as represented by the last integral in (8.1). The heat loss due to radiation is not only larger than that due to lateral export, but is also dynamically more effective because it occurs at a relatively high altitude of low pressure. Variation in this altitude would affect the efficiency of the circulation.

Radiational cooling caused by the emission of infra-red radiation from water vapour and clouds is diffused throughout the middle and upper troposphere. The mean height of the layer which is being cooled depends on the height to which water vapour is being transported by convection and this, in turn, is influenced by the height of the tropopause.

As discussed by GOODY (1949), the equilibrium temperature and height of the tropopause depends on the surface temperature and on the relative abundance of water vapour, carbon dioxide and ozone. At middle and high latitudes, ozone has little effect. The equilibrium position and temperature of the tropopause there depends mainly upon the balance between radiative heating caused by CO_2—the mixing ratio of which is constant with height—and cooling caused by water vapour. Conditions are different over the tropics. The radiation due to CO_2 and H_2O is small at the tropical tropopause because the cold, tenuous air can only contain a small mass of these gases. The ozone amount in the tropical stratosphere, though also smaller than in higher altitudes, is not reduced in the same large ratio. It has, therefore, been suggested by Goody, that the effect of ozone—and of its variations—is probably more important in the tropics than at higher latitudes. A decrease of the stratospheric ozone mixing ratio would tend to lower the temperature and raise the height of the tropical tropopause. According to PLASS (1956), it would also tend to lower the equilibrium temperature at the top of clouds and hence intensify the cooling rate within the tropopause.

Most of the ozone in middle and high latitudes has been transported there by horizontal air currents and is not in photo-chemical equilibrium. Near the Equator, equilibrium conditions are more closely approached and the height and temperature of the ozone layer is more likely to be effected directly by possible changes in solar ultra-violet radiation.

Changes in the radiation balance produced by temporary changes of ozone amount in the lower stratosphere would cause changes in the height of the troposphere and in the cooling rate below. The resulting change in the efficiency η should cause after a time δ a possible sudden change in the circulation. This may conceivably be associated with SHAPIRO's (1956) findings of a decrease in weather persistence correlation about 2 weeks after severe increases in geo-magnetic activity. The latter are likely to be preceded by temporary changes in ultra-violet radiation. It is stressed that this suggested solar-water relationship would need observational backing before it could be raised to the rank of a hypothesis.

It is of some interest to note that changes in the CO_2 mixing ratio will also affect the height of the tropopause. In particular, an increase of the CO_2 mixing ratio would tend to reduce the height of the tropopause and the rate of radiational cooling in the troposphere. This would tend to lower the efficiency and intensity of the direct meridional circulation.

9. The effect of land surfaces in the tropics

The supply of sensible heat from the heated land surfaces to the atmosphere can be comparatively large, particularly during the day. SUTTON (1953) quotes observations obtained by Maude in the Gobi Desert during June, which indicate a heat flow of up to 27×10^4 erg cm^{-2} sec^{-1} from the surface to the atmosphere. On the average, during the hours of daylight the upwards heat flow was 14×10^4 erg cm^{-2} sec^{-1} and during night it was reversed, giving a mean upwards flow of 6×10^4 erg cm^{-2} sec^{-1} for the whole day.

June conditions over the Gobi Desert are extreme. The mean upwards flux of sensible heat energy from tropical and sub-tropical land masses, averaged over all seasons is unlikely to exceed 3×10^4 erg cm^{-2} sec^{-1} at the most. If it is considered that land covers only 25.8 per cent of the global surface between 30° S and N, it can be concluded that the energy obtained by the atmosphere from the tropical land surfaces is at most one tenth, and probably less, than that obtained as latent and sensible heat from the oceans.

Although heating from the land surfaces initiates the great monsoon circulations, most of the energy supplied to the atmosphere from the continents is probably again dissipated over the continents. It would be very difficult, if not impossible, to obtain estimates of frictional stress over the various tropical land surfaces. The prevailing existence of light variable winds suggests that this stress must be considerable and that it largely balances the generation of kinetic energy.

If there is a small excess of heat supply associated with the land surfaces, it would add another small negative term to the right hand side of equation (6.1). An overall balance requires in this case a somewhat higher value of the median velocity $\hat{u}$ or lower values of the efficiency η. However, on present indications this effect does not seem to be important.

10. Conclusions and further problems for investigation

The marked persistence and stability of the trade wind circulation has been shown to be due to an equilibrium between the generation and dissipation of energy. The velocity distributions shown in Figures 1 and 2 are probably an expression of this equilibrium. However, no attempt has been made in the present paper to explain the actual shape of the velocity distribution curves and this would be an interesting problem for further study.

The generality of such a study would be limited by basing it on frequency distributions and climatic data as had been done above. Substantial departures from the area/velocity configuration shown in Figure 1 could well occur over limited periods of time, without being detectable by climatological methods. The steadiness of the trade wind circulation suggests that these departures might not be very large, but synoptic investigation into this matter would be desirable. An assessment in synoptic termsof the distribution of wind strength in space and time, and of the energy level of the global trade wind circulation, would be laborious but feasible with modern computational methods.

The assumption of a constant value of Δe—the vapour pressure difference between the sea and the air—introduces a simplification of the actual conditions. A decrease of the evaporation should lead to an increase in the sea surface temperature, which would favour a renewal of the upwards moisture flux and hence the maintenance of a stable circulation. Consideration of changes in Δe might therefore lead to modifications in the present treatment, but they could not affect the main deductions.

A stable circulation can be affected by oscillations which would tend to have a preferred period, comparable to the mean time which elapses between evaporation of the water molecules in the trade wind zone and their precipitation in the equatorial trough. At present the values, which this time interval may assume, can only be guessed. Quantitative studies of this matter would be of interest. The ozone distribution at low latitudes has been described by KARANDIKAR and RAMANATHAN (1949). Little observational material appears to have been published, however, about variations of the vertical distribution in time or about the relation of these variations to the height of the tropopause and the infrared cooling rate below. Observation and investigation of these phenomena may possibly be required for a full understanding of the evaporation-precipitation cycle at low latitudes and of its fluctuations.

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REFERENCES

BROCKS, K., 1955: Wasserdampfschichtung über dem Meer und Rauhigkeit der Meersoberfläche. *Arch. Met. Geophys. Biokl.*, **A8** p. 354.

DEACON, E. L., SHEPPARD, P. A., and WEBB, E. K., 1956: Wind profiles over the sea and the drag at the sea surface. *Aust. J. of Physics*, **9** p. 511.

GOODY, R. N., 1949: The Thermal Equilibrium at the Tropopause. *Proc. Royal Soc. Series A*, **197** p. 488.

JACOBS, W. C., 1951: Large scale aspects of energy transformation over the oceans. *Compendium of Meteorology*, Boston, A.M.S., p. 1057.

KARANDIKAR, R. V., and RAMANATHAN, K. R., 1949: Vertical distribution of atmospheric ozone in low latitudes. *Proc. Indian Acad. of Sciences.* **29** p. 330—348.

MANDELBAUM, H., 1956: Evidence for a critical wind velocity for air-sea boundary processes. *Trans. Amer. Geophys. Un.* **37** p. 685.

MILLER, J. E., 1951: Energy Equations. *Compendium of Meteorology*. Boston A.M.S. p. 483.

MONTGOMERY, R. B., 1940: Observation of vertical humidity distribution above the ocean surface and their relation to evaporation. *Pap. Phys. Ocean Meteor. M.I.T./ Woods Hole Ocean Inst.* **7**: 4.

MUNK, W. H., 1947: A critical wind speed for air-sea boundary processes. *J. Mar. Res.* **6** p. 203.

PRIESTLEY, C. H. B., 1951: Physical interactions between tropical and temperate latitudes. *Q. J. R. Met. Soc.* **77**, p. 200.

RIEHL, H., 1954: *Tropical meteorology*. New York, McGraw-Hill Book Co. p. 6, 11, 74.

RIEHL, H., YEH, T. C., and LA SEUR, 1950: A study of variations of the general circulation. *J. of Met.* **7** p. 181.

ROSSBY, C.-G., and MONTGOMERY, R. G., 1935: The layer of frictional influence in wind and ocean currents. *Pap. Phys. Ocean Meteor., M.I.T./ Woods Hole Ocean Inst.* **3**: 3.

SHAPIRO R., 1956: Further Evidence of a solar-weather effect. *J. of Met.* **13** p. 335.

SHEPPARD, P. A. and OMAR, M. H., 1952: The wind stress over the ocean from observations in the trades. *Quart. J. R. Met. Soc.* **79**.

STARR, V. P., and WHITE, R. M., 1954: Balance require- *pheric General Circulation*, Boston, M.I.T., Dep. of ments of the general circulation. *Studies of the Atmos-* Met. 186.

SUTTON, O. G., 1953: *Micrometeorology*, New York, McGraw Hill Book Co., p. 201, 245.

SVERDRUP, H. U., 1951: Evaporation from the ocean. *Compendium of Meteorology*, Boston, A.M.S., p. 1071.