The Formation of Rain by Coalescence in Very Shallow Cumulus

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Abstract

It is demonstrated theoretically that showers may be released by coalescence from shallow cumulus, only 1 km deep, lasting for one hour and having cloud-droplet spectra typical of those measured in clouds formed in clean, maritime air. In clouds formed in more heavily polluted continental air and which have higher concentrations of smaller cloud droplets, the coalescence process is delayed and retarded so that, under otherwise comparable conditions, the time required for shower formation is considerably increased.

Introduction

In a recent paper in this journal, SQUIRES (1958) has pointed to the differences which exist between the cloud-droplet spectra of non-freezing cumuli formed in maritime air and those of similar clouds which have developed in continental air masses. The 'continental' cumuli contain higher total concentrations of droplets, have narrower droplet spectra with smaller maximum and average droplet sizes, and possess greater colloidal stability. Squires attributes these differences to the fact that clouds formed over land will be supplied with higher concentrations of condensation nuclei in the 'Aitken' and 'Large' size groups so that smaller peak supersaturations will be achieved than in maritime clouds containing updraughts of similar strength.

It is fairly well established that rain falls more readily from small cumuli formed in warm maritime air masses than from clouds of similar dimensions and duration occurring in the interiors of continents, and that these showers are often released by the coalescence process. It is interesting to enquire to what extent the development of the coalescence mechanism is determined by those differences of microstructure noted by Squires.

In an earlier paper, MASON and GHOSH (1957) investigated the formation of large cloud droplets in small cumulus having properties very similar to those described as 'continental' by Squires, but containing some giant salt nuclei, their constitution being typical for small cumulus formed over southern England. It was shown that the appearance of droplets of radius $> 30\,\mu$, in rather feeble cumulus, only $1^1/_2$ km deep, may be accounted for by the growth on giant hygroscopic nuclei of mass $> 10^{-11}$ g in a cloud of mean water content 0.5 gm^{-3} composed of droplets of mean-volume radius $7\,\mu$, provided that it lasted for at least one hour.

More recent calculations, based on accurate theoretical values of collision efficiencies between drops of $R \leq 30\,\mu$ for droplets of all smaller sizes, which have only just become available (HOCKING, 1959), indicate that the rates of droplet growth obtained by Mason and Ghosh, which were based on the inaccurate data of PEARCY and HILL (1957), are rather too high and that, in the conditions considered, it would take nearly 90 minutes instead of 1 hour for nuclei of 10^{-11} g and 10^{-10} g to grow to droplets of 30 μ and 50 μ radius respectively. Under the same conditions, giant salt nuclei of $m = 10^{-9}$ g present in maritime air in concentrations of order 1/litre would, in one hour, reach radii of 100 μ but would require a further $^1/_2$ hr to grow to 350 μ radius and become small raindrops. These times of $1^1/_2$ hours are rather long for the life-times of small, isolated, 'continental' cumulus and although radar echoes and light showers are sometimes observed (see e.g. FETERIS and MASON, 1956), many such clouds will dissipate without precipitating.

However, there are well-documented reports from such places as the Caribbean (BYERS and HALL, 1955), Hawaii (SQUIRES and WARNER, 1957) and Central Sweden (LUDLAM and SAUNDERS, 1956) that light showers sometimes fall from isolated, warm, 'maritime' cumulus little more than 1 km in depth. The life-times of these clouds probably do not exceed one hour.

It will now be demonstrated theoretically that rain may form by coalescence in such shallow clouds, but that precipitation develops more slowly in clouds having a typical 'continental' constitution in the sense described by Squires.

Calculations of Drop Growth in a Model 'Maritime' Cloud

The cloud under discussion is assumed to have a base temperature of 10° C (pressure 900 mb), an average liquid-water content of 0.4 gm^{-3} composed of a low concentration (30/cm^3) of droplets of mean-volume radius 15 μ. These characteristics are typical of the measurements made in small maritime cumulus (SQUIRES, 1958).

The raindrops are assumed to originate on giant salt nuclei of mass 10^{-9} g, present in concentrations of order 1/litre as indicated by the measurements of WOODCOCK (1953, 1957) entering the cloud base as droplets of sea spray.

The cloud, as a whole, is required to last for at least one hour during which time it may grow to a height of about 1 km above its base, at which level it may be limited by an inversion or a dry, stable layer.

There are assumed to exist, inside the cloud, stirring motions capable of transferring droplets from one level to another and of allowing some droplets to remain for longer periods and therefore to have longer effective fall paths inside the cloud than would be the case in a constant, steady updraught.

It is assumed that our cloud forms by rather weak convective lifting of the air, at an average rate of $^1/_2$ m/sec for $^1/_2$ an hour, to reach a depth of 900 m at the end of this period during which the cloud droplets are assumed to have obtained a mean-volume radius of 15 μ. If the droplets are assumed to originate on salt nuclei of mass 10^{-13} g which, over a disturbed sea, exist in concentrations of a few tens per cm^3, this implies that the air has an average supersaturation during their growth of 0.1 per cent. Superimposed on the steady ascending motion of the cloudy air we picture the existence of eddying motions which effectively stir the cloud so that, except close to the cloud boundaries, the liquid-water content and drop-size distribution may be regarded as nearly uniform.

We now consider the growth, among the population of cloud droplets, of a few giant salt nuclei of $m = 10^{-9}$ g which have entered the cloud base as droplets of sea water of radius 20 μ. Their subsequent growth by condensation under a supersaturation of 0.1 % and by coalescence with the smaller cloud droplets is shown in Fig. 1 (a). After one hour, these droplets will have acquired radii of 375 μ, a terminal velocity of a little more than 3 m/sec, and fall out of the cloud as small raindrops. Even if the air is assumed to be just saturated relative to liquid water i.e. to have zero supersaturation, the droplet radius after one hour will be only slightly less at 340 μ, as shown in Fig. 1 (b).

The growth rate of a salt nucleus of $m = 10^{-10}$ g entering the cloud base as a spray droplet of radius 10 μ, but otherwise subjected to exactly the same conditions specified for the first calculation, is represented by Fig. 1 (c). After one hour the drop would attain a radius of only 110 μ and 80 minutes would be required for it to reach 350 μ.

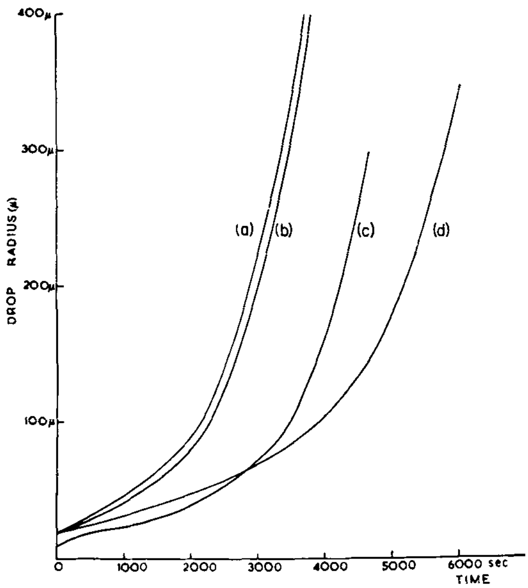

Fig. 1. The times taken for drops to grow by condensation and coalescence under the following conditions.
(a) $w = 0.4$ gm^{-3}, $r = 15\,\mu$, $m = 10^{-9}$ g, $R_0 = 20\,\mu$, supersaturation $\sigma = 0.1$ %.
(b) As for (a), except $\sigma = 0$.
(c) $w = 0.4$ gm^{-3}, $r = 10\,\mu$, $m = 10^{-10}$ g, $R_0 = 10\,\mu$, $\sigma = 0.1$ %.
(d) $w = 0.4$ gm^{-3}, $r = 8\,\mu$, $m = 10^{-9}$ g, $R_0 = 20\,\mu$, $\sigma = 0$.

The calculations represented by Fig. 1, which may easily be repeated for other values of the cloud parameters, enable us to fix the time-scale for the development of precipitation in our model cloud in which values for the temperature, supersaturation, water content and droplet size are specified. We have seen that raindrops of nearly 1 mm diameter may be produced providing that the cloud lasts for one hour and that drizzle drops (diameter $\simeq 300\,\mu$) may be formed after about $^3/_4$ of an hour.

Having fixed the time scale (maximum 1 hour) and the vertical dimensions of the cloud (about 1 km), it is now necessary to formulate a pattern of vertical motion which will allow the incipient raindrops to remain in the cloud for a time sufficient to reach a radius of, say, $350\,\mu$ at which they have a terminal velocity of 3 m/sec. It turns out that it is not possible to accommodate the trajectory of drop which will grow to radius $350\,\mu$ in a cloud only 1 km deep if it is assumed to ascend, grow and fall out in a steady, continuous updraught. One might argue that providing the cloud sustains vigorous stirring motion it is reasonable to expect that at least some of the larger drops will be retained sufficiently long for them to grow to precipitation size in much the same way as MASON (1952) demonstrated for layer clouds. But the formation of convective clouds is associated with buoyant thermals rising upwards through the cloud mass and it is the magnitudes of these updraughts, in relation to the other characteristics of our cloud model, which call for some discussion. If the updraughts are very weak the growing drops will fall out before reaching raindrop size; on the other hand, strong upcurrents may well carry the drops up to the cloud summit before they have reached a size (about $150\,\mu$ in radius) at which they can survive evaporation of the cloud towers.

The following is suggested as a reasonable pattern of vertical motion which, in conformity with the rate of droplet growth shown in Fig. 1 (a), would enable drops of radius about $350\,\mu$ to develop within one hour in a cloud only 1 km deep. The cloud is formed initially by a steady updraught of $^1/_2$ m/sec in which a sea-spray droplet having radius $20\,\mu$ on entering cloud base will have grown to $75\,\mu$ 1,700 seconds later and after having risen 450 m. At this stage, when it is about to fall back towards the cloud base, it is pictured as being caught up in a second thermal ascending at 1 m/sec in which, after a further 650 seconds, it attains a radius of $120\,\mu$ at a height of 670 m. By this time, the depth of the visible cloud will have reached 1,100 m unless restricted by an inversion or dry, stable layer. If, at this stage, the updraught pauses and the drop falls freely to the cloud base under its terminal velocity it will arrive there, 450 seconds later, with radius $190\,\mu$; to allow further growth one might visualize it being carried up in a third thermal with updraught in excess of 2 m/sec. But, if the updraught associated with the second thermal is assumed to persist in the lower two-thirds of the cloud while the cloud top is prevented from rising farther, our drop, having reached radius $120\,\mu$ and terminal velocity 1 m/sec at a height of 670 m, will now reach cloud base with a minimum radius of $300\,\mu$ after a total time of 3,300 sec. But some drops may be delayed by eddying motions and so will ultimately fall

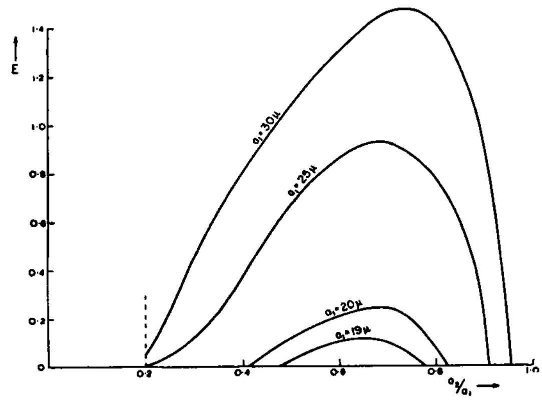

Fig. 2. The collision efficiency E of drops of radius a_1, for smaller droplets of radii a_2, as calculated by Hocking.

out of the cloud with larger radii after the correspondingly longer times indicated in Fig. 1 (a).

Raindrop Formation in a Model 'Continental' Cumulus

Similar calculations have been made for a model cloud having the same liquid-water content and other characteristics as those just described except that the mean cloud-droplet radius is now reduced from 15 μ to 8 μ, a value more typical of clouds formed in continental air masses. The degree to which the growth rate of the nascent raindrops is sensitive to the average size of the cloud droplets is seen by comparing Figs. 1 (b) and 1 (d), the time required for growth to 350 μ radius being increased from 60 minutes to 100 minutes.

This is because Hocking's calculations of collision efficiencies, which are based on an accurate solution for the flow pattern round two spheres each having Reynolds number less than 0.5, predict a value of 0.18 for droplets of radius 15 μ colliding with droplets of radius 20 μ and a rapid increase to $E = 1.08$ as the drop radius increases to 30 μ, while for droplets of radius 8 μ, the corresponding values are zero and 0.30.

Conclusions

On the basis of the above calculations it seems entirely reasonable that drops of radius 350 μ should develop in shallow maritime clouds within periods of about one hour, and, if present in concentrations of 1/litre, will produce precipitation at the rate of 1 mm/hour.

Because the growth of drops by coalescence sets in earlier and proceeds more rapidly if the average cloud-droplet radius is large, shower formation is favoured in maritime clouds but the whole process will be considerably delayed in more heavily polluted air where the average cloud-droplet size is considerably smaller.

The greater colloidal stability of small cumuli forming over the continents relative to that for similar clouds formed over the oceans may therefore be at least partially explained by the observed differences in microstructure, although differences in dynamical properties, not yet well established, may also be important.

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