Some Computations of Water Heights and Currents in the Baltic

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Abstract

The Skagerack, the Kattegatt, the Danish Sounds and the Baltic, all together here called the Trans-Baltic, are treated as a one-dimensional channel with homogenous water. Water heights and velocities are computed from the linear hydrodynamical equations, including terms of acceleration, wind stress, air pressure and friction. It is made numerically with help of the electronic computor BESK. The computations are mainly concentrated to a case 5/12—12/12 1932. The same treatment is also made for a closed basin consisting of the Baltic proper + the Gulf of Finland, altogether here called the Eastsea.

Introduction

Hansen (1956) published numerical computations of water heights and currents in German rivers and in the North Sea. In the former case a tidal wave progressing from the mouth was examined. In the latter case the storm surge of January 1953, "the Holland-Orkan", was investigated.

Since then this work has been continued at the International Meteorological Institute in Stockholm. Some troubles of the numerical stability in Hansen's non-linear model have been overcome at least for a linear model. Fisher (1959) gives the results of these things together with computations with the new model. His scheme in two dimensions has here been translated to a one-dimensional model.

The equations and the numerical scheme

We assume the movements to be mainly longitudinal (x-direction). The contribution of the Coriolis acceleration is therefore only a compensating pressure gradient in the lateral direction (y-direction); the equation of motion in the y-direction is used only when the computed water heights are compared with real heights on one side of the channel.

We assume furthermore the water to be homogenous, which means that the pressure gradient $-\frac{1}{\varrho}\frac{\partial p}{\partial x}$ can be substituted by $-g\frac{\partial h}{\partial x}$; ϱ is the density, g is the acceleration of gravity and h is the variation of the sea surface (Fig. 1). If ϱ is put equal to one, the equations of motion and continuity have the following appearance:

$$\left\{\begin{aligned} &\frac{\partial u}{\partial t}=-g\frac{\partial h}{\partial x}-\frac{\partial p_a}{\partial x}+\frac{\partial \tau}{\partial z}\\ &\left(fu=-g\frac{\partial h}{\partial y}\right)\\ &\frac{\partial u}{\partial x}=-\frac{\partial w}{\partial z}\end{aligned}\right.$$

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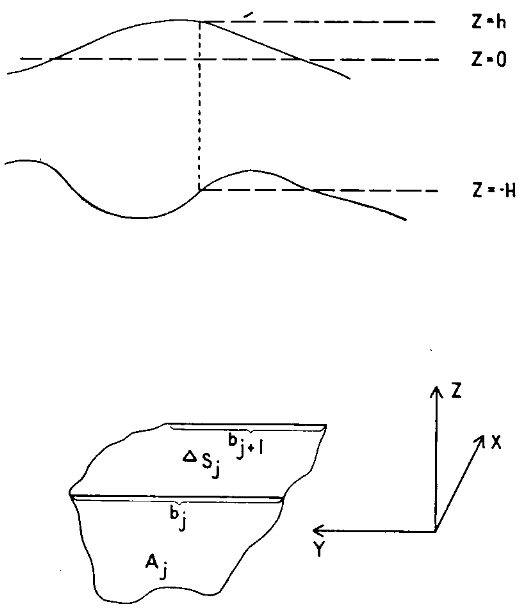

Fig. 1.

where u = the velocity in the x-direction
w = the velocity in the z-direction
τ = turbulent stress component
p_a = atmospheric pressure

These are the equations of any water particle. But we wish to consider the total transport through every cross section. Therefore the integration $\int_{-H}^{h}\int_{0}^{b_j} - - \, dz\, dy$ is carried out everywhere:

$$\int_{-H}^{h}\int_{0}^{b_j} dz\, dy = A_j; \quad \frac{1}{A_j}\int_{-H}^{h}\int_{0}^{b_j} u\, dz\, dy = \bar{u};$$

$$\int_{0}^{b_j} dy = b_j$$

Then we get for every cross section j:

$$\begin{cases} \dfrac{\partial \bar{u}}{\partial t} = -g\dfrac{\partial h}{\partial x} - \dfrac{\partial p_a}{\partial x} + \dfrac{b}{A}(\tau_h - \tau_{-H}) \\ \left(f\bar{u} = -g\dfrac{\partial h}{\partial y}\right) \\ b\dfrac{\partial h}{\partial t} + \dfrac{\partial}{\partial x}(A\bar{u}) = 0 \end{cases}$$

If the water is inhomogenous the most important difference is that the pressure term has to be supplemented with a term $\dfrac{g}{H}\dfrac{\partial Q}{\partial x}$ where $Q = \varrho \int_{-H}^{0}\int_{-z}^{0} \delta\, dz\, dz$ and δ is the anomaly in specific volume. If this term is neglected we have an error in h of order of magnitude $\dfrac{Q}{H}$. (Note that there is already in the real water level values a steady state error of this kind, Witting's "dynamische Störung".)

If we put the bottom stress τ_{-H} equal to $R\bar{u}$ (will be discussed later) and differentiations are converted to differences, FISCHER's (1959) numerical scheme may be written in the following way:

$$u_j^{n+1} = \alpha_1 u_j^n + \alpha_2 \left(u_{j+1}^n + u_{j-1}^n\right) - \\ - \frac{g\cdot \Delta t \cdot b_j}{\Delta s_{j-1} + \Delta s_j}\cdot\left(h_{j+1}^n - h_{j-1}^n\right) - \frac{\partial p_a}{\partial x}\Delta t$$

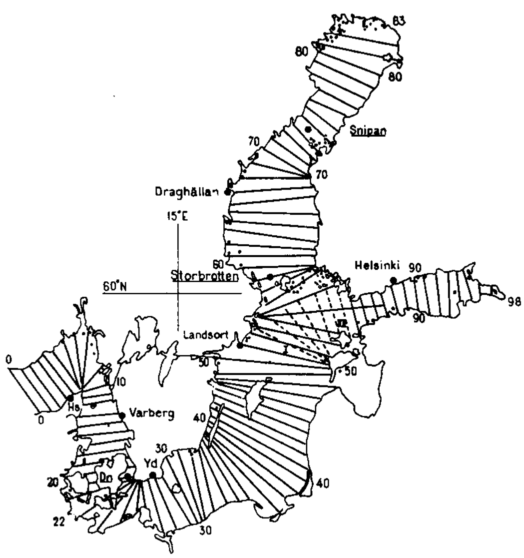

Fig. 2. Map of the Trans-Baltic. Some numbers of sections are indicated. Hs = Hirtshals, Dn = Drogden, Yd = Ystad. Names of lightships are underlined.

$$+\frac{b_j \cdot \Delta t \cdot \tau_{hj}^{n+1}}{A_j} - \frac{R \cdot b_j \cdot \Delta t \cdot u_j^n}{A_j}$$

$$h_j^{n+1} = \alpha_1 h_j^n + \alpha_2 \left(h_{j+1}^n + h_{j-1}^n\right) - \frac{\Delta t \cdot \left(A_{j+1} \cdot u_{j+1}^{n+1} - A_{j-1} \cdot u_{j-1}^{n+1}\right)}{\Delta s_{j-1} + \Delta s_j}$$

u_j^n means $\bar{u}$ at the timestep n and at the section j.

α_1 and α_2 are smoothing terms governing numerical stability; $\alpha_1 + 2\,\alpha_2 = 1$. (According to later personal communication with Dr Fischer, it is not necessary to use such a smoothing when there are no Coriolis accelerations in the system.)

The Trans-Baltic. Some earlier investigations

The Trans-Baltic consists mainly of four different basins (see Fig. 3), separated by the three sills: the Kattegatt + the Danish Sounds, the Åland sill and the Kvarken sill. Even in the deeper parts the depths are small (mean values 40—60 m) except in the Skagerack (220 m).

The water of the Trans-Baltic is horizontally and vertically stratified and the steady state density distribution is governed by a yearly outflow of about 470 km³ river water. The topography of the Baltic and the mixing seems to be such to give a compensating salt water mass in the Danish Sounds of about the same amount, 470 km³/year ($\bar{u} \approx 7$ cm/s) going inwards. Therefore the "river current" should be about 14 cm/s in the Danish Sounds. Most of this river water seems to leave the Trans-Baltic as a rather concentrated current along the whole Swedish and parts of the Norwegian coast.

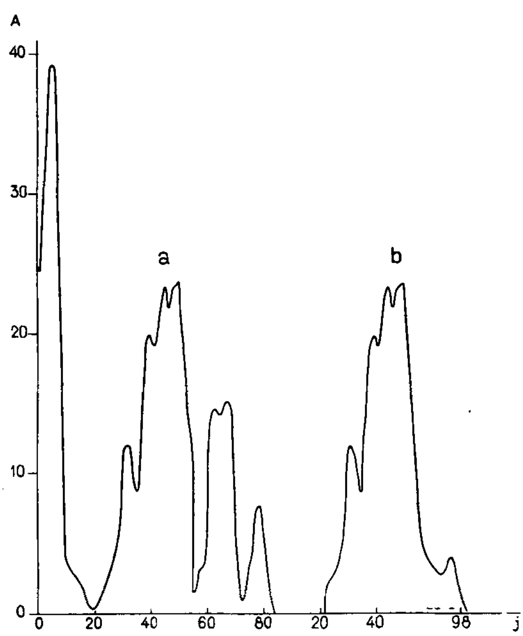

Fig. 3. Cross section area A km². a) Trans-Baltic, sections 0—83. b) Closed Eastsea, sections 22—51, I—VIII, 84—98.

In the different basins of the Trans-Baltic there is a weak cyclonic horizontal circulation. It is not impossible that this is wind-driven.

The biggest transports are given by the wind surges. According to Hela (1944) such transports in the Danish Sounds give current velocities $\bar{u}$ of 25 cm/s as a common value. This article deals mainly with movements of such a type. (It should, however, be made clear that the numerical model may be used for many other purposes, e.g. for tidal waves, either coming from the North Sea or generated in the region itself.)

From the discussion above is clear, that it is a rather great simplification to treat the Trans-Baltic as a channel with homogenous water. Nevertheless many features of these waters, especially the ratio between its length and its width (about 7 : 1), makes it worth while to try such a treatment as a first approximation. But even so, there are many possible ways of proceeding:

1) Witting (1911) has computed the tides in the Eastsea considered as a channel of constant depth. He used, however, a seiche period, which he computed taking the variable depth into consideration. Because the tides in the Baltic are very small, it was hard to get good agreement between theory and reality.

2) Hellström (1941) solved water level problems for the closed Baltic by assuming steady state:

$$0 = -g\frac{\partial h}{\partial x} - \frac{\partial p_a}{\partial x} + \frac{b}{A}\tau_h$$

The whole Baltic is cut into sections perpendicular to the wind direction. The results are rather good for "non-oscillatory" cases.

3) Neumann (1941) showed that the oscillations in the Eastsea mostly are seiches in this basin (period T = 27.6 hrs) considered closed

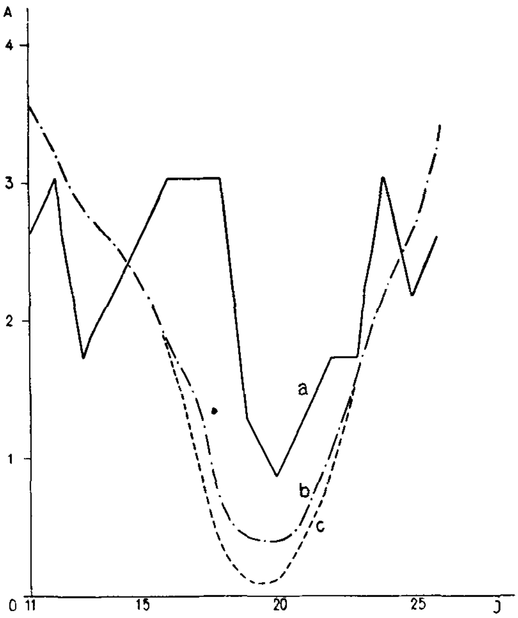

Fig. 4. Cross section area A km² for sections 11—26.
a) direct sea chart values.
b) smoothed values, minimum 0.4 km².
c) smoothed values, minimum 0.1 km².

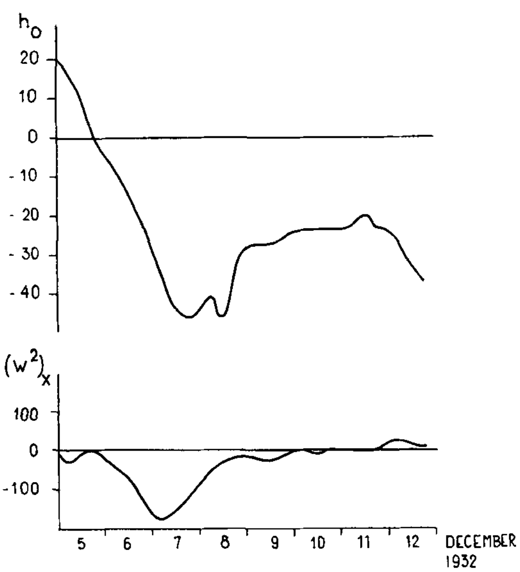

Fig. 5. Water heights at Hirtshals h_0 cm. $(W^2)_x$ m²/s² for sections 0—8.

(closed at section No. 22, Fig. 2). He examined a lot of cases, the first one of which was 10—15/12 1932.

4) NEUMANN (1941) also computed the period T of the closed and branched Baltic (Fig. 2 sections 22—83, except 55, and VI, VII and 84—98). The theoretical value was 39.2 hrs, but is not very often met with in nature according to Neumann.

The numerical integration for the Trans-Baltic

A code has been made for sections 0—83 and 84—98, where the latter part is influenced by the former ($h_{84} = h_{55}$) but not vice versa. The runnings are noted I. The water level value h_0 is put equal to observed water heights at Hirtshals (there were no data available for Hanstholm), and $u_{83} = u_{98} = 0$. The case of 10—15/12 1932 is treated. The same case is also made for a closed Eastsea. The runnings are noted II. $u_{22} = u_{98} = 0$. The basin consists of the sections 22—51, I—VII and 84—98.

Constants for I and II.

1) Δs, A and b (see Fig. 3 concerning A) of the sections 26—98 and I—VII have been taken from NEUMANN (1941). For the rest of the sections they have been determined from sea charts. All the values have been smoothed by making sliding mean values. Especially sensitive are the A-values of the shallow parts. Fig. 4 shows sections 11—26 treated in three different ways. The minimum value 0.4 km² is a value close to that one quoted by HELA (1944). The A-values in this group have been used in a running noted $I_{0.4}$. A running with $A_{\min} = 0.1$ km² is noted $I_{0.1}$.

2) The wind stress τ_h is put

$\tau_h = 0.5 \times 3.2 \times 10^{-6} (W^2)_x$ MTS units, where W is the geostrophically computed wind speed. The constant 3.2×10^{-6} is taken from SVERDRUP et al. (1946), mainly computed from steady state windstau measurements in the Baltic. The figure 0.5 is a factor converting geostrophic wind velocities to surface velocities.

Every 6th hour seven new τ_h- and seven new $\frac{\partial p_a}{\partial x}$-values are fed in for the sections 0—8, 9—22, 23—33, 34—60, 61—77, 78—83 and 84—98 for code I. For code II three new values are fed in for 22—33, 34—51—I—VII, and 84—98. For every timestep new values are interpolated from the fed in ones.

3) The bottom stress, τ_{-H}, is put equal to $R\bar{u}$. It may be better, however, to call this term a damping term instead of defensing it really to represent the bottom stress. R is put equal to 0.207×10^{-3} m sec^{-1}. This value is computed by dividing NEUMANN'S (1941) damping factor 0.5×10^{-5} sec^{-1} by the mean value of b/A for a closed Eastsea.

4) The timestep Δt is chosen 15 min., deduced approximately from the Courant-Friedrich-Lewy criterion and adjusted by experience. The runnings with the codes I take about 40 minutes and with II 25 minutes on the machine.

Results

The results are given in the figures 6—13 of the runnings $I_{0.4}$, $I_{0.1}$ and II as dashed lines.

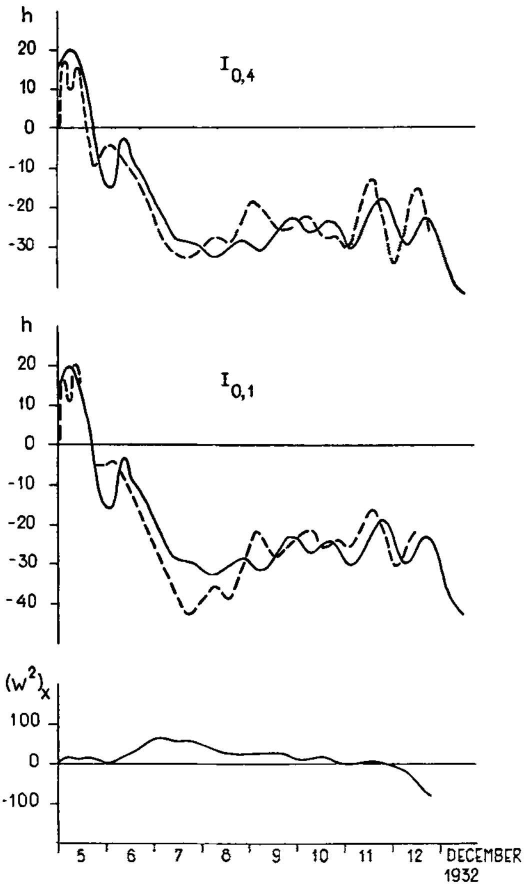

Fig. 6. Varberg, h cm. $(W^2)_x$ m^2/s^2 for the sections 9—22. Dashed lines represent computed values for the cases $I_{0.4}$ and I_0.

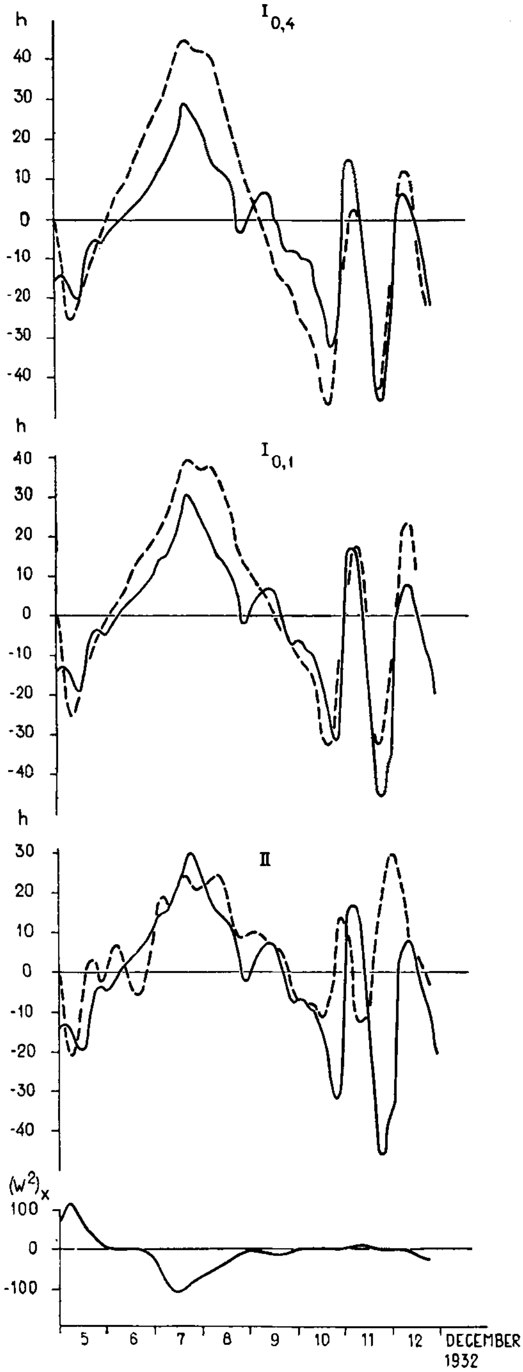

Fig. 7. Ystad, h cm. $(W^2)_x$ m^2/s^2 for the sections 23—33. Dashed lines represent computed values for the cases $I_{0.4}$, $I_{0.1}$ and II.

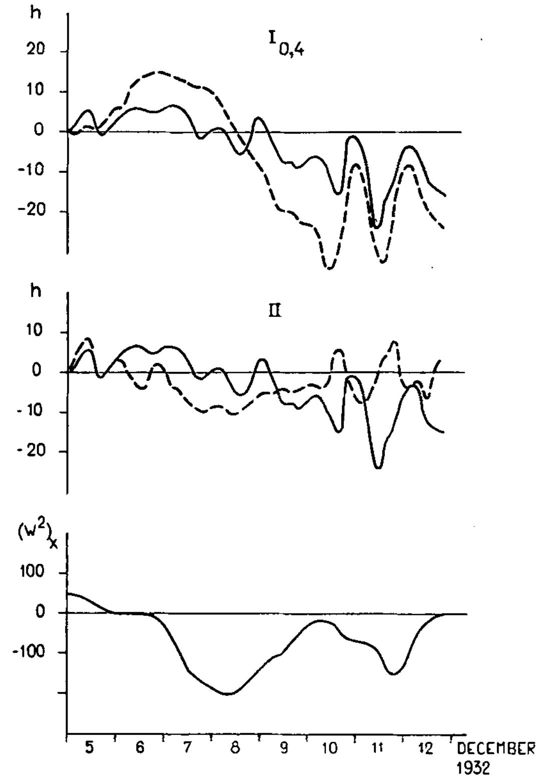

Fig. 8. Landsort, h cm. $(W^2)_x$ m^2/s^2 for the sections 34—60. Dashed lines represent computed values for the cases $I_{0.4}$ and II.

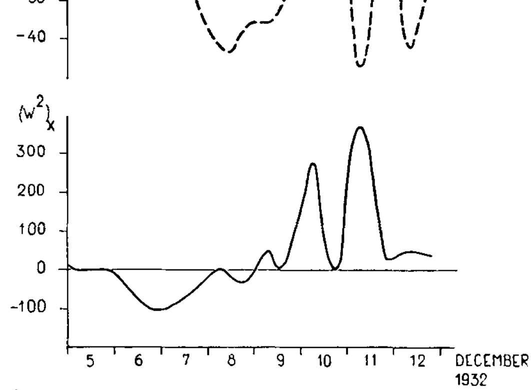

Fig. 9. Draghällan, h cm. $(W^2)_x$ m^2/s^2 for the sections 61—77. Dashed line represents computed values for the case $I_{.04}$.

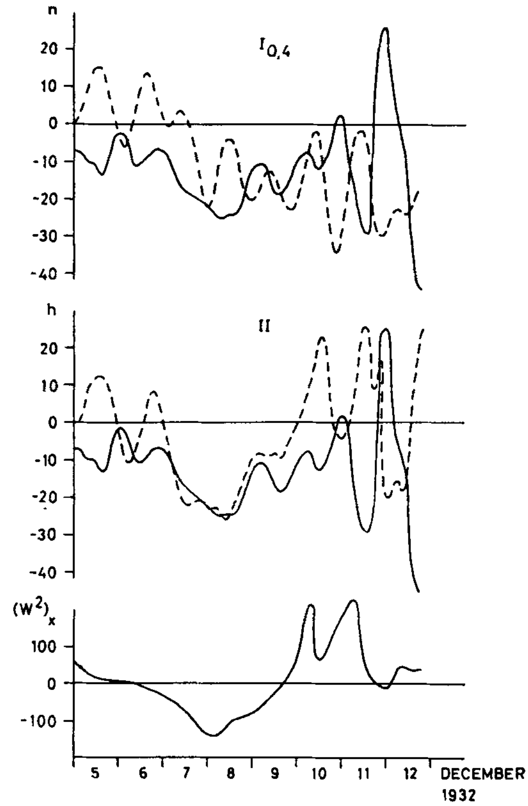

Fig. 10. Helsinki, h cm. $(W^2)_x$ m^2/s^2 for the sections 84—98. Dashed lines represent computed values for the cases $I_{0.4}$ and II.

The values of h have been corrected for the geostrophic effect and are compared with corresponding points on the Swedish coast and at Helsinki. The Swedish values are referred to a level 14 cm above mean sea level and are corrected for the anemo-baric disturbance. Concerning the velocities, the measured values for Snipan and Storbrotten are mean values of surface and 25-meter data. For Drogden only the surface value is given. All real values are represented by full-drawn lines.

Runnings I: For our case of 1932 with its rather moderate wind velocities the water heights in the Skagerack and mostly in the Kattegatt to the greatest extent seems to be dependent on the level of the North Sea. A short special case (III) has been run without

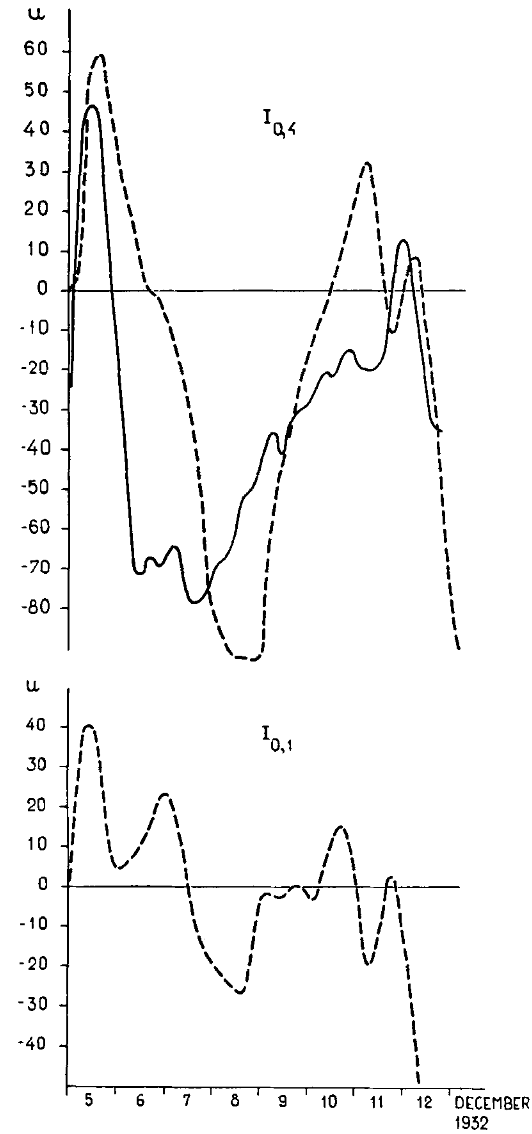

Fig. 11. Drogden, u cm/s. Fulldrawn line represents the measured surface velocity. Dashed lines give computed mean velocities for the cases $I_{0.4}$ and $I_{0.1}$.

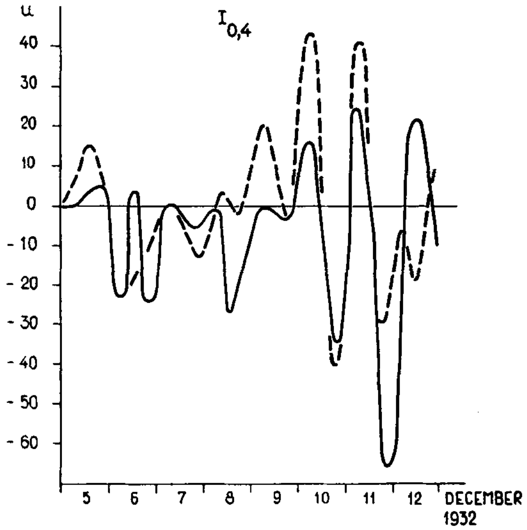

Fig. 12. Snipan, u cm/s. Fulldrawn line = mean value of u, measured at 0 m and 25 m depth. Dashed line = computed mean velocity for the case $I_{0.4}$.

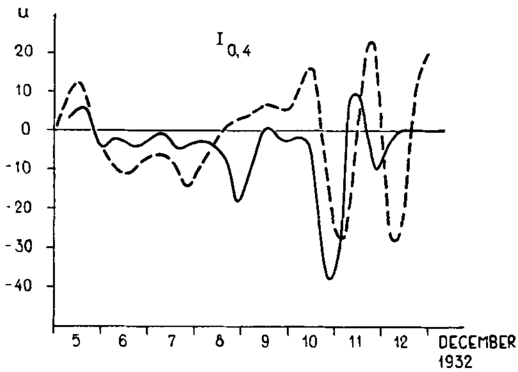

Fig. 13. Storbrotten, u cm/s. Fulldrawn line = mean value of u, measured at 0 m and 30 m depth. Dashed line = computed mean velocity for the case $I_{0.4}$.

any wind at all, which is presented in Fig. 14. The result is seen to be nearly the same for sections 0 and 13, whereas for the Baltic sections there is nearly no change in water level.

We have a special interest in the transports in the Danish Sounds and for that purpose runnings $I_{0.4}$ and $I_{0.1}$ are compared with each other and with other ways of computation

	Transports in km³ per 24 hrs. Mean values over 5 days
$I_{0.4}$	11.3
$I_{0.1}$	0.065
HELA (1944): a water level sinking of 1 cm/24 hrs	3.65
JACOBSEN (1925): 5/30 times the surface velocity at the lightship Drogden (here 53 cm/s)...	8.85

Running II was made to get the seiches according to NEUMANN (1941). Since, however, the long-wave motion is better reproduced by the running I than II for most parts outside the Gulf of Finland, it is probable, that the waves are not seiches but wind-

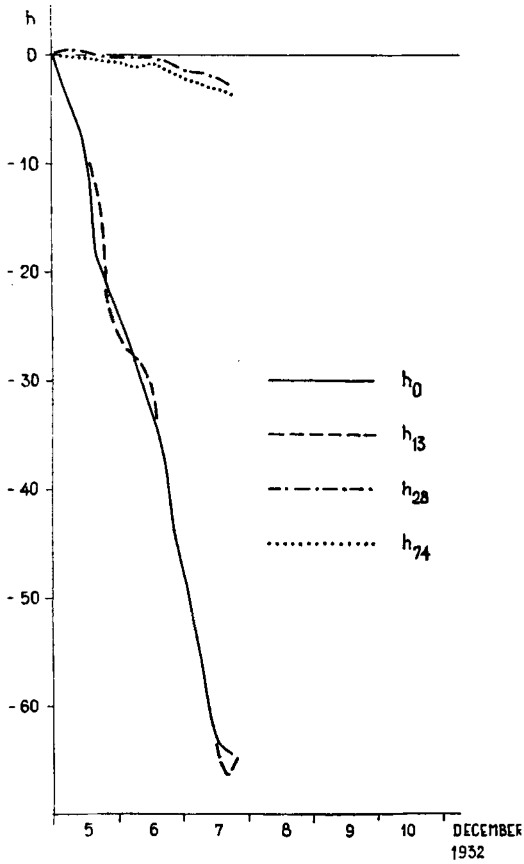

Fig. 14. Running III; h_0 cm is given value.

forced waves in this special case. In no running there is a good result for the Gulf of Finland. It is to be hoped that a model for a branched Baltic will solve this problem better.

Conclusions

This article will show some of the possibilities of a one-dimensional treatment of the Trans-Baltic with the aid of modern computors. Given here is only the very first step. If possible a continuation with e.g. the following steps is planned:

1) A more careful investigation of the cross sections at the sills. Trial with different mesh sizes;

2) a model for a branched Baltic;

3) a 2- and 3-layer model for the Trans-Baltic channel to give the depths of the density discontinuities and the velocities in the different layers;

4) a lot of cases to be run.

REFERENCES

FISCHER, G., 1959: Ein numerisches Verfahren zur Errechnung von Windstau und Gezeiten in Randmeeren. *Tellus* **11**, pp. 60—76.

HANSEN, W., 1956: Theorie zur Errechnung des Wasserstandes und der Strömungen in Randmeeren nebst Anwendungen. *Tellus* **8**, pp. 287—300.

HELA, I., 1944: Über die Schwankungen des Wasserstandes in der Ostsee. *Havsforskn. Skr.* **134**, 108 pp.

HELLSTRÖM, B., 1941: Vattenståndsvariationerna i Östersjöbäckenet. *Tekn. Tidskrift* **8**.

JACOBSEN, L. P., 1925: Die Wasserumsetzung durch den Öresund, den Grossen und den Kleinen Belt. *Medd. fr Komm. f. Havund. Hydr.* **II**: **3**.

NEUMANN, G., 1941: Eigenschwingungen der Ostsee. *Aus d. Arch. d. Deutsch. Seewarte* **61**: **4**.

REID, R. O., 1957: Forced and free surges in a narrow basin of variable depth and width, a numerical approach. *The A & M College of Texas.*

SVERDRUP, H., ET AL., 1946: The Oceans.

WITTING, R., 1911: Tidvattnen i Östersjön och Finska Viken. Helsingfors.