On Pyrheliometric Measurements

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Abstract

Pyrheliometric measurements are now introduced in the meteorological practice to an extent which makes a condensed survey over the present status desirable.

In the following I have combined the main content of the survey on the pyrheliometric scales, given by the International Radiation Commissions, with a description of the present modernized type of the Ångström Pyrheliometer. I have added a critical examination of its function and the sources of error which ought to be avoided at its operation. A detailed survey of its construction seems justified by the fact that it is the only standard instrument at present available in the market, where institutes can easily determine its physical constants, thus controlling the basis for *absolute measurements* of solar radiation. I have added an Annex containing a simple theory of the "edge effect" in the pyrheliometer. The main content of the annex coincides practically with the theory I gave already in 1914, but I repeat it here with some minor corrections together with a discussion of the factors influencing the effect and also of the results of practical experiments.

The important result is derived, that the edge correction is to some extent dependent on the internal convection within the pyrheliometer, and may from this reason vary between about 1.3 and 3.8 percent of the measured radiation. In the new constructions, now available in Sweden, this error is eliminated.

I. Pyrheliometric scales

In meteorological services and institutes radiation measurements have normally been standardized by one of two types of instrument, the Abbot Silver disc pyrheliometer or the Ångström compensation pyrheliometer. The silver disc instrument is calibrated by reference to a calorimeter maintained by the Astrophysical Observatory of the Smithsonian Institution, the sun being used as a source. The Ångström compensation pyrheliometer is now calibrated at Stockholm at the Meteorological and Hydrological Institute by reference to a standard Ångström instrument, earlier used for such calibrations at the Physical Institute at Uppsala. The calibration is checked and controlled in every special case by comparison with the instrument factor obtained by measuring the physical constants of the sensitive part of the instrument (see description given below).

1. The Smithsonian Pyrheliometric Scale

The history of the Smithsonian scale of radiation may be studied in the publications, especially the annals, of the Astrophysical Observatory of the Smithsonian Institution. It is important to note that in 1932, following the introduction of an improved calorimeter, the Institution announced that the Scale of 1913 has an error, measurements in this Scale being high by 2.5 percent; this has been confirmed in 1934, 1947 and 1952. Nevertheless, the Smithsonian Institution has continued to standardize instruments in terms of the Scale of 1913 in order to preserve continuity in series of measurements and particularly in the publication of its solar constant determinations. The correction of −2.5 % has, as far as is known, not been applied in meteorological practice. For further details see especially Annals of the Astrophysical Observatory of Smithsonian Institution, Vol. 7, 1954, p. 25.

2. *The Ångström pyrheliometric Scale*

The Ångström Scale is based on the compensation pyrheliometer constructed as to its main principle, by K. Ångström in 1893 and recommended in 1905 by the International Meteorological Organization for meteorological radiation measurements. The Scale is, and has been, embodied in a small group of specimen instruments, other instruments being substandards, not used absolutely. (As regards *absolute* measurements with the Ångström pyrheliometer, see description below.) When the Ångström instrument is used absolutely it is necessary to apply a certain correction to the readings, mainly on account of the so-called "edge effect" (A. Ångström 1914). The present estimate, following recent measurements in Stockholm by F. Lindholm (Lindholm 1958) of the magnitude of the required correction, is slightly above 2 percent, measurements on the uncorrected scale being too low. This correction has never been applied when substandard instruments have been calibrated in Sweden; in meteorological practice measurements have always been referred to the original Ångström scale, before the new international scale of 1956 was introduced.

3. *Comparison of the existing pyrheliometric Scales*

There have been frequent intercomparisons of the Ångström and Smithsonian scales, through substandard instruments representing either scale. Many comparisons of substandard instruments have been made with the sun as source. The advantage of such comparisons lies in the fact that they are referred to the same source as the one which is the object of the field measurements. The difficulty of such comparisons is that different areas of circumsolar sky are covered by the different instruments, and the intensity of radiation therefrom is widely variable with time and place, so that the silver disc and the Ångström instrument are not strictly comparable if the sun is used as source. As the circumsolar sky radiation is generally increasing with an increase in the scattering power of the atmosphere, one would expect that the difference between two radiometers, of which the one has a wider opening than the other, would vary with the scattering. A variation in the percentic difference with turbidity, airmass and height of the station above sea-level must therefore occur. As the difference due to these causes generally must be rather small in the case of the relation between readings with the Silver disc and the Ångström instrument respectively, the field measurements have not, however, given a clear indication of it.

The difficulty introduced by the circumsolar sky radiation is eliminated in laboratory comparisons with a lamp as a source, but other difficulties are here introduced, especially in the case of the Silver disc, e.g. it is not easy to maintain an even distribution of radiation over the comparatively large sensitive surface.

4. *Summary of results of the comparisons*

The generally accepted mean difference between measurements of solar radiation by instruments standardized by the Smithsonian Institution on the 1913 scale and by instruments standardized on the uncorrected Ångström scale may be expressed by the relation:

$$\frac{\text{SI}}{\text{Å}} = 1.035$$

Individual determinations show, however, considerable scatter. The mean of laboratory determinations, on artificial sources, give:

$$\frac{\text{SI}}{\text{Å}} = 1.028$$

A single laboratory determination of the difference between the (British) National Physical Laboratory (NPL) standard Scale of radiation and a substandard representing the uncorrected Ångström Scale shows readings on the latter Scale to be the lower in the relation:

$$\frac{\text{NPL}}{\text{Å}} = 1.005$$

A series of intercomparisons at Davos (1934) between a substandard Ångström instrument and a Potsdam absolute pyrheliometer (PAP) of calorimeter type, using the sun as source, show a relation:

$$\frac{\text{PAP}}{\text{Å}} = 1.010$$

The Smithsonian Institution considers the 1913 scale to give too high values in the relation.

$$\frac{\text{SI}}{\text{True Scale}} = 1.025$$

Let us designate the Smithsonian Scale of 1913, less 2.5 percent the corrected Smithsonian Scale. On the other hand the edge effect seems to demand a correction of about +2 percent to the Ångström Scale. Let us designate the uncorrected Ångström Scale multiplied by the factor 1.02, the Ångström corrected Scale. The comparison of substandard instruments using the sun as source suggests that readings on the Ångström corrected Scale are in the mean 1.0 percent higher than on the Smithsonian corrected scale. Comparison on laboratory sources suggests that this figure is 1.7 percent. The single comparison of Guild suggests that measurements on the Ångström corrected Scale are 1.5 percent higher than measurements on the N.P.L. Scale. The Potsdam comparisons show that measurements on the Ångström corrected Scale are 1 percent higher than those referred to the Potsdam absolute pyrheliometer. The mean difference of 0.7 percent between comparisons using the sun and those using laboratory sources is of the magnitude to be expected from considerations of the apertures of standard and substandard instruments and an average distribution of circumsolar radiation.

Conclusions. In view of the facts and consideration given above the International Radiation Commissions at a meeting at Davos, September 1956

Recommended the adoption of a new pyrheliometric Scale to be known as the International Pyrheliometric Scale 1956. Measurements made according to the original Ångström Scale should be increased by 1.5 percent. As the difference between the Smithsonian Scale of radiation as revised in 1913, and the original uncorrected Ångström Scale has been found to be 3.5 percent on the average for measurements of solar radiation, measurements made according to the Smithsonian Scale of 1913 should be reduced by 2.0 percent.

II. Present construction and use of the K. Ångström compensation pyrheliometer

1. *General principle*

The receiver of the instrument consists of two thin strips of manganin, made as identical in every way as possible. One strip is exposed to the sun radiation, while the other one is screened from the sun. Through the screened strip an electric current is passed, the intensity of which is regulated so that heating of the two strips is the same.

To secure this thermojunctions connected through a sensitive galvanometer are attached to the central points at the back side of the two strips. A current through the screened strip is adjusted, so that the galvanometer shows no deflection.

Let q be the radiation per second per square centimeter, a the width of the strips, β the absorbing power of their blackened surface, r their resistance in ohm per unit of length, and i the intensity of the compensation current in ampere.

We then have:

$$a\beta q = \frac{ri^2}{4.186} \qquad (1)$$

from which:

$$q = \frac{ri^2}{4.186 \cdot a \cdot \beta} \text{ gram calories/second} \qquad (2)$$

or

$$Q = \frac{60ri^2}{4.186 \cdot a \cdot \beta} \text{ gram calories/minute} \qquad (3)$$

If the intensity of radiation be expressed in watts per square centimeter, we have

$$q' = \frac{ri^2}{a \cdot \beta} \qquad (4)$$

In order to measure the radiation we have to determine β, a and r, a determination which can be made once for all. As the radiation is generally given in gram calories per square centimeter and minute, we put:

$$\frac{60 \cdot r}{4.186 \cdot a \cdot \beta} = K \qquad (5)$$

where K is "the constant" of the instrument. Knowing the constant we may, through measuring the compensation current, obtain the radiation from the equation:

$$Q = Ki^2 \qquad (6)$$

As the strips are exposed in practically the same way to the air and have the same temperature, corrections for cooling and convection are in this method eliminated. (On a slight influence of convection on the edge correction see the annex.)

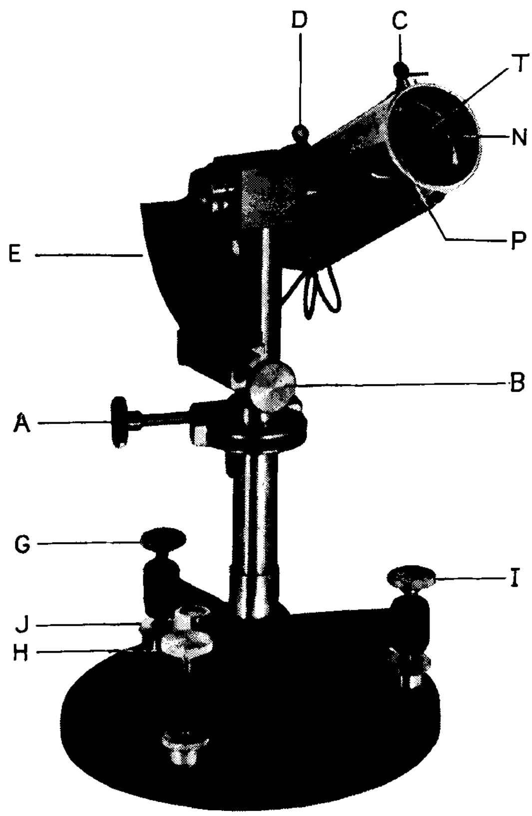


FIG. 1. FRONT VIEW OF PYRHELIOMETER. FILTER HOLDER REMOVED.

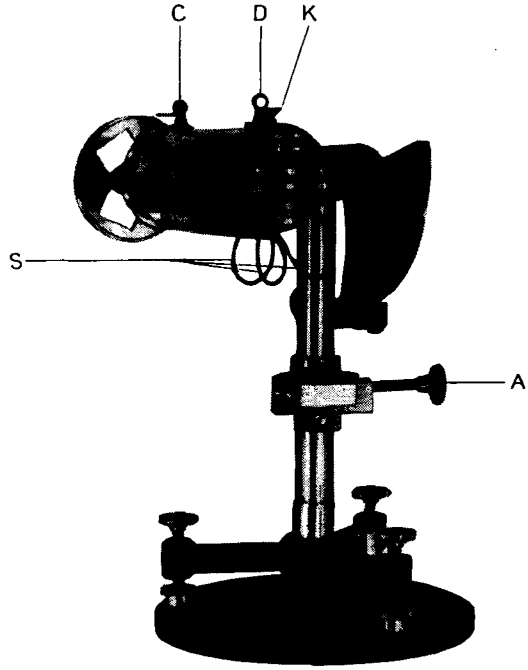


FIG. 2. BACK VIEW OF PYRHELIOMETER WITH. FILTER HOLDER IN PLACE.

2. *Details of construction*

The instrument consists of a cylindrical metal tube – see Figs. 1 and 2 – in the rear of which the manganin strips are mounted on a strip-holder. The tube itself is mounted on a holder and can by means of the two knobs (A) and (B) be directed against the sun. The right direction is controlled by means of the diopters (C) and (D). The height angle of the sun may be read to a tenth of a degree on the scale (E). A condition for obtaining a correct height angle is that the whole instrument has been adjusted to the horizontal plane by means of the screws (G, H, I) and the libel (J).

The stripholder at the back of the tube can be removed for inspection by loosing the screw (K) and decoupling the three wires (S) (see fig. 2).

The three wires (S) are connected to the screen (T) by which the strips can be screened from the sun, in such a way that by the alternate screening of the strips *the current is auto-*

matically passed through the strip which is in shadow.

On the back side of the two manganin strips, two thermocouples (Q) and (R) of copper-constantan, electrically isolated from the strips, are mounted. The thermocouples are connected with two contact screws on the back side of the strip holder. To these same screws the wires to the sensitive galvanometer shall be connected. Two other screws on the back side of the strip holder serve as connections for the wires of the circuit which includes electric cells, milliammeter and slide resistance.

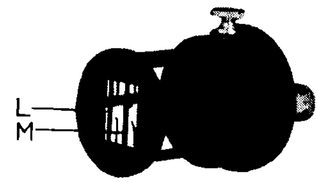
3. *Auxiliary instruments*

a) *The sensitive galvanometer* which serves to show the temperature difference between the strips, is used as a zero instrument. It shall be able to indicate whether temperature differences larger than about 0.01 of a degree C occur, but we are not concerned with the absolute value of its sensitiveness. Its inner resistance ought not to be too large. A sensitiveness of $1 \cdot 10^{-8}$ to $5 \cdot 10^{-8}$ amperes pro division, and an inner resistance of about 10 ohms are suitable. For many conditions a so called "light spot" galvanometer (for instance the type Norma, delivered by Philips) can be recommended. The pointer of the common ammeter type is here replaced by a small light spot crossed by a fine dark line, the position of which can be read on the scale. A number of other types can be used provided the sensitiveness is what has been suggested.

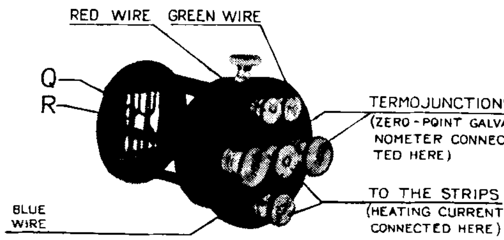
b) *The milliammeter* which serves to measure the compensation current through the screened strip is a highly important instrument. From the equation (6) we obtain:

$$\frac{dQ}{Q} = \frac{2di}{i} \qquad (7)$$

which shows that if we wish to measure the radiation correctly on half a percent we must measure the current correctly within a quarter of a percent. Most milliammeters of high quality enable us to do that. A frequent source of error, not always payed sufficient attention to, is *the temperature dependence of the milliammeter*. The instruments of the highest quality are practically free from a temperature dependence, but often one is forced to work



3A FRONT VIEW.



3B BACK VIEW.

FIG. 3 STRIP HOLDER.

with more simple instruments. Especially in such cases it is important not to subject the ammeter to large variations of temperature. This can to some extent be avoided if the instrument is kept in shadow. Further one has the possibility to determine the temperature coefficient and apply a correction for it.

It is equally important to control, not too seldom, the accuracy of the milliammeters used with compensation pyrheliometers as to control the pyrheliometers themselves.

The milliammeter ought to be able to measure currents up to about 0.4 ampere.

c) *The slide resistance.* When the compensation current is closed, it is of importance that we begin with a rather high resistance and slowly change it to a value which brings the galvanometer deflection back to zero. With an inner resistance of the milliammeter of about 10 ohm, the resistance of the slide resistance ought to be variable within the interval 10 to about 100 ohm. As variations can be obtained also by using smaller electromotive force in the circuit, one need not to be too strict about these limits. It is important,

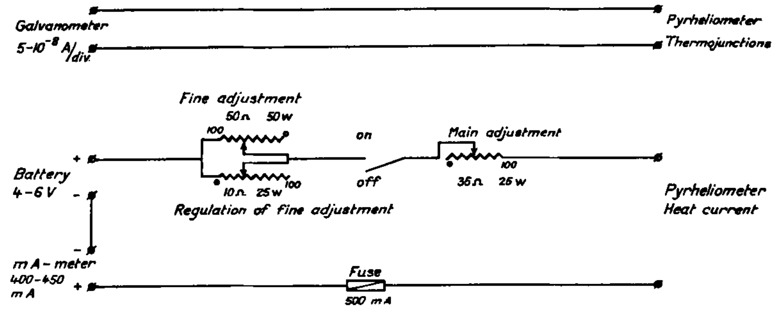


Fig. 4 A *Circuit diagram*

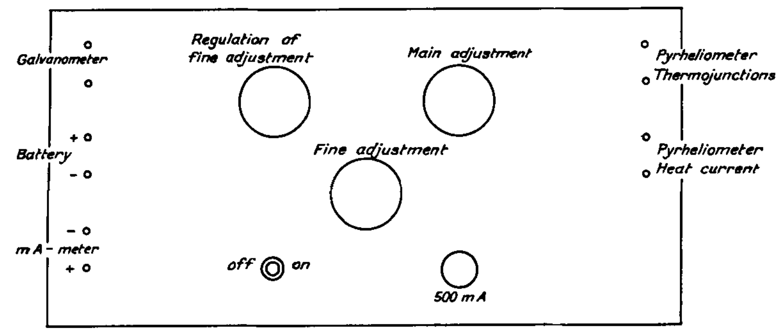


Fig. 4 B. *Switchboard drawing*

however, that resistance and electromotive force are balanced in such a way that the maximum current which can be obtained is not so high that the strips are burned by heating.

It is important that the slide resistance is such that smooth variations can be obtained. A combination of two resistances, parallel or in series, is advisable, one for a coarse, the other for fine regulation of the current.

4. *Determination of the constant K*

Before the delivery the constant K of every instrument is determined through comparison with a standard or substandard instrument. The constant of the standard pyrheliometer has been determined through measuring the quantities r, a and β defined above. As the instruments are expected to give readings which are correct within less than 0.5 percent, the accuracy of the determination of r, a and β for the standard should for every one of these factors be correct within about 0.1 percent.

The strips are cut off in a dividing engine and their width given on its scale. The width is also controlled under the microscope. The resistance between two knife edges resting at a distance apart, which has been measured, is determined. As the resistance of the manganin varies with less than 0.1 percent for a temperature difference $-40°$ to $+50°$, no regard needs to be taken to the temperature of the strips. The constant K, may therefore be regarded as independent of temperature, and no correction needs to be introduced on its account.

Direct observations by K. Ångström at the original construction and also by others have shown that the absorbing power a of the surface blackened by the method of Crova, as described above, may be put equal to 0.980, with an accuracy, probably, of $\pm$ 0.2 percent.

The constant K_s of the standard being com-

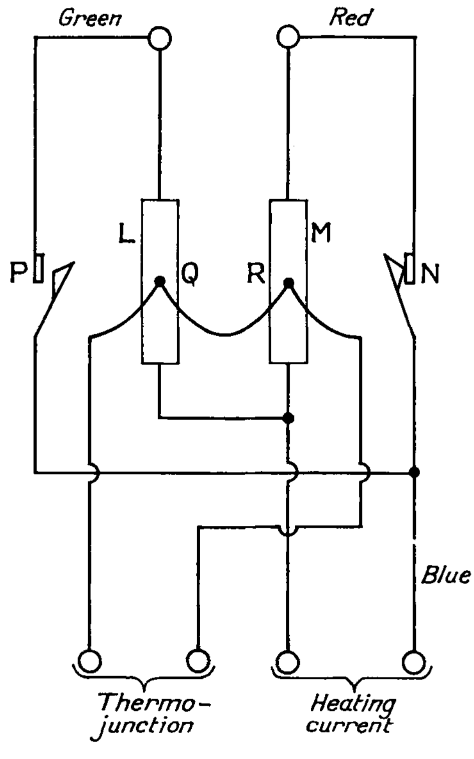


Fig. 5. *Electric circuit diagram of Pyrheliometer*

(when strip M is shaded)

puted on the basis of these measurements, the constant K_n of an individual instrument is determined from the equation:

$$K_s\, i_s^2 = K_n\, i_n^2 \tag{8}$$

from which:

$$K_n = K_s \left[\frac{i_s}{i_n}\right]^2 \tag{9}$$

where i_s is the current measured in using the standard, i_n the current measured in using the instrument, the constant of which is to be determined. From obvious reasons it is of advantage to use the same milliammeter for both determinations, provided the sun radiation is not so variable that simultanity of the both observations is to be given priority. In general K_n is determined from several series of alternating measurements of i_s and i_n. Every single computation of K_n is in general made from mean values of i_s and i_n, and the finally adopted value of K_n is itself a mean value of several such computations of K_n.

At the constant determinations made at the Swedish Meteorological and Hydrological Institute, the constant of the individual instrument is always determined also in a second way, namely on the basis of measurements of a and r, while p is assumed to be equal to the absorption of the strips of the standard instrument, viz. equal to 0.980. In this way a constant K_v is obtained. It serves as a control of the correct functioning of the instrument. This is as a rule approved for delivery only if K_v does not differ from K_n with more than 1 percent. The same rule was applied at earlier constant determinations at the Physical Institute of Uppsala (prior to 1922). But in all cases the value of K_n is given priority. *It is the comparison with the standard instrument or with certain substandards, which determines the value of the constant to be accepted.*

5. *Correction to the constant experimentally determined*

Already in 1914 it was pointed out by the present author (A. ÅNGSTRÖM 1914), that a small correction ought to be applied to the readings with the Ångström compensation pyrheliometer, on account of the fact that in the standard instrument at Uppsala and also in the secondary instruments, which were then copies of the standard, the strip illuminated by the sun is not heated by the sun radiation to its whole length, as is, on the other hand, the strip heated electrically. The diaphragm screenes off the edges of the illuminated strip to an extention of about 1.5 to 2.0 mm at both ends. The consequence of this is that it takes less current energy per unit length of the strip to heat the screened strip to the same temperature as the illuminated one, than it would take if the sun radiation was allowed to fall on the whole length of the strip. At that early time I gave a simple theory, showing the error to be expected. It is presented in Annex I together with some new applications.[1]

The edge effect has after 1914 been subjected to a number of investigations among which the ones of MARTEN (1916) and LINDHOLM (1958) may be especially mentioned. For an instrument used by myself at Algeria (No. 158) I found experimentally an edge effect of 1.3 percent, Marten gives a value as high as 2.8 and recent measurements by Lindholm seem to support the last named value. If one would consider the edge effect as the only cause of a deviation from a true scale, one would consequently have to apply a correction of + 2 percent or even slightly more to the constant K_n determined as described above. There are however probably also some other small sources of error working in an opposite direction. On the bases of all comparisons made between the Ångström and Smithsonian Scales, and especially with reference to the laboratory experiments at the National Physical Laboratory, London, the International Radiation Commission recommended (1956) that for all measurements given in the uncorrected Ångström Scale a correction of + 1.5 percent should be applied after the 1st of January 1957. This implies that to constants K_n determined through comparisons with the standards in Stockholm according to the procedure described in § 4 of the present paper, this same correction ought to be applied.

All certificates on pyrheliometer constants issued by the Meteorological and Hydrological Institute at Stockholm are now (after January 1 st, 1957) in agreement with this recommendation.

6. *Operation of K. Ångström compensation pyrheliometer*

The instrument is adjusted against the sun by aid of libel and diopters. The screen is put straight forward allowing radiation to fall simultaneously on the both strips. No heating current is then applied to either strip. Take the reading of the galvanometer connected to the thermo-couple as *zero position.*

[1] In my previous paper I used a value for the heat conductivity of manganin taken from Smithsonian Physical Tables (1910). This value is ten times too large. The correct value is now introduced.

Now turn the screen 90°, for instance to the left so that the left strip is shadowed, the right one exposed to the sun. At the same time the contact to the left strip is automatically closed. The intensity of the current is regulated as to bring back the galvanometer deflection to the zero position. When that has been made, the intensity of the current is read at the milliammeter. Next turn the screen 180°, in order to shadow the right strip. Automatically the current is switched over to that strip. The current is regulated for no deflection on the galvanometer and a new reading of its intensity is taken. Finally the screen is turned to the left again and the procedure of regulating and reading repeated. If the readings are L_1, R and L_2, respectively, the current i_m is determined from the equation:

$$i_m = \frac{L_1 + L_2 + 2R}{4} \qquad (10)$$

and the radiation computed from:

$$Q = K i_m^2 \qquad (11)$$

where K is the constant given in the certificate. If the certificate does not state that the constant refers to the International Pyrheliometric Scale of 1956, but is issued earlier and given in the Ångström Scale, add 1.5 percent to the constant in order to reduce it to the international Scale.

The height of the sun is obtained to a tenth of a degree from the scale attached to the instrument.

When high accuracy is demanded the solar height h should be computed from the true local time according to the equation:

$$\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t \qquad (12)$$

where φ is the latitude of the station, δ is the declination of the sun and t is the local time angle. At the time 1^h p.m. for instance the time angle is $\frac{360°}{24} = 15°$,

It is desirable that the indications on the elevation scale are controlled now and then through computations according to equation (12).

Under all circumstances it is important that the time is read at the same moment as pyrheliometer readings are taken. At high sun elevations it may in general be sufficient to read the watch, when the reading R is made at the ammeter. In general the time ought to be given in hours and minutes.

Comparison of instruments. When comparisons are made for the purpose of controlling the constants or for new standardizations it is especially important that they are made under the most favorable conditions as regards installment of the instruments and as regards sky and sun conditions. An exceptionally clear sky is desirable, mostly because the radiation is then, for solar heights above 30°, rather constant, with only a regular and slow variation. It is also desirable that the solar height is not very low. Solar heights below about 20° are not suited for standardization purposes.

At all installments of actinometric instruments for general purposes the neighbourhood of smoke creating industries ought to be avoided. The milliammeter and slide resistance ought if possible be placed within an observatory or in the shadow of some shelter; not only for protecting them against excessive heating but also to avoid as much as possible the heating of the contact connections of the electric wires.

I am indebted to my successor as Director of the Meteorological and Hydrological Institute of Sweden for the permission to use a number of figures included in the Institute's instruction for the use of the Ångström pyrheliometer.

ANNEX

On the edge effect at the Ångström compensation pyrheliometer.

By ANDERS ÅNGSTRÖM

The so called "edge effect" involves a correction to the readings of the Ångström compensation pyrheliometer. The correction is of the order of about 2.0 per cent in the case of the standard pyrheliometer, used to define the Ångström Pyrheliometric Scale. It arises through certain features of the construction, involving that the manganin strip exposed to the sun is not illuminated to its whole length but that the strip to a length of about 1.5—2.0 mm from the edges is screened by a diaphragm, while the other strip, which is in shadow, is heated by the electric current to its whole length. The consequence of this is that it takes less energy per unit length of the strip to heat it by the electric current, than corresponds to the heating by the sun radiation for the same temperature at the centre of the strips.

It is easy to give for an idealized case, which corresponds to the essentials of the actual conditions, a theory of the energy and temperature distribution at the two strips. The advantage of such a theory is that it clearly shows what order of size may be expected as regards the correction and also that it gives a very clear idea about the factors which influence the correction and about the variations to which it may be subjected under various conditions of exposure.

We make the assumption that the strips are perfectly absorbing, with a width a, a thickness b and a conductive power for heat c. We further assume the strips to be soldered to great metal blocks, the temperature of which are independent of the temperature of the strips. The strips have the total length $2\ l$ and the strip illuminated by the sun is in shadow to a length d from the edges. I further assume that the temperature t_0 at the centre of the strip is measured, and that this temperature is uninfluenced by the measuring device. Let it also be assumed that the thickness b of the strips is negligible in comparison with l and a and that the temperature in every cross-section of the strip is the same.

We further assume that the convection and radiation to the surroundings from the different parts of the heated strips can be expressed through the cooling law of Newton, viz, it is proportional to the temperature difference between the surroundings and the part of the strip, which we consider. The temperature of the surroundings we assume equal to the temperature of the blocks to which the strips are soldered. To simplify we let this temperature be the zero-point of our temperature scale. Then the convection from various parts of the strip is expressed by the quantity kt, where t is the variable temperature and k a cooling constant.

Some of these assumptions represent an idealization. The blocks to which the strips are soldered may to some extent adopt a temperature slightly above the "surroundings", the conduction of heat through the wires of the thermojunction influences to some degree the temperature distribution in the strips, and finally it seems probable that k is to some extent variable along the strip in a way which probably is dependent on a number of external conditions. This, however, does not worry us so much, as the chief aim of the theory is to make clear what factors are involved and what is the order of size of their influence. There are no reasons to suppose that their influence in the practical case will be very different from what it is in the idealized one.

Let us investigate the variation of temperature of the strip as a function of the distance x from the centre of the strip.

If the energy produced by the current is W per unit area of the screened strip we get:

$$Wa\,dx = k\cdot t\cdot a\cdot dx - a\cdot b\cdot c\,\frac{d^2t}{dx^2}\,dx \qquad (1)$$

from which

$$\frac{d^2t}{dx^2} - \frac{k}{b\cdot c}\cdot t + \frac{W}{b\cdot c} = 0 \qquad (2)$$

The general solution has the form

$$t = Ae^{+\mu x} + Be^{-\mu x} + S \qquad (3)$$

where we have

$$S = \frac{W}{k}, \text{ and } \mu = \sqrt{\frac{k}{b\cdot c}} \qquad (4)$$

The constants A and B are determined by the conditions:

$$t = 0 \text{ for } x = l; \text{ and } \frac{dt}{dx} = 0 \text{ for } x = 0$$

the last condition following simply from symmetry.

We thus get

$$A = B = -\frac{S}{e^{+\mu l} + e^{-\mu l}} \qquad (5)$$

For the temperature t_0 at the centre, where $l = 0$, we get from (3) and (5)

$$t_0 = \frac{W}{k}\left[1 - \frac{2}{e^{+\mu l} + e^{-\mu l}}\right] \qquad (6)$$

This temperature t_0 is equal to the temperature t_{01} at the centre of the illuminated strip.

For the illuminated part of that strip we have:

$$t_1 = A_1\,(e^{+\mu x} + e^{-\mu x}) + S_1 \text{ where} \qquad (7)$$

$$S_1 = \frac{Q}{k} \qquad (8)$$

where Q is the radiation per unit surface of the illuminated strip.

For the shadowed part of the exposed strip we have:

$$\frac{d^2t_2}{dx^2} - \frac{k}{b\cdot c}\cdot t_2 = 0 \qquad (9)$$

which gives

$$t_2 = Me^{+\mu x} + Ne^{-\mu x}$$

A_1, M and N shall fulfill the conditions:

$$t_2 = 0 \text{ for } x = l;\ t_2 = t_1 \text{ for } x = l - d$$

and

$$\left(\frac{dt_1}{dx}\right)_{x = l - d} = \left(\frac{dt_2}{dx}\right)_{x = l - d}$$

This gives us for the temperature of the illuminated part of the exposed strip:

$$t_1 = -\frac{S_1}{2}\left[\frac{e^{+\mu d} + e^{-\mu d}}{e^{+\mu l} + e^{-\mu l}}\right](e^{+\mu x} + e^{-\mu x}) + S_1 \qquad (10)$$

and for the temperature at the centre:

$$t_{01} = \frac{Q}{k}\left(1 - \frac{e^{+\mu d} + e^{-\mu d}}{e^{+\mu l} + e^{-\mu l}}\right) \qquad (11)$$

We have $t_{01} = t_0$ and by dividing (11) by (6) we get:

$$\frac{Q}{W} = \frac{\psi(l) - 1}{\psi(l) - \psi(d)} \qquad (12)$$

where

$$\psi(l) = \frac{e^{+\mu l} + e^{-\mu l}}{2}$$

and similar for $\psi(d)$

For μ we had the expression $\mu = \sqrt{\frac{k}{b\cdot c}}$

In the case of the type of the Ångström pyrheliometer, to which most *measurements* of the edge correction refer, the value of b is 20 microns; for c we have measurements, for instance by Jäger and Dieselhorst, giving a value of 0.0519 gram calories per second and square centimeter for a temperature gradient of 1° per cm, at a temperature of 18°. At 100° the value is considerably larger namely 0.0631. A change of c with about 8 percent for a temperature difference of 30° seems probable, which evidently means a change in μ of about 4 percent for the same interval of temperature.

The factor, the evaluation of which involves the greatest difficulty is k. The combined cooling effect of radiation and convection is without doubt variable within rather wide limits, dependent on the wind velocity, which must to some extent influence the air circulation within the pyrheliometer tube. Mac Farlane gives a value for k of 0.000287 at 18° C (gram calories per square centimeter and second) with an increase to about 0.000319 at a temperature of 40°. This value, however, refers to the cooling of a black sphere placed in the centre of a spherical enclosure which has a blackened surface, the medium between the surfaces being moist air. The air was not ventilated, and aside from the thermal convection it must have been mainly at rest. The value of Mac Farlane is therefore probably smaller than the value of k corresponding to the conditions within the pyrheliometer. If, in spite of that, we introduce the values of b, c and k given above and corresponding to a temperature of 18°, we find, allowing for the convection and the radiation from front and back surface of the strip (= 2 k), a value for μ of 2.35. As the ratio $\frac{k}{c}$ enters into μ, it may be assumed that the value of μ is much less dependent on the temperature than c and k. The temperature dependence may therefore here be neglected. On the other hand it seems probable that the value of μ in our actual case is different from 2.35, especially on account of k being different from what has been assumed.

Accurate measurements of the edge correction have now in fact been carried out by MARTEN after my first article on the subject was published in 1914. In the case of the pyrheliometer Nr 140 III, then belonging to the Meteorological Observatory at Potsdam, MARTEN determined d to be equal to 0.2 cm, and found, by changing the length of the shadowed part of the strip from 0.1 to 0.4 cm mean values on the edge correction, given in the Table I, under δ^{obs}. Here δ is defined through:

Table 1

d	δ^{obs}	δ^{comp}
0.1	0.7	0.65
0.2	2.8	2.65
0.3	6.5	6.3
0.4	11.9	12.1

$$\delta = \left[\frac{Q}{W} - 1\right] \cdot 100 \qquad (13)$$

or

$$Q = W\,(1 + 0.01\ \delta) \qquad (14)$$

From (12) and (13) we get:

$$0.01\ \delta^{comp} = \frac{\psi\,(l) - 1}{\psi\,(l) - \psi\,(d)} \qquad (15)$$

If we in this formula introduce a value on μ equal to 2.4, we get the values of δ^{comp} given in the last column of Table I.

The variation of δ^{comp} with d (in centimeter) is also demonstrated by fig. 6 through a full drawn line. The measured values of Marten are represented by crosses. The value for k found

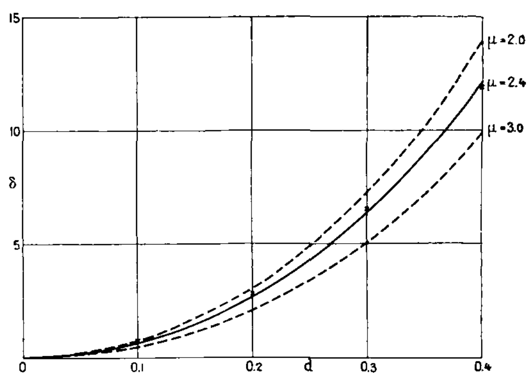


Fig. 6. Edge correction δ in per cent in its dependence of shadowed length d (μ=2.4, μ=2.0, μ=3.0). Curves according to theory, crosses according to measurements by Marten.

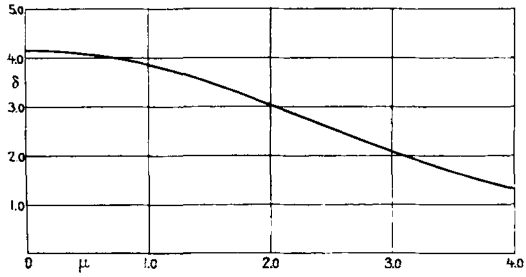


Fig. 7. Dependence of edge correction δ on convective constant μ (d = 0.2 cm, l = 1.0 cm).

through introducing the measured values of Marten into our formula seems, with regard to what has been stated earlier, to be a reasonable one. It is rather astonishing to find how well the theoretical formula then represents the actual conditions. This gives us reason for assuming that the formula may also give a trustworthy idea of the dependence of the edge correction on other variables than d, especially then on the dependence of δ on k.

This seems at present to constitute a rather important question. Can δ for a given instrument, where d is keeping a fixed value of let us say 0.2 cm be regarded as practically constant? For all determinations of the difference between the Ångström and the Smithsonian Scale this is a rather fundamental question. Fig. 7 now gives the values of δ computed from my formula for different values of μ, under the assumption $d = 0.2$, $l = 1.0$.

The limit value $\delta_{\mu=0}$ is computed from the formula and given by:

$$0.01\ \delta_{\mu=0} = \frac{l^2}{l^2 - d^2} - 1 = \frac{d^2}{l^2 - d^2} = 0.0415$$

It must here be emphasized that the zero-value of μ is quite imaginary, in the respect that μ can not be reduced below the value corresponding to radiation alone. In fact k is composed of a radiation term and a convection term. It seems to be reason to assume that the cooling is to about two third due to convection. But even the convection can not be reduced below the value corresponding to the free convection due to the rise of the air from the heated strip on account of differences in density. It therefore seems probable that μ can under no conditions of ventilation be below about 50 percent of its normal value, viz. not below about 1.0.

The important result to be derived from the formula now is that δ can only be regarded as a constant for the case that μ does not vary between too wide limits. Already the measurements of Marten showed that in his different series, δ, for $d = 0.2$, varied between the limits 2.6 and 3.2 percent. But it seems rather probable that in practical field work the variation of k, and consequently also of μ, is such, that much greater variations can occur. Let us assume that under extreme conditions μ takes on values as low as 1.0, and as high as 4.0, viz. about the half and the double of the normal value. This gives us a variation of the edge correction from 3.85 at the minimum value of μ, to 1.31 for the maximum, which means a difference of almost 2.5 percent in the measured radiation value. Such a variation may well be produced if the pyrheliometer in the first case is well screened against wind and in the second is openly exposed to a wind blowing directly into the tube. It seems rather probable that we have here an explanation to the rather large differences obtained by, for instance, Courvoisier in comparing substandards of the two standard Scales. In general one has, at observatories, much better opportunities to get installments of pyrheliometers, where the wind influence is reduced to a minimum than at field measurement in the open. This may mean a difference of more than ±1 percent in extreme cases.

What here has been said emphasizes the importance of reducing the edge correction to a minimum. For d equal to 0.05 cm, and technically such a reduction of d seems practicable, the edge correction for $\mu = 2.4$ is only 0.2 percent and its variation corresponding to extreme values of μ seems then to be reduced to less than ±0.2 percent. In general it may only be a fraction of this value.

At the construction of new instruments by A. Lindblad[1] or at the Swedish Meteorological and Hydrological Institute, attention is now payed to the conditions which here have been analysed. In the instrument of recent construction the screened edges of the illuminated strip are less than 0.2 mm in length. This means that the edge correction is practically eliminated.

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