The Spatial Variation of Liquid Water and Droplet Concentration in Cumuli

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Abstract

Measurements of liquid water content and droplet concentration in cumuli over path lengths of about one metre have been used to investigate the structure of these clouds. The measurements indicate that the cloud process is far from adiabatic, and that a dominating role is played by the admixture of drier air from outside the cloud. They suggest, further, that the main source of this dry air may be the region above the cloud top.

1. Introduction

Measurements of the properties of cumuli have led a number of investigators to the conclusions that the classical model of an adiabatically expanding parcel of air is inadequate, and that significant interaction must occur between the cloud and its environment. It is apparent that the processes involved in the life cycle of a cumulus are of great complexity, and the volume of data so far available does not seem to form an adequate basis for the construction of any complete theory.

Some measurements of droplet spectra have been made in cumuli, and certain aspects of these results, which appear relevant to the problem of interaction between the cloud and its surroundings, are discussed.

2. Observations

Cloud droplet spectra have been measured in various types of cloud, mostly cumuli, using the aircraft sampler described by Squires and Gillespie (1952). Ten samples can be taken at one loading. The minimum interval between samples is about 3 seconds, corresponding to about 200 metres of travel.

The errors in the measurement of droplet concentrations arise from two sources: variations in the action of the exposing mechanism, and errors in air speed. The sampling device has been calibrated at intervals during the measurements, and has shown fluctuations amounting to 3 % in the exposure time of a given point on the sampling rod. Air speed indicators are reliable to about 5 %, and in cloud the aircraft speed changes at times by this amount from the mean value. It seems therefore that the droplet concentrations should have an accuracy of about 10 %.

Until 1952, a sampling layer of magnesium oxide was used. Subsequent investigation showed that this was unsatisfactory for the measurement of droplet diameters and since then a layer of soot has been used. The diameters of the holes made by droplet of a given size in the soot layer have been found by projecting, at aircraft speed, a small sampling piece coated with soot through a curtain of droplets, produced, in most cases, by a spinning disk. The ratio of hole diameter to drop diameter (d) is known with an accuracy which varies with droplet size; above $d = 20\ \mu$, the probable error is under 5 %. Below $d = 20\,\mu$ it increases with decreasing drop size to about 15 % at $d = 5\,\mu$; this increase does not appear to be due to a lack of reproducibility in the ratio, but almost wholly to the difficulty of producing and measuring a group of uniform droplets in this size range. The accuracy of liquid water contents therefore depends on the sizes of the drops occurring in the cloud. Over the range of diameters $d > 10\,\mu$, which usually contributes most of the liquid water, the probable error in diameter does not exceed 10 %. It seems reasonable therefore to assign an overall accuracy of about 30 % to the measurements of liquid water content. However, the comparative accuracy of the measurements is probably better than this.

For this reason, the mean value of n or w found from the samples in one pass through a cumulus is not reliable. Thus, when the mean value of w is plotted against h for each cloud, the resulting graph is confused, and shows no clear trend of w with height. Since WARNER (loc. cit.) has shown that w begins to decrease one to two thousand feet below the cloud top, it is not satisfactory to lump together observations from several clouds of differing total depths. In the case of the ratio w/w_a, however, this is admissable, as this ratio decreases steadily with increasing height. The mean variation of w/w_a with h is shown in Table 1. This variation agrees with that shown by WARNER, but these values of w/w_a are consistently smaller by about 0.1. This discrepancy is not as serious as it seems, since the values of w used here to give a mean value are measured at randomly distributed points, whereas WARNER used the "peak value" of w from each traverse.

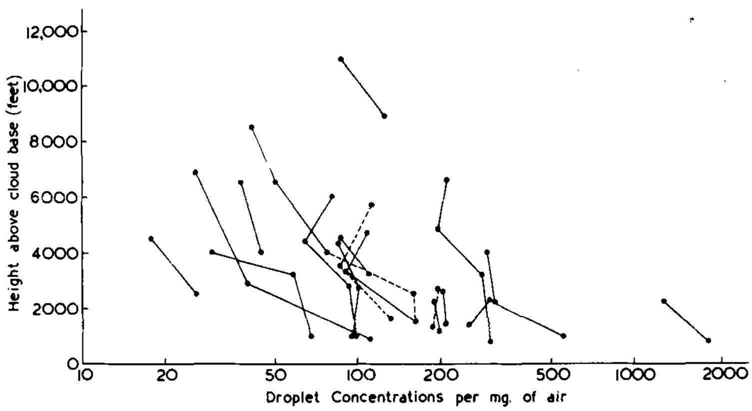

Fig. 1. The variation of droplet concentration with height above cloud base in individual clouds. Dashed lines are used in places for the sake of clarity.

3. Variations in the vertical

According to the extensive measurements of WARNER (1955) the liquid water content (w g m^{-3}) in cumuli increases upwards to within one or two thousand feet below the cloud top. However, he found that at all levels it was less than the value which would have resulted from the adiabatic expansion of air which was just saturated at the level of the cloud base (w_a). He further showed that the ratio w/w_a decreases with increasing height above the cloud base (h). The present measurements confirm some of these conclusions. As will be shown later by some examples, the characteristics of the droplet spectrum change erratically and indeed almost randomly when successive samples are spaced at about 200 m.

Both ZAITSEV (1950) and WEICKMANN and KAMPE (1953) have reported a rapid decrease in droplet concentration with increasing height. The present series of observations indicates a trend in this direction, although not so marked or consistent as that found by these authors. The mean droplet concentration per milligram of air (N_h) has been found for each pass through an isolated cumulus. When these mean values are plotted against the height above cloud base, it appears on the whole that N_h decreases as h increases; there are

Table 1. The variation of the ratio w/w_a with height above cloud base

Height above cloud base (feet)	No. of observations	Median value of w/w_a
0—1000	24	0.33
1000—2000	119	0.26
2000—3000	66	0.18
3000—4000	62	0.17
4000—5000	23	0.12
5000—6000	0	—
6000—7000	27	0.08

however exceptions. These exceptions may be real, or they may be due to the fact that, with the rapid and erratic variations in droplet concentrations, many more samples would have been needed to secure a fair average at each level. In order to smooth out these fluctuations, the data for each cloud have been grouped in height intervals of 2,000 feet (h = 0 - 2,000; 2,000 - 4,000 etc.). For each interval of h, the mean value of N_h has been found for the observations in each cloud, and in Fig. 1 this is plotted against the centroid of the h-values in the interval. This figure is based on 455 samples. Points referring to observations in a single cloud are connected by straight lines. Out of the 31 lines in the figure, 24 (about 77 %) indicate a decrease in N_h with increasing h. Out of the eighteen clouds represented only two (11 %) showed an increase in droplet concentration from the lowest to the highest level plotted in Fig. 1. In both these cases the height interval was only about 1,000 feet, so that even if the rate of decrease of N_h had been similar to that found in other clouds, the change between the two levels would have been small, and easily masked by random fluctuations.

Concentrations have been expressed for the present purpose in number per milligram of air instead of in the usual volume units. Thus the apparent trend for droplet concentrations to decrease upwards owes nothing to the mere expansion of the air as it rises. This decrease must therefore be due to some process which eliminates droplets from the cloud. Concentrations expressed in numbers per cm³ naturally show the same trend as is seen in Fig. 1, and to a slightly greater extent because of the upward decrease of air density.

The observations at the various levels were not, of course, simultaneous. However, the practice followed was to begin passes at the higher levels and work down. The droplet concentration in a cumulus is more likely to

Table 2.

Showing the random nature of the variation of droplet concentration per cm³ (n) and liquid water content (w g m⁻³) in samples spaced about 200 m apart in horizontal traverses through some cumuli: **n** in bold face, *w* in italics.

Date	Height in feet of: Cloud base	Cloud top	Level of traverse	1	2	3	4	5	6	7	8	9	10
23 Oct. 1954	2,200	9,000	6,250	**28**	**42**	**24**	**75**	**4**	**5**				
				0.46	*0.11*	*0.57*	*1.40*	*0.25*	*0.03*				
6 Nov. 1954	800	6,000	1,800	**63**	**43**	**55**	**133**	**117**	**74**	**82**			
				0.05	*0.14*	*0.18*	*0.45*	*0.44*	*0.25*	*0.38*			
26 Nov. 1954	3,600	8,500	7,000	**66**	**79**	**58**	**82**	**70**	**238**	**57**	**128**	**62**	**164**
				0.66	*0.74*	*0.71*	*1.00*	*0.71*	*2.14*	*0.62*	*1.50*	*0.79*	*2.69*
1 Dec. 1954	2,500	6,500	5,600	**107**	**19**	**19**	**14**	**10**	**15**	**6**	**14**		
				1.29	*0.20*	*0.25*	*0.11*	*0.10*	*0.26*	*0.06*	*0.05*		
13 Mar. 1956	2,500	7,500	6,500	**82**	**59**	**52**	**73**	**199**	**140**	**77**	**67**	**260**	
				0.15	*0.13*	*0.13*	*0.17*	*0.63*	*0.41*	*0.13*	*0.17*	*0.80*	
2 Nov. 1956	7,200	12,000	9,800	**24**	**231**	**141**	**505**	**117**	**270**	**127**	**177**	**165**	**549**
				0.02	*0.15*	*0.15*	*0.74*	*0.17*	*0.35*	*0.19*	*0.26*	*0.21*	*0.73*

Table 3. The relative variation of droplet concentration on entering a cumulus (250 observations)

Sample number	Mean relative droplet concentration
1	1.02
2	1.08
3	1.02
4	0.88
5	0.96
6	1.16
7	1.10
8	1.04
9	0.89
10	0.99

decrease with time than to increase; hence this lack of simultaneity would not contribute to the effect shown in Fig. 1, but rather would tend to mask it.

4. Variations in the horizontal

When successive samples taken on a single horizontal pass through a cumulus are compared, it is found that the variations in the droplet concentration per cm^3 (n) and in the liquid water content (w) are strongly correlated. This is illustrated in Fig. 2 by scatter diagrams for some individual passes. A regression line through the origin corresponds to a constant mean droplet volume. The corresponding mean radius $\bar{r}$ is entered on each diagram. It will be seen that the correlation is especially clear when this mean radius is small.

The scatter diagrams of Fig. 2 serve also to illustrate the magnitude of the fluctuations which occur in a horizontal traverse. As mentioned earlier, these fluctuations are nearly random. In order to illustrate this further, the values of n and w found in typical traverses through some cumuli are shown in Table 2.

Discussing his measurements of liquid water content, WARNER (loc. cit.) states that, in many clouds, the transition from clear air to dense cloud is abrupt. A similar conclusion is indicated by the present measurements. In order to investigate the variation of n on entering a cloud, the mean value of n for each traverse was found (n_h). The ratio n/n_h was then computed for each sample. This was done for 26 traverses in which nine or ten samples were taken in uninterrupted succession at intervals of about 200 m. The mean value of the ratio n/n_h was then found for all the first samples taken in the cloud, for all the second samples, and so on. These mean values are given in Table 3. In the majority of cases the samples, spread over some 2 km, represent nearly the whole width of the cloud, and in all cases at least half of it. The practice was to take the first sample one to two seconds after the aircraft entered the cloud. Cloud entry was judged by watching the wing-tip of the aircraft; this carried a black de-icer boot against which intervening cloud could be seen distinctly. Table 3 indicates that, beyond a point about 100 m inside the cloud, there is on the average no further increase in droplet concentration.

In the case of liquid water content, the volume of data available is much less, since w cannot be found from the observations where a magnesium oxide layer was used. In

Table 4. The relative variation of liquid water content on entering a cumulus (164 observations)

Sample number	Mean relative liquid water content
1	0.99
2	0.71
3	0.88
4	1.22
5	0.89
6	1.16
7	0.74
8	1.02
9	1.16
10	1.02

Table 5. Frequency table of values of the ratio w/w_a (321 observations)

Range of w/w_a	Percentage frequency
0—0.1	23
0.1—0.2	31
0.2—0.3	19
0.3—0.4	8
0.4—0.5	6
0.5—0.6	4
0.6—0.7	3
0.7—0.8	2
0.8—0.9	0
0.9—1.0	2
over 1.0	1

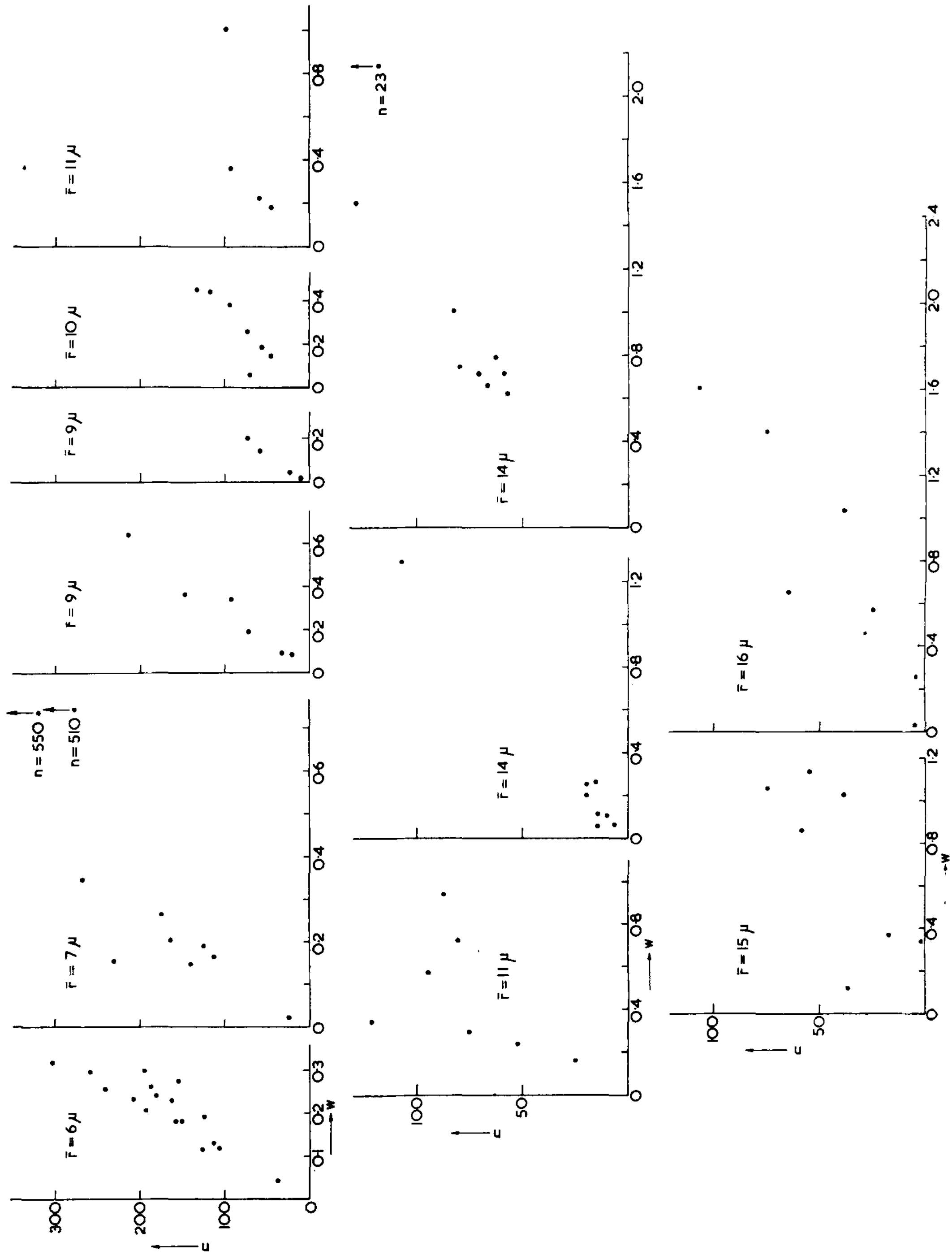

Fig. 2. Scatter diagrams illustrating the association between the fluctuations of droplet concentration (n per cm^3) and liquid water content (w g per m^3) in horizontal traverses through cumuli.

order to investigate the variation of w on entering a cumulus, all traverses have been utilized where at least seven samples were taken. Otherwise the data have been handled in the same way as for n. The result is shown in Table 4. It will be seen that the total number of observations is smaller, and that this sequence is rather more irregular than that of Table 3; however it appears unlikely that w increases on the average by more than 10 % from the first sample to the middle of the cloud and it is quite possible that there is no average increase.

5. The range of variation of the liquid water content

It is of some interest to consider the frequency of values of w/w_a, irrspective of position in the cloud; this is shown in Table 5. Since the sample is taken from a strip of air which is never more than 2 metres in length, and usually less than 1 metre, it may be considered as a point measurement of w. Table 5 indicates that w exceeds w_a only in about 1 % of the samples. The maximum value found for w/w_a was 1.16; this exceeds unity by less than the possible error of the measurement. Even if it is assumed that the values of w are consistently too low by 30 %, so that all values of w/w_a greater than about 0.8 in Table 5 really correspond to $w \geq w_a$, it would appear that the adiabatic liquid water content was attained in only 3 % of the samples. It seems likely therefore that this happens only in a few percent of the volume of an average cumulus. The overall mean value of w/w_a was 0.18.

Occasionally, completely null samples were found inside a cumulus. To all appearances, the aircraft was completely surrounded by cloud but it is possible that, in some cases, samples were taken in a clear space between two adjacent cumuli, or between two towers rising out of a common base. This was certainly not always the case. Out of nearly a thousand samples taken in cumuli, about 8 % were null, and rather more than half of these were isolated null samples; that is, the sample about 200 m away on each side of it showed cloud droplets. It is most unlikely that any large proportion of these isolated null samples were due to the kind of error described above. This means that, even in the interior of cumuli, the liquid water content is sometimes zero; in fact, w is at least as likely to fall to a zero value as it is to rise to the adiabatic value, w_a.

6. Summary of results

Summarizing the results given above, some of which have already been reported by the authors mentioned, it has been found that:

(a) The liquid water content (w) of cumuli, even when measured over a path length of only about one metre, is nearly always less than the adiabatic value w_a. In the clouds observed, w attained the value w_a only over a few per cent of the volume of a cloud, on the average; it fell to a zero value, well inside the cloud, at least as often. The average value of w/w_a was about 0.2.

(b) On the average, both the droplet concentration (n) and w/w_a decrease upwards from the cloud base.

(c) On a horizontal traverse, the variations of n and w are large and erratic, but they rise and fall together. This association between the variations of n and w seems to be closer when the average droplet size is small.

(d) On entering a cumulus from the clear air the droplet concentration rises, on the average, to its mean value for the whole traverse within a distance of about 100 m from the cloud boundary. The liquid water content also rises sharply over this distance, but it is possible that there may be a slight remaining increase from this point towards the centre of the cloud, amounting to perhaps 10 %; the data are however not sufficient to decide whether this further increase occurs or not.

7. Discussion

The fact that w is nearly always less than w_a implies that an admixture of drier air occurs throughout all but a few percent of the volume of a cumulus. This dry air may enter from the sides or the top, but not from the bottom, since the almost horizontal base of a cumulus indicates that all air passing through this level is saturated; the variations in cloud base height correspond to variations in w_a which are small compared with the observed values of the deficit of liquid water, $(w_a - w)$. At times the admixed air apparently can penetrate far into the cloud without becoming saturated, for a region devoid of water can remain so only if the air is unsaturated, so that water droplets diffusing into it will evaporate. The upward decrease of the ratio w/w_a implies that the effects of this admixture increase upwards.

It is plausible that this admixture of dry air is also the cause of the variation of droplet concentration, some of the smaller droplets evaporating when dry air mixes with cloud air. There is however the alternative possibility, that these variations are caused by the coalescence of droplets. DAS (1956) has computed numerically the evolution as a result of coalescence of several droplet spectra, some of them containing large droplets. His results show that over periods up to an hour the change in the droplet spectrum, and in droplet concentration, would be very slight. These calculations were based on an analysis similar to that of SCHUMANN (1940), using a value of unity for the collection efficiency. The calculations of PEARCEY and HILL (1957) indicate that for the small droplet sizes often found near the bases of cumuli, for which the Reynolds number is much less than 1, the collection efficiency is usually much less than unity.

It seems reasonable therefore to take the results of DAS as giving an upper limit for the effectiveness of gravitational coalescence in modifying the droplet spectrum as a whole. This negative result found by DAS has no bearing on the problem of the formation of a relatively small number of large drops or raindrops by coalescence, since he has excluded some droplets at both the upper and lower ends of the spectrum from his calculations; moreover, as TELFORD (1955) has shown, statistical fluctuations can be important in the formation of the relatively small number of raindrops.

It would appear therefore on purely a priori grounds that the variations in droplet concentration are not attributable to coalescence, but to the evaporation of some of the smaller droplets when dry air mixes with cloudy air. This view is clearly in agreement with the observational results given above. The variations of w in a horizontal traverse cannot be due to coalescence; it is plausible, therefore, that the variations of n, which are strongly associated with those of w, are also due to the admixture of dry air. Clearly, droplets are more likely to be evaporated completely by mixing when the average drop size is small. Thus the fact that the association between the variations of n and w seems to be stronger when the average drop size is small also points in this direction. Finally, the mere erratic nature of the variations of n across a cloud suggests that they are more likely to be due to a process like the turbulent admixture of dry air than to an internal process such as coalescence.

If it is true that the horizontal variations of n are due to admixture, it seems plausible that the vertical variation, which is similar in magnitude to the erratic variations in the horizontal, could be due to the same cause. Like w/w_a, the droplet concentration per milligram of air, N, decreases upwards, and this too could be due to the increasing effects of the admixture of dry air in the higher parts of the cloud.

The admixture of drier air from outside the cloud would seem therefore to play an important part in the structure of cumuli. As for the source of this air, observations discussed so far indicate only that its effects increase upwards. On the other hand the data of Tables 3 and 4 give an indication which does not, on the face of it, accord well with the theory originated by STOMMEL (1947), according to which the departure of the observed lapse rate in cumuli from the wet adiabatic lapse rate is due to the lateral entrainment of dry air from around the sides of the cloud. If this were so, one would expect to find a marked gradient of w and n from the boundary of the cloud towards its centre. The examples given in Table 2 illustrate the fact that there is no consistent trend in this direction. Tables 3 and 4 show that, when data from a number of cumuli are averaged together, there is little or no increase in n or w beyond a point about 100 m inside the cloud boundary.

However, MALKUS (1949) has pointed out that as a result of vertical wind shear, cumuli are often asymmetrical. Since the traverses used in Tables 3 and 4 were made in directions which were randomly distributed with respect to the direction of the wind shear vector, this asymmetry could contribute to the apparent uniformity of average cloud properties along a traverse. Nevertheless, it does not seem likely that asymmetry due to wind shear could so completely mask the central maximum of liquid water content and droplet concentration which the entrainment theory would imply.

In the light of these measurements, an alternative mechanism suggests itself. The sharp increase in n and w near the cloud boundary and the upward increase in the effects of admixed air are both explained readily if it is supposed that the dry air comes into the cumulus, not from the sides, but from the top. A parcel of dry air which is enveloped by the cloud during its growth, or which diffuses into the cloud top by turbulence, is in an unstable situation. It will be cooled by the inward diffusion of liquid water from all sides, and must subside through the cloud. Since the driving force is supplied by the evaporation of liquid water, this mechanism will be the more effective, the wetter the cloud. The increased turbulence found in cumuli derives its energy from density differences, and it is likely therefore that vertical mixing predominates over the horizontal mixing which is postulated in the entrainment theory.

On this view, a cumulus will contain some dry air at all stages of its growth, and at all levels. Its structure may be likened to that of a sponge, the spaces in which represent parcels of dry air. Some of these parcels may not have moved far from the positions they occupied when the growing cloud enveloped them; others may have subsided a considerable distance. Evidently this model offers a simple explanation also for the rapid fluctuations which are found when flying through cumuli, and the occasional presence of clear regions even well inside the cloud.

8. Conclusion

The specific conclusions concerning the spatial variations of the liquid water content and droplet concentration have been given in detail in Section 6 above. The discussion of Section 7 confirms the conclusions of other investigators that interaction between the cloud and its surroundings is of great importance; the fact that the liquid water content averaged only about one fifth of the adiabatic value shows that, in cumuli, the adiabatic model bears little resemblance to reality. The existence of regions inside the clouds where liquid water was completely absent also indicates very definitely that they must contain some air which could not have originated below the condensation level. At the most, only a few per cent of the volume of an average cumulus consists of unmixed air which has risen adiabatically from cloud base.

It appears that the values of the liquid water content, and the spatial variations of the properties of the cloud, can be qualitatively explained by postulating an admixture of dry air in a proportion which increases steadily from the base upwards. This postulate could be satisfied by several models, and in particular by that of the lateral entrainment of dry air; however, the results concerning the average variation of properties across cumuli do not seem to agree well with expectations based on this theory.

The measurements suggest that most of the dry air may enter by turbulent diffusion through the cloud top, both during the growing and mature phases. The mechanism by which such parcels of dry air could subside through the cloud is physically clear. The random variations which would result from the operation of this process throughout the cloud's life correspond well with the fine structure in turbulence, temperature, droplet concentration and liquid water content which is observed.

9. Acknowledgements

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