On the Theory of Magnetic Storms and Aurorae

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Abstract

A. Chapman-Ferraro's theory of magnetic storms is discussed. It is pointed out that the fundamental assumption about a non-magnetized beam is in conflict with cosmic ray evidence. It is further shown that there is a fundamental objection against any theory of the main phase based on a non-magnetized beam.

B. The objections, which Cowling and Chapman have raised against the electric field theory, are discussed, and the discharge mechanism during a magnetic storm is analysed. When a magnetized beam enters the earth's magnetic field, it is heated due to the compression. In the hot plasma the earth will act as a "probe" so that it gets a negative charge in relation to the plasma. As a result of ambipolar diffusion to the earth, the ionosphere in the auroral zones will be bombarded by high energy protons and electrons.

It is shown that a mechanism of this type is not subject to the objections which have been raised.

A. Discussion of Chapman-Ferraro's theory

Chapman and Ferraro have made a very careful study of what happens when a beam of ionized non-magnetized gas approaches the earth. There is little reason to doubt that their calculations—as far as it has been possible to carry them out—really tell us what would happen under the assumed circumstances. On the other hand the following arguments indicate that the beam probably has other properties than they have assumed and, as a consequence, phenomena fundamentally different from those which they treat are likely to be responsible for magnetic storms and aurorae.

§ 1. Beam properties from cosmic ray observations

Cosmic ray evidence indicates that the storm-producing beam must be associated with a magnetic field or an electric field or both. It has been shown by JOHNSON (1938), TREIMAN (1953) and HAYAKAWA (1955) that the Forbush decrease in connection with magnetic storms cannot be due to changes in the magnetic cut-off of the earth. This is further confirmed by the fact that the cosmic radiation decreases even at stations close to the geomagnetic pole, as shown by SINGER (1954). It seems inevitable to conclude that the cosmic ray change is produced directly by the beam.

There are only two ways known in which a change in cosmic ray intensity can be produced by the beam *viz.*, 1) by a magnetic field which is strong enough to produce forbidden cones or 2) by an electric field.

Measurements show that the cosmic rays of a rigidity of $H\varrho = 10^8$ gauss cm exhibit a storm variation. The breadth of the beam near the earth could not exceed 10^{13} cm. Hence the magnetic field which is necessary in order to produce variations is $H \geq 10^{8-13} = 10^{-5}$ gauss. A smaller field can deflect the particles but —according to Liouville's theorem—not change the intensity, unless an electric field is present.

An electric field may change the cosmic ray intensity by acceleration or retardation of the particles. In order to explain a decrease of 1 % in the cosmic radiation, an electric voltage difference of about 10^8 volt is necessary (ALFVÉN 1954, LUNDQUIST 1954). If the breadth of the beam is $\leq 10^{13}$ cm, the field is $E \geq 10^{-5}$ volt/cm. With a beam velocity $v = 10^8$ cm/sec, this field reveals the existence of a magnetic field $H \geq 10^{-5}$ gauss.

The conclusion of this discussion, which has been carried out more in details elsewhere (loc. cit.), is that as soon as a cosmic ray storm variation of 1 % or more is observed, the beam must contain a frozen-in magnetic field of at least the order of 10^{-5} gauss. (If the beam has an irregular turbulent structure the value becomes larger.) In the rather rare cases when a decrease of 10 % is observed the field should be at least of the order 10^{-4} gauss. Certainly there are many storms, in connection with which no decrease as large as 1 % is observed. This may indicate that the breadth of the beam is smaller than 10^{13} cm, that the field H is smaller than 10^{-5} gauss, or that the field is irregular.

In a critical review of the electric field theory Chapman (1952) writes that the field in the beam cannot be more than 10^{-6} gauss. This value is obtained under the assumption that the field in the beam is the same as the solar dipole field at the distance of the earth (an assumption which I have also made in my earlier publications). However, as the magneto-hydrodynamic coupling in interplanetary space is certainly very good, it seems impossible to reconcile the existence of beams with an undisturbed solar dipole field. Instead we are led to assume that the field falls off much more slowly than a dipole field. A possible model of the interplanetary magnetic field has been given elsewhere (ALFVÉN 1956). As we do not know with certainty according to which law the field falls off, the magnetic field near the earth's orbit cannot be derived directly from the value of the magnetic field at the sun. The cosmic ray value of the field seems not to be in conflict with any other observations.

As the earth's field at the distance of the ring current is of the order 10^{-3} gauss, a beam field which is much smaller than this may appear to be negligible. This is not necessarily correct because when approaching the earth the beam is braked with the result that the gas of the beam is compressed. This process causes an increase in magnetic field at the same time.

CHAPMAN (1952) considers a beam field of 10^{-6} gauss to be negligible. As we have seen, this conclusion is doubtful. Moreover, cosmic ray evidence indicate that the beam field often is 10^{-5} gauss or even more.

§ 2. *Beam properties and the conservation of magnetic flux*

During an auroral display the ionosphere is hit by clouds of particles which obviously arrive from above, at least in part. The energy of these particles may be of the order of 10^5 eV, which means a rigidity of $H\varrho = 10^3$ gauss cm for electrons and $H\varrho = 4 \cdot 10^4$ gauss cm for protons. In the earth's field they spiral along the lines of force. Even near the equatorial plane, where the magnetic field reaches a minimum of about 10^{-3} gauss for a line through the auroral zone, the radius of curvature does not exceed 10^6 cm for the electrons and $4 \cdot 10^7$ cm for the protons.

All theories of magnetic storms agree in postulating a ring current in the equatorial plane during the main phase. Although the pictures about the formation and the structure of it differ, the theories also seem to agree that the current is due to the differential motion of electrons and ions in a cloud of ionized gas. The thickness of the ring is estimated by Chapman and Ferraro to about $3 \cdot 10^8$ cm. The gas temperature may be rather low so that the rigidity of the particles does not exceed the values mentioned above. In the electric field theory the ring is asymmetric, consisting of electrons with energies of up to 10^5 eV and protons with probably lower energies.

Thus auroral observations indicate, and a reasonable interpretation of magnetic disturbances confirm, that during a storm there are extended clouds of ionized gas in the earth's magnetic field. One of the fundamental problems in magnetic storm theory is how these clouds have been brought into the earth's magnetic field, and a still more fundamental question is how a magnetic field has been brought into the gas clouds, in other words, how the clouds have been magnetized.

In order to realize the importance of this question, let us consider a closed line limiting a region which is much larger than the radius of curvature of the particles. If the line follows the motion of the cloud, the magnetic flux inside it must remain constant under very general conditions (high conductivity, etc.), which are likely to be satisfied in most problems in cosmical physics. If a cloud expands isotropically by a linear factor α, the density decreases as α^{-3}. At the same time the magnetic field decreases as α^{-2}. Hence the particle density n is proportional to $H^{3/2}$;

$$n \sim H^{3/2} \tag{1}$$

This holds only for an isotropic expansion or contraction. An expansion along the field lines may change n without affecting H.

Tracing backwards the history of the magnetized clouds in the terrestrial magnetic field, we should expect that they have initially been part of the beam. If we accept this, it seems natural that the clouds were magnetized already when they were parts of the beam. In other words, the beam should have a frozen-in magnetic field.

The invariance of the flux offers no difficulty to the electric field theory. In fact, the theory is based on the flux invariance in a volume of magnetized gas, which can be followed from the solar corona the whole way to the border of the forbidden region near the earth. In the solar atmosphere the beam is formed of ionized gas, which is already magnetized by the general magnetic field (or by a sunspot field).

When the beam is emitted, the density as well as the magnetic field decreases very much. When the gas enters the terrestrial field a compression occurs, which is connected with the increase in field strength.

In Chapman-Ferraro's theory the demand for the flux invariance seems to constitute a difficulty, which may be superable in the case of the solar phenomena but is much more serious for the terrestrial phenomena. Chapman considers the beam to be formed of solar gas (an idea which—as mentioned above—has been taken up by the electric field theory) and he gives a value $\alpha^3 = 10^5$—10^8 for the volume expansion (loc. cit.). If the expansion is isotropic the magnetic field should decrease by a factor $\alpha^2 = 10^{10/3}$—$10^{16/3}$. Thus in order to come down to a beam field of 10^{-6}, which Chapman considers to be negligible, the beam must originate in a region where the field is much lower than 1 gauss, or the expansion must be strongly anisotropic. Both alternatives are not very probable but by no means unreasonable. However, as the gas in the beam becomes compressed when it is braked by the earth's magnetic field, the beam field increases, and there is no reason to assume that a primary field of 10^{-6} gauss is negligible.

§ 3. *The main phase*

The flux invariance introduces a much more serious difficulty in the treatment of the phenomena in the earth's magnetic field. As far as the strict calculations by Chapman and Ferraro go, the ionized matter of the beam cannot penetrate into the earth's magnetic field. What happens according to their analysis, which no doubt is correct, is that the gas of the beam pushes the field before it, but remains non-magnetized itself. FERRARO (1952) has made a calculation where he divides the approaching stream into a series of sheets and shows that the front can advance in a magnetic field. It seems that this process rather indicates that the field is pushed away than that the gas becomes magnetized. In any case he has not shown clearly how the flux invariance can be avoided.

Chapman and Ferraro suggest that electrons and ions start from the front of the beam and produce the ring current, but they do not claim to have shown how this process occurs, not even that the process is possible in principle. The ring current flows in a gaseous ring which is *permeated by the earth's magnetic field*. This is essential, because a non-magnetized gas ring will no doubt be strongly diamagnetic with the result that it breaks into pieces and is expelled from the dipole field very rapidly. At this point the theory encounters the fundamental difficulty that the non-magnetized gas of the beam must be transformed into the magnetized gas of the ring. How serious this objection is, can be shown by calculating the time constant T of the magnetization of a gas cloud. We have

$$T = \sigma\left(\frac{l}{c}\right)^2$$

where σ is the conductivity and l the linear dimension of the cloud and c the velocity of light. The conductivity is

$$\sigma = 1.4 \cdot 10^7 \, T^{3/2}$$

where T is the temperature. Chapman has not given any value for the temperature of the beam, but he has estimated the temperature of the interplanetary gas to $2 \cdot 10^5$ degrees. This may give the order of magnitude for the temperature of his beam and ring. With $l = 3 \cdot 10^8$ cm we find

$$T = 10^{11} \text{ sec.} = 3{,}000 \text{ years.}$$

This is the time in which the field in a cloud decays, but it is also a measure of the time required to magnetize a non-magnetic cloud by putting it into a magnetic field.

This does not mean with certainty that a magnetized cloud could not be produced in a shorter time—experience teaches us that theorems of the impossibility of a certain process are dangerous in astrophysics. In fact a magnetized cloud in the earth's magnetic field could be produced by ionization of a non-ionized gas which is situated in the earth's field. This is the process which Hulbert's ultraviolet light theory of magnetic storms makes use of, so this theory does not violate the theorem of flux constancy. Another possibility is that the cloud is divided into very small units, which are magnetized, or that the temperature or degree of ionization is very low in the ring-cloud. None of these possibilities seem to be attractive, because we have to change T by eight powers of ten in order to make it comparable to the real time of formation of the ring current. Until the contrary has been shown, it seems legitimate to conclude, that the theorem of flux invariance prevents the main phase of a storm to develop from the initial conditions assumed by Chapman and Ferraro.

The difficulties of Chapman-Ferraro's theory are increased still more by the fact that their postulated ring current is unstable, as shown by Herlofson (1957). Even if some mechanism could be found, by which the ring current could be produced, this would not be of much help because the ring current will break up in less than 30 seconds, according to Herlofson.

The instability demonstrated by Herlofson does not occur for a ring current of the type of the electric field theory because of the trochoidal motion, which is stable.

The following point of view may also be of interest. In Störmer's theory of the aurora one of the difficulties was that particles with a reasonable energy (10^5 eV) could reach the earth only in very small regions around the geomagnetic poles. By assuming that the beam is non-magnetized Chapman and Ferraro meet the same difficulty, only in a slightly different form. The non-magnetized gas can reach singular points at the axis and very small region around them. Certainly the pressure which the beam exerts on the field may change the shape of this field so that the singular points are displaced (the "horns"), but in principle the theory does not allow the occurrence of magnetized clouds in the earth's magnetic field far away from the field lines through singular points.

Our conclusion is that *if* the earth is approached by a cloud which has the properties which Chapman and Ferraro assume, there is no doubt that those phenomena will occur which they have found by their admirably stringent analysis: A hollow will be formed and the earth's magnetic field will be disturbed in a certain way. On the other hand, there is no support for their suggestion that a ring current will be formed and the main phase of a storm will develop. On the contrary, there are strong arguments for the view that a state resembling the main phase would *not* be produced by a beam of the assumed type.

It should further be observed that no convincing arguments have been given that beams with the assumed properties exist. Certainly at the time when Chapman and Ferraro made their theory it was natural to assume that the beam was non-magnetized, but our increased knowledge of magneto-hydrodynamic phenomena in astrophysics makes this assumption look improbable today. As shown above it is much more likely that the beam is magnetized.

§ 4. *The initial phase*

If independent arguments had been given for the existence of beams of Chapman-Ferraro's type, and if there had been a reasonable probability of making a theory of the main phase along their lines, their theory of the initial phase would have been attractive. As these premises are not satisfied, there are good reasons to doubt that the phenomena they treat has anything to do with a magnetic storm and it is legitimate to scrutinize what support there is for their claim that their theory does describe the initial phase of a storm.

According to the theory the arrival of a beam should disturb the magnetic field and currents should be produced at the front surface of the beam, which should result in an increase in the horizontal component.

This is all which the theory has given so far.

§ 5. *Summary of the criticism of Chapman and Ferraro's theory*

No one can study Chapman-Ferraro's theory without the deepest admiration for the stringent analysis they have given and no one can avoid to conclude that they really describe what would happen, if the conditions they assume were realized. Yet at the same time at least the present author cannot avoid the feeling that their theory has nothing to do with a magnetic storm. To summarize what is said above, the reasons for this view are:

1. Cosmic ray observations indicate that the beam has other properties than supposed by Chapman and Ferraro.
2. The initial phase is not adequately described by the theory.
3. A development of the theory into a theory of the main phase will probably be impossible without violating the theorem of flux invariance.
4. As Herlofson has shown Chapman-Ferraro's ring current is unstable so that even if it had been possible to create it, it would break up in less than 30 seconds.

B. The electric field theory

§ 6. The electric field theory of magnetic storms was presented in a paper 1939 (Part I) and developed in two papers 1940 (Part II and III). These three papers give a brief sketch of a theory of the emission of a beam from the sun and a more detailed outline of the phenomena near the earth during the main phase of a magnetic storm.

The theory of the emission of the beam cannot be upheld today. The development in magneto-hydrodynamics has shown that magnetic fields may be frozen-in, and this changes essentially the picture of the properties of the beam when it is emitted from the sun.

On the other hand, the theory of the phenomena near the earth seems to be essentially correct. The new knowledge accumulated since the first presentation of the theory has given good support to it, and only some details call at present for a revision. Even the numerical values given in the cited papers seem acceptable today. However, a more careful treatment of some of the phenomena is urgently needed especially as parts of the theory seem to have caused some misunderstanding. It is the purpose of the following paragraphs to elucidate some obscure points.

The scale-model experiment by MALMFORS (1946) gave a good support to the general lines of the theory. Some of his results (about the self-sustained discharge) seemed to call for a modification of the theory. This was the state when a new review of the theory was written (ALFVÉN 1950). However, the subsequent experiments by BLOCK (1955) supported the original theory.

In 1953 the theory was developed so that it also gives at least an outline of the initial phase. In this connection the existence of an inner auroral zone was predicted, and independent investigations by NIKOLSKIJ (1955) seem to give some confirmation of this.

§ 7. *Objections by Cowling and Chapman*

COWLING (1942) has made several objections against the electric field theory. The most important of these are:

1. If the particles move with $\mu = eV/H = $ const. as is supposed in the theory, they cannot reach the ionosphere unless their energy is extremely large.

This objection will be discussed in detail in § 8. The result is that the objection is correct under the condition that the particles really move exactly according to the idealized mathematical model, but that already very small disturbances are enough to allow the particles to reach the ionosphere. In other words, the objection hits the earlier formulation of the theory but not its physical content.

2. If positive charges entering the forbidden zone build up space charges strong enough to drive charges to the auroral zones, they must be strong enough to modify the electric field near the forbidden region seriously.

This objection can be met with a reference to Block's experiment, which shows that even when space charge is of great importance, as it is in the light discharge, the basic pattern of the motion is not changed. However, COWLING (1957) does not accept such a reference to an experiment. I think that this attitude is partly justified, because in a final theory we want to express all phenomena in clear mathematical formulae and not by a reference to an experiment. On the other hand, as we are still far from a definite theory, a reference to an experiment is allowed. The experiment demonstrates that an assumed process is physically possible, and this shows that even if the objection hits the present formulation of the theory, it does not hit the essential physical picture. In other words, the experiment makes probable that we may find another formulation, which is free from the objection. An attempt to satisfy Cowling's justified request for a more clear picture is made in § 10. The discussion leads to a more detailed theory of the aurorae.

3. Cowling has objected against the mechanism of ejection of the beam according to the paper of 1939. This objection is fully justified. As pointed out in § 6, this part of the theory cannot be upheld today.

4. Cowling has also objected against the shape of the forbidden region on the night side. This criticism of a detail in the theory will be discussed further in a following paper.

Against the electric field theory Chapman (1952) has made a generally formulated objection which he considers so fatal that he does not consider it worth to take up any particular part of the theory to a serious discussion. This objection is that "the theory goes astray at the outset" because "Alfvén ignored the powerful electrostatic forces existing between the positive and negative charges" (loc. cit. p. 207). If Chapman's objection is only a summary of some of Cowling's objections, I hope that the present paper will clarify the matter. If not I must invite Chapman to specify his criticism so that it is possible to discuss it.

§ 8. *Isotropy of electron velocities*

Electrons invading the earth's magnetic field under the influence of an electric field will gain energy from the electric field, so that their velocity perpendicular to the magnetic field increases. (This is only another way of describing that when the cloud enters the magnetic field it is compressed and that the increase in the magnetic field accelerates the electrons by betatron action.) During this motion their velocity parallel to the field remains constant. Consequently, when they arrive to the neighbourhood of the forbidden region, their velocity vector is almost parallel to the equatorial plane. This means that the amplitude of oscillation perpendicular to the equatorial plane is small. Hence an electron cannot go from the equatorial plane along the lines of force to the upper atmosphere of the earth.

From this COWLING (1942) concludes that a discharge from the equatorial plane to the ionosphere is impossible, and this is one of his most important objections against the theory.

This objection would no doubt be valid if no other effects were of importance. However, already a very small disturbing effect is enough to make the electron velocity distribution more or less isotropic or even Maxwellian with the result that electrons will move along the field lines and reach the earth.

As the electrons need a few hours in order to reach the neighbourhood of the forbidden region and as their velocity is of the order of 10^{10} cm/sec, the path is of the order 10^{14} cm. Hence the question is whether the electrons could move 10^{14} cm without changing the angle between their velocity and the magnetic field.

Taking account of the high energy of the electrons (10^5 evolt) and the low density in space, we could say with certainty that the ordinary mean free path is much larger than 10^{14} cm, so that no change in velocity is accomplished by collisions with other particles. In spite of this it seems reasonable to assume (as was done already in the earlier presentations of the theory) that there exist magnetic field irregularities strong enough to accomplish a change in velocity. An electron will complete 10^7 turns with a radius of curvature of 10^6 cm. If the angle between H and the velocity vector changes statistically by only 0.1 degrees for each turn, e.g. due to irregularities in the electric or magnetic field, the changes will add up as random contributions and produce an almost completely isotropic velocity distribution. The fact that the aurora often has an irregular and rapidly changing structure, and the fact that magnetograms often show very large oscillations favour the view that the path of the electrons is likely to be disturbed.

In fact, a numerical calculation shows that only if extreme assumptions are made concerning the regularity of the fields, this conclusion can be avoided.

Recent results by GABOR, ASH and DRACOTT (1955) give additional evidence of scattering processes. In their classical investigations on plasmas, Langmuir and Mott-Smith have shown that in a discharge tube the plasma sets up a thin boundary sheath which separates the plasma from the wall or from a probe. In the boundary sheath there is an electric field which accelerates the ions towards the wall but retards the electrons, so that in case the wall is an insulator, the same number of electrons and of ions reach it. Inside the plasma the electric field is small. The electron temperature is usually much higher than the ion temperature.

The plasma theory of Langmuir is founded on the assumption that the velocity distribution of the electrons is Maxwellian. Experimental investigations show that this assumption is usually rather well satisfied. In a plasma at so high pressure that the mean free path is small compared to the linear dimensions, this is a natural result of the collisions between the particles in the plasma. However, it is observed experimentally that even at so low pressures that theoretically the collisions are too rare to give any considerable energy exchange, the velocity distribution is Maxwellian to a high degree of approximation. This is what Gabor, Ash and Dracott call Langmuir's paradox.

Gabor, Ash and Dracott have made a series of experimental investigations in order to make this paradox understandable. Their result is that the energy exchange takes place in an oscillating boundary layer, the existence of which they have demonstrated with an electron beam used as a probe. They conclude that "plasma-boundary oscillations are likely to arise at almost any boundary, not necessarily abrupt or permanent, which is in contact with a plasma however slightly off equilibrium. These oscillations constitute a powerful mixing, entropy-increasing mechanism."

It seems reasonable that their results are applicable also to the boundary between our ionized gas and the forbidden region. Hence we should expect that an oscillating sheath is produced which makes the electron distribution Maxwellian at least near the forbidden region. Whether even far outside the forbidden region a similar phenomenon takes place is more difficult to decide. It is a possibility worth considering that during the whole time when the cloud invades the magnetic field the electron distribution is isotropic. Under this assumption BLOCK (1955) has calculated the motion of the ionized cloud in the terrestrial magnetic field. This motion is rather similar to the motion with μ = const. For the present discussion it is, therefore, not very important whether we assume approximate isotropy in the whole space or only close to the forbidden region.

We may summarize the arguments for a redistribution at least near the forbidden region in the following way:

1. If an electron when it completes a circle with a radius of curvature of 10 km, changes its velocity vector in an irregular way by as little as 0.1 degrees, a good isotropy will be produced.

2. Observed fluctuations in the aurora and magnetic records make it likely that the path of the electrons will be disturbed in an irregular way.

3. Oscillations of Gabor's type near the forbidden region will easily produce a Maxwellian distribution.

It should be stated clearly that none of these arguments is absolutely convincing. Our present knowledge of a plasma is not sufficient to rule out Cowling's objection. However, the arguments seem to be so plausible that it is worth while to continue our analysis under the assumption that at least near the forbidden region the velocity distribution is Maxwellian. If the distribution is isotropic but not Maxwellian, essentially the same results will be obtained.

§ 9. The voltage of the earth in relation to the plasma

We shall now study what happens in a flux tube through an element F, if the velocity distribution of the electrons is Maxwellian near the equatorial plane.

The pressure of the ionized gas, which essentially is due to the electrons, will tend to make the gas expand along the field lines, so that the whole tube will be filled by the ionized gas. The time scale of this process is

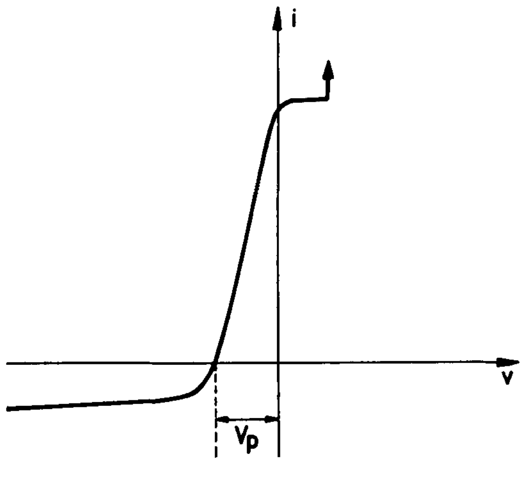

Fig. 1.

difficult to estimate. Near the forbidden region the length of a field line from the equatorial plane to the ionosphere is of the order 10^{10} cm. A velocity of the front of 10^6, 10^7 or 10^8 cm/sec would give 10^4, 10^3 or 10^2 seconds.

The electron temperature T_e is

$$T_e = \frac{2}{3}\frac{eV_e}{k}$$

where k is Boltzmann's constant. eV_e is of the order 10^4—10^5 eV. The mean free path of the electrons is very long (10^{14} cm) compared to the dimensions of the flux tube (cf. § 8). It is difficult to determine the velocity of the ions. As earlier we assume that the kinetic energy of the spiralling motion of the ions is much smaller than the electron energy. The number of neutral atoms is probably so small that it is negligible.

Suppose that in the ionized gas we put a plane conducting electrode perpendicular to the magnetic field. This "probe" will be hit by electrons and ions moving parallel to the magnetic field, and the current to it may be determined according to Langmuir's theory of a probe in a plasma (Fig. 1). If the probe is insulated so that the electron current must be equal to the ion current (ambipolar diffusion) the probe is negative in relation to the plasma, and the voltage difference V_p is a few times the voltage $V_e = \frac{3}{2}\frac{kT}{e}$ which corresponds to the electron temperature T. The voltage drop is confined to a thin sheath with thickness of the order

$$\Delta \equiv \sqrt{\frac{V_e}{en}}$$

and the electric field is small in the rest of the plasma.

The probe will be hit by the Maxwell tail of the electrons (energies above eV_p) and by positive ions. The ions will impinge on the probe with the kinetic energy eV_p (to which should be added their thermal energy which we have assumed to be small).

In nature we have no solid probe. Instead the invading plasma will at first be limited by a free surface or perhaps a shock front. Later it will reach the ionosphere, and the ionosphere will to some extent replace a solid probe. There is a considerable difference, however, because the ionosphere will emit electrons if acted upon directly by the plasma sheath. The analysis of the details of these complicated phenomena must be reserved for a future investigation. The only certain conclusion which can be drawn now is, that even if the solid probe is replaced by an element of the ionosphere, there must still be a voltage drop of the order V_p, because if this were not present the plasma electrons would leave the flux tube in a very short time and an unreasonable high positive space charge would be produced. The voltage drop will accelerate positive ions. Hence the ionosphere will be hit by electrons and positive ions with energies of the order of 10^4—10^5 eV in a typical case. It is reasonable to identify these particles with the protons observed by Gartlein and Meinel (1952) in some types of aurorae.

Let us now consider the case when we have two flux tubes containing exactly similar plasmata and being both in contact with the ionosphere, which is supposed to be an infinitely good conductor. In both flux tubes there will be an ambipolar diffusion to the ionosphere. The plasmata in both flux tubes will be charged to the voltage $+ V_p$ in relation to the ionosphere.

Let the two flux tubes be situated at the border of the forbidden region, one on the day side and the other symmetrically at the night side (Fig. 2). Let us further suppose, in accordance with the theory, that there is a net flow of positive ions into the flux tube at

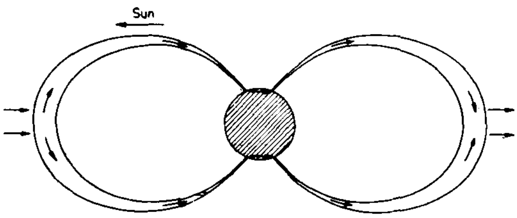

Fig. 2.

the dayside and a net outflow of positive ions from the flux tube at the night side. Both flows take place especially near the equatorial plane. The drift of the electrons does not produce any net flow into the tubes.

The increase of positive charge in the flux tube on the dayside will increase its voltage to $V_p + \Delta V_p$ in relation to the ionosphere. In a similar way the flux tube at the night side will attain a lower voltage $V_p - \Delta' V_p$. This causes a current from the day side to the ionosphere and further on to the night side. This is the current over the auroral zone in the electric field theory.

An ordinary probe is hit by both electrons and positive ions not only in the case of ambipolar diffusion but also when a net current (below a certain limit) flows to the probe. It is reasonable that a similar phenomenon occurs in our case. This means that we may expect the auroral zone to be hit by both electrons and ions on the dayside as well as the night side. On the day side the number of ions is larger than the number of electrons and on the night side the reverse is the case, so that a current over the auroral zone is produced. As mentioned above it is beyond the scope of this paper to analyze the very interesting details of the interaction between the plasma and the ionosphere, but some important features may be pointed out. It is possible that a plasma sheath may be situated very high up in the ionosphere, or perhaps in space far above the ionosphere, and that from this sheath the ionosphere will be bombarded by positive ions and electrons. This state would produce a quiet auroral arc, excited by ions and electrons impinging on a "passive" ionosphere. It is also possible that under certain conditions (e.g. when a large current is drawn) the plasma sheath is broken up so that the voltage drop occurs in an irregular way in the ionosphere itself. This would perhaps give the vivid auroral forms in which the ionosphere plays an active role in the discharge. The different auroral forms may be accounted for as due to the direct action of the electric field. This would be in accordance with some of the views expressed by CHAMBERLAIN (1956) and by LEBEDINSKY (1956).

§ 10. Influence of space charge

One of Cowling's most important objections against the electric field theory is that in order to produce a discharge over the auroral zones a large space charge is necessary and this space charge will seriously disturb the drift motion in and near the equatorial plane. This objection is quite plausible from a qualitative point of view, but a quantitative analysis, which we shall make here, will show that it is not serious. This is due to two circumstances:

a) The drift motion is very insensitive to field disturbances.

b) Only a small space charge is necessary in order to produce a discharge over the auroral zones.

In order to find how much the drift motion is disturbed by a space charge, let us put the voltage V in the equatorial plane

$$V = V^1 - Ex \tag{1}$$

in the undisturbed case (see Fig. 3). Here V^1 is the voltage at the y-axis. Electrons entering the earth's magnetic field along the asymptote $x = x_0$ will pass the line $x = x_1$ at a point $(x_1 y)$ given by

$$\frac{H}{H_0} = \frac{V_0 + E(x_0 - x_1)}{V_0} \tag{2}$$

$$\left(\frac{\lambda}{R}\right)^3 = \frac{E(x_0 - x_1)}{V_0} \tag{3}$$

where H_0 is the magnetic field at infinity (in the beam), V_0 the energy of the spiralling motion of the electrons in the beam, $\lambda = \sqrt[3]{\frac{a}{H_0}}$ and a the dipole moment of the earth. Further,

$$H = H_0\left[1 + \left(\frac{\lambda}{R}\right)^3\right] \tag{4}$$

is the field at the point $(x_1 y_1)$.

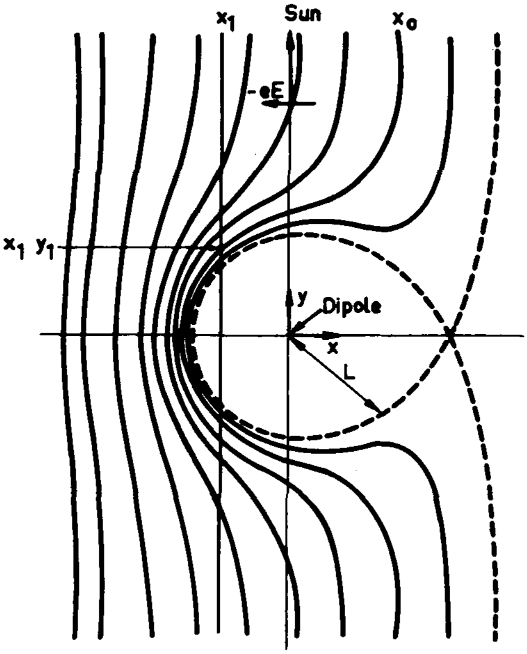

Fig. 3.

Suppose that an accumulation of ions or electrons in the environment of $(x_1 y_1)$ changes the voltage of this region by ΔV. The result is that the electrons will cut the line $x = x_1$ at another point $(x_1 y^1)$ given by

$$\left(\frac{\lambda}{R}\right)^3 = \frac{E(x_0 - x_1) + \Delta V}{V_0} \qquad (5)$$

As this formula shows, the value of R is insensitive to changes in V. In fact, if ΔV is as much as 50 % of $E(x_0 - x_1)$, this produces a change in R by only about 15 %. If we project R along the field lines to the auroral zone, we find that the change in the polar distance is only 7 %, i.e. about 1.5°, in latitude, or 160 km.

If space charge is accumulated in a systematic way so that all along the border of the forbidden region a similar charge is accumulated, a systematic displacement of the auroral zone is produced, which is so small that with the present accuracy of the theory it will be impossible to detect it. If on the other hand the space charge is only local, so that the voltage is changed only in the environment of the point $(x_1 y_1)$ a local displacement of the aurorae by 160 km will be produced. A displacement of this size is a normal occurrence at most displays, especially in connection with the formation of draperies.

Hence we have found that a change in voltage by a factor of 1.5 produced by space charge, will give no remarkable change in the drift motion. This means that we could allow space charge to produce a voltage of 5—50 kV in the equatorial plane in a typical case ($eV = 10^4$—10^5 e volt), and still consider the theory to be valid as a first approximation.

We shall next see whether an electromotive force of 5—50 kV is enough to produce a discharge over the auroral zones.

According to the theory the drift of the ions is confined to a plane x = const., which is the plane of fig. 2. The ions arrive from the left in the figure with the drift velocity

$$v_L = cE/H_L \approx v_0 \left(\frac{L}{\lambda}\right)^3 \qquad (6)$$

where $H_L = H_0\left[1 + \left(\frac{\lambda^3}{L}\right)\right] \approx H_0 \frac{\lambda^3}{L^3}$ is the field at the border of the forbidden region, which is situated at about the distance L from the dipole, and v_0 is the velocity of the beam. With $v_0 = 2 \cdot 10^8$ cm, $L/\lambda = 4$ (compare ALFVÉN 1955, p. 55) we obtain

$$v_L = 3 \cdot 10^6 \text{ cm/sec.} \qquad (7)$$

For protons this translational velocity corresponds to a kinetic energy of only 4 eV.

According to the theory the ions cannot continue their motion towards the night side (in fig. 2) because when they reach the border of the forbidden region, their space charge is no longer compensated by electrons. Instead they must go parallel to the field lines to the ionosphere in the auroral zone. The velocity v_A of this motion must be at least of the same order as v_L. In order to confine the motion towards the auroral zones to a resonably thin layer we should require that v_A is at least a few times v_L, say

$$v_A \geq 5\, v_L \qquad (8)$$

or, with the numerical value from (7)

$$v_A \geq 1.5 \cdot 10^7 \text{ cm/sec.}$$

The kinetic energy of protons with this velocity is 100 eV. This means that the space charge need only produce a potential of 100 volt near the equatorial plane in order to send away the ions towards the auroral zones with a sufficient energy.

Thus we see that only a very small fraction of the voltage (5—50 kV) from a reasonable space charge is enough to send the ions away from the equatorial plane towards the auroral zones. As the mean free path of the ions in the plasma is long compared to the distance from the equatorial plane along a field line to the auroral zone, the motion will not be impeded by collisions. On the other hand, the increase in magnetic field when we approach the ionosphere will brake the motion and reflect the ions in case their spiralling motion is large enough. Suppose for a moment that the ion temperature is zero and the space charge field acts only in the direction of the magnetic field. Then the effect of the magnetic inhomogeneity is negligible. The ions move along the magnetic field lines to the plasma sheath, and when passing this they become accelerated before they impinge on the ionosphere. Thus in this case only a very small fraction of a reasonable space charge voltage is enough to produce a discharge to the auroral zone.

If the ion temperature differs from zero and the space charge also accelerates the ions in the plane perpendicular to the magnetic field, the discharge will probably require a larger electromotive force. Only a detailed analysis, which it is beyond the scope of this paper to give, can show how high ion temperature is permissible without impeding the discharge. As at present the ion temperature is unknown, it would not be possible to check an obtained value of this quantity.

A similar discussion shows that also the electrons in the plasma are easily set into motion even by small fields.

Our ignorance of the ion temperature and the fact that the theory of a hot plasma in contact with the ionosphere is still undeveloped makes it impossible to state that the conditions during a storm necessarily are such as to make a discharge possible. However, as in that case which we have treated only 100 volt out of available 5—50 kV are necessary for the discharge, we may be confident that the conditions often are such as to allow a discharge of positive charge to the ionosphere.

An accumulation of negative charge in or near the equatorial plane at the night side may give rise to a slightly different phenomenon. As the electrons are easily movable a very small field is enough to send them away towards the auroral zones. When they reach the plasma sheath, most of them are turned back by the opposing voltage, but they transfer the voltage from the equatorial plane to the sheath. Hence an accumulation of negative space charge in the equatorial plane will diminish the voltage V_p over the sheath (compare Fig. 1), with the result that a current of electrons flows to the ionosphere.

What has been said makes it reasonable that a space charge, which is small enough not to disturb seriously the drift in the equatorial plane may suffice to produce a discharge over the auroral zones. To this should be added the support we get from the similarity with Block's experiment, and the fact that the mechanism we are considering gives a fairly good picture of the basic phenomena observed during auroral displays and magnetic storms.

§ 11. *Remarks on frozen-in field lines of force*

In a fluid with infinite electric conductivity the magnetic field lines can often be considered as "materialized" or "frozen-in". A discussion of the behaviour of the field lines (the "spaghetti" method) gives a good survey of many problems, such as magneto-hydrodynamic waves travelling along the field lines, changes in the field geometry when the fluid moves, etc. It is of interest to note that in the present problem the limitations of this spaghetti method are obvious. In the two-dimensional case (motion in the equatorial plane) which is treated in the electric field theory, the motion of the field lines of the initial beam can be followed. When the ionized gas passes the λ-region or approaches the L region, the electrons move in a way which is different from the ions. Especially in the L region, this difference is large: the electrons move along almost circular arcs around the dipole whereas the ions move in straight lines up to the forbidden region. At least in a stationary state it is not very important whether we consider the field lines to follow the motion of the electrons or of the ions. In the former case we could follow the motion of the field lines from the initial beam all way round the earth and further on in the continuation of the beam.

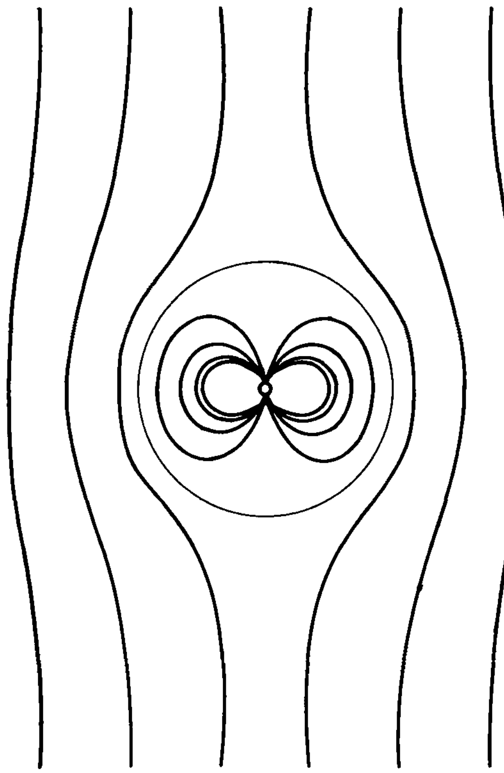

Fig. 4.

In the latter case the field lines move up to the dayside of the forbidden region, but as the matter frozen on them is discharged there, we lose track of them. On the nightside of the forbidden region new field lines are materialized and move on outwards.

Hence there is an ambiguity but no real difficulty in the two-dimensional case. If from this we turn to the three-dimensional case an apparent difficulty occurs (Fig. 4). As long as the ionized matter is outside the sphere $R = \lambda \sqrt[3]{2}$, the field lines go from the equatorial plane to infinity (or in reality in some way to the sun), whereas inside the sphere they go to the earth. When the beam has passed the distance $\lambda \sqrt[3]{2}$, field lines from the earth should go to the cloud, and if the same field line is frozen into the ionosphere at the same time as in the cloud, the advance of the cloud would be impossible.

In order to solve this paradox, let us consider two conducting bodies A and B in a homogeneous magnetic field H (Fig. 5). Suppose that A moves downwards perpendicular to the plane of the paper and B upwards. The polarisation $v \times H$ produces electric fields E with opposite directions in A and B.

If space between A and B is a good electrical conductor, currents will be produced which tend to brake the motion. In case of infinite conductivity we could regard the lines of force to be frozen in both in A, in B and in the medium between, and a well-known type of discussion will show that the breaking could be envisaged to be produced by elastic field lines.

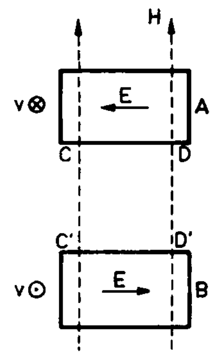

Fig. 5.

On the other hand, if space between A and B is electrically insulating, no current will result from the polarizations, and the motions will not be braked. The voltage difference V_C between C and C', and the difference V_D between D and D', will not be equal: $V_C \neq V_D$. We could say that as space between A and B is insulating, the lines of force could not be identified, and it is of no use to speak of a continuation of a certain line of force from A to B.

The same holds also in the case that the medium between A and B is conducting under the condition that there is a double layer, which takes up the voltage difference produced by the polarisation. This is what occurs in the case of a plasma invading the earth's magnetic field. Between the plasma and the ionosphere there is a plasma sheath which is able to take up a voltage difference of the order of 10^5 volt. This makes possible a relative motion between the plasma and the ionosphere. This means that the lines of force are "cut" by this plasma sheath so that a "slip" in the motion is allowed. In the mathematical treatment it is important to introduce the temperature of the ionized gas and the voltage difference which is produced by the inhomogeneous temperature.

I wish to thank Professor Cowling for discussions and correspondence. His criticism has helped me to clarify the ideas and develop the theory. (This does by no means imply that Professor Cowling accepts the theory even in its present form.)

Like most of my papers the last years, this paper has been written in close contact with Dr. Herlofson. I have also profited from numerous discussions with Dr. Lehnert and Dr. Åström.

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