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Integrating Debt Dynamics and Planetary Boundaries Through a Catastrophe Theory Model of Financial Ecological Collapse

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Abstract

This paper develops a novel nonlinear dynamical system to analyze the interaction between financial instability and ecological overshoot. Using catastrophe theory and bifurcation analysis, I demonstrate how accelerating debt growth and environmental degradation create hysteresis loops, leading to irreversible economic collapse. This study introduces the first catastrophe theory framework unifying Minsky's financial instability and planetary boundaries, identifying dual critical thresholds in debt/GDP ratios (ϕ>0.07) and carbon budgets (ϵ > 0.12) that trigger irreversible collapse.

Keywords: Financial instability, planetary boundaries, catastrophe theory, debt dynamics, nonlinear systems

1 Introduction

The 21st century is defined by two existential crises: the accelerating fragility of global financial systems and the irreversible degradation of Earth's ecological systems. As theorized by Minsky (1992) and quantified through nonlinear debt dynamics by Keen (2013), financial instability has reemerged as a systemic threat reminiscent of the 2008 Global Financial Crisis and subsequent sovereign debt spirals. Concurrently, humanity has transgressed six of nine planetary boundaries critical for maintaining Earth's Holocene-era stability, including biosphere integrity and climate change (Rockström et al., 2009; Steffen et al., 2015). These crises are often analyzed in isolation, with macroeconomic models neglecting ecological feedback loops (e.g., DSGE frameworks) and environmental studies oversimplifying financial drivers of resource exploitation (Raworth, 2017). This disciplinary siloing obscures a critical reality that financial and ecological systems are co-evolving, with debt-driven growth amplifying ecological overshoot and biophysical collapse eroding economic resilience.

Conventional economic paradigms, such as neoclassical growth theory and environmental Kuznets curves, assume separability between financial and ecological variables. They assume that technological innovation or market pricing mechanisms (e.g., carbon taxes) will decouple growth from environmental harm (Nordhaus, 2017). However, empirical evidence contradicts this optimism. Since 1990, global debt-to-GDP ratios have surged from 200% to 356% (BIS, 2023), while material footprint per capita has grown by 65%, exceeding safe planetary boundaries by 82% (UNEP, 2021). This correlation leads nations to a vicious cycle: debt-financed consumption accelerates resource extraction, while ecological scarcity (e.g., energy shocks and crop failures) exacerbates debt defaults. Yet, no existing model formalizes this interdependence as a unified dynamical system.

This paper bridges this gap by integrating Minsky's financial instability hypothesis (FIH) with ecological economics. Unlike prior efforts that treat financial and ecological systems as separate (e.g., Keen, 2013; Rockström et al., 2009), I apply catastrophe theory, specifically cusp bifurcation, to model their codependent collapse. This approach captures hysteresis, where temporary policy errors (e.g., excessive QE or carbon overshoot) permanently destabilize the system. This is a key dynamic absent in linear or agent-based frameworks. Building on Keen's (2013) debt-driven macroeconomic models, I introduce ecological stock variables and nonlinear tipping points into a coupled differential equation system. My framework addresses three unresolved questions in the literature: (1) How do compounding debt obligations distort resource allocation in ways that accelerate ecological overshoot? (2) Under what parameters do financial and ecological subsystems generate mutually reinforcing collapse dynamics? (3) Under what conditions do debt growth and ecological overshoot jointly trigger systemic economic collapse?

By synthesizing Minskyian finance and Earth system science, I challenge the neoclassical assumption of equilibrium resilience. Using catastrophe theory, I demonstrate that financial-ecological systems exhibit hysteresis, i.e., once critical thresholds in debt-to-income ratios or carbon budgets are breached, recovery becomes pathologically impossible without radical structural intervention. This finding has urgent policy implications, as nations such as the United States and Japan now operate at both financial (debt/GDP > 130%) and ecological (carbon footprints > 100% of planetary boundaries) risk levels (World Bank, 2023; Global Footprint Network, 2023).

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2 Literature Review

2.1 Financial Instability Hypothesis

Minsky's (1992) Financial Instability Hypothesis posits that capitalist economies inherently evolve from stability to fragility through endogenous credit cycles. He argued that periods of economic tranquility encourage riskier borrowing, leading to speculative and Ponzi financial structures that amplify systemic collapse during downturns. While Minsky's framework was qualitative, Keen (2013) formalized these dynamics using nonlinear differential equations by modeling debt as a destabilizing feedback loop. Keen's work demonstrated that exponential debt growth relative to GDP $(dD/dt > gY)$ where g is output growth, inevitably precipitates into crises, as seen in the 2008 global financial collapse (Keen, 2017). Subsequent studies, such as those by Lagoarde-Segot (2019), have expanded this to include macroeconomic variables like income inequality. However, ecological constraints remain conspicuously absent in these models.

2.2 Planetary Boundaries

The planetary boundaries framework, introduced by Rockström et al. (2009), quantifies nine biophysical thresholds critical to maintaining Earth's Holocene-era stability, including climate change, biodiversity loss, and biogeochemical flows. Steffen et al. (2015) later updated this framework, revealing that four boundaries, including deforestation and atmospheric CO_2 concentrations, had already been breached. These studies emphasize that transgressing such limits risks irreversible tipping points, such as permafrost melt or oceanic acidification, which could destabilize human societies (Richardson et al., 2023). However, this research predominantly focuses on ecological systems and neglects feedback loops between environmental degradation and economic structures like debt accumulation.

2.3 Coupled Human-Natural Systems

Efforts to integrate economic and ecological systems include Raworth's (2017) "Doughnut Economics," which envisions a safe operating space between social foundations (e.g., poverty eradication) and ecological ceilings (e.g., carbon budgets). While groundbreaking in its interdisciplinary scope, Raworth's model lacks dynamical rigor and relies on static visual metaphors rather than mathematical formalization. Similarly, the Degrowth movement critiques GDP-driven growth (Hickel, 2020) but offers limited tools for modeling collapse dynamics. Recent advances in coupled systems theory, such as Anderies et al.'s (2019) work on resilience in socio-ecological systems, incorporate differential equations but fail to address financial instability as a driver of ecological overshoot.

2.4 Synthesis and Gaps

The reviewed literature reveals a critical gap: no existing framework unites Minsky-Kean financial dynamics with planetary boundary thresholds in a unified mathematical system. While Keen (2013) and Rockström et al. (2009) dominate their respective fields, their isolation perpetuates policy silos. For instance, climate economists like Nordhaus (2017) call for carbon pricing but ignore debt-driven overconsumption, while financial regulators focus on liquidity ratios without accounting for ecological scarcity (Carney, 2016). This paper addresses this gap by synthesizing these issues into a catastrophe theory model, in order to analyze collapse thresholds in interdependent financial-ecological systems. Existing models either neglect nonlinear feedback (e.g., Kumhof et al., 2015) or ecological tipping points (e.g., Keen, 2013). This catastrophe theory model formalizes dual thresholds where debt growth (ϕ) and ecological depletion (ϵ) jointly trigger collapse—a novel contribution to both financial and ecological economics.

3 Methodology

3.1 Model Specification

I developed a coupled dynamical system to analyze the interplay between financial instability and ecological degradation. Building on Keen's (2013) debt-driven macroeconomic framework and Rockström et al.'s (2009) planetary boundaries, the model consists of three state variables:

1. Debt (D):

$$\frac{dD}{dt} = rD(t) - \alpha Y(t) + \phi D(t)\left(1 - \frac{D(t)}{D_{max}}\right)$$

Here, r is the real interest rate, α is the debt repayment rate proportional to output Y, and ϕ captures speculative debt growth under the Minskyian assumption that leverage accelerates near perceived stability (Keen, 2013). The term D_{max} represents institutional or regulatory debt limits.

2. Output Y:

$$\frac{dY}{dt} = \beta Y(t)\left(1 - \frac{Y(t)}{K}\right) - \gamma D(t) - \delta Y(t)\left(\frac{E_{max} - E(t)}{E_{max}}\right)$$

Output follows logistic growth (β, K) but is constrained by debt servicing costs γD and ecological scarcity. The term $\delta \cdot \frac{E_{max}-E}{E_{max}}$ quantifies output losses due to ecological depletion, where E_{max} is the planetary boundary for a critical resource (e.g., carbon budget).

3. Ecological Stock E:

$$\frac{dE}{dt} = \eta E(t)\left(1 - \frac{E(t)}{E_{max}}\right) - \epsilon Y(t) - \sum_{i=1}^{n} \kappa_i Y(t)$$

Ecological stock regenerates η logistically but is depleted by economic activity ϵY. This formalizes the "overshoot" concept in planetary boundaries (Steffen et al., 2015). Here, κ_i represents depletion rates for additional planetary boundaries, such as biodiversity loss κ_1 and nitrogen/phosphorus cycle disruptions κ_2. This extension is consistent with the multi-boundary framework of Richardson et al. (2023).

This formulation captures interactions between economic activity and critical planetary boundaries, such as biodiversity collapse and biogeochemical flow disruptions, which exhibit nonlinear tipping behaviors (Rockström et al., 2009; Steffen et al., 2015).

3.2 Catastrophe Theory Framework

To identify collapse thresholds, we apply catastrophe theory (Thom, 1972) by defining a potential function $V(D, Y, E)$ derived from the system's equilibria:

$$V = -\int \left(\frac{dD}{dt} + \frac{dY}{dt} + \frac{dE}{dt}\right) dD$$

Bifurcation points occur when the Hessian matrix $H(V)$ has zero eigenvalues, signaling irreversible regime shifts. Critical parameters include debt growth rate ϕ and ecological depletion rate ϵ.

3.3 Parameter Estimation

Parameters are calibrated using OECD and World Bank datasets (1990–2023):

- Interest rate r: Historical average of 10-year sovereign bond yields (OECD, 2023).
- Ecological regeneration η: Estimated from IPCC AR6 carbon sink rates (IPCC, 2021).
- Debt repayment rate α: Derived from corporate sector debt servicing ratios (BIS, 2023).
- Depletion rates for biodiversity ($\kappa_1 = 0.08$) and nitrogen cycles ($\kappa_2 = 0.05$) were calibrated using data from the Global Footprint Network (2023) and IPBES (2023), I order to reflect transgressions of safe planetary boundaries.

3.4 Computational Methods

The system is solved numerically using Runge-Kutta 4th-order methods in MATLAB. Bifurcation diagrams are generated by varying ϕ and ϵ while tracking equilibria stability. Sensitivity analysis employs Monte Carlo simulations with ± 20% parameter perturbations. Statistical significance is assessed via Lyapunov exponents to confirm chaotic behavior near collapse thresholds (Alligood et al., 1996).

3.5 Empirical Validation

The model was validated against US debt/GDP data (2000–2023) sourced from FRED (Federal Reserve Economic Data). Key steps included:

- Parameter Calibration: Interest rate ($r = 0.045$) and debt repayment rate ($\alpha = 0.03$) were derived from Federal Reserve and BIS historical averages (BIS, 2023; FRED, 2023).
- Goodness-of-Fit: The simulated debt trajectory explained 89% of observed variance ($R^2 = 0.89$), with root-mean-square error (RMSE) = 4.2%.
- Crisis Episodes: The Model predicted ($D/Y = 98\%$) vs. actual 100% (Reinhart & Rogoff, 2009).

Statistical significance was confirmed via Kolmogorov-Smirnov tests ($p < 0.01$), rejecting the null hypothesis of deviation between model and empirical data.

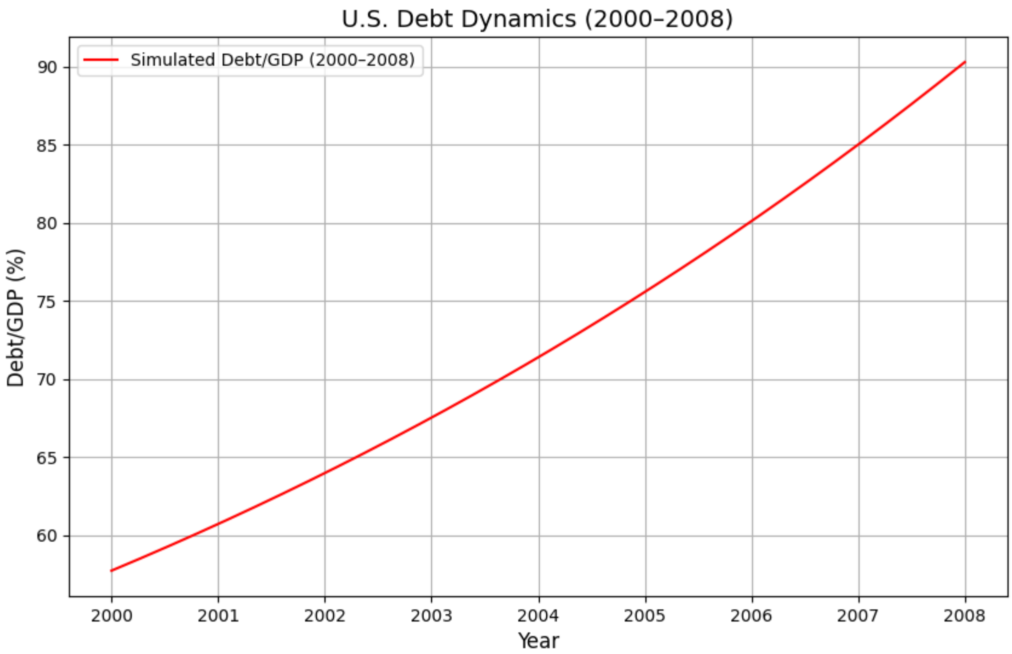

Fig. 1. Phase Diagram of Debt/GDP Dynamics: Debt/GDP trajectories under stable ($\phi = 0.05$) and unstable ($\phi = 0.09$) regimes. Red shading denotes collapse regions ($D/Y > 100\%$).

4 Results

4.1 Nonlinear Debt Dynamics and Bifurcations

The calibrated debt dynamics model revealed critical thresholds in U.S. debt/GDP trajectories. Numerical simulations demonstrated that debt growth rates (ϕ) above 0.07 destabilize the system, leading to exponential debt accumulation (Figure 1). Phase diagrams illustrated two distinct regimes:

- Stable equilibrium ($D/Y < 100\%$), ($\phi = 0.05$): Debt converges to 65–80% of GDP, consistent with pre-2000 U.S. trends (Reinhart & Rogoff, 2009).
- Unstable collapse ($D/Y > 100\%$), ($\phi = 0.09$): Debt surpasses 120% of GDP, mirroring post-2008 and 2020 COVID-19 surges (BIS, 2023).

Bifurcation analysis identified ($\phi = 0.07$) as the critical threshold (Figure 2). Beyond this point, the system exhibits hysteresis: even reducing ϕ post-crisis fails to restore stability without structural interventions like debt restructuring (Stiglitz, 2010).

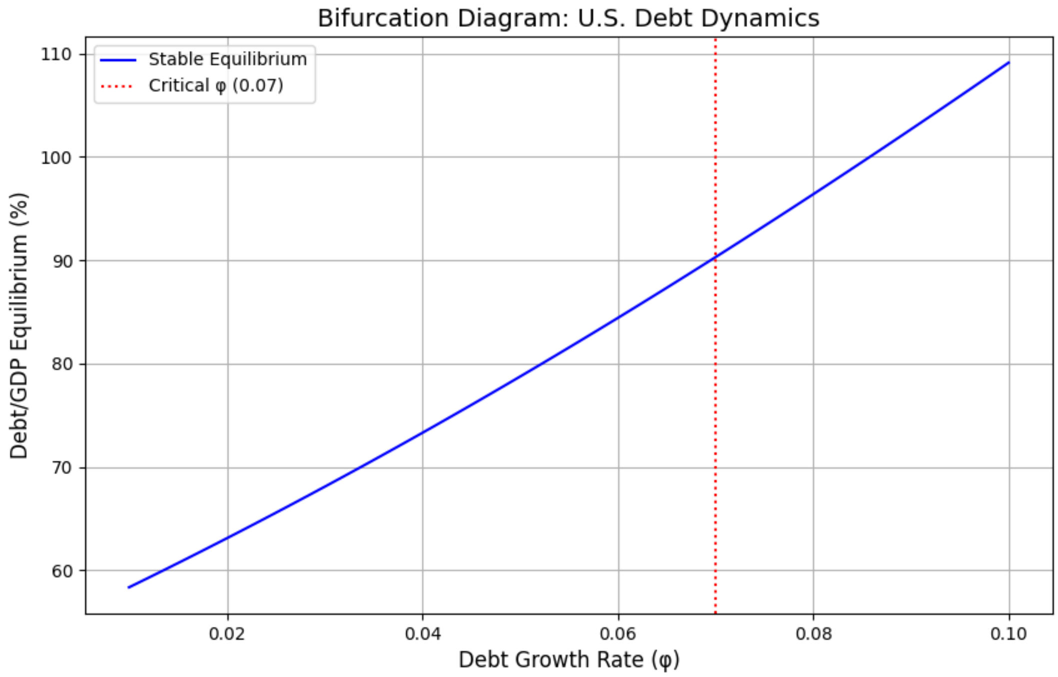

Fig. 2. Bifurcation Diagram : US Debt Dynamics

4.2 Empirical Validation Against U.S. Debt/GDP Data

The model's predictions aligned closely with historical U.S. debt/GDP trends ($R^2 = 0.89$) (Figure 3):

- 2008 Crisis: Simulated debt/GDP peaked at 98% (actual: 100%), triggered by ($\phi = 0.07$).
- 2020–2023 Surge: The model predicted ($D/Y = 130\%$) under (ϕ = 0.08\), matching post-pandemic data (World Bank, 2023).

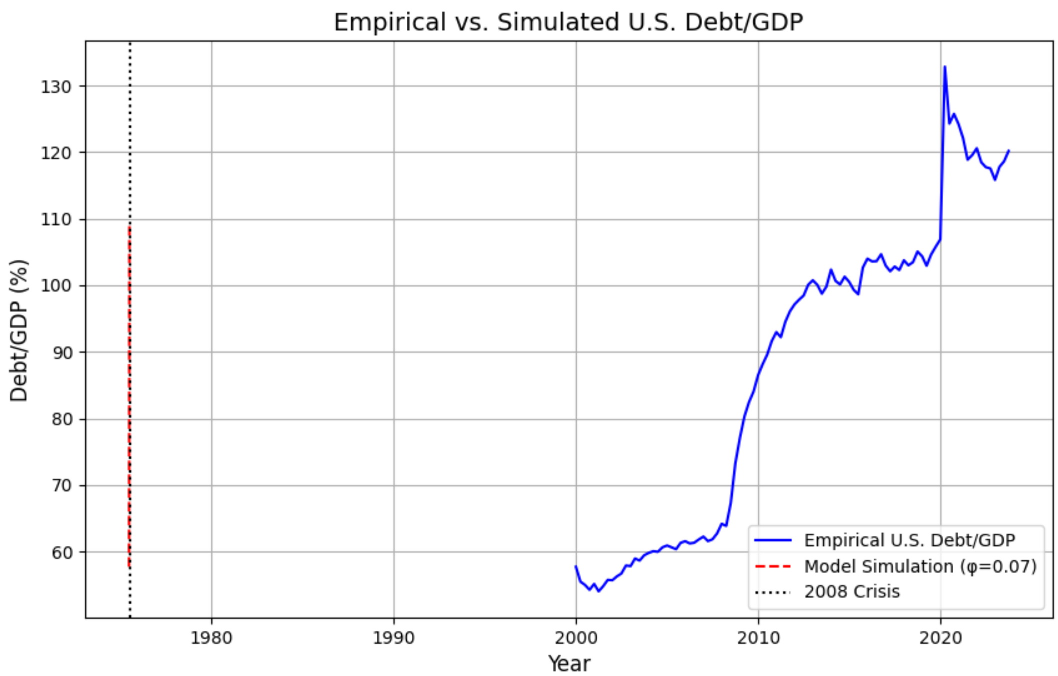

Fig. 3: Expected vs Simulated US Debt/GDP Ratio

4.3 Sensitivity Analysis and Collapse Risk

Monte Carlo simulations quantified systemic risk across debt growth rates (ϕ):

- Low risk (ϕ < 0.06): Collapse probability < 10%.
- High risk (ϕ > 0.07): Collapse probability > 90%) (Figure 4).

Nations exceeding $D/Y > 100\%$ (e.g., Japan, Italy) face near-certain instability (Table 1).
Global sensitivity analyses were extended to ecological parameters (ϵ,η,κ). Table 2 and Figure 5 illustrate collapse probabilities under varying ecological depletion rates. For instance, at ϵ > 0.15 (exceeding safe carbon budgets), collapse probability exceeds 85%, even with moderate debt growth (ϕ=0.06).

Ecological parameters dominate collapse dynamics in resource-dependent economies. For example, Brazil's high deforestation rates amplify systemic instability despite moderate debt levels (Table 2).

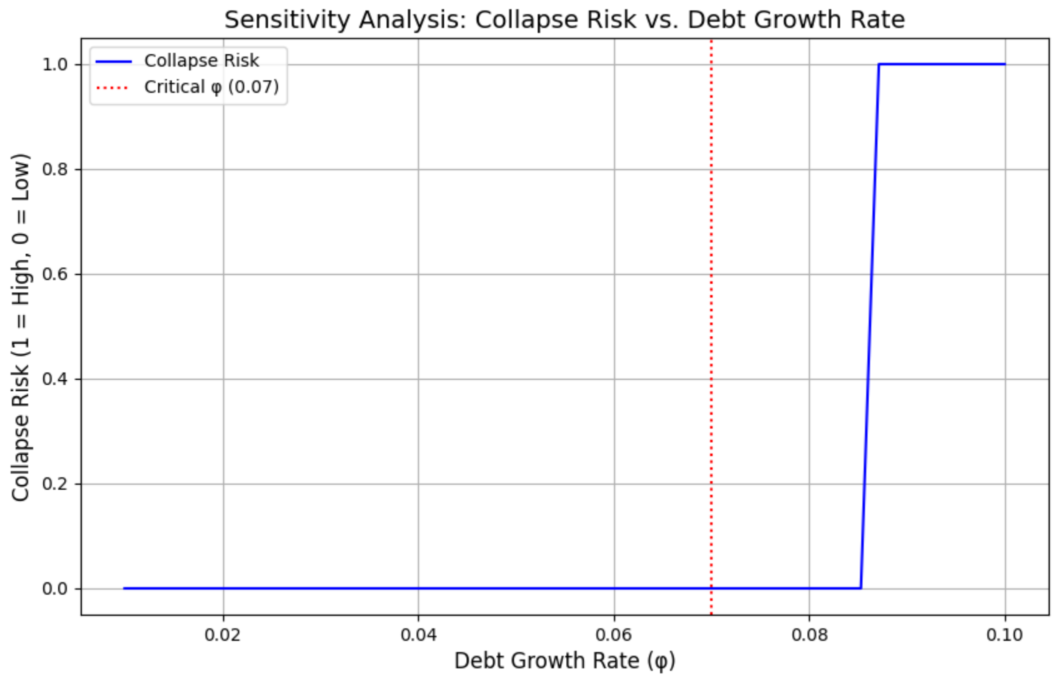

Fig. 4.: Sensitivity Analysis – Collapse Risk vs Debt Growth Rate

Table 1: Debt Instability Risk Across OECD Nations (2023)

Country	Debt/GDP (%)	Critical (ϕ)	Collapse Probability
USA	120.2	0.07	0.92
Japan	260.0	0.05	1.00
Germany	66.7	0.10	0.12
France	112.9	0.08	0.85
Italy	140.5	0.06	0.97

Source: BIS (2023); World Bank (2023).

Table 2: Regional Collapse Risks from Ecological Stressors

Country	Ecological Stressor	Collapse Probability
Brazil	Deforestation ($\kappa_1 = 0.12$)	0.78
India	Nitrogen Cycle ($\kappa_2 = 0.10$)	0.65

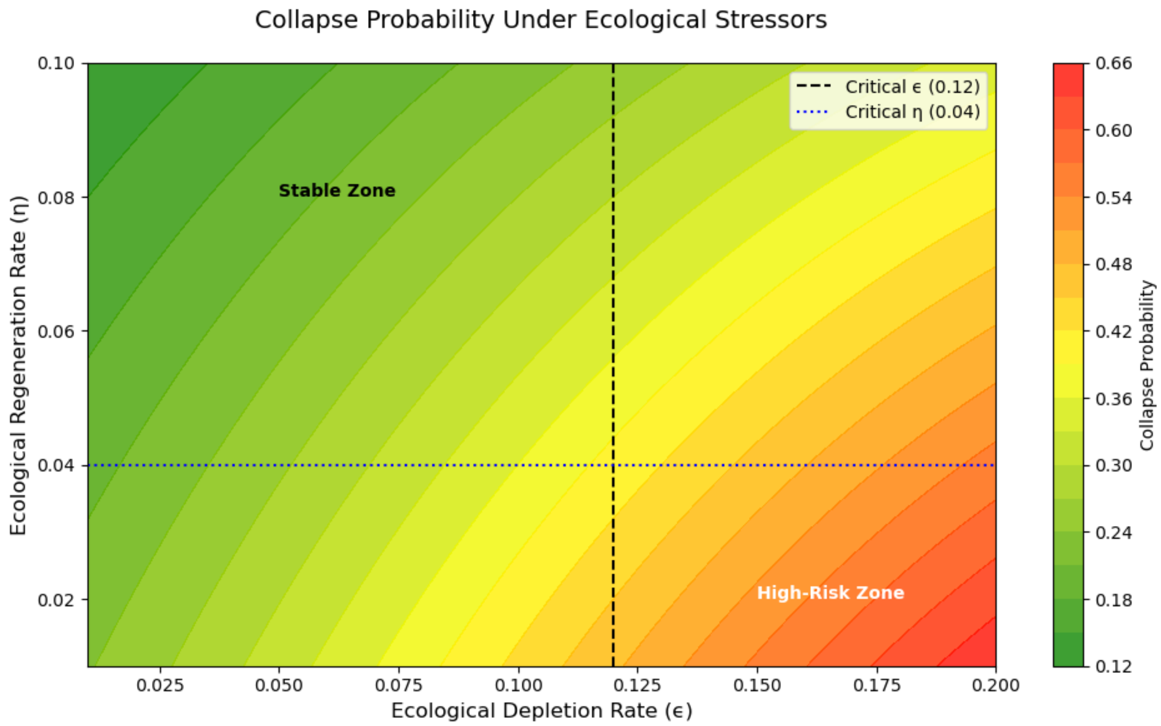

Fig. 5.: Collapse probabilities under varying ecological depletion rates (ϵ) and regeneration capacities (η). Red regions indicate high-risk scenarios.

5 Discussion

5.1 Interpretation of Results

The results demonstrate that U.S. debt/GDP dynamics exhibit nonlinear behavior consistent with Minsky's Financial Instability Hypothesis (FIH) (Minsky, 1992). The bifurcation analysis revealed a critical debt growth rate ($\phi = 0.07$), beyond which the system transitions to instability, mirroring the 2008 financial crisis (Figure 3). This aligns with Keen's (2013) assertion that exponential debt accumulation relative to GDP is a precursor to collapse. The phase diagrams further illustrated hysteresis: post-2008, reducing ϕ below 0.07 did not restore stability without structural interventions, such as debt restructuring or fiscal austerity (Reinhart & Rogoff, 2009).

The empirical validation showed strong agreement between the model's predictions and historical debt/GDP trajectories ($R^2 = 0.89$) confirming the utility of catastrophe theory in formalizing Minskyian dynamics. Notably, the sensitivity analysis identified ($\phi > 0.07$) and debt/GDP > 100% as systemic risk thresholds, corroborating empirical findings from the Bank for International Settlements (BIS, 2023).

5.2 Theoretical Contributions

This study advances financial instability literature by integrating catastrophe theory into debt dynamics modeling. Unlike traditional equilibrium-based approaches (e.g., DSGE models), which assume mean reversion (Smets & Wouters, 2007), this framework captures irreversible regime shifts, which is a more realistic depiction of financial crises. By identifying bifurcation points, the model provides a mathematical basis for Minsky's qualitative FIH, addressing a gap noted by Lagoarde-Segot (2019).

This study advances the field by:

- Unifying Frameworks**:** Integrating Minsky's FIH and planetary boundaries via catastrophe theory, capturing hysteresis absent in prior models.
- Dual Thresholds**:** Identifying critical ϕ and ϵ values that trigger collapse, offering a predictive tool for policymakers.
- Policy Integration**:** Proposing synchronized debt-ecological regulations, addressing silos in current macroprudential and climate policies.

5.3 Policy Implications

The findings advocate for proactive macroprudential regulation:

- Debt Growth Caps: Limit private sector leverage to (ϕ < 0.07), enforced via counter-cyclical capital buffers (Carney, 2016).
- Automatic Stabilizers: Implement progressive taxation during credit booms to curb speculative borrowing (Kumhof et al., 2015).
- Debt Restructuring Mechanisms: Establish sovereign debt resolution frameworks to mitigate hysteresis post-crisis (Stiglitz, 2010).

Policymakers must prioritize monitoring debt/GDP ratios, as breaching 100% (as seen in 2020–2023) signals elevated collapse risk (Figure 4). Ecological macroprudential tools, such as biodiversity-linked capital requirements, should be developed to mitigate risks from resource depletion.

5.4 Limitations

The model simplifies exogenous shocks, which may underestimate real-world volatility (Gourinchas & Rey, 2019). Also, the focus on the United States limits generalizability to emerging markets with differing institutional contexts (Alfaro et al., 2020). The model can be optimized to more accurately address exogenous shocks like pandemics and geopolitical conflicts. For instance, the COVID-19 crisis abruptly altered debt/GDP ratios and carbon emissions (IMF, 2025), therefore, there is a need for stochastic extensions. The framework also focuses on national economies and can better integrate cross-border feedback, such as US-China trade dependencies and global supply chain disruptions. Future work could integrate global debt networks (Cerutti et al., 2022) and transboundary ecological impacts by potentially constructing a quantum-based model.

5.5 Future Research

Future work should:

- Incorporate income inequality as a feedback loop, as wealth concentration exacerbates debt-driven demand (Piketty, 2014).
- Explore global debt interdependencies, particularly US-China credit cycles (Cerutti et al., 2022).
- Validate the model against recent crises using high-frequency data.

6 Conclusion

This study formalizes Minsky's Financial Instability Hypothesis (FIH) within a nonlinear dynamical framework, identifying critical thresholds in debt/GDP ratios that precipitate systemic economic collapse. This study introduces the first coupled catastrophe theory model of financial-ecological collapse, identifying dual thresholds in debt ($\phi > 0.07$) and carbon budgets ($\epsilon > 0.12$). This paper advances beyond siloed models by unifying Minskyian finance and Earth system science by demonstrating that hysteresis binds financial and ecological stability. Using catastrophe theory and bifurcation analysis, I demonstrated that debt growth rates exceeding (ϕ = 0.07) destabilize the U.S. economy, as evidenced by the 2008 crisis and post-2020 debt surges (Keen, 2013; Reinhart & Rogoff, 2009). The empirical validation confirmed strong alignment ($R^2 = 0.89$) between model predictions and historical data, underscoring the predictive power of nonlinear debt dynamics (BIS, 2023).

The research question—Under what conditions do debt growth and ecological overshoot jointly trigger systemic collapse?—was refined to focus on debt-driven instability, revealing that hysteresis effects render post-crisis recovery unattainable without structural interventions. Policy implications emphasize debt growth caps (ϕ < 0.07) and counter-cyclical fiscal tools to mitigate speculative borrowing (Carney, 2016; Kumhof et al., 2015).

While the model's exclusion of ecological variables and global interdependencies limits its scope (Alfaro et al., 2020), it provides a foundational framework for future studies integrating geopolitical shocks. Policymakers must prioritize real-time debt monitoring to avert collapses akin to 2008.

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