

SOME ASPECTS OF THE DISCRETE CODE-BASED OPTIMIZATION OF A TELECOMMUNICATIONS TOWER

Gediminas BLAŽEVIČIUS ^{1*}

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Abstract

This paper focuses on the optimization of the discrete sizing and shape of a telecommunications tower. The tower is modelled as a 3D truss. The finite element code is written in MATLAB with software, including a genetic optimisation algorithm. The strength, buckling, slenderness, global stability, and displacement constraints are included, following the Eurocode requirements. The elements are selected from industrial circular hollow and angle steel section catalogues. The paper discusses the performance of the genetic algorithm, the effect of various model constraints, its global stability, and the wind load on the final tower design. Emphasis is placed on the convenience of implementing the optimisation in a practical design problem. The results demonstrate several peculiarities of algorithm properties. The sizing variable is treated as a continuous or discrete variable. A full spectrum of the dependency of the optimised design on the sizing variable is presented. The designs are validated with commercial software. All the data for the repeatability of the results is supplied in the appendices.

Address

^{1*} Department of Applied Mechanics, Faculty of Civil Engineering, Vilnius Gediminas Technical University, Vilnius, Lithuania

* **Corresponding author:**
gediminas.blazevicius@vilniustech.lt

Key words

- Optimization of truss sizing and shape
- Shear reinforcement,
- Telecommunications tower,
- Eurocode.

1 INTRODUCTION

The expansion of mobile networks, especially with the advent of 5G, necessitates more telecommunication infrastructure to handle increased data traffic and provide reliable service coverage (Salahdine et al., 2023). The increased need for mobile connectivity also increases the need to build new telecommunications towers. While multiple solutions are available in urban areas, building free-standing towers or masts in rural areas is the only option (Oughton et al., 2018). The need for reliable and efficient (i.e., optimised) telecommunications tower designs remains important. Another reason to carefully design new structures arises from the environmental aspects.

By consuming energy, natural resources and producing greenhouse gas and waste, the construction industry has a significant impact on the environment. A considerable amount of this consumption is related to structural building materials (Webster et al., 2012).

In recent years much attention has been paid to finding more ways to reduce the environmental impact of buildings through improving design and production processes. In mathematical terms, this means solving an optimization problem, i.e., finding the best solution in terms of the desired objective, while satisfying all the necessary conditions.

While research in optimization techniques and applications is active for decades, its implementations in practical structural design are still relatively rare. Optimization tools are complex and require special skills and adoption for every particular problem, while structural engineers tend to rationalize, i.e., to simplify designs for cost efficiency in their jobs and reduce labour costs in manufacturing. Designing for a minimum of material rather than a minimum cost could significantly reduce steel usage (Moynihan and Allwood, 2014), which would lead to an equivalent reduction in embodied carbon emissions. Optimization techniques could help to balance the use of material and to rationalize the design.

Nevertheless, it remains a challenge to find an optimized (let alone truly optimal) solution for practical civil engineering design problems due to several aspects. First, these problems are usually discrete (one needs to pick from an available cross section of catalogues). Second, standard requirements (e.g., the Eurocode) are very complex and defined in nonlinear (and non-smooth) functions. And third, these problems are relatively large in terms of variables and computational cost (e.g., a metaheuristic algorithm would need to run for 6-12 h to get a decent result (Dillen et al., 2021), and one would need to run it dozens of times). Therefore, investing in an optimization technique pays off for the design of structures that are very expensive to build or for those that would be replicated several times. An example of the latter is a lattice telecommunications tower. Developing an algorithm that could account for various wind load conditions, geometric structural parameters, and produce optimized results could significantly help to reduce the carbon footprint.

In terms of structural research on telecommunications towers focuses on several areas: optimization techniques; analysis of nonlinear effects and standard requirements; design and improvements of connections; manufacturing processes and experimental research on laboratory or full-scale models (Szafran and Rykaluk, 2016). Optimization problems are divided into three categories: optimization of sizing, shape and topology. If the aim is to determine the optimal dimensions of structural components (e.g., cross sections of a tower), it is called size optimization. Shape optimization means that the goal is to optimise specific geometric parameters (node positions). Finally, topology optimization means that the entire optimal geometry and material (or distribution of discrete bars) is to be determined. While optimizing the shape and topology usually involves continuous variables, optimizing practical sizing in structural engineering involves discrete variables, i.e., the cross section can only take certain specific values from the industry-produced member catalogues. Many practical problems could involve optimizing combination of shape, sizing and topology, which could be performed sequentially or simultaneously. This paper will focus on optimizing a discrete sizing and shape.

Depending on the type of optimization, there are numerous algorithms to choose from. Typically, they are divided into three groups, i.e., gradient-based, heuristic, and metaheuristic (Stolpe, 2016). Again, combinations of several algorithms can be used in practice or even hybrid methods can be developed (Dillen et al., 2021). While gradient-based algorithms might

have a guaranteed global optimum, their practical applications are best suited for continuous design variables and, as the name indicates, differentiable functions. For larger discrete problems (a greater number of variables) and complex nonlinear and non-smooth constraint functions, metaheuristic methods are usually better suited. However, these methods have been criticized for their drawbacks, tuning parameters for one specific problem (which makes it difficult to assess an algorithm's effectiveness); no guarantees for global optimality (which typically is completely unknown); inability to evaluate the quality of a solution; and long computational time. Nevertheless, on many occasions, these algorithms are the only ones applicable as they can interact with any "black box" solution generator. In other words, if an analytic software can read input data and produce a solution, it can be paired with an optimization algorithm. This also means that they are probably the most user friendly and still have great potential to be used in practical structural design.

Scientific literature contains a limited number of articles on discrete, Eurocode-based structural optimization. In recent years the focus has been on the development of optimization algorithms themselves and avoiding very particular problem solutions. One first reason for that is that in order to judge a new algorithm, it needs to be compared to other by running it on a well-known benchmark problem, which is lacking in this area. Secondly, it is difficult to include all the design requirements in an optimisation problem, and the ones that are left out might significantly influence the final design (i.e., the above-mentioned rationalization, design of connections, resistance to fire, insulation, architectural aesthetics, etc.). In this regard, optimization of lattice telecommunications towers has distinct properties, which make them worth analysing from scientific as well as practical design aspects. From a practical point of view, a tower has a relatively simple structure and very few non-loadbearing features (cables, ladder and transmission hardware). For this reason, an optimization problem can take into account almost all the requirements and produce a close-to-final design. Yet from a structural point of view, it is not even close to simple, particularly because the loading (the wind) depends on the structure (sizing and shape), and it is very sensitive to the local and global buckling (Tugilimana et al., 2018).

This paper aims to analyse some practical aspects of implementing an off-the-shelf metaheuristic (genetic) optimization algorithm on a realistic telecommunications tower design. It will emphasize the effect of some key Eurocode requirements on the solution and will aim to evaluate its quality.

2 FORMULATION OF THE PROBLEM

The object of this study is the telecommunications tower shown in Fig. 1. The total height of the tower is 40 meters. It is made of three legs connected by a crossed bracing. While multiple lattice tower designs exist (Tsavdaridis et al., 2020; Jasim and Galeb, 2002), this type is rather popular in the Baltic region (Szafran and Rykaluk, 2016). While the general optimisation of topology is outside the scope of this

paper, the shape of such a tower should be a consideration for every designer. Due to technological reasons (attachment of the communications equipment) the top 6 m section has a fixed geometry, but the variable-width bottom part could be optimised. Here, we only chose one sizing parameter, i.e., the distance between the supports (“ a ” in Fig. 1). As should be obvious and will be proved by the results, it has a significant effect on the tower’s stability. The tower’s legs are designed from circular, hollow sections (CHS). There are a total of 7 groups of these elements for every two sections and the top section. The bracing elements are designed from equal leg angle sections (LE). There are a total of 13 different LE section groups to design. The catalogues for these sections are shown in Appendix A. The CHS catalogue contains 165 different sections, while the angle section catalogue contains 99 options. Therefore, the total number of different available discrete sets is $n = 167^7 \cdot 99^{13} = 2.92^{41}$, which clearly demonstrates the size of the problem. It makes enumeration close to impossible.

Two types of loading are analysed, i.e., self-weight and wind. Both loads significantly depend on the geometry of the tower, which makes the design process nonlinear. The self-weight is defined by the cross-sectional areas (to be determined) and the lengths of the bars (dependent on the variable of the shape). The wind load heavily depends on the surface areas of the section facing the wind and the total envelope area of the

structure. Both of these again depend on the unknowns, i.e., sections and shape. Apart from the structural elements, there should be several non-structural elements, which are not shown here, but are incorporated in the loading calculations. The top section carries antennas and transmitters weighing a total of 0.8 kN and having a surface area that can gather 39.08 kN of concentrated wind load (distributed in four model nodes). These values are approximated in the current wind conditions, the height of the assembly and the surface area. Other non-structural elements are the ladder and cables positioned in the centre of the tower (for illustrations, see (Szafran and Rykaluk, 2016)). These are considered as ancillaries of the fixed area, but the actual loads depend on the altitude above ground. The cables and the ladder contribute to the wind loading, but their effect on the tower’s structural behaviour is ignored. The wind loads are determined according to the Eurocodes (European Committee for Standardisation (CEN), 2004a; 2006), following the so-called “general method”. The standard implies its use; nevertheless, some authors find it produces rather conservative results (Szafran, 2015). Every section of the tower (a total of 13, see Fig. 1) is treated separately in the wind analysis. All the wind parameters and calculation details are given in Appendix B. For a graphical loading illustration, see Fig. 3.

All the structural elements are modelled as 3D truss bars with six degrees of freedom (Fig. 2).

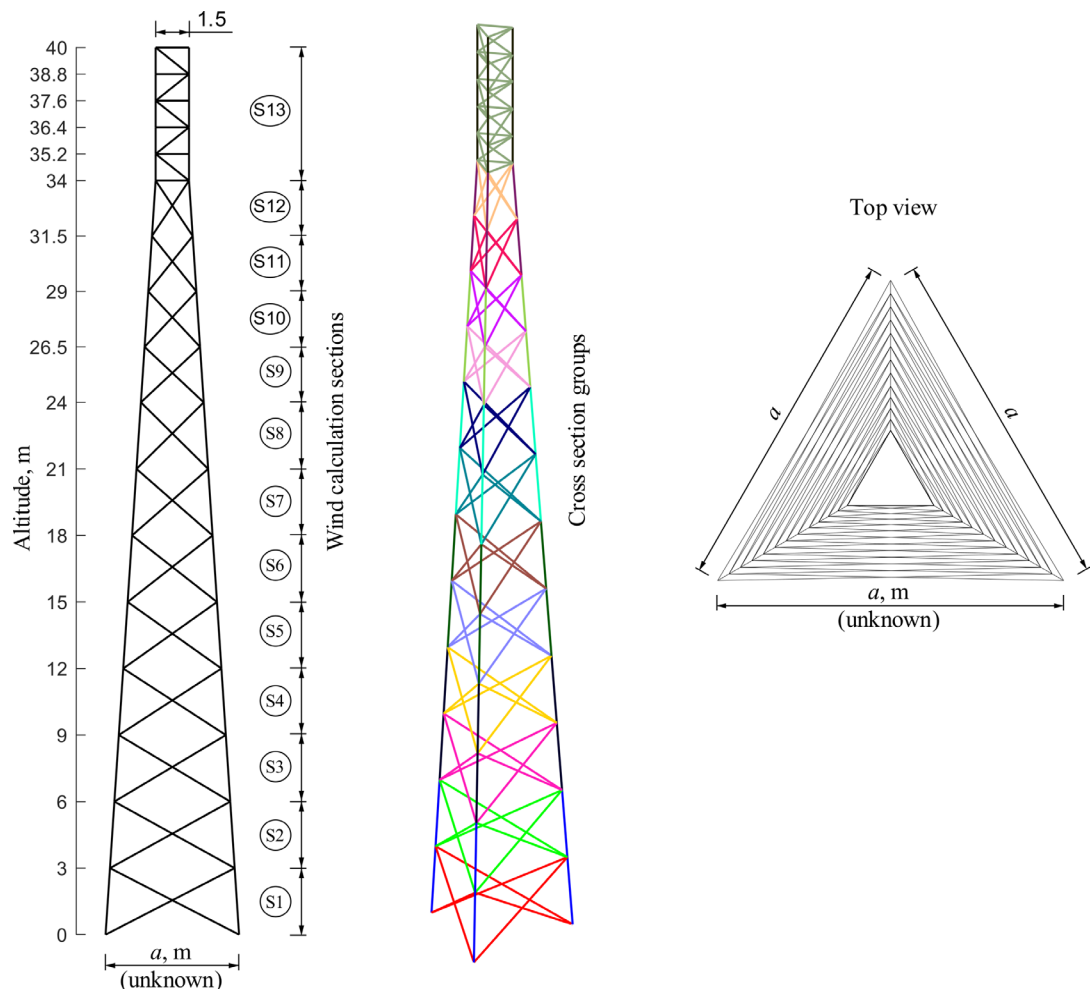


Fig. 1 Structural model of tower and the grouping of cross sections

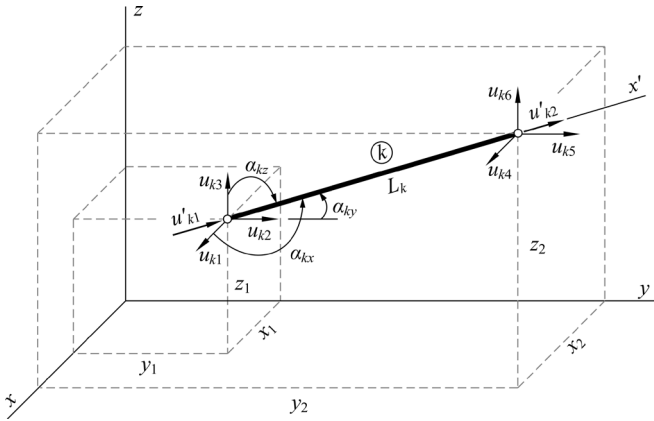


Fig. 2 3D truss finite element

The stiffness matrix of the finite element in global coordinate system K_k for any element k is defined as follows:

$$K_k = \frac{(EA)_k}{L_k} \begin{bmatrix} C_x^2 & C_x C_y & C_x C_z & -C_x^2 & -C_x C_y & -C_x C_z \\ C_x C_y & C_y^2 & C_y C_z & -C_x C_y & -C_y^2 & -C_y C_z \\ C_x C_z & C_y C_z & C_z^2 & -C_x C_z & -C_y C_z & -C_z^2 \\ -C_x^2 & -C_x C_y & -C_x C_z & C_x^2 & C_x C_y & C_x C_z \\ -C_x C_y & -C_y^2 & -C_y C_z & C_x C_y & C_y^2 & C_y C_z \\ -C_x C_z & -C_y C_z & -C_z^2 & C_x C_z & C_y C_z & C_z^2 \end{bmatrix}$$

Where $C_x = \cos \alpha_{kx} = (x_2 - x_1) / L_k$; $C_y = \cos \alpha_{ky} = (y_2 - y_1) / L_k$; $C_z = \cos \alpha_{kz} = (z_2 - z_1) / L_k$; and L_k is the length of element k . As the global vertical axis z is directed from the ground up, the self-weight force vector in the global coordinate system for every element is: $F_{m,k} = -0.5g \rho_k A_k L_k [0 \ 0 \ 1 \ 0 \ 0 \ 1]^T$. Where g is the gravitational constant, ρ_k is the material density, and A_k is the area of cross-section. For the global buckling analysis, the geometric stiffness matrix is as follows (Němec et al., 2016):

$$K_{g,k} = \frac{N_k}{L_k} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Where N_k is the element's axial force.

The aim is to solve the optimization problem formulated as follows:

$$\min m(\mathbf{x}_d, \mathbf{x}_c) = \rho \sum_{i=1}^{156} A_i L_i \quad (a)$$

subject to

$$g_{strength}(\mathbf{x}_d, \mathbf{x}_c) = |\sigma| - f \leq 0 \quad (b)$$

$$g_{buckl,EC3}(\mathbf{x}_d, \mathbf{x}_c) = -N_{b,Rd} - N \leq 0 \quad (c)$$

$$g_{slenderness}(\mathbf{x}_d, \mathbf{x}_c) = \lambda - \lambda_{lim} \leq 0 \quad (d)$$

$$g_{global\ stability}(\mathbf{x}_d, \mathbf{x}_c) = \gamma_{cr} - \gamma_{lim} \leq 0 \quad (e)$$

$$g_{displacement}(\mathbf{x}_d, \mathbf{x}_c) = |u| - u_{lim} \leq 0 \quad (f)$$

$$\mathbf{x}_d = [x_1, x_2, \dots, x_{20}]^T \in \{\mathbf{x}_{d1}, \mathbf{x}_{d2}, \dots, \mathbf{x}_{dn}\}.$$

The objective function (a) expresses the total mass of the structure made of 156 bars (Fig. 1). The element length L_i depends on the continuous variable x_c , which defines the distance between supports a , while cross section A_i is chosen from the discrete set of available sections and therefore is defined by the discrete variables x_d . The discrete variables are numbered as follows: x_1 defines the cross sections for the columns of sections S1 and S2; x_2 is the bracing for S1, x_3 is the bracing for S2, x_4 are the S3 and S4 columns, etc. Finally, x_{19} defines columns of S13 and x_{20} defines all the other elements of S13.

The strength conditions (b) are checked for all the elements. Following EC3, for elements under tension, the axial force should not exceed the design capacity: $N_{Ed} \leq N_{tRd} = (A_f \gamma_y) / \gamma_{M2}$. For elements under compression, $N_{Ed} \leq N_{cRd} = (A_f \gamma_y) / \gamma_{M0}$ ($\gamma_{M0} = 1$, $\gamma_{M2} = 1.25$).

Apart from strength conditions, the buckling condition (c) is applied for all the elements under compression. This follows the Eurocode (European Committee for Standardization (CEN), 2004b) requirement that an acting axial force should not exceed the design buckling resistance force: $N_{Ed} \leq N_{b,Rd} = (\chi A_f \gamma_y) / \gamma_{M1}$ (steel grade S355, $\alpha_{LE} = 0.34$, $\alpha_{CHS} = 0.21$, $\gamma_{M1} = 1$). The angle elements are assumed to buckle on the weakest axis (not parallel to the angle sides). The effective length factors are assumed to be 1 for all the elements except for the crossed bracing, where it is taken as half of the distance that the Eurocode allows (European Committee for Standardization (CEN), 2006). It is assumed that there is a bolted connection where the elements intersect and one of the crossing members is in tension. Yet in the FEM model, there is no node. It is assumed that this bolted connection has no effect on the resulting axial forces. It should be noted that for stocky elements, a potentially more precise flexural-torsional buckling algorithm could be applied (Hussain et al., 2018); however, in the current study, it is not considered.

Practical design standards also limit the slenderness of the elements. In some cases, these are stricter than the buckling conditions; therefore, they must be included in the problem's formulation (Belevičius et al., 2017). Conditions (d) assign the elements' slenderness following the Eurocode (European Committee for Standardization (CEN), 2006) recommendations: for the tower legs $\lambda_{lim} = 120$, and $\lambda_{lim} = 180$ for the bracing elements. Again, for the crossed bracing, the effective length is halved.

Apart from the local buckling of the elements, the global stability of the structure must be considered as well (Poulsen et al., 2020). This is achieved with model constraints (e). Three dimensional (3D) lattice towers are sensitive to global buckling and may collapse due to a global mode even sooner than the bifurcation of one element. This paper includes a standard eigenvalue buckling analysis for the entire FEM model under the same loading combination as for the strength conditions. As the structure is modelled from 3D truss elements, global buckling at the bifurcation point happens due to nodal displacements and element rotations (Kočvara, 2002). Because the truss elements do not define any flexural deformations as frame elements, this approach can overestimate the real global buckling factor γ_{cr} . To compensate, the global buckling limit γ_{lim} should be set to a higher than unit value ($\gamma_{lim} > 1$). If the design is being carried out in an elastic state (as in this paper), the current Eurocode 3 (European Committee for Standardization (CEN), 2004b) requires the setting $\gamma_{lim} = 10$. The second generation of steel design Eurocodes will include more requirements for this limit (European Committee for Standardization (CEN), 2023). This safety factor is frequently disregarded in favour of the theoretical limit of $\gamma_{lim} = 1$. The effect of this factor will be evident from the optimization results shown in the following section.

Some researchers have suggested solving the buckling of a truss as a frame structure (Torii et al., 2015). This paper argues that that is not necessary, because, firstly, most connections in such telecommunications towers are bolted, i.e., essentially not transferring bending moments. Secondly, it is a triangulated structure, which results in the occurrence of very small bending moments; thus, the truss model is an adequate approximation for an internal force solution. Then, if there is specific knowledge about the connections, the buckling conditions can be adjusted through effective length factors. Third, all these approximations lead to a more conservative (safer) design while the model is simpler and faster to implement and solve.

The last model constraints (f) include limitations for the deformations of the tower. Displacements of the top of the tower are constrained in all directions to not exceed 20 cm. This is an arbitrary limit chosen only to demonstrate its effect on the final design. The actual limit is not strictly defined by the standards but should be adjusted according to the sensitivity of the tower equipment (transmitters, antennas, sensitive 5G equipment etc.). This model has limitations, as some specific effects, such as the icing of an element or dynamic vibrations, were not included in the mathematical model.

There are multiple ways to approach this optimization problem. Due to its size (the sets of discrete options) and complexity of the constraints, global deterministic methods, such as branch-and-bound, might be a significant challenge (Stolpe, 2016). Another approach would be the use of metaheuristics. The goal of this paper is to test if off-the-shelf optimization algorithms could help in code-based practical design applications and would be relatively simple to apply. Metaheuristics are advertised as suitable for these demands. In this paper the MATLAB's genetic algorithm (GA) has been chosen as a tool on hand, which could treat discrete optimization with nonlinear constraints. As the literature

demonstrates (Lagaros et al., 2009), there is no definite “best” metaheuristic algorithm for all problems, and the goal of this paper is to analyse the convenience of applicability, not the algorithms themselves, thus the choice of MATLAB.

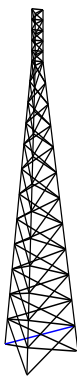
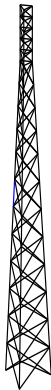
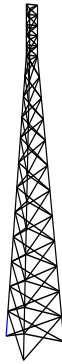
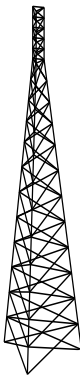
Another big question is the tuning of the algorithm parameters. Here again, we can find recommendations, but every problem proves to be different and demands careful tuning if we aim to “promote” a specific algorithm. In this test, we prioritize convenience. The parameters were changed based on the first solutions for the simplified problems. Specifically, the problem, which is called “Problem #1” in the next section, only contains strength constraints (formulation (a)-(b)). This problem produces results that could be evaluated relatively easily by an engineer. It is obvious when some cross section groups do not reach at least 80-90% of the strength limit. One can look at the section catalogue and see if there is another strong enough section with a smaller cross-section area. Having the full FEM code allows for a quick test. And the first GA solutions ended up far from fully loaded structures. This allowed rough tuning some of the GA parameters, which ended up as: maximum generations = 1600, population size = 1500, elite count = 150, maximum stall generations = 200, and crossover fraction = 0.9. All the other parameters were left as default. Experienced readers might suggest these numbers are too big. Yet with smaller populations and fewer generations, the repetitive runs would end up scattered, and multiple constraints would be left with large safety margins (sub-optimal). Surely this slows down computations significantly, especially the stall generations. The time needed to solve problems will be indicated with the results in the next section.

3 RESULTS

Five types of optimisation problems have been solved to demonstrate how every type of constraint changes the final solution. As briefly mentioned in the previous chapter, Problem #1 only includes the strength constraints. Problem #2 additionally includes the buckling of the elements. Each formulation adds another type of constraint. The solutions to these problems are summarised in Table 1. To get more practical insights in to the optimiser behaviour, all the problems were solved in two modes: first, when the support spacing a is treated as a continuous variable, and two, when a is a discrete variable from the set $a \in \{3, 3.2, 3.4, \dots, 13\}$ m.

The better solution of the two is marked with the shaded cell in Table 1. All the other results are from that specific solution. For two problems, i.e., #2 and #5, a full spectrum of locked leg spacing solutions was determined. This was done by fixing the distance a and finding the optimized solution. Every optimisation run was repeated at least 3 times. Surprisingly, this resulted in better solutions with different leg spacings (shown in the shaded cells). For example, it was found that the best solution of Problem #5 is at $a = 10.2$ m, even though the solution with a discrete set (including this point) did not find the same result and ended up at $a = 9.2$ m. This is discussed in more detail in Section 3.1.

Tab. 1 General results for all the problems analysed

Problem Number and formulation	#1 (a)-(b)	#2 (a)-(c)	#3 (a)-(d)	#4 (a)-(e)	#5 (a)-(f)
Constraints	Only strength costraints	Strength and EC3 buckling	Strength, buckling and slederness	Strength, buckling, slenderness and global stability with	Strength, buckling, slenderness, andglobal stability and displacement
Resultant geometry from the best-found solution					
Global buckling multiplier	0.58	2.29	1.74	10.00	10.01
Best solution with continuous variable a	1.276 t mass and 9.79 m leg spacing	3.686 t 4.26 m	3.921 t 3.65 m	9.222 t 7.78 m	9.988 t 8.86 m
Best solution with discrete variable a	1.290 t 9.5 m	3.648 t 5.6 m	3.827 t 4.3 m	9.241 t 7.8 m	10.031 t 9.2 m
Best solution with fixed leg spacing	(Not analysed)	3.639 t 5.6 m	(Not analysed)	(Not analysed)	9.793 t 10.2 m
Resultant wind force and its altitude for the best solution	146.29 kN at 24.94 m altitude	175.05 kN at 23.94 m	172.8 kN at 24.11 m	204.08 kN at 22.12 m	218.69 kN at 21.16 m

Tab. 2 Cross sections from the best solutions of all five problems

Var. No	Section type	Problem #1	#2	#3	#4	#5
1	CHS	114.3x3.6	193.7x5	219.1x5	168.3x5	168.3x5
2	L	20x20x3	70x70x6	75x75x6	150x150x12	150x150x10
3	L					
4	CHS	114.3x3.2	168.3x5	193.7x5	168.3x4	168.3x5
5	L	20x20x3	70x70x6	70x70x6	150x150x12	150x150x10
6	L				150x150x10	120x120x11
7	CHS	88.9x3.6	168.3x4	193.7x5	168.3x3.2	139.7x5
8	L	20x20x3	60x60x6	70x70x6	120x120x11	120x120x10
9	L		60x60x5	60x60x5	110x110x10	100x100x10
10	CHS	88.9x3.2	168.3x3.2	168.3x4	139.7x3.6	114.3x6
11	L	20x20x3	60x60x5	60x60x5	80x80x10	90x90x8
12	L				75x75x8	80x80x8
13	CHS	76.1x3.2	139.7x3.2	139.7x3.6	114.3x3.6	88.9x6
14	L	20x20x3	50x50x4	50x50x5	60x60x6	70x70x6
15	L				60x60x8	60x60x5
16	CHS	60.3x3.6	114.3x3	114.3x3.2	88.9x6	88.9x6.3
17	L	20x20x3	50x50x4	50x50x4	80x80x10	60x60x8
18	L				100x100x12	80x80x10
19	CHS	42.4x3.2	48.3x5	60.3x3	48.3x5	48.3x5
20	L	25x25x3	60x60x6	60x60x6	60x60x6	60x60x6

The results clearly demonstrate how every constraint adds to the previous one and increases the overall mass of the structure. Problem #1 results in a relatively wide tower as a wider support dimension reduces the internal forces in the structure. To the contrary, Problem #2 produces an overall narrower tower as the buckling of the elements favours shorter elements. By adding slenderness (Problem #3), this effect is manifested even more as some elements need to be increased or shortened. The narrowing tower and redistributing the cross sections cause a smaller, but higher acting, resultant wind force. The global buckling multipliers demonstrate that in theory, #3 is a stable structure ($\gamma_{cr}=1.74$), but as mentioned before, the safety factor should be increased to $\gamma_{lim}=10$. Thus, Problem #4 significantly increases the overall structural mass and produces a wider optimised tower. Adding displacement constraints in Problem #5 increases the mass even more while still favouring a wider structure

A detailed analysis of the optimised cross sections (Table 2) also demonstrates the significance of all the model constraints. If only strength conditions are included in the problem formulation (#1), several cross sections are identical. This is a result of very similar axial forces, while a shorter length has no influence until the buckling is included. It is worth mentioning that some sections in the solutions of Problems #1 – #4 are less than a 4 mm thickness. This might cause problems in the welding and node designs. In such cases the optimisation problem should include additional *structural* constraints, or the entire cross section catalogue should be limited to selected elements. Nevertheless, we can observe that Problem #5 results in larger cross sections as the other conditions are naturally

stricter. Also, this paper focuses on a generalised approach and aims to test how the current optimization techniques cope with full databases of sections.

To verify the author's written algorithm, Problem #5 solution was imported into a commercial FEM analysis software (Autodesk Robot Structural Analysis 2025) and checked for Eurocode compliance (Fig. 3). This software checks strength, buckling, and slenderness conditions. It also reports displacements and the global buckling factor. All these results matched exactly with the MATLAB solution and thus validated the written program. The detailed results are omitted for the sake of brevity.

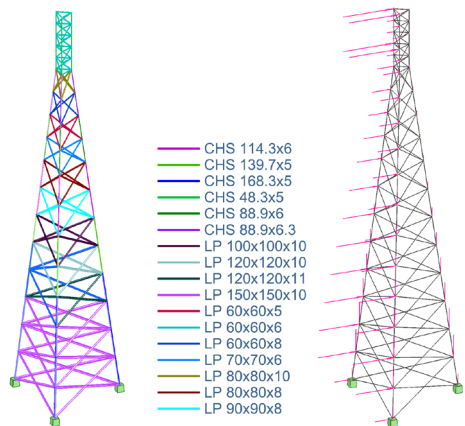


Fig. 3 Autodesk Robot cross section and loading schemes for the best solution of Problem #5 (9.793 t mass and 10.2 m distance between the supports)

3.1 Detailed analysis of the effect of the shape variable

Fig. 4 demonstrates the solution spectrum for Problem #2. The blue dots are solutions when the distance between the supports a has a fixed value. A yellow triangle is the best solution with distance a as a continuous variable. A green rhombus is the best solution for the problem when a is a variable with a discrete set. Each problem was solved three times; thus, some blue dots are at the same vertical axis position. With the GA parameters selected, one solution takes around 1 h 40 mins. Making this graph took a total of 7 days and 4 hours. The calculations were made on a virtual machine comparable to a fast personal computer. The best overall solution was found when distance a is fixed. It is quite interesting that if a is chosen from a discrete set (3, 3.2, 3.4 ... 10), the algorithm finds the same variable value (5.6 m), but it does not converge to the same discrete set and ends up with a slightly worse overall solution. If distance a is a continuous variable, the best-found solution is very different (4.26 m vs 5.6 m), but very close in terms of the objective function (3.686 t vs 3.648 t; $\sim 1\%$ worse).

Fig. 5 demonstrates the spectrum of the equivalent solution for Problem #5. For this problem one run of GA takes around 2 h 30 mins. A total of 16 days to make this graph. One run of the problem, where a is a discrete (or continuous) variable, takes around 3 hours. Again, the best overall solution is found with the fixed distance a . Notably for this case, the solution with a discretely variable a does not end up in the same place (10.2 m vs. 9.2 m). The problem with continuously varying a finds yet another good solution at $a = 8.86$ m. While having very different geometries, all the solutions result in very close objective function values (9.988, 10.031 and 9.793 t, which is $\sim 2\%$ variation).

An analysis of these graphs suggests a few observations, if not conclusions. It seems that in this problem, the optimum area is quite wide, and GA struggles to converge to the same solutions when the shape variable is included. Having in mind

that analysing the entire spectrum of variable a takes a lot more time, an engineer might prefer to solve the problem all at once (with a as either a discrete or continuous variable). Yet the results suggest that this could lead to various solutions with similar objective function values, but defining very different structural shapes (architecture). These differences could lead to multiple engineering design consequences not included in the problem formulation (e.g., bigger land acquisitions, foundation design, landscaping, fences). Therefore, it seems advisable to restrict the structural shape as much as possible. This demonstrates that for practical engineering problems, well-considered pre-design decisions by humans are indispensable. Focusing on only one criterion (the weight of the structure), could lead to undesirable solutions.

3.2 Wind load variations

To have comparable values of the wind effects, the resultant wind force is determined for all the optimised solutions (Table 1). The resultant wind force is an equivalent force to all the horizontal nodal forces induced by wind (see Fig. 3). As explained in Section 2, wind forces greatly depend on the structural geometry. The greater the cross sections and overall envelope area of the tower, the greater the resultant wind force. Figure 6 shows the resultant forces for all five problem formulations. A greater force requires greater sections to withstand it. This adds another nonlinearity to the optimization problem. Figure 7 clearly shows that for narrower towers (shorter foot spacing a), the wind force is relatively higher due to the wider sections. The same happens for the wider towers, i.e., as the mass and sections increase, the wind force follows. This figure should be analysed in parallel with Figure 5, which shows that too narrow or too wide a footing means a (relatively) greater overall mass. It is worth noting that very wide towers (e.g., $a = 13$ m) result in bigger loads than very narrow ($a = 3$ m), but in terms of mass, the opposite is true (see Fig. 5: a wide optimised tower is lighter than a narrow one).

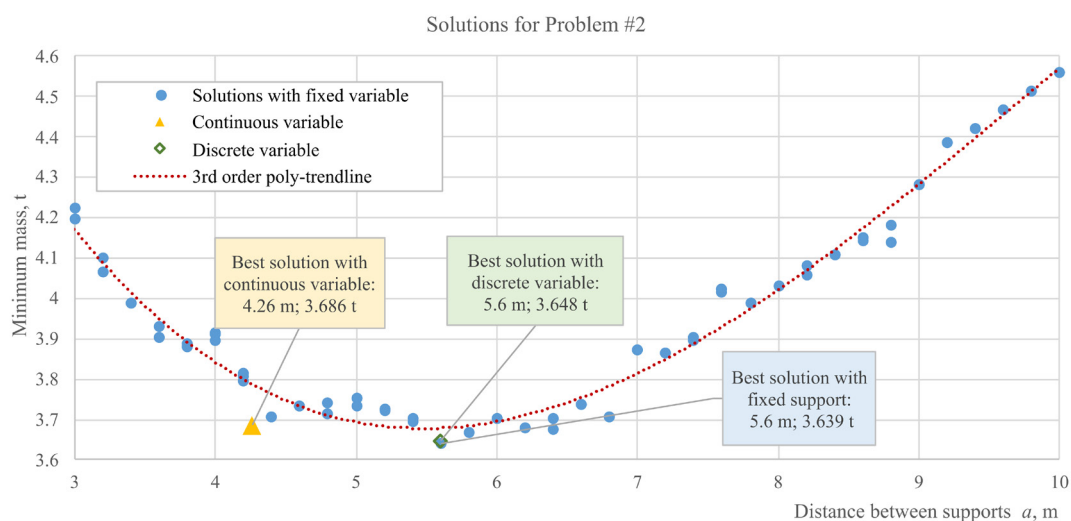


Fig. 4 Solutions of optimization problem #2 (strength and buckling constraints)

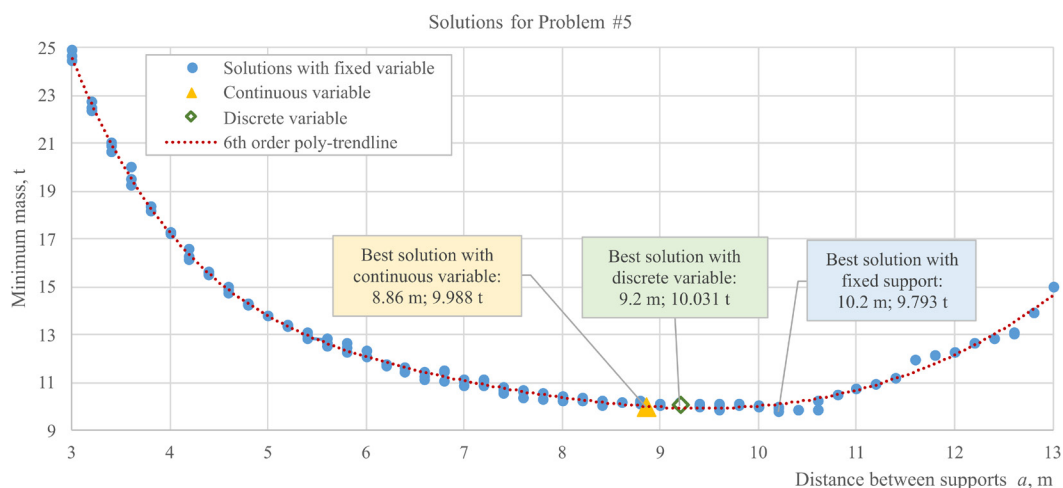


Fig. 5 Solutions of optimization problem #5 (all the constraints)

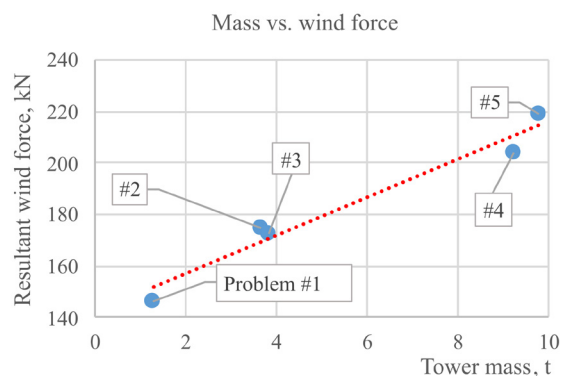


Fig. 6 Resultant wind force vs tower mass for all five problems

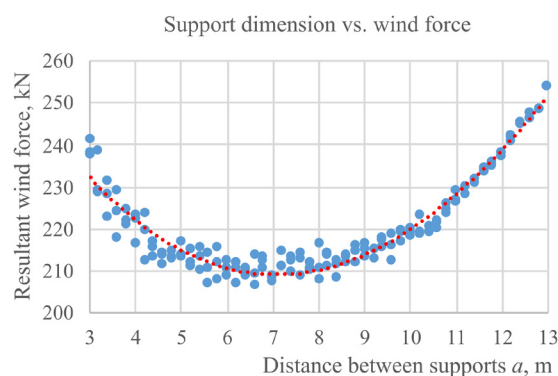


Fig. 7 Resultant wind force vs. distance between supports for problem #5

4 DISCUSSION

The conclusions from the computations results are:

- An evaluation of the global elastic stability limit should be included in the optimisation formulation with a carefully selected safety factor.
- The GA results might vary depending on whether the shape variable is treated as continuous or discrete.
- Restricting the shape variable might help find better solutions. It would be advised to run fixed geometry optimisation cycles in the vicinity of an expected optimum.
- Wind loads highly depend on the structural parameters of the telecommunications towers. Therefore, the conventional iterative design process would be exhausting and would probably end up with a conservative design. This is where automated FEM models with an optimisation algorithm demonstrate their ability to save material and time.

- Nevertheless, writing code for the optimization and running it takes a considerable amount of time. For a simple one-off structural design, it is still arguably inefficient. But it is a worthwhile tool for designing expensive or replicated structures, such as telecommunications towers.

Practical engineering does not require a mathematically perfect (i.e., globally optimal) solutions; it requires fast and reliable tools to improve designs. Optimization should be regarded as a tool for transitioning from an initial concept to a sound design, which is finalized by an engineer. To make effective use of optimization, engineers should constrain the design space as much as possible, i.e., group structural components for manufacturing simplicity or use their experience to set strict geometric limits. Narrowing the problem in this way reduces computation time and, more importantly, filters out unrealistic or undesirable solutions. Optimization algorithms must be guided by engineers to ensure the process is efficient and leads toward a realistic design.

REFERENCES

- BELEVIČIUS, R., JATULIS, D., RUSAKEVIČIUS, D. and MAČIŪNAS, D. *Optimization of Rigidly Supported Guyed Masts*. Advances in Civil Engineering. 2017. Vol. 2017, pp. 1–9. <https://doi.org/10.1155/2017/4561376>. [viewed 2025-10-16].
- DILLEN, Wouter; Geert LOMBAERT and Mattias SCHEVENELS. *A hybrid gradient-based/metaheuristic method for Eurocode-compliant size, shape and topology optimization of steel structures*. Engineering Structures. Jul. 2021. Vol. 239, pp. 112–137. <https://doi.org/10.1016/j.engstruct.2021.112137>. [viewed 2025-10-16].
- EUROPEAN COMMITTEE FOR STANDARDIZATION (CEN), 2004a. *Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions*. 2004. Brussels.
- EUROPEAN COMMITTEE FOR STANDARDIZATION (CEN), 2004b. *Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings EN 1993-1-1:2005*. 2004. Brussels.
- EUROPEAN COMMITTEE FOR STANDARDIZATION (CEN), 2006. *Eurocode 3 - Design of steel structures - Part 3-1: Towers, masts and chimneys - Towers and masts*. 2006. Brussels.
- EUROPEAN COMMITTEE FOR STANDARDIZATION (CEN), 2023. *Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings EN 1993-1-1:2023*. 2023. Brussels.
- HASANÇEBİ, O., ÇARBAŞ, S., DOĞAN, E., ERDAL, F. and SAKA, M.P. *Performance evaluation of metaheuristic search techniques in the optimum design of real size pin jointed structures*. Computers & Structures. Mar. 2009. Vol. 87, nos. 5–6, pp. 284–302. <https://doi.org/10.1016/j.compstruc.2009.01.002>. [viewed 2025-10-16].
- HUSSAIN, A., LIU, Yao-Peng, CHAN, Siu-Lai. *Finite Element Modeling and Design of Single Angle Member Under Bi-axial Bending*. Structures. Nov. 2018. Vol. 16, pp. 373–389. <https://doi.org/10.1016/j.istruc.2018.11.001>. [viewed 2026-03-17].
- JASIM, Nabeel Abdulrazzaq and Alaa Chaseb GALEB. *Optimum design of square free-standing communication towers*. Journal of Constructional Steel Research. Mar. 2002. Vol. 58, no. 3, pp. 413–425. [https://doi.org/10.1016/S0143-974X\(01\)00055-4](https://doi.org/10.1016/S0143-974X(01)00055-4). [viewed 2025-10-16].
- KOČVARA, M., 2002. *On the modelling and solving of the truss design problem with global stability constraints*. Structural and Multidisciplinary Optimization. Apr. 2002. Vol. 23, no. 3, pp. 189–203. <https://doi.org/10.1007/s00158-002-0177-3>. [viewed 2025-10-16].
- LAGAROS, Nikos D., Vagelis PLEVRIS and Nikos Ath. KALLIORAS. *The Mosaic of Metaheuristic Algorithms in Structural Optimization*. Archives of Computational Methods in Engineering. Nov. 2022. Vol. 29, no. 7, pp. 5457–5492. <https://doi.org/10.1007/s11831-022-09773-0>. [viewed 2025-10-16].
- MOYNIHAN, Muir C. and Julian M. ALLWOOD. *Utilization of structural steel in buildings*. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. 8 Aug. 2014. Vol. 470, no. 2168: 20140170. <https://doi.org/10.1098/rspa.2014.0170>. [viewed 2025-10-16].
- NĚMEC, I., TRCALA, M., ŠEVČÍK, I. and ŠTEKBAUER, H. *New Formula for Geometric Stiffness Matrix Calculation*. Journal of Applied Mathematics and Physics. 2016. Vol. 04, no. 04, pp. 733–748. <https://doi.org/10.4236/jamp.2016.44084>. [viewed 2025-10-16].
- OUGHTON, Edward, Zoraida FRIAS, Tom RUSSELL, Douglas SICKER and David D. CLEEVELY. *Towards 5G: Scenario-based assessment of the future supply and demand for mobile telecommunications infrastructure*. Technological Forecasting and Social Change. Aug. 2018. Vol. 133, pp. 141–155. <https://doi.org/10.1016/j.techfore.2018.03.016>. [viewed 2025-10-16].
- POULSEN, Peter Noe, John Forbes OLESEN and Mads BAANDRUP. *Truss optimization applying finite element limit analysis including global and local stability*. Structural and Multidisciplinary Optimization. Jul. 2020. Vol. 62, no. 1, pp. 41–54. <https://doi.org/10.1007/s00158-019-02468-4>. [viewed 2025-10-16].
- SALAH DINE, Fatima, Tao HAN and Ning ZHANG. *5G, 6G, and Beyond: Recent advances and future challenges*. Annals of Telecommunications. Oct. 2023. Vol. 78, nos. 9–10, pp. 525–549. <https://doi.org/10.1007/s12243-022-00938-3>. [viewed 2025-10-16].
- STOLPE, Mathias. *Truss optimization with discrete design variables: a critical review*. Structural and Multidisciplinary Optimization. Feb. 2016. Vol. 53, no. 2, pp. 349–374. <https://doi.org/10.1007/s00158-015-1333-x>. [viewed 2025-10-16].
- SZAFRAN, Jacek. *Analytical determination of aerodynamic resistance of the skeletal telecommunication towers*. Struct. Environ. 2015. Vol. 7, pp. 5–14.
- SZAFRAN, Jacek and Kazimierz RYKALUK. *A full-scale experiment of a lattice telecommunication tower under breaking load*. Journal of Constructional Steel Research. Apr. 2016. Vol. 120, pp. 160–175. <https://doi.org/10.1016/j.jcsr.2016.01.006>. [viewed 2025-10-16].
- TORIL, André J., Rafael H. LOPEZ and Leandro F. F. MIGUEL. *Modeling of global and local stability in optimization of truss-like structures using frame elements*. Structural and Multidisciplinary Optimization. Jun. 2015. Vol. 51, no. 6, pp. 1187–1198. <https://doi.org/10.1007/s00158-014-1203-y>. [viewed 2025-10-16].
- TSAVDARIDIS, Konstantinos Daniel, Andreas NICOLAOU, Anand Dipak MISTRY and Evangelos EFTHYMIU. *Topology optimisation of lattice telecommunication tower and performance-based design considering wind and ice loads*. Structures. Oct. 2020. Vol. 27, pp. 2379–2399. <https://doi.org/10.1016/j.istruc.2020.08.010>. [viewed 2025-10-16].

REFERENCES

TUGILIMANA, Alexis, Rajan FILOMENO COELHO and Ashley P. THRALL. *Including global stability in truss layout optimization for the conceptual design of large-scale applications.* Structural and Multidisciplinary Optimization. Mar. 2018. Vol. 57, no. 3, pp. 1213–1232. <https://doi.org/10.1007/s00158-017-1805-2>. [viewed 2025-10-16].

WEBSTER, Mark D., Helena MERYMAN, Adam SLIVERS, Tonatiuh RODRIGUEZ-NIKL, Lionel LEMAY, Kathrina SIMONEN, Harinee TRIVEDI, Lindsey MACLISE, Dirk KESTNER, Kenneth BLAND, Won LEE and Emily LORENZ. *Structure and Carbon – How Materials Affect the Climate.* ASCE: SEI Sustainability Committee; Carbon Working Group. 2012

Appendix A. Cross section catalogues

No	Size	A, mm ²	I _y , cm ⁴	No	Size	A, mm ²	I _y , cm ⁴	No	Size	A, mm ²	I _y , cm ⁴
1	20x20x3	112	0.16	34	120x120x12	2750	151	67	203x203x19	7360	1174
2	25x25x3	142	0.33	35	120x120x13	2970	162.2	68	203x203x22.2	8500	1350
3	25x25x4	185	0.43	36	120x120x15	3390	184.2	69	203x203x25.4	9680	1522
4	30x30x3	174	0.58	37	130x130x12	3000	193.7	70	203x203x28.6	10800	1692
5	30x30x4	227	0.75	38	150x150x10	2930	256	71	250x250x17	8210	1997
6	35x35x4	267	1.22	39	150x150x12	3480	302	72	250x250x18	8670	2104
7	40x40x4	308	1.84	40	150x150x14	4030	346.9	73	250x250x19	9140	2212
8	40x40x5	379	2.25	41	150x150x15	4300	368.9	74	250x250x20	9640	2341
9	45x45x4.5	390	2.94	42	150x150x18	5100	433.8	75	250x250x21	10100	2447
10	50x50x4	389	3.69	43	160x160x14	4320	423.8	76	250x250x22	10600	2551
11	50x50x5	480	4.51	44	160x160x15	4610	450.8	77	250x250x23	11000	2655
12	50x50x6	569	5.31	45	160x160x16	4900	477.6	78	250x250x24	11500	2759
13	60x60x5	582	7.97	46	160x160x17	5180	504.1	79	250x250x25	11900	2861
14	60x60x6	691	9.38	47	180x180x13	4550	571.6	80	250x250x26	12400	2963
15	60x60x8	903	12.11	48	180x180x14	4880	611.3	81	250x250x27	12800	3065
16	65x65x7	870	13.78	49	180x180x15	5210	650.5	82	250x250x28	13300	3166
17	70x70x6	813	15.16	50	180x180x16	5540	689.4	83	250x250x30	14100	3340
18	70x70x7	940	17.4	51	180x180x17	5870	727.8	84	250x250x31	14540	3439
19	75x75x6	873	18.74	52	180x180x18	6190	766	85	250x250x32	14970	3538
20	75x75x8	1140	24.25	53	180x180x19	6510	803.8	86	250x250x33	15410	3636
21	80x80x8	1230	29.72	54	180x180x20	6840	841.3	87	250x250x34	15840	3734
22	80x80x10	1510	36.23	55	200x200x15	5810	903	88	250x250x35	16300	3859
23	90x90x7	1220	38.02	56	200x200x16	6180	957	89	300x300x25	14270	4930
24	90x90x8	1390	42.87	57	200x200x17	6550	1011	90	300x300x26	14820	5115
25	90x90x9	1550	47.63	58	200x200x18	6910	1064	91	300x300x27	15370	5294
26	90x90x10	1710	52.32	59	200x200x19	7270	1117	92	300x300x28	15910	5475
27	100x100x8	1550	59.47	60	200x200x20	7630	1169	93	300x300x29	16460	5650
28	100x100x10	1920	72.65	61	200x200x21	7990	1221	94	300x300x30	17000	5828
29	100x100x12	2270	85.42	62	200x200x22	8350	1273	95	300x300x31	17540	5999
30	110x110x10	2120	97.72	63	200x200x23	8710	1324	96	300x300x32	18070	6184
31	110x110x12	2510	115	64	200x200x24	9100	1375	97	300x300x33	18610	6351
32	120x120x10	2320	128.3	65	200x200x25	9410	1426	98	300x300x34	19140	6532
33	120x120x11	2540	139.8	66	200x200x26	9760	1476	99	300x300x35	19670	6696

Circular hollow sections “CHS”

No	Size	A, mm ²	I, cm ⁴	No	Size	A, mm ²	I, cm ⁴	No	Size	A, mm ²	I, cm ⁴	No	Size	A, mm ²	I, cm ⁴
1	21.3x3.2	182	0.77	43	114.3x3.6	1250	192.0	85	244.5x6.3	4710	3346	127	406.4x20	24300	45430
2	26.9x3.2	238	1.70	44	114.3x4	1385	211.0	86	244.5x8	5940	4160	128	406.4x25	30000	54700
3	33.7x2.6	254	3.09	45	114.3x5	1720	257.0	87	244.5x10	7370	5073	129	406.4x32	37600	66430
4	33.7x3	289	3.44	46	114.3x6	2040	300.1	88	244.5x12	8761	5935	130	457x6.3	8916	22643
5	33.7x3.2	307	3.60	47	114.3x6.3	2140	313.0	89	244.5x12.5	9110	6147	131	457x8	11279	28432
6	33.7x3.6	340	3.91	48	139.7x3.2	1372	319.6	90	244.5x16	11500	7533	132	457x10	14000	35090
7	33.7x4	373	4.19	49	139.7x3.6	1538	356.5	91	244.5x20	14100	8957	133	457x12	16768	41535
8	42.4x2.6	325	6.46	50	139.7x4	1704	392.7	92	244.5x25	17200	10520	134	457x12.5	17500	43140
9	42.4x3	371	7.24	51	139.7x5	2120	481.0	93	273x5	4208	3779	135	457x16	22200	53960
10	42.4x3.2	394	7.62	52	139.7x6	2519	564.0	94	273x6	5030	4485	136	457x20	27500	65680
11	42.4x3.6	439	8.32	53	139.7x6.3	2640	589.0	95	273x6.3	5280	4696	137	457x25	33900	79420
12	42.4x4	483	8.99	54	139.7x8	3310	720.0	96	273x8	6660	5852	138	457x32	42700	97010
13	48.3x2.5	360	9.46	55	139.7x10	4070	862.0	97	273x10	8260	7154	139	457x40	52400	114900
14	48.3x3	427	11.0	56	168.3x3.2	1659	565.5	98	273x12	9834	8392	140	508x6.3	9925	31231
15	48.3x3.2	453	11.6	57	168.3x3.6	1862	631.6	99	273x12.5	10200	8697	141	508x8	12560	39260
16	48.3x3.6	505	12.7	58	168.3x4	2064	696.7	100	273x16	12900	10710	142	508x10	15600	48520
17	48.3x4	557	13.8	59	168.3x5	2570	856.0	101	273x20	15900	12800	143	508x12	18689	57507
18	48.3x5	680	16.2	60	168.3x6	3058	1008	102	273x25	19500	15130	144	508x12.5	19500	59760
19	60.3x2.5	454	19.0	61	168.3x6.3	3210	1053	103	323.9x5	5007	6366	145	508x16	24700	74910
20	60.3x3	540	22.2	62	168.3x8	4030	1297	104	323.9x6	5989	7569	146	508x20	30700	91430
21	60.3x3.2	574	23.5	63	168.3x10	4970	1564	105	323.9x6.3	6290	7929	147	508x25	37900	110900
22	60.3x3.6	641	25.9	64	168.3x12	5889	1809	106	323.9x8	7940	9910	148	508x32	47900	136100
23	60.3x4	707	28.2	65	168.3x12.5	6115	1867	107	323.9x10	9860	12160	149	508x40	58800	162200
24	60.3x5	869	33.5	66	193.7x5	2960	1320	108	323.9x12	11752	14312	150	508x50	71900	190900
25	76.1x2.5	578	39.2	67	193.7x6	3536	1559	109	323.9x12.5	12200	14850	151	559x20	33849	123093
26	76.1x3	689	46.1	68	193.7x6.3	3710	1630	110	323.9x16	15500	18390	152	559x25	41919	149746
27	76.1x3.2	733	48.8	69	193.7x8	4670	2016	111	323.9x20	19100	22140	153	559x32	52953	184510
28	76.1x3.6	820	54.0	70	193.7x10	5770	2442	112	323.9x25	23500	26400	154	559x40	65186	220787
29	76.1x4	906	59.1	71	193.7x12	6846	2838	113	355.6x6.3	6910	10542	155	559x50	79913	261297
30	76.1x5	1120	70.9	72	193.7x12.5	7120	2934	114	355.6x8	8740	13200	156	610x20	37052	161408
31	76.1x6	1321	81.7	73	193.7x16	8930	3554	115	355.6x10	10900	16220	157	610x25	45923	196807
32	76.1x6.3	1381	84.8	74	219.1x5	3360	1928	116	355.6x12	12947	19130	158	610x32	58077	243278
33	88.9x2.5	678	63.3	75	219.1x6	4015	2281	117	355.6x12.5	13500	19850	159	610x40	71592	292185
34	88.9x3	809	74.7	76	219.1x6.3	4210	2386	118	355.6x16	17100	24660	160	610x50	87920	347394
35	88.9x3.2	862	79.2	77	219.1x8	5310	2960	119	355.6x20	21100	29790	161	660x20	40192	205984
36	88.9x3.6	964	87.9	78	219.1x10	6570	3598	120	355.6x25	26000	35680	162	660x25	49848	251636
37	88.9x4	1070	96.3	79	219.1x12	7804	4198	121	406.4x6.3	7915	15841	163	660x32	63101	311885
38	88.9x5	1320	116.0	80	219.1x12.5	8110	4345	122	406.4x8	10008	19864	164	660x40	77872	375732
39	88.9x6	1562	134.9	81	219.1x16	10200	5297	123	406.4x10	12500	24480	165	660x50	95770	448443
40	88.9x6.3	1634	140.2	82	219.1x20	12500	6261	124	406.4x12	14861	28922				
41	114.3x3	1048	162.5	83	244.5x5	3760	2697	125	406.4x12.5	15500	30030				
42	114.3x3.2	1116	172.4	84	244.5x6	4493	3197	126	406.4x16	19600	37450				

Appendix B. EC Wind load algorithm. Calculations are performed for every segment of the tower (see Fig. 1)

PART I. Peak velocity pressure at a particular height $q_p(z)$

Terrain parameters	Category I, $z_0 = 0.01m$, $z_{\min} = 1m$, $z_{\max} = 200m$, $z_{0,II} = 0.05m$.		
Basic wind velocity	$v_{b,0} = 24m/s$	Orography factor	$c_0(z)$
Air density	$\rho = 1.25 \text{ kg/m}^3$	Turbulence factor	$k_I = 1$
Seasonal factor	$c_{dir} = 1$	Directional factor	$c_{season} = 1$
Basic wind velocity	$v_b = c_{dir} \cdot c_{season} \cdot v_{b,0} = 24m/s$	Basic velocity pressure	$q_b = 0.5 \cdot \rho \cdot v_b^2$
Altitude above the ground of the top of a segment	z		

Terrain factor, depending on the roughness length z_0	$k_r = 0.19 \cdot \left(\frac{z_0}{z_{0,II}}\right)^{0.07}$
Terrain roughness factor	$c_r(z) = k_r \cdot \ln\left(\frac{z}{z_0}\right)$ for $z_{\min} \leq z \leq z_{\max}$, $c_r(z) = c_r(z_{\min})$ for $z \leq z_{\min}$.
Turbulence intensity	$I_v(z) = \frac{k_I}{c_0(z) \cdot \ln\left(\frac{z}{z_0}\right)}$ if $z_{\min} \leq z \leq z_{\max}$, $I_v(z) = I_v(z_{\min})$ if $z \leq z_{\min}$.
Mean wind velocity	$v_m(z) = c_r(z) \cdot c_0(z) \cdot v_b$
Peak velocity pressure at a height z	$q_p(z) = [1 + 7 \cdot I_v(z)] \cdot \frac{1}{2} \cdot \rho \cdot v_m^2(z)$

PART II. Total wind force coefficient in the direction of the wind over section c_f

Overall envelope area of the tower segment	A_{segment}	Projected area of flat-sided ancillaries for one linear meter of tower height	$A_{A,f} = 0.16 \text{ m}^2/m$
Total projected area when viewed normal to the face of the flat-sided section members	A_f	Projected area of circular-sided ancillaries for one meter of tower height	$A_{A,c} = 0.20 \text{ m}^2/m$
Total projected area when viewed normal to the face of the circular-section members in the sub critical regimes	A_c	Total projected area of ancillaries	$A_A = A_{A,f} + A_{A,c}$
Wind incidence factor	$K_\theta = 1$	Solidity ratio	$\varphi = (A_S + A_A)A_{\text{segment}}$
Overall normal drag coefficient for flat-sided sections	$c_{f,A,0} = 2$	Coefficients for triangular structures	$C_1 = 1.9, C_2 = 1.4$
Reduction factor	$K_A = 0.8$	Total projected area of structural members	$A_S = A_f + A_c$
Angle of wind incidence to the longitudinal axis of a linear member	$\psi = 90^\circ$	Wind force coefficient of structural part	$c_{f,S} = K_\theta \cdot c_{f,S,0}$

The force coefficients for sections composed of flat sided elements	$c_{f,0,f} = 1.76 \cdot C_1 [1 - C_2 \cdot \varphi + \varphi^2]$
The force coefficients for sections composed of circular elements	$c_{f,0,c} = C_1 \cdot (1 - C_2 \cdot \varphi) + (C_1 + 0.875)\varphi^2$
Value of the overall normal force coefficient	$c_{f,S,0} = c_{f,0,f} \frac{(A_f + A_{A,f})}{(A_S + A_A)} + c_{f,0,c} \frac{(A_c + A_{A,c})}{(A_S + A_A)}$
Wind force coefficient of the ancillaries	$c_{f,A} = K_A \cdot c_{f,A,0} \cdot \sin^2(\psi)$
Total wind force coefficient in the direction of the wind over a section	$c_f = c_{f,S} + c_{f,A}$

PART III. Standard equivalent gust wind load in the direction of the wind on the tower's body $F_{T,W}(z)$

Structural factor (assumption regarding structural type)	$c_s c_d = 1.05$	Segment reference area	$A_{ref} = A_S + A_A$
Mean wind load in the direction of the wind on the tower segment	$F_{m,W}(z) = \frac{q_p(z)}{1 + 7 \cdot I_v(z)} c_f A_{ref}$		
Standard equivalent gust wind load in the direction of the wind on the tower's body. Where h is the total height of the tower	$F_{T,W}(z) = F_{m,W}(z) \left[1 + \left(1 + 0.2 \left(\frac{z_m}{h} \right)^2 \right) \cdot \frac{(1 + 7 \cdot I_v(z)) \cdot c_s c_d - 1}{c_0(z)} \right]$		