

DEVELOPMENT AND VALIDATION OF A VON MISES PLANAR TRUSS MODEL FOR STUDYING STRUCTURAL NONLINEARITIES

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Abstract: The paper focuses on developing a simple symmetric planar truss structure that could be effectively used to understand and study nonlinearities in large structures. A simple two-member truss model was defined and used for the study. A simple analytical model was derived using the theories of first and second order, which were then solved using the numerical formulations. To validate the model, an experimental setup was created and mounted on a tensile testing machine to apply required loads on the system. The results obtained from the test are used to validate the numerical model. Different parametric studies were conducted using the validated model to further understand the effects of the different variables on the model's behaviour.

KEYWORDS: Planar truss, von Mises truss, theories of first and second order, numerical methods, experimental methods, nonlinearities, buckling, snap-through, spring supports

1 Introduction

Structural Engineering predominantly deals with understanding and analysis of mechanical systems for ensuring their safety, performance, and material efficiency. Most research in the field focused on linear models and their analysis for easier development and solutions. Methods such as joints, sections, and matrix stiffness formulations efficiently analyse these systems under small deformations and linear elastic assumptions. However, in cases where the geometric changes become appreciable, like in the case of large deformations or buckling, or when the material exhibits constitutive nonlinear behaviour, like plasticity, the corresponding model departs from linearity and must be treated within the framework of nonlinear mechanics; see [1] to [3].

Nonlinearity in structures mainly manifests through geometric nonlinearity and material nonlinearity, often coupled to produce complex phenomena such as snap-through, post-buckling, and path dependency. Recognizing these mechanisms is essential both for accurate prediction of structural performance and for safe and optimal design of structures across civil, mechanical, and aerospace applications; see [4] to [6].

One simple and effective way to model, solve, and teach the nonlinearities in a structure is to employ planar truss structures. The simplicity of the trusses makes them an ideal structure

for modeling and learning nonlinear behaviors from different models. Many specialized literary sources offer details about truss structures, including their modeling and various methods for solving different nonlinearities, such as finite element analysis and computational techniques that enhance understanding of their behavior under complex loading conditions. Historically, truss theory developed from statics and strength of materials, with the assumptions of small deformations and linear elastic behavior to exploit the axial nature of the members and the forces associated with them. The necessity to use them in nonlinear formulations became clear when engineers started encountering structures with limit points and snap-through effects; see [3].

Landmark advances in the field came with the formulation of tangent stiffness matrices and incremental-iterative strategies like the Newton (Newton-Raphson) methods being adapted to follow equilibrium in the deformed configuration. Methods such as arc-length continuation methods, which can trace unstable branches beyond limit points, further improved the field. Contemporary studies reinforced these developments by providing analytical solutions for planar trusses with combined geometric and material nonlinearities and by validating such solutions against nonlinear finite element computations, thereby anchoring nonlinear truss analysis within both theory and practice; see [2].

Analytical modelling and the numerical methods used to solve subsequent models have significantly diversified the solution landscape for nonlinear planar truss analyses. Incremental-iterative solution techniques like the Newton-Raphson method remain the backbone of the numerical modeling of truss structures. While methods like the Newton-Raphson augmented by tangent stiffness updates enable convergence at each load step, methods like the arc-length continuation method allow tracking of equilibrium paths through limit points, snap-through, and snap-back nonlinearities; see [3].

For dynamic and frequency-domain problems, reduced-order modeling (ROM) has emerged to treat large truss assemblies with nonlinear joints, combining multi-harmonic description functions, condensed hybrid elements, and continuation strategies tailored to complex algebraic systems; see [5]. With the help of Lagrangian formulations to account for the kinematic nonlinearity, classical matrix stiffness methods still underpin assembly and solution. Collectively, these methodologies provide computational tools to predict and design for large deformation, path-dependent behavior, and multi-physics coupling with planar truss systems; see [3] to [5].

Another closed form analytical expression was developed to predict the buckling behavior of open coiled helical springs, providing a more efficient alternative to purely numerical and experimental approaches; see [7]. The study derives stability criteria by accounting for geometric and material parameters, offering improved insight into critical load estimation for spring structures. This contribution is particularly relevant to nonlinear truss analysis, as helical springs can be idealized as discrete structural elements exhibiting significant geometric nonlinearity. The closed-form solution enhances the understanding of instability phenomena and can inform the development of more accurate and computationally efficient models for nonlinear structural systems.

Along with analytical formulations, experimental investigations also play a crucial role in validating the models and revealing the richness of nonlinear truss behavior beyond analytical and numerical models. Laboratory studies on adaptive planar truss beams have demonstrated controlled large shape changes via actuated members, enabling stress homogenization and significant redistribution of internal forces; see [8]. Measurement with the help of strain sensors and optical tracking showed strong correlation with nonlinear numerical predictions. Experimental programs on modified von Mises planar trusses, canonical prototypes for snap

through and buckling, have explicitly compared first and second order analytical models with measured responses, documenting buckling, snap through, and error estimates and highlighting the necessity of nonlinear modeling to capture observed instability phenomena; see [9]. Such experimental designs are crucial for calibrating and assessing the performance of control strategies in adaptive trusses, particularly where joint clearances, friction, and hysteresis introduce additional nonlinearities; see [8].

Data-driven and AI-assisted methods are shaping recent developments in nonlinear planar truss mechanics. Recent computational mechanics work proposes data-driven stable algorithms and sequential linear programming within large-deformation frameworks to capture geometric instabilities and multiple equilibrium states, like snap through and post-buckling, with high accuracy and controllability; see [10].

Parallel machine learning approaches are being recently explored to speed up nonlinear solvers, represent nonlinear behavior with learned truss elements, and augment or replace iterative procedures to improve convergence and computational efficiency in complex structures; see [11]. There are also new approaches on the experimental side, using enhanced test beds for nonlinear joints and adaptive trusses, which could support tighter model validation loops; see [5] and [8]. As these developments mature, we can expect integrated models that combine physics-based nonlinear formulations, reduced-order surrogates, and data-driven models to deliver accurate, scalable, and real-time analysis and design tools for planar truss systems.

Nonlinear planar truss analysis is always considered relevant to modern engineering practices. Trusses form essential components in structures, including bridges, roads, towers, and space structures. As structures become larger, lighter, and multifunctional, nonlinear dynamic behavior and vibration control become central to reliability and performance; see [6]. Since the imperative to ensure structural safety under extreme events requires steps for progressive collapse mechanisms, nonlinear analysis integrated with reliability-based optimization helps us in demonstrating tangible reductions in collapse probability. Furthermore, in advanced sectors like aerospace, joints with clearances and complex hysteretic behavior create pronounced nonlinearities that directly affect the frequency response and coupling, making accurate nonlinear modeling indispensable for design and control; see [3] to [5]. Thus, the study of nonlinear planar trusses underpins robust, efficient, and safe structural design across multiple domains.

Another notable approach in solving nonlinear truss structures was done using probabilistic methods. A notable work in this area is the integration of Simulation-Based Reliability Assessment (SBRA), which is a Monte-Carlo driven framework used to quantify the probability of plastic deformation in truss members; see [12]. By applying SBRA directly to nonlinear mechanical models, the study illustrates that reliability outcomes are highly sensitive to the fidelity of the underlying structural analysis and that linearized approaches often underestimate likelihood of failure. The study bridges deterministic nonlinear mechanics with modern probabilistic assessment, providing transparent, educational examples that clarify how nonlinearities propagate through uncertainty quantification. These studies form a methodological foundation for subsequent reliability analyses of slender truss systems where geometric effects and stochastic variability must be jointly considered.

The paper focuses on the nonlinearities arising from geometric deformations with special emphasis on planar truss structures, thereby understanding and accurately modeling the nonlinear behavior, which is essential in predicting structural performance, avoiding failures, and designing more efficient systems. The study starts with developing an analytical model utilizing the concepts of first- and second-order theories for a two-member planar truss system

with spring support; see [13]. The ensuing nonlinear model is then solved using iterative numerical approaches like the Newton-Raphson method with the help of programming languages like Python. To bridge theory and practice, the study is furthered by performing an experimental analysis of the proposed model using the experimental setup developed in-house, and the results are compared with those obtained from the numerical solvers. The study concludes with some parametric studies aimed at obtaining a more sophisticated grasp of the nonlinear truss model behavior, which could aid in the development of reliable computational tools for engineering analysis and design.

2 Methodology

2.1 Analytical Model

The analytical model consists of a two-ideal-member (without imperfections) truss structure supported at the end with a spring system; see [9] and [14]. The model is an ideal case of a modified von Mises truss system; see Fig. 1.

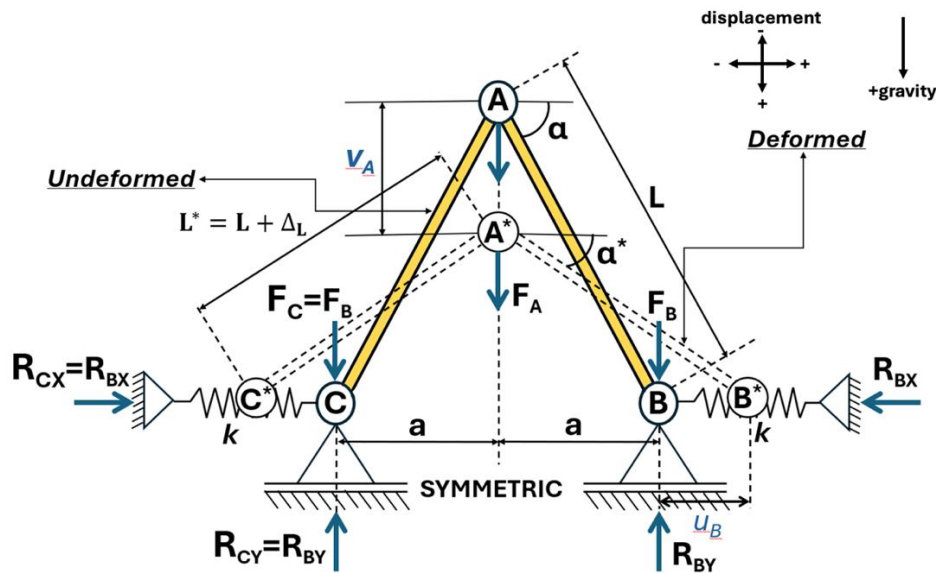


Fig. 1 Von Mises truss structure loaded with force F and elastic supports

The expressions for the analytical solution are derived using the Method of Joints for each joint to obtain a relation between the force applied to Joint A and the displacement resulting from this force in Joints B and C. Following the symmetric nature of the model, assumptions can be made regarding the similarities in the forces $F_B = F_C$ (N) and reaction forces $R_B = R_C$ (N) being applied at the two joints B and C as inferred from the figure due to the similar nature of the displacements at these joints. Several geometrical and material properties, including the forces due to the weight of the system components, the dimension of the members and the material properties of the members, are considered to be known initially.

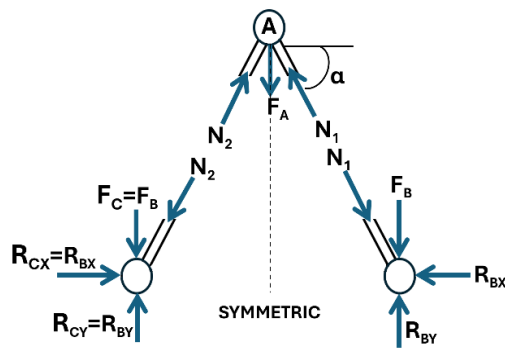


Fig. 2a: Undeformed geometry
(Theory of 1st order)

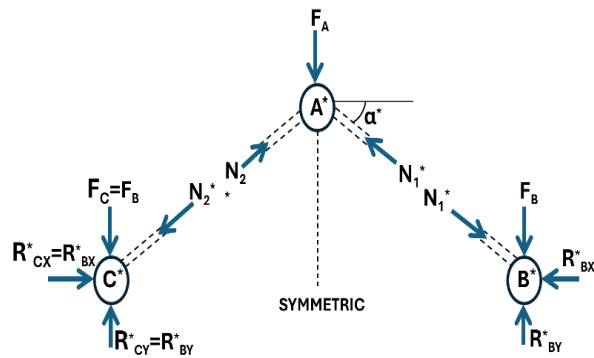


Fig. 2b Deformed geometry
(Theory of 2nd order)

Fig. 2 Loading and Reactions of von Mises Truss

2.1.1 Theory of first order

First-order theory [13] follows the assumption of the linear dependence between the load and the deformation occurring in the system, providing nearly no visible change in the angle α between the members. Hence, changes of α can be neglected. Given an external load F (N) acting on the truss, the component of forces acting on each joint, which will be a combination of forces due to the weight of the joints, truss members, and springs, can be derived as follows:

$$F_A = F + G_A + G_{\text{ROD}}, \quad F_B = G_B + \frac{1}{2}G_{\text{ROD}}, \quad F_C = F_B \text{ (i. e. symmetry)}$$

Here the forces F_A , F_B , and F_C (N) represent the combined forces acting at the joints A, B, and C, respectively. The force F is the external loading applied on the system and the forces G_A , G_B (N) denote the dead loads applied due to the weight of the joint parts, including the springs that are attached to the joints. Finally, load of G_{ROD} represents the weight of the truss members with original length L (m).

According to Fig. 2a, the equilibrium conditions following the first-order theory for joint A where the external loading is applied can be given as follows:

$$\begin{aligned} \Sigma F_x \text{ at point A} &= 0 \Rightarrow N_1 \cos \alpha - N_2 \cos \alpha = 0 \Rightarrow N_1 = N_2 = N \\ \Sigma F_y \text{ at point A} &= 0 \Rightarrow F_A - N_1 \sin \alpha - N_2 \sin \alpha = 0 \Rightarrow N = \frac{F}{2 \sin \alpha} \end{aligned}$$

where N_1 and N_2 (i.e., N) are the normal (axial, internal) forces.

The formulation, which was derived under the assumption of the first-order theory and symmetry, provides a linear dependency and can be effective only in the case of small deformation models where the effect of deformation on the change of the geometric properties can be ignored. The model's inability to compare with an experimental model indicates the necessity of modifying the formulation to include the geometric nonlinearities present in the physical model. These effects can be incorporated using the theory of second order [13], which can provide a more reliable analytical model that accurately reflects the geometric nonlinearities and improves the model's predictive capabilities.

2.1.2 Theory of second order

Theory of second order assumes the system undergoes larger deformations resulting in a change in the original configuration due to the deformation that occurred, thereby resulting in a change of geometric properties, including the length of the members (i.e., L changes to $L +$

Δ_L) and the angle between the members (α to α^*). The deformation will result in a change of positions for the joints and a change in the forces acting on the members.

According to Fig. 2b, the new equilibrium for the system (denoted by $*$) that follows the second-order theory at the new position (A^*) can be denoted as follows:

$$\Sigma F_X \text{ at point } A^* = 0 \Rightarrow N_1^* \cos \alpha^* - N_2^* \cos \alpha^* = 0 \Rightarrow N^* = N_1^* = N_2^*$$

$$\Sigma F_Y \text{ at point } A^* = 0 \Rightarrow N_1^* \sin \alpha^* + N_2^* \sin \alpha^* - F_A = 0 \Rightarrow N^* = \frac{F_A}{2 \sin \alpha^*}$$

Following this, we can derive the equilibrium condition of the remaining points B^* and C^* (similar due to the condition of symmetry) as follows:

$$\Sigma F_X \text{ at point } B^* = 0 \Rightarrow N^* \cos \alpha^* - R_{BX}^* = 0$$

$$\Sigma F_Y \text{ at point } B^* = 0 \Rightarrow N^* \sin \alpha^* + F_B - R_{BY}^* = 0$$

$$\Sigma F_X \text{ at point } C^* = 0 \Rightarrow N^* \cos \alpha^* - R_{CX}^* = 0$$

$$\Sigma F_Y \text{ at point } C^* = 0 \Rightarrow N^* \sin \alpha^* + F_C - R_{CY}^* = 0$$

The horizontal components of the reaction forces at the joints are influenced by two external forces, mainly by the spring force from the spring attached at the joints and depend on the stiffness of the spring k (N/m) and the Coulomb friction force associated with the displacement of the joints in the horizontal direction, which depends on the coefficient of Coulomb friction f (1). The displacement due to the deformation also plays a key role in developing the analytical model with a vertical displacement v_A (m) acting on the joint A and a horizontal displacement $u_B = u_C$ (m) acting at the joints B and C respectively.

By assuming that the bending of the members is not present, the relations for displacement from Fig. 3 are as follows:

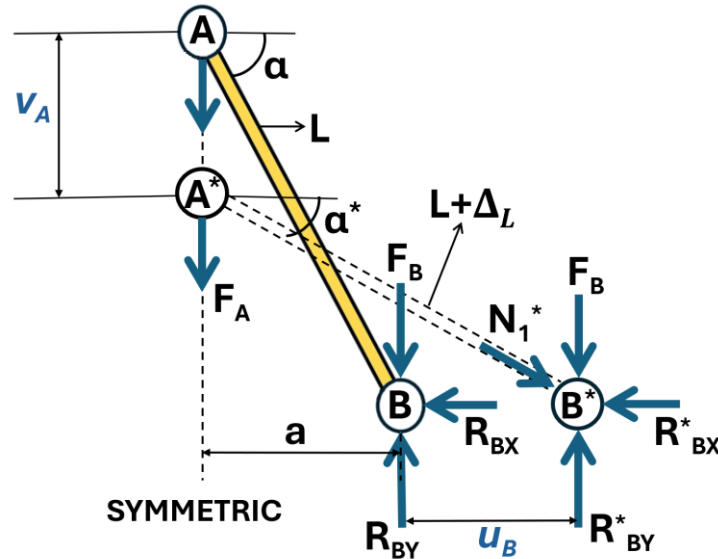


Fig. 3 Method of Joints

By assuming that the bending of the members is not present, the relations for displacement from Fig. 3 as follows:

$$a + u_B = (L + \Delta_L) \cos \alpha^*$$

$$L \sin \alpha = v_A + (L + \Delta_L) \sin \alpha^*$$

As a first step, the system is modeled without any material nonlinearities and is therefore expected to follow a linear dependency between stress and strain among the members (Hooke's

Law). This equation will allow us to establish the relationship between the material properties and the axial elongation Δ_L experienced by the members because of the applied load. Using Hooke's Law, we obtain

$$\Delta_L = \left(\frac{N^* L}{EA} \right)$$

where E is the Young's modulus and A is the cross-sectional area of the members.

All these equations form the basis of the analytical model developed. The model primarily depends on three variables: the external loading on the truss members, the angle between the truss members and the deformation caused by the application of the load. These variables mainly depend on factors including frictional forces due to horizontal displacement and the spring force from the attached springs at the end of the truss members. The analytical model developed using all these equations and variables can be solved with the help of a numerical solver like the Newton-Raphson method, which provides faster convergence and higher accuracy. The developed model can also be solved using other numerical methods, such as the Bisection method or the Secant method, or even used to make comparisons between different numerical methods to evaluate their efficiency and accuracy in solving the model.

2.2 Numerical Formulation

The analytical model and the obtained equations indicate that the relationship between the applied load and the resulting displacement due to deformations is influenced by multiple factors. Analytical solutions become harder as the dependent variables increase, thereby creating the necessity of solving the model using numerical methods, particularly when dealing with complex interactions among the variables that affect the system's response. From the analytical model, the force applied on the system and the displacement due to the applied force can be expressed as a function of different variables as shown:

$$F = f(k, L, EA, \alpha^*, \alpha, F_B, f)$$

$$v_A = f(k, L, EA, \alpha^*, \alpha, F_B, f)$$

In this case, many of these variables are known (prescribed) and remain constant throughout the loading phase, which will help us simplify the relationship:

$$F = f(\alpha^*)$$

$$v_A = f(\alpha^*)$$

The nonlinear dependence of force and displacement on the angle between the members is solved in turn, giving us the force and displacement dependence due to the applied force.

Numerical methods like the Newton-Raphson method are used to solve for the nonlinear dependency between force and displacement. For our study, we make use of the programming language Python [15] and different libraries available within the language to create a code that was used to solve the numerical formulations obtained from the analytical model; see [16] to [18]. With most of the variables known from the experimental setup, the corresponding relation for the force and the displacement was derived and solved to obtain the force–displacement dependency, which was then used to compare with the result from the experiments performed.

2.3 Experimental Setup

A major part of the study is designing, configuring, and setting up the experimental apparatus. This will be used to test different planar truss configurations with different member lengths and spring types to get reference data for comparing the results from the analytical

model. The experimental setup was designed to have the appropriate flexibility for modifying various system parameters, including the initial length of the truss member, the starting angle between the members, the span of the truss structure, the spring used at the joints and so on. This flexibility helps the experimental setup to be robust and adapt to different requirements if needed. The setup was designed in such a way that it could be mounted on a tensile testing machine, which provides the vertical loading needed at the joint of the two truss members (Fig. 4). The experimental setup will be used to perform various experiments involving truss members with the purpose of understanding the structure's behavior under load and the influence of geometric nonlinearities on the beams. The apparatus could be used for studying various nonlinear effects occurring in the beams, including buckling, snap through, and so on. The experimental models could also serve as a baseline for developing different analytical models and validating the numerical methods used to solve them.

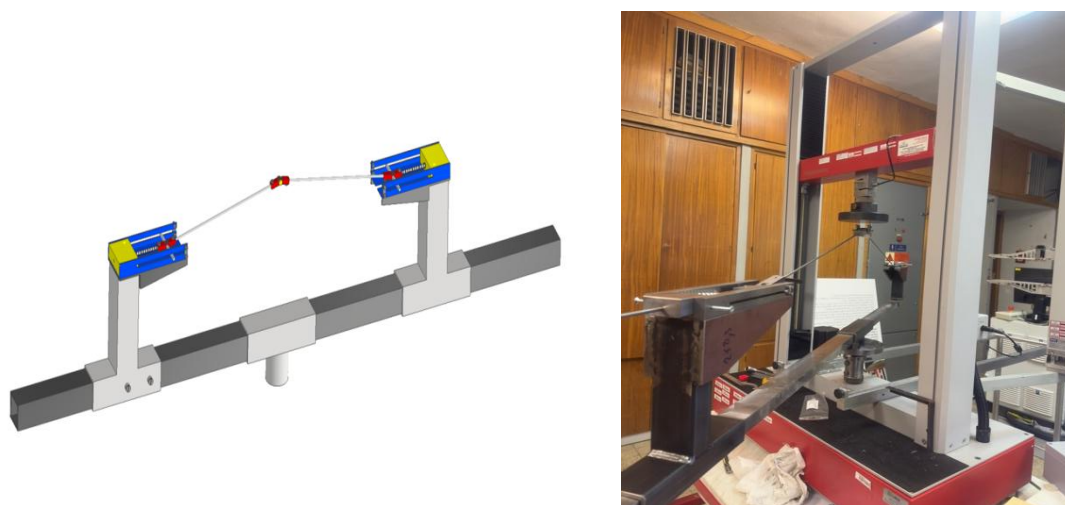


Fig. 4 Experimental Setup

The accuracy of the results obtained from the numerical method determines the effectiveness of the analytical model and the possibility of using similar models to understand the nonlinearities associated with the truss structures better. The aim of the experimental results is to validate the numerical results and assess how closely the developed model represents the nonlinearities present in the physical model. The validation of the numerical model using the results from experimentation will provide a baseline from which further numerical models can be developed with increasing complexities. The experimental models have been kept simple for the initial study with the possibility of introducing further variations in the future, including unsymmetrical truss members, varying loads and stiffness and so on. The following configurations were tested using the developed experimental setup and to validate the corresponding numerical methods. The experiments focused on symmetric planar truss models with different configurations of springs used to provide support to the elements.

4 Results

The above configurations were used to test the relationship between the load on the planar truss and the resulting deformation, which is caused by the deflection of the spring system at the joint of the truss members. The experimental setup mounted on the tensile testing machine is applied with loading from the tensile testing machine on the joint connecting the two truss members. Each experimental configuration was tested twice to ensure reliability of the results and to avoid potential errors that could arise during the testing. An example of the results obtained from the experimental setup is given in Fig. 5:

Table 1. Dimensions for Experiments

	$K \text{ (Nm}^{-1}\text{)}$	$L \text{ (m)}$	$A \text{ (m}^2\text{)}$	$a \text{ (m)}$	f	$E \text{ (Pa)}$
<i>Experiment 1</i>	3450	0.533	1.936×10^{-5}	0.5	0.7	2.08×10^{11}
<i>Experiment 2</i>	1850	0.533	1.936×10^{-5}	0.5	0.7	2.08×10^{11}
<i>Experiment 3</i>	1280	0.533	1.936×10^{-5}	0.5	0.7	2.08×10^{11}
<i>Experiment 4</i>	718	0.533	1.936×10^{-5}	0.5	0.7	2.08×10^{11}
Forces (N) were measured by weight.						
G_A			6.129			
G_B			5.363			
G_{ROD}			0.885			

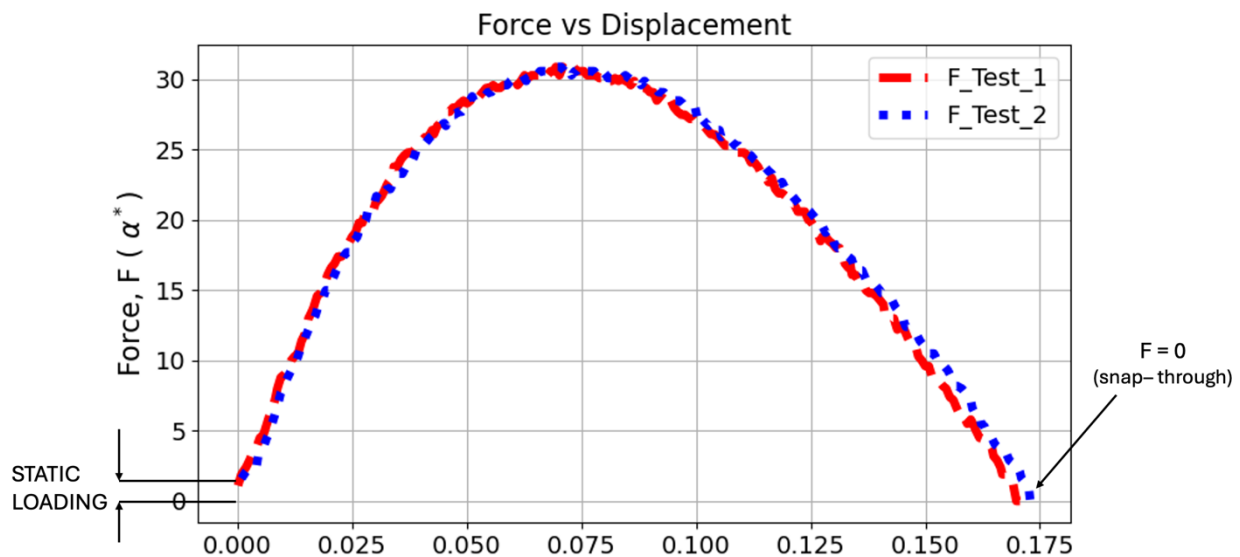


Fig. 5: Results curve for Experiment 1

The results obtained show an expected trajectory obtained for a typical Force - Displacement curve. As the loading is applied to the truss members, the corresponding deformations at the joints increase along with the deflection of the supporting springs. The magnitude of the curve keeps on increasing until they reach a peak value where the effect of the force is reversed by the sudden change in the equilibrium state of the truss members because of the snap-through effect. The angle between the truss members reaches a value greater than 180 degrees, thereby reversing the effects of forces on the deformation of the spring system.

Modifications to the experimental results were required to ensure clarity and precision. The test exhibited an early displacement caused by the forces resulting from the weight of the structure. It generated an initial force that counteracts the reaction forces within the components of the beam. The results also indicated some instabilities at various points, such as when contact occurred between the loading element and the truss structure and when the truss exhibited sudden displacement due to its own gravitational weight after reaching a certain point without assistance from the loading.

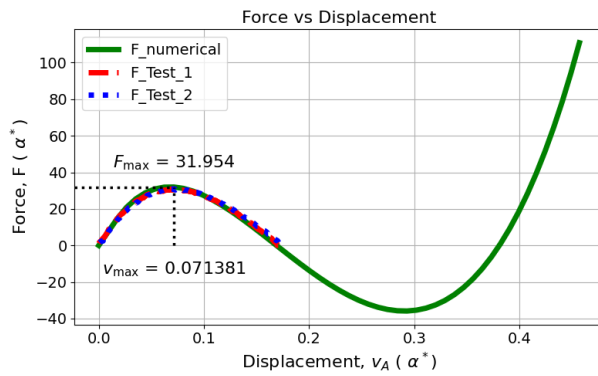


Fig. 6a – Experiment 1

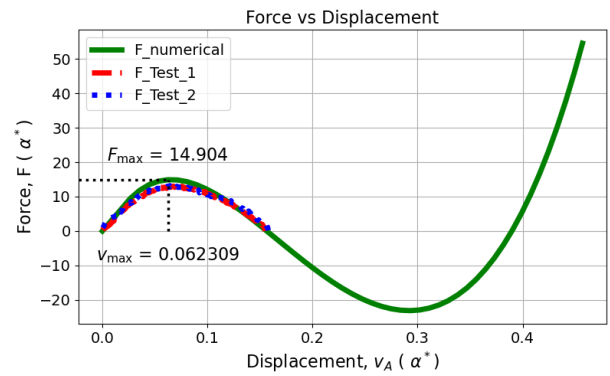


Fig. 6b – Experiment 2

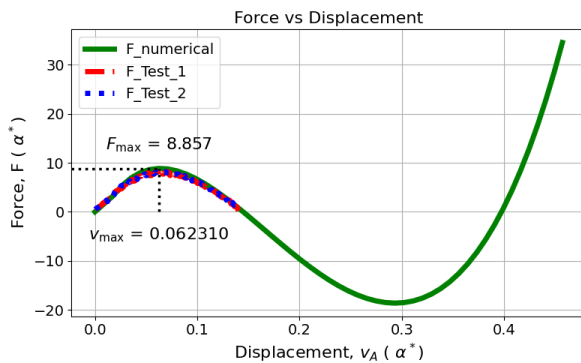


Fig. 6c – Experiment 3

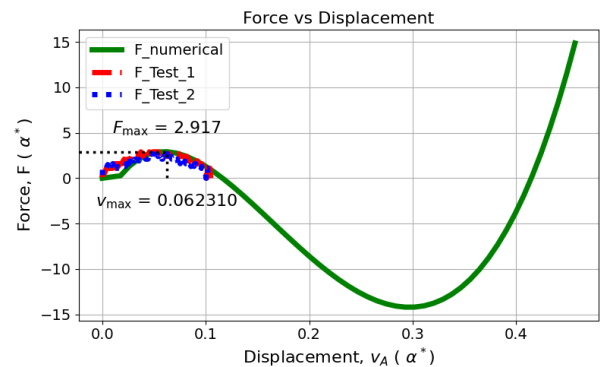


Fig. 6d – Experiment 4

Fig. 6 Comparison of Numerical and Experimental Results (Inputs are defined in Table 1)

The results from the experimental tests are compared with those from the developed numerical model, and they show good correlation. The numerical model was able to capture the behavior of the experimental curves well and provided further behavior outside the range of the experimentation, including predictions of snap-through behavior and insights into potential applications in engineering design. The snap-through behavior is captured properly and the error estimations are also good. The curves show closeness to the experimental values well, especially in the models with higher spring stiffness. It was observed that the numerical models with lower stiffness show linear behavior at the initial displacements (Fig. 6d). This is because at lower displacements and low angles, the model follows Theory of first order more and hence provides a curve with a linear nature. But as the displacement and the angle increased, it was able to capture the nonlinear nature of the model much better, demonstrating a significant deviation from the linear behavior observed at lower displacements and angles, which indicates that the model's accuracy improves with greater complexity in the parameters being analyzed.

Based on the results, it is evident that the force curve before the snap-through has a smaller peak compared to the force curve after changing the equilibrium condition. Several factors could explain this behavior. One of the key reasons behind it might relate to the geometrical nonlinearity. A number of factors contribute to this, namely change in member orientations during deformations, different internal force distribution and the ability of some members to carry higher effective loads after snap-through. Another reason relates to the energy stored in the system for two different equilibrium conditions. The first peak corresponds to overcoming the energy barrier. However, after the change in equilibrium, the structure settles into a different configuration that may require higher restoring force to reverse. Other factors, such as imperfections, asymmetry, and load path dependence, can contribute to a higher peak value after the snap-through occurs, potentially leading to increased stress and structural failure if not properly managed.

The dependencies of the loading force on the angle can be easily established using Fig. 7. The system begins at the initial point when it is unloaded ($F = 0$). The system is then gradually compressed with a positive force, resulting in a positive increase in the angle until it reaches the peak value. The force rapidly decreases after reaching its peak, abruptly altering the equilibrium conditions and triggering the snap-through effect. The new equilibrium follows the change in the angle closely and the corresponding section of the curve will have a negative peak that is higher than the first peak.

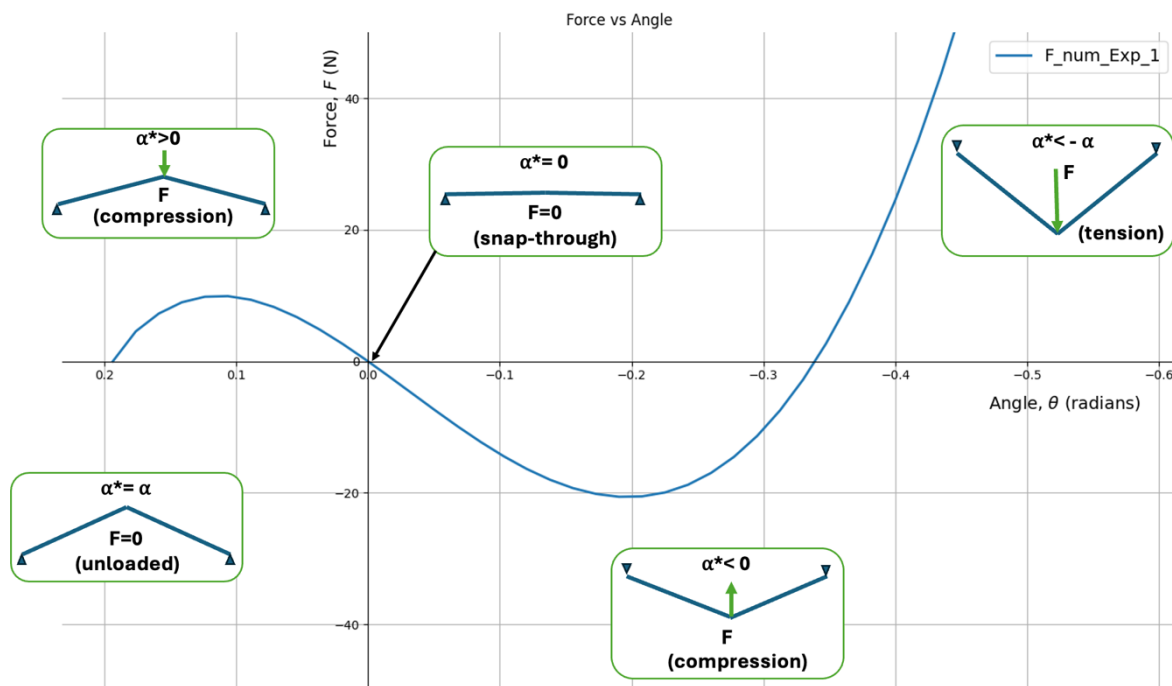


Fig. 7 Dependencies of loading force on angle

An error estimation is done to ensure that the validation of the numerical model with the experimental results is reliable and does not deviate much from the actual results obtained from the tests (Table 2). An error estimation within a satisfactory tolerance range is always a positive indication of the correlation model. The error estimation for the above experiments shows a positive trend with a satisfactory comparison to the experimental data, especially for cases with higher spring stiffness.

Table 2. Error Estimations

Error Estimation (%)	$F_{\max}$	$V_{\min}$	F_{mean}	F_{median}	F_{sd} (Standard Deviation)
Experiment 1	1.5	3.09	3.3	1.3	2.49
Experiment 2	6.81	3.47	5.69	0.24	12.34
Experiment 3	6.5	1.23	2.01	4.53	27.22
Experiment 4	4.18	34.72	11.74	7.10	65.19

The overall error estimation is satisfactory with the exception of some values, which vary with different configurations and models. Overall, the values show a positive trend going forward. The deviation of the results may be due to several factors that differ from the actual experimental results obtained from the numerical model. Some of them include clearance of joints, homogeneous weight of the rods versus point loads, bending of the long rods, assumption of the material to be homogeneous and isotropic and so on.

A clear dependency of the structure with different variables can be better understood with the help of parametric studies. These parametric studies could shed more light on how the system behaves when there is a change in a particular variable and could help in predicting the behavior of the system and designing it better.

The primary variable that varied in the experiments above is the spring stiffness and hence it is important to understand how the variation of the stiffness affects the overall forces acting on the system and the corresponding deformations due to it. A parametric model is made using the stiffness values of some of the known metals used for spring manufacturing. The results obtained can be seen in Fig. 8.

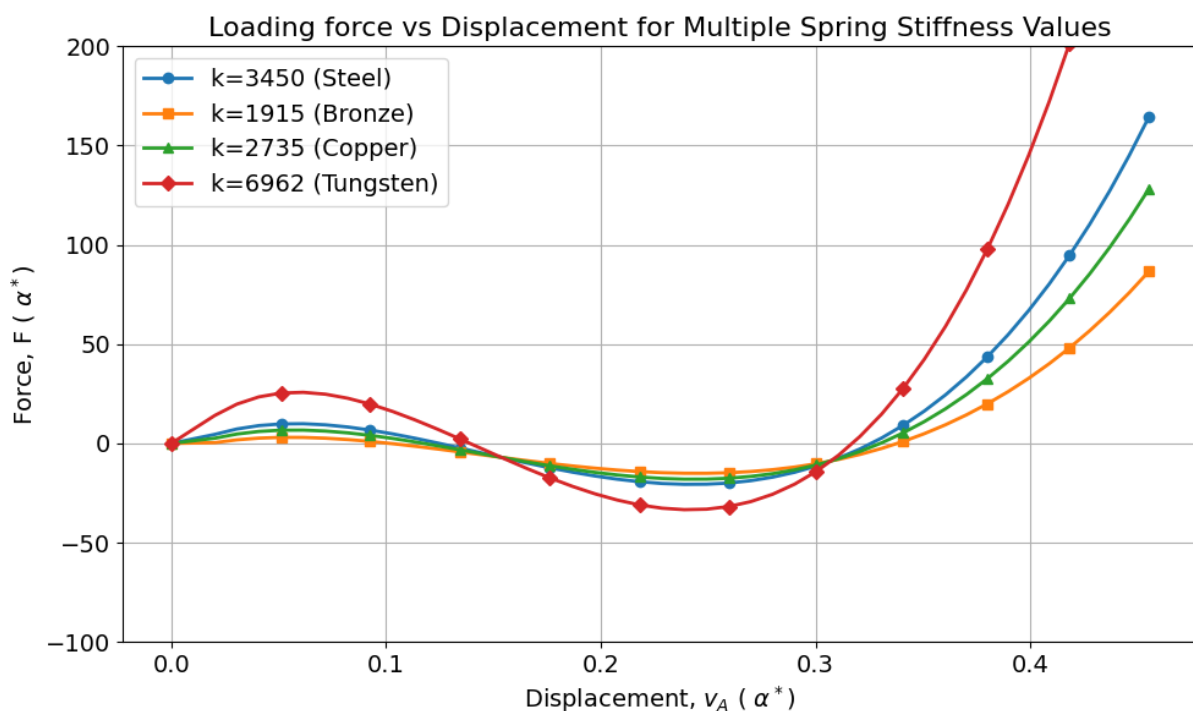


Fig. 8 Dependencies of force on stiffness of springs (Parameters from Exp. 1)

The curves follow the expected pattern, with the peak force increasing with the increase in stiffness, as the stiffer springs will reduce the deformations, causing the force to be on the higher side. The maximum force, F_{max} , directly depends on the stiffness and increases as the stiffness of the spring increases. The maximum force also depends on the length of the truss members but has an inverse effect between them. As the length of the member increases, the maximum force goes down. This can be clearly visible from Fig. 9 which does a parametric study of calculating the maximum forces and plotting it against the stiffness of the springs for different lengths of the truss members.

The curve showed the dependency of the maximum force on stiffness for different lengths. However, the dependency of the maximum force on the length of the truss members looks very different and that can be another parametric study that could give a clearer picture regarding the dependency of different variables. As discussed earlier, increasing the length of the truss

members inversely affects the maximum force on the system. The maximum force tends to be reduced as the length of the truss members increases, as shown in Fig. 10.

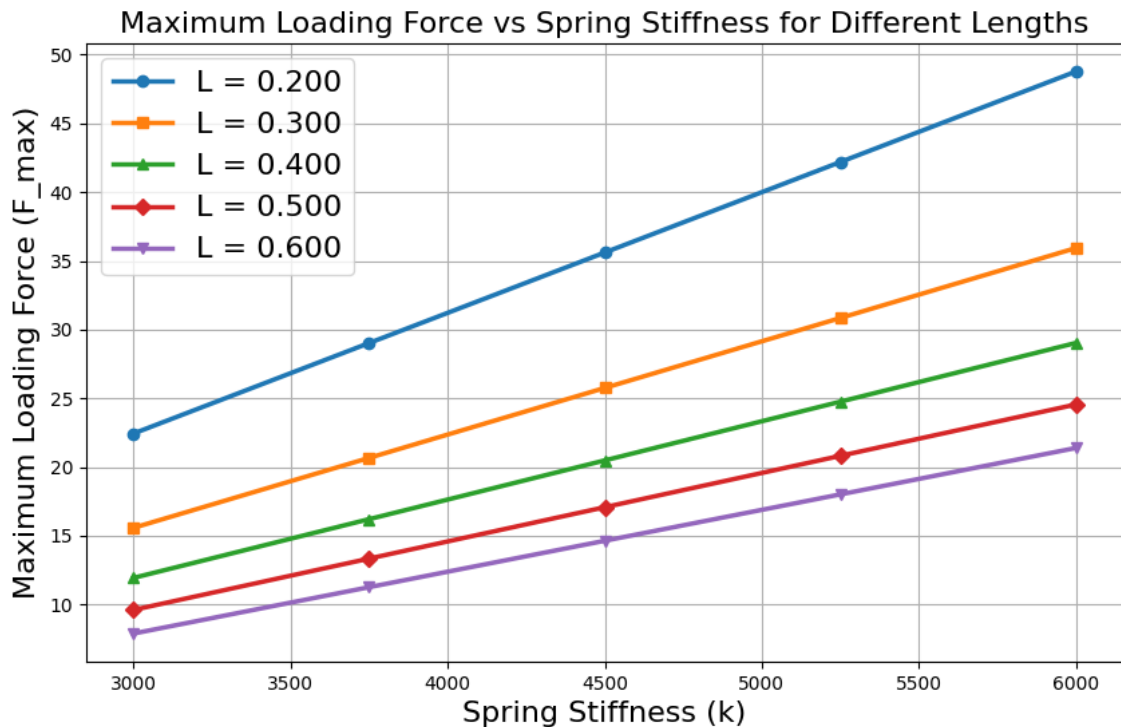


Fig. 9 Dependencies of maximum force and stiffness (Parameters from Exp. 1)

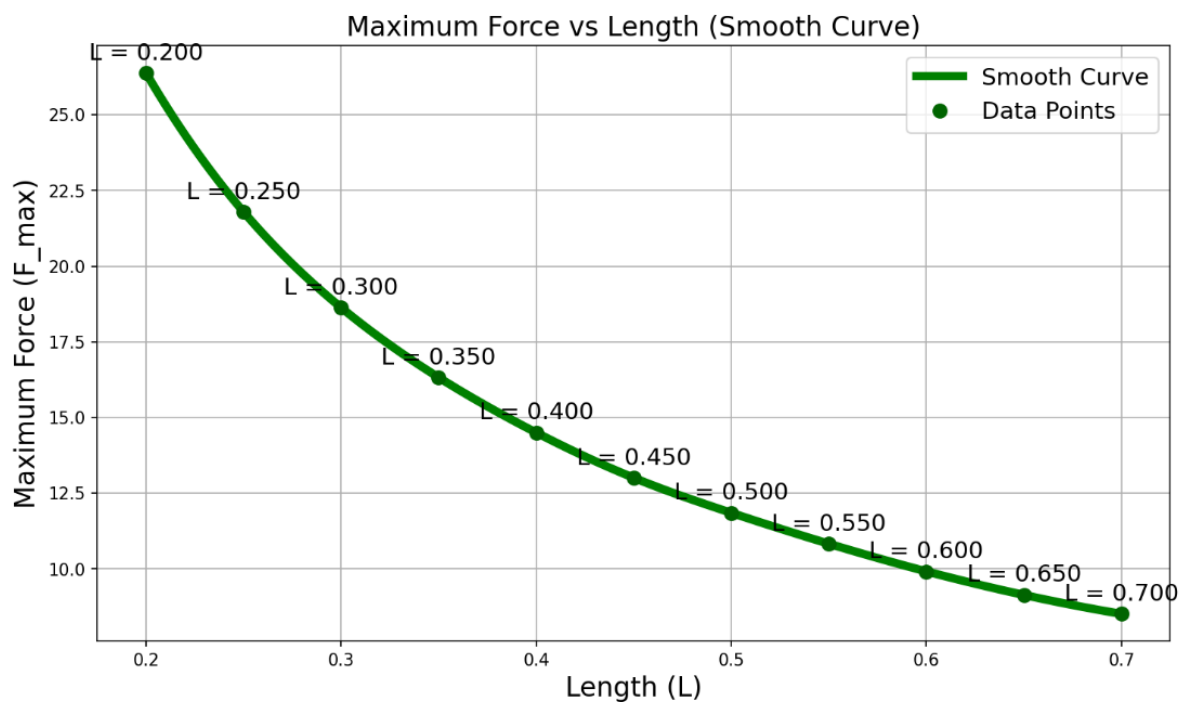


Fig. 10 Dependencies of maximum force and member length (Parameters from Exp. 1)

Other factors, like spring stiffness and truss member length, can also affect system behavior. However, the effects of these other parameters are less pronounced compared to the stiffness of the spring and the length of the truss members, which primarily determine the overall behavior of the system. Understanding this parametric effect is critical to creating a robust and reliable structure, which is the most crucial part of developing a structural system. A simple system like a planar truss can effectively achieve an in-depth understanding of nonlinear systems, as it allows for the exploration of fundamental principles such as load distribution and material behavior under various conditions.

Conclusion

The focus of the study resides in comprehending the behavior of truss structures with different nonlinearities. By analyzing the theoretical frameworks used to model planar trusses, the paper provides a foundation for understanding how these structures respond to large displacements and deformations. A simple analytical model was developed that could encompass the nonlinear behavior of a system, including the snap-through effect. The developed model was then solved using a numerical formulation, such as the Newton–Raphson method, which aided in understanding the system's behavior and the deformation that occurs in the model when external loads are applied. The developed model is further validated using an experimental setup developed in-house. A number of experiments were conducted using various spring systems and the results obtained from the test were used to validate the computational model developed. An error estimation is also performed to ensure that the developed model is robust and reliable. The model's extensive scope allows for a more profound understanding of the system's behavior. For this purpose, a parametric study was also conducted, which gave us more insight on the model behavior in relation to the different variables in the system. The study could serve as a baseline for further research in the field and could enhance the understanding of the nonlinearities affecting the system, thereby improving the design of nonlinear systems. The study can be furthered by introducing more complexities such as unsymmetric members, variable loading, and so on, which would allow researchers to explore how these factors influence system behavior and improve the accuracy of their models. The use of probabilistic methods could also be an option for future research, which could help in the better prediction of models and reliability analyses, particularly in scenarios involving uncertainties and varying parameters that affect the outcomes.

Acknowledgments

This work was supported by Specific Research SP2026/027 “Advanced Numerical and Experimental Approaches in Mechanics and Biomechanics”.

References

- [1] Greco, M., Carlos, E. R. V. "Analytical Solutions for Geometrically Nonlinear Trusses", *Revista Escola De Minas* 62 (2), pp. 205 – 214, **2009**. DOI: 10.1590/s0370-44672009000200012
- [2] Cunha, Pedro F. L. "Analytical Solution for a Geometrically and Material Nonlinear 2D Truss", Zenodo (CERN European Organization for Nuclear Research), **2023**. DOI: 10.5281/zenodo.10516457
- [3] Hrinda, G. A. "Geometrically Nonlinear Static Analysis of 3D Trusses Using the Arc-length Method", *WIT Transactions on Modelling and Simulation I*, pp. 243 – 252, **2007**. DOI:10.2495/cm070261

- [4] Hrinda, G. "Snap-through instability patterns in truss structures", In the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Orlando, FL, USA, 12–15 April 2010, AIAA 2010-2611, Orlando, FL: American Institute of Aeronautics and Astronautics, **2010**. DOI:10.2514/6.2010-2611
- [5] Liu, F., Jin, D., Li, X. et al. "Reduced-order modelling and solution method for nonlinear frequency response analysis of large space truss structures", *Nonlinear Dyn* 112, pp. 10127 – 10145, **2024**. DOI: 10.1007/s11071-024-09572-1
- [6] Liu, M., Cao, D., Wei, J. "Survey on Equivalent Continuum Modelling for Truss Structures and Their Nonlinear Dynamics and Vibration Control", *J. Vib. Eng. Technol.* 10, pp. 667 – 687, **2022**. DOI: 10.1007/s42417-021-00398-4
- [7] Vebil, Y. "A Closed-Form Buckling Formula for Open-Coiled and Properly Supported Circular-Bar Helical Springs." *Strojnícky časopis – Journal of Mechanical Engineering* 68 (3), pp. 33 – 48, **2018**. DOI: 10.2478/scjme-2018-0025
- [8] Reksowardojo, Arka P., et al. "Experimental Testing of a Small-Scale Truss Beam That Adapts to Loads Through Large Shape Changes", *Frontiers in Built Environment* 5, **2019**. DOI: 10.3389/fbuilt.2019.00093
- [9] Sudhakar, A., Čepica, D., Trefil, A. "Comparison of Analytical and Experimental Approaches for Modified von Mises Truss Behavior", *Engineering Mechanics International Conference 2025*, 31 (45), pp. 205 – 208, **2025**. DOI: 10.21495/em2025-205.
- [10] Huang, M., Du, Z., Cui, T. et al. "Data-driven-based stable analysis algorithm for nonlinear truss structures with geometric instabilities", *Comput Mech* 77, pp. 451 – 465, **2026**. DOI: 10.1007/s00466-025-02672-8
- [11] Shin, S., et al. "Nonlinear Analysis of Truss Structures Based on Machine Learning", *The 2023 World Congress on Advances in Structural Engineering and Mechanics (ASEM23)*, conference proceedings, **2023**. Available at: [www.i-asem.org/publication_conf/asem23/1.SM/\[0237\].pdf](http://www.i-asem.org/publication_conf/asem23/1.SM/[0237].pdf).
- [12] Frydryšek, K., Jančo, R. "Simple Planar Truss (Linear, Nonlinear, and Stochastic Approach) ", *Strojnícky časopis – Journal of Mechanical Engineering* 66 (2), pp. 5 – 12, **2016**. DOI: 10.1515/scjme-2016-0013
- [13] Frydryšek, K., Jahodová, M., Morávková, Z. "Comparative Study of Simple Truss Solved According to the Theories of 1st and 2nd Order", *Engineering Mechanics* 25, pp. 117 – 120, **2019**. DOI: 10.21495/71-0-117
- [14] Frydryšek, K. "Some Selected Tasks of Elasticity and Plasticity 4 (Basic Nonlinear Mechanics of Deformable Bodies in Examples)", *Department of Applied Mechanics, Faculty of Mechanical Engineering, VSB–Technical University of Ostrava, Ostrava*, pp. 1 – 139, **2016**. ISBN 978-80-248-4152-6
- [15] "Python Software Foundation", *Python*, version 3.8.5., Python Software Foundation, Year **2024**, Available at: www.python.org.
- [16] Harris, C.R., Millman, K.J., van der Walt, S.J. et al. "Array programming with NumPy.", *Nature* 585, pp. 357 – 362, **2020**. DOI: 10.1038/s41586-020-2649-2
- [17] Hunter, J.D., "Matplotlib: A 2D Graphics Environment", *Computing in Science & Engineering* 9 (3), pp. 90 – 95 , **2007**. DOI: 10.5281/zenodo.592536

- [18] McKinney, Wes. "Data Structures for Statistical Computing in Python", Proceedings of the 9th Python in Science Conference, edited by Stéfan van der Walt and Jarrod Millman, pp. 51 – 56, **2010**.