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PREDICTION OF COMPOSITE PANEL OPERABILITY UNDER NON-STATIONARY HEATING BY THE EXTERNAL ENVIRONMENT

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Abstract: A rectangular composite sandwich panel that is convectively heated by the external environment is considered. A physical and mathematical model consisting of two stages is proposed to predict the operability of such a panel depending on the heat transfer conditions. In the first stage, the unsteady temperature field is determined from the relations of the unsteady heat conduction problem. In the second stage, the parameters of the stress-strain state of the panel are determined from the relations of the quasi-static thermoelasticity problem. Under the conditions of a panel hinged at the edges of the considered panel, general solutions to the problems of thermal conductivity and quasi-static thermoelasticity are found using the Fourier and Laplace integral transforms. For numerical analysis, a variant of a composite sandwich panel of the ceramic-metal-ceramic structure was chosen. The temperature field, radial deflections, normal forces and stresses, and bending moments in the sandwich panel under consideration are analyzed as a function of both geometric parameters and the dimensionless Biot criterion.

KEYWORDS: composite sandwich panel, ceramic-metal-ceramic, convective heating, temperature, stress-strain state, thermal conductivity, quasi-static thermoelasticity, Biot criterion

1 Introduction

In various fields of modern technology, layered structural elements are often used to operate at high temperatures and in aggressive environments. The layering properties of these structures are used to insulate them from the external environment, or vice versa, to intensify heat transfer in order to strengthen and increase the reliability of structures. Therefore, the study of the stress-strain state of layered structural elements under conditions of their convective heating by the external environment is an urgent engineering and technical task.

The behavior of layered structural elements has been studied, in particular, in [1–3]. Exact solutions to the problems of thermoelasticity of layered panels based on three-dimensional equations were constructed in [4], [5]. Refined models based on two-dimensional equations are presented in [6]–[8]. Analytical solutions for the bending of heterogeneous and composite plates under thermal and power loads were obtained in [9]–[13]. In [14], the equation of interconnected thermoelasticity was used to analyze the effect of the coefficient of coupling on the nonlinear behavior of plates. In [14], [15], the finite element method was used to study thermoelastic processes in layered plates. The thermomechanical behavior of a nonferromagnetic homogeneous plate under the action of electromagnetic pulses of different durations is given in [16]. The temperature resistance of composite material plates was studied in [17]. Models and methods for studying inhomogeneous thin-walled structures are presented

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in [1], [2], [18], [19]. Practical problems requiring the involvement of the stress-strain state theory for the composite sandwich panels during particular external influences have been addressed in numerous studies, particularly in [20], [21], [22]. The paper [20] considers the impact of adding a micro-mixing chamber with inclined walls and a rectangular rib on heat transfer and pressure drop in a microchannel heat sink. In [21], the authors investigate the mechanical characteristics of composites reinforced with jute fibers and steel fillers to explore their potential as advanced engineering materials. The accuracy and efficiency of Navier and Rayleigh-Ritz methods for predicting the stress-strain state of laminated composite plates under various external influences, compared to finite element analysis, are investigated in [22-24].

The aim of this paper is to study the temperature and stress-strain state of a composite sandwich panel of the ceramic-metal-ceramic structure when it is heated by an external environment with convective heat exchange. The obtained results will be implemented to improve the processes of lapping and polishing of metal parts with the help of ceramic working bodies (laps, polishers, etc.) while performing further research based on [25-27].

2 Physical and mathematical model

A rectangular composite sandwich panel consisting of three layers is considered. The materials of these layers are homogeneous and isotropic. The sandwich panel is assigned to the orthogonal coordinate system $Oxyz$ (Fig. 1). The panel has the dimensions $a \times b$ (width a, length b) and a constant thickness $2h$, i.e., the points of the panel space belong to the orthogonal coordinate system $Oxyz$ and occupy the region $[0, a] \times [0, b] \times [-h, h]$. Consider further a three-layer sandwich panel, the outer layers of which have the same thickness $h_1 = h_3$, and the inner layer has a thickness of h_2, that is, the total thickness is $h = 2 \cdot h_1 + h_2$.

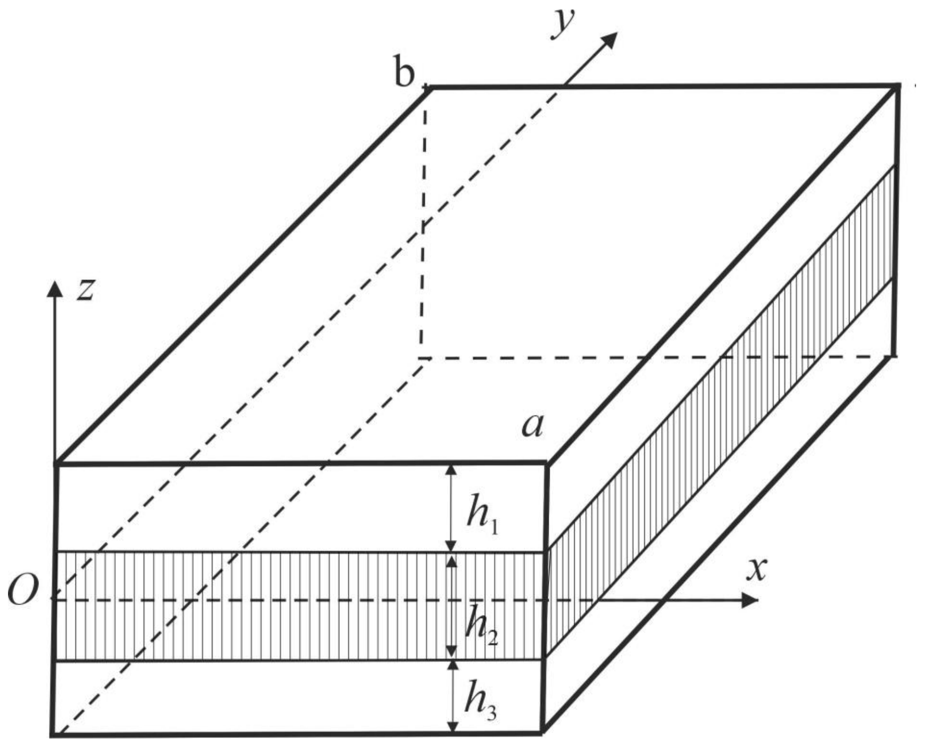

Fig. 1 Three-layer sandwich panel being studied

This panel is subjected to convective heat exchange with the external environment. To determine its stress-strain state, a physical and mathematical model consisting of two stages is proposed. In the first stage, the unsteady temperature field is determined from the relations of the unsteady heat conduction problem. In the second stage, the parameters of the stress-strain state of the panel are determined from the relations of the quasi-static thermoelasticity problem.

2.1 Determining the temperature field

Suppose that at the initial time the temperature of the panel is zero. Starting at time $\tau > 0$, the panel is heated by an external medium whose temperature on the surface $z = +h$ is $t_c^+ (x, y, \tau) = t_c (x, y)\, t^+(\tau)$, and on the surface $z = -h$ is $t_c^- (x, y, \tau) = 0$. The function $t^+(\tau)$ describes the time change in the temperature of the medium on the surface $z = +h$. Convective heat exchange with a constant heat transfer coefficient α_z occurs between the external environment and the surfaces $z = \pm h$. The temperature field of the panel $t (x, y, \tau)$ is determined from the system of two-dimensional heat conduction equations with a linear dependence of temperature on the transverse coordinate [7]:

$$A^\lambda \Delta T_1 - 2\alpha_z T_1 + B^\lambda \Delta T_2 - A^c \frac{\partial T_1}{\partial \tau} - B^c \frac{\partial T_2}{\partial \tau} = -\alpha_z t_c^+,$$
$$B^\lambda \Delta T_1 + D^\lambda \Delta T_2 - \left(\frac{A^\lambda}{h^2} + 2\alpha_z\right) T_2 - B^c \frac{\partial T_1}{\partial \tau} - D^c \frac{\partial T_2}{\partial \tau} = -\alpha_z t_c^+, \tag{1}$$

where $T_j = \frac{2j-1}{2h^j} \int_{-h}^{h} t z^{j-1} dz$ $(j = 1,2)$ are the integral temperature characteristics, and

$$\{A^\lambda, B^\lambda, D^\lambda\} = \int_{-h}^{h} \lambda(z) \left\{1, \frac{z}{h}, \left(\frac{z}{h}\right)^2\right\} dz,$$
$$\{A^c, B^c, D^c\} = \int_{-h}^{h} c_v(z) \left\{1, \frac{z}{h}, \left(\frac{z}{h}\right)^2\right\} dz, \tag{2}$$
$$\Delta = (\partial_{11}^2 + \partial_{22}^2);\ c_v = c\rho;\ \partial_1 = \frac{\partial}{\partial x};\ \partial_2 = \frac{\partial}{\partial y};$$

$\lambda(z)$ is the thermal conductivity coefficient; $c(z)$ is the specific heat capacity; $\rho(z)$ is the specific density; τ is the time variable; $t(x, y, \tau)$ is the temperature field function.

The system Eq. 1 for the integral temperature characteristics and at the edges of the panel is solved with homogeneous boundary conditions:

$$x = 0, a: T_1 = T_2 = 0, \qquad y = 0, b: T_1 = T_2 = 0,$$
$$\{A^c, B^c, D^c\} = \int_{-h}^{h} c_v(z) \left\{1, \frac{z}{h}, \left(\frac{z}{h}\right)^2\right\} dz, \tag{3}$$

and zero initial conditions:

$$\tau = 0: T_1 = T_2 = 0. \tag{4}$$

Using the integral Laplace transform in time τ and the finite Fourier transform in coordinates x and y, we obtain the general solutions of the system Eq. 1 in the form:

$$T_1 = \frac{1}{2C^*} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{i \neq j=1}^{2} \frac{Q_{nm} Z_j(\tau') Bi}{p_j - p_i} \left[\left(C_3 p_j - g_4\right) - \left(C_2 p_j - g_2\right)\right] \sin\frac{\pi n}{a} x \sin\frac{\pi m}{b} y.$$
$$T_2 = \frac{1}{2C^*} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{i \neq j=1}^{2} \frac{Q_{nm} Z_j(\tau') Bi}{p_j - p_i} \left[\left(C_1 p_j - g_1\right) - \left(C_2 p_j - g_3\right)\right] \sin\frac{\pi n}{a} x \sin\frac{\pi m}{b} y. \tag{5}$$

Here

$$g_1 = \Lambda_1(\mu_n^2 + \mu_m^2) + Bi;\ g_2 = g_3 = \Lambda_2(\mu_n^2 + \mu_m^2);\ g_4 = \Lambda_3(\mu_n^2 + \mu_m^2) + Bi + \Lambda_1;$$

$$Bi = \frac{\alpha_z h}{\lambda_0};\ \mu_n = \frac{h\pi n}{a};\ \mu_m = \frac{h\pi m}{b};\ \tau' = \frac{\lambda_0}{c_v^0 h^2}\tau;\ C^* = C_1 C_3 - (C_2)^2;$$

$$\{\Lambda_1, \Lambda_2, \Lambda_3\} = \frac{1}{2h\lambda_0}\{A^\lambda, B^\lambda, D^\lambda\};\ \{C_1, C_2, C_3\} = \frac{1}{2hc_v^0}\{A^c, B^c, D^c\};$$

λ_0 and c_v^0 are some characteristic coefficients of thermal conductivity and heat capacity, respectively; p_1 and p_2 are the roots of the quadratic equation:

$$C^* p^2 + [C_1 g_4 + C_3 g_1 - C_2(g_3 + g_2)]p + g_1 g_4 - g_2 g_3 = 0;$$

$$Q_{mn} = \frac{4}{ab}\int_0^a \int_0^b t_c(x,y)\, sin\frac{\pi n}{a}x\, sin\frac{\pi m}{b}y dxdy, \tag{6}$$

$$Z_j(\tau') = \int_0^{\tau'} t^+(v) e^{-p_j(\tau'-v)} dv, \quad (j = 1, 2). \tag{7}$$

2.2 Determination of the stress-strain state

To investigate the stress-strain state of the panel caused by the temperature field Eq. 5, we use the basic equations of thermoelasticity based on the first-order shear theory [7], written for isotropic plates through the generalized displacements u_i, w, γ_i $(i = 1, 2)$. Physical equations for stresses at any point of the plate are the following:

$$\begin{gathered}
\sigma_{11} = \frac{E(z)}{1-\nu^2}[\partial_1 u_1 + \nu\partial_2 u_2 + z(\partial_1\gamma_1 + \nu\partial_2\gamma_2) - (1+\nu)\alpha_t(z)t];\\
\sigma_{22} = \frac{E(z)}{1-\nu^2}[\partial_2 u_2 + \nu\partial_1 u_1 + z(\partial_2\gamma_2 + \nu\partial_1\gamma_1) - (1+\nu)\alpha_t(z)t];\\
\sigma_{12} = \frac{E(z)}{2(1+\nu)}(\partial_2 u_1 + \partial_1 u_2), \qquad \sigma_{13} = \frac{E(z)}{2(1+\nu)}(\gamma_1 + \partial_1 w),\\
\sigma_{23} = \frac{E(z)}{2(1+\nu)}(\gamma_2 + \partial_2 w).
\end{gathered} \tag{8}$$

where $E(z)$ and $\alpha^t(z)$ are the elastic modulus and the coefficient of thermal linear expansion, which depend on the coordinate z; ν is the Poisson's ratio, which is assumed to be constant; u_i, w are the displacements of points in the median surface; γ_i are the angles of rotation of the normal.

The physical equations for the forces and moments in the median surface are as follows:

$$\begin{gathered}
N_1 = A(\partial_1 u_1 + \nu\partial_2 u_2) + B(\partial_1\gamma_1 + \nu\partial_2\gamma_2) - A^t T_1 - B^t\frac{T_2}{h},\\
N_2 = A(\partial_2 u_2 + \nu\partial_1 u_1) + B(\partial_2\gamma_2 + \nu\partial_1\gamma_1) - A^t T_1 - B^t\frac{T_2}{h},\\
M_1 = B(\partial_1 u_1 + \nu\partial_2 u_2) + D(\partial_1\gamma_1 + \nu\partial_2\gamma_2) - B^t T_1 - D^t\frac{T_2}{h},\\
M_2 = B(\partial_2 u_2 + \nu\partial_1 u_1) + D(\partial_2\gamma_2 + \nu\partial_1\gamma_1) - B^t T_1 - D^t\frac{T_2}{h},
\end{gathered} \tag{9}$$

$$N_{12} = \frac{1-\nu}{2}\big(A(\partial_1 u_2 + \partial_2 u_1) + B(\partial_1 \gamma_2 + \partial_2 \gamma_1)\big),$$

$$M_{12} = \frac{1-\nu}{2}\big(B(\partial_1 u_2 + \partial_2 u_1) + D(\partial_1 \gamma_2 + \partial_2 \gamma_1)\big),$$

$$Q_1 = k' A \frac{1-\nu}{2}(\gamma_1 + \partial_1 w),$$

$$Q_2 = k' A \frac{1-\nu}{2}(\gamma_2 + \partial_2 w).$$

Here

$$\{A, B, D\} = \frac{1}{1-\nu^2}\int_{-h}^{h} E(z)\{1, z, z^2\}dz\,;$$

$$\{A^t, B^t, D^t\} = \frac{1}{1-\nu}\int_{-h}^{h} E(z)\alpha_t(z)\{1, z, z^2\}dz\,;$$

N_i, N_{12}, Q_i are normal, shear, and cutting forces, respectively; M_i, M_{12} are bending and torsional moments; k' is the shear coefficient [28].

The equilibrium equations are as follows:

$$\begin{aligned}
&A\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right)u_1 + A\frac{1+\nu}{2}\partial_{12}^2 u_2 + B\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right)\gamma_1 \\
&\qquad + B\frac{1+\nu}{2}\partial_{12}^2\gamma_2 = A^t\partial_1 T_1 + \frac{B^t\partial_1 T_2}{h}, \\
&A\frac{1+\nu}{2}\partial_{12}^2 u_1 + A\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right)u_2 + B\frac{1+\nu}{2}\partial_{12}^2\gamma_1 \\
&\qquad + B\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right)\gamma_2 = A^t\partial_2 T_1 + \frac{B^t\partial_2 T_2}{h}, \\
&\left[\frac{1-\nu}{2}Ak'(\partial_1^2 + \partial_2^2)\right]w + \frac{1-\nu}{2}Ak'\partial_1\gamma_1 + \frac{1-\nu}{2}Ak'\partial_2\gamma_2 = 0, \\
&B\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right)u_1 + B\frac{1+\nu}{2}\partial_{12}^2 u_2 - \frac{1-\nu}{2}Ak'\partial_1 w \\
&\qquad + \left[D\left(\partial_{11}^2 + \frac{1-\nu}{2}\partial_{22}^2\right) - \frac{1-\nu}{2}k'A\right]\gamma_1 + D\frac{1+\nu}{2}\partial_{12}^2\gamma_2 \\
&\qquad = B^t\partial_1 T_1 + \frac{D^t\partial_1 T_2}{h}, \\
&B\frac{1+\nu}{2}\partial_{12}^2 u_1 + B\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right)u_2 - \frac{1-\nu}{2}k'A\partial_2 w + D\frac{1+\nu}{2}\partial_{12}^2\gamma_1 \\
&\qquad + \left[D\left(\partial_{22}^2 + \frac{1-\nu}{2}\partial_{11}^2\right) - \frac{1-\nu}{2}k'A\right]\gamma_2 = B^t\partial_2 T_1 + \frac{D^t\partial_2 T_2}{h}.
\end{aligned} \tag{10}$$

Under the conditions of a panel hinged at the edges, the boundary conditions have the form:

$$\begin{aligned}
&x = 0, a: \quad u_1 = w = \gamma_1 = 0, \qquad N_1 = M_1 = 0, \\
&y = 0, b: \quad u_2 = w = \gamma_2 = 0, \qquad N_2 = M_2 = 0.
\end{aligned} \tag{11}$$

The system of differential equations Eq. 10, together with the boundary conditions Eq. 11, constitutes the boundary value problem of temperature stresses for isotropic inhomogeneous rectangular plates in generalized displacements. This problem is solved by the finite double Fourier transform method in the x and y coordinates. Based on the generalized displacements found, we write down the temperature stresses, forces, and moments using the formulas Eq. 8 and Eq. 9.

3 Solution of the problem for a three-layer sandwich panel

Based on the above general relations, let's consider a three-layer sandwich panel. Let the physical and mechanical characteristics of its layers be equal to

$$q^{(1)} = q^{(3)} = \{\lambda^{(1)}, c_v^{(1)}\}, \qquad q^{(2)} = \{\lambda^{(2)}, c_v^{(2)}\}.$$

Then, the expressions for the integral characteristics of the quantities $A^q = \{A, A^t, A^\lambda, A^c\}$, $B^q = \{B, B^t, B^\lambda, B^c\}$ and $D^q = \{D, D^t, D^\lambda, D^c\}$ due to the characteristics of the constituent layers of the panel according to [3] are determined by the following formulas:

$$\begin{gathered} A^q = 2h\left[2q^{(2)} - \frac{h_1}{h}\left(q^{(2)} - q^{(1)}\right)\right], \qquad B^q = 0, \\ D^q = \frac{2h^3}{3}\left\{q^{(1)} + \left(q^{(2)} - q^{(1)}\right)\left(1 - \frac{h_1}{h}\right)^3\right\}. \end{gathered} \tag{12}$$

The following expressions of the distribution functions of the external temperature were used for numerical studies:

$$t_c(x, y) = t^* N(x) N(y), \tag{13}$$

$$t^+(\tau) = 1 - e^{-\beta^* \tau}. \tag{14}$$

Here

$$\begin{gathered} N(x) = S_-\big(x - (x_0 - a_0)\big) - S_+\big(x - (x_0 + a_0)\big); \\ N(y) = S_-\big(y - (y_0 - b_0)\big) - S_+\big(y - (y_0 + b_0)\big); \end{gathered}$$

$S_\pm(x)$ are asymmetric unit functions; $2a_0$ and $2b_0$ are the dimensions of the heating region; (x_0, y_0) are the coordinates of the center of this region; $t^*, \beta^* = \text{const}$.

From the relations Eq. 6 and Eq. 13, we obtain the expressions for the Fourier coefficients Q_{nm}:

$$Q_{nm} = \frac{16t^*}{mn\pi^2} sin\frac{\pi n a_0}{a} sin\frac{\pi n x_0}{a} sin\frac{\pi m b_0}{b} sin\frac{\pi m y_0}{b}.$$

In the case of uniform heating over the entire surface of the plate $\left(x_0 = \frac{a}{2}, y_0 = \frac{b}{2}, a_0 = \frac{a}{2}, b_0 = \frac{b}{2}\right)$ we have $Q_{nm} = \frac{4t^*}{mn\pi^2}(1 - cos\,\pi\, n)(1 - cos\,\pi\, m)$.

The time function

$$Z_j(\tau') = \left[\frac{1}{p_j}\left(1 - e^{-p_j\tau'}\right) + \frac{1}{\beta^* - p_j}\left(e^{-\beta^*\tau'} - e^{-p_j\tau'}\right)\right]$$

is calculated using the formulas Eq. 7 and Eq. 14.

4 Numerical analysis of temperature and stress-strain state

Metal (Ti-6Al-4V) and ceramics (ZrO_2) with the following physical and mechanical characteristics were chosen as the materials of the constituent layers of the ceramic-metal-ceramic sandwich panel [13]. The metal is characterized by the following parameters:

$$\nu = 0.321;\ E_m = 66.2\ \text{GPa};\ \alpha_m^t = 10.3 \cdot 10^{-6}\ \frac{1}{\text{K}};\ \lambda_m = 18.1\ \frac{\text{W}}{\text{mK}};\ c = 808.3\ \frac{\text{J}}{\text{kg} \cdot \text{K}}.$$

The ceramics is characterized by the following parameters:

$$\nu = 0.333;\ E_c = 117\ \text{GPa};\ \alpha_c^t = 7.11 \cdot 10^{-6}\ \frac{1}{\text{K}};\ \lambda_c = 2.036\ \frac{\text{W}}{\text{mK}};\ c = 615.6\ \frac{\text{J}}{\text{kg} \cdot \text{K}}.$$

The values of the other parameters are as follows:

$$\frac{h}{a} = 0.025;\ \frac{a}{b} = 1;\ k' = \frac{5}{6};\ \tau' = 3;\ E_0 = 10^2\ \text{GPa};$$
$$\alpha_0^t = 10^{-5}\ \frac{1}{\text{K}};\ \lambda_0 = 1\ \frac{\text{W}}{\text{mK}};\ c_0 = 6 \cdot 10^2\ \frac{\text{J}}{\text{kg} \cdot \text{K}}.$$

The dimensionless temperature $t' = \frac{t}{t^*}$, temperature characteristics $T_i' = \frac{T_i}{t^*}$, radial deflections $w' = \frac{w}{ht^*\alpha_0^t}$, normal forces $N_1{}' = \frac{N_1}{t^*hE_0\alpha_0^t}$, bending moments $M_1{}' = \frac{M_1}{t^*h^2E_0\alpha_0^t}$ and normal stresses $\sigma_1' = \frac{\sigma_{11}}{t^*E_0\alpha_0^t}$ were calculated in the center of a square three-layer sandwich panel of the ceramic/metal/ceramic (C/M/C) type with a uniform distribution of ambient temperature over the surface. Figures 2-5 show the graphical dependencies of these dimensionless quantities (temperature characteristics, normal stresses, normal forces, bending moments, and radial deflections) for the Biot criterion Bi = 1, $\beta^* = 0.5$ and different values of the ratio h_2/h_1 of the thickness of the inner layer to the outer layer.

For a sandwich panel of the C/M/C structure, the influence of the metal layer increases with an increase in the ratio h_2/h_1. Therefore, the values of T'_i decrease, while the moments M'_1, forces N'_1, and stresses σ'_1 increase (see Figs. 2-5).

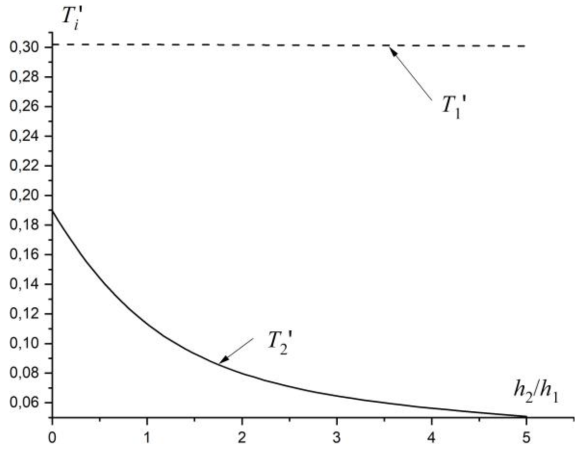

Fig. 2 Temperature characteristics T'_i depending on the ratio h_2/h_1 (Bi = 1, $\beta^* = 0.5$)

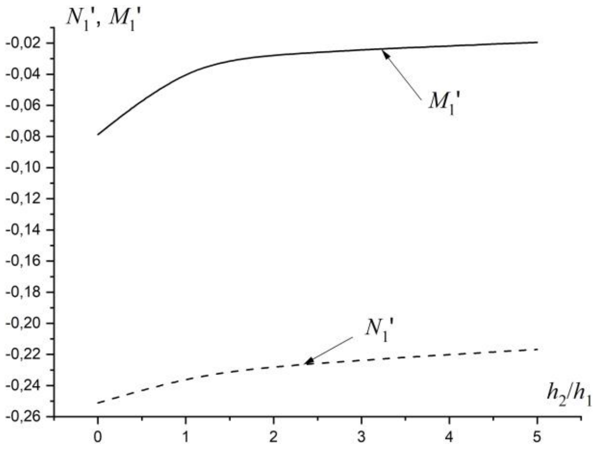

Fig. 3 Normal forces $N_1{}'$ and bending moments $M_1{}'$ depending on the ratio h_2/h_1 (Bi = 1, $\beta^* = 0.5$)

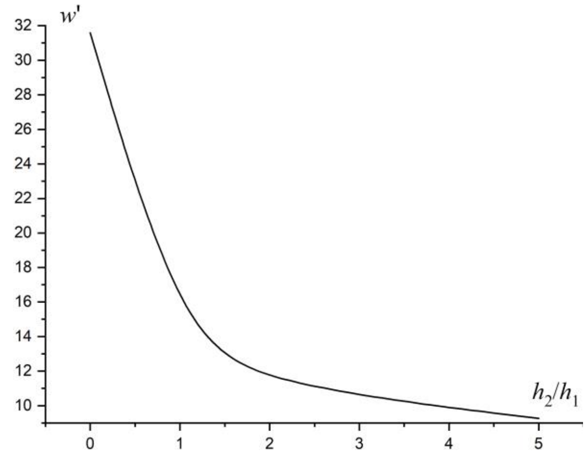

Fig. 4 Radial deflections w' depending on the ratio h_2/h_1 ($\mathrm{Bi} = 1$, $\beta^* = 0.5$)

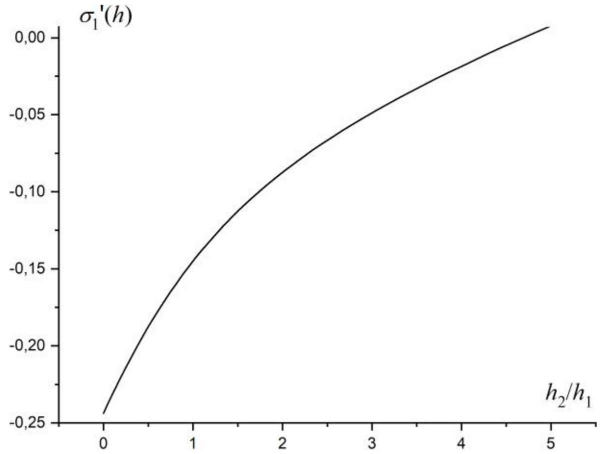

Fig. 5 Normal stresses σ_1' depending on the ratio h_2/h_1 ($\mathrm{Bi} = 1$, $\beta^* = 0.5$)

Figures 6-9 illustrate the dependence of the temperature characteristics T'_i, and the parameters w', N'_1, M'_1, σ'_1 of the stress-strain state of the sandwich panel for different values of the dimensionless heat transfer coefficient Bi. The calculations were carried out for the values $h_2/h_1 = 1$, $\beta^* = 0.5$. It is obtained that with an increase in the coefficient Bi, the values T'_i, and w' increase, and the values of N'_1, M'_1, σ'_1 decrease.

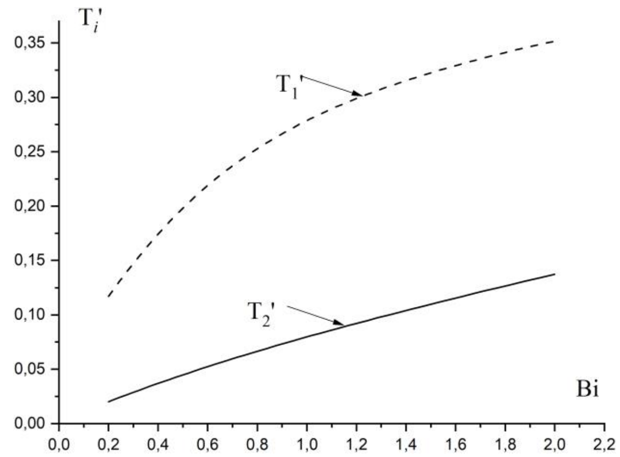

Fig. 6 Temperature characteristics T'_i at different values of the heat transfer coefficient Bi ($h_2/h_1 = 1$, $\beta^* = 0.5$)

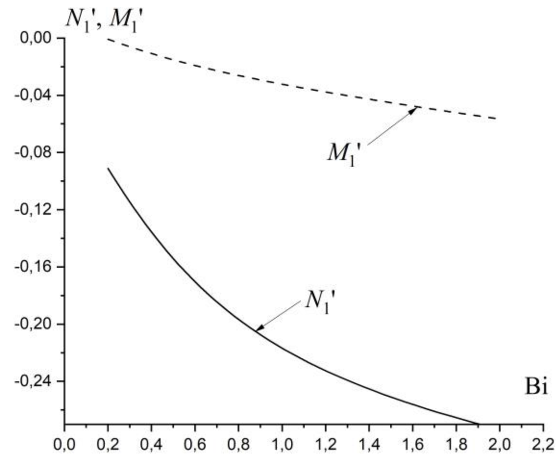

Fig. 7 Normal forces N_1' and bending moments M_1' at different values of the coefficient Bi ($h_2/h_1 = 1$, $\beta^* = 0.5$)

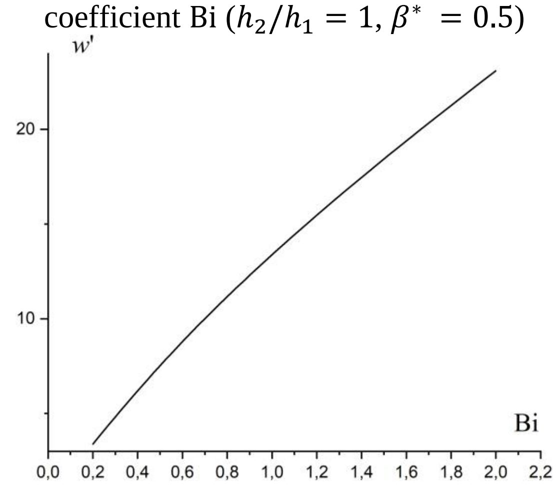

Fig. 8 Radial deflections w' at different values of the heat transfer coefficient Bi ($h_2/h_1 = 1$, $\beta^* = 0.5$)

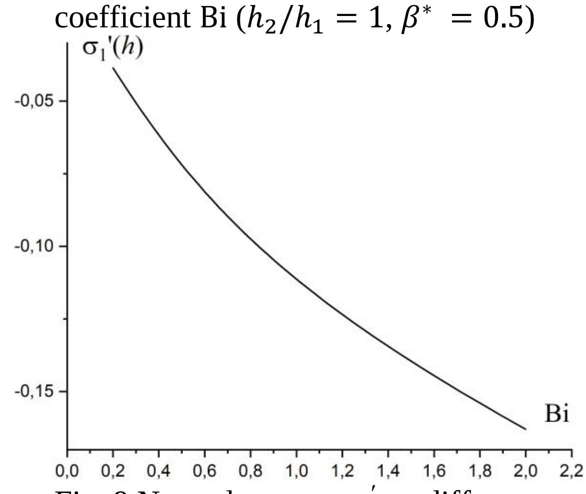

Fig. 9 Normal stresses σ_1' at different values of the heat transfer coefficient Bi ($h_2/h_1 = 1$, $\beta^* = 0.5$)

Figures 10-13 show the values of the temperature characteristics T'_i, and the parameters w', N'_1, M'_1, σ'_1 of the stress-strain state of the sandwich panel for the ratio $h_2/h_1 = 1$, Bi $= 1$ and different values of the coefficient β^*, which characterizes the rate of reaching the steady-state heating regime for all the considered values.

From the analysis of the dependencies shown in Figs. 10-13, it is established that the characteristics of temperature T'_i and deflection w' increase with an increase in the coefficient β^*, while the forces, moments, and stresses decrease.

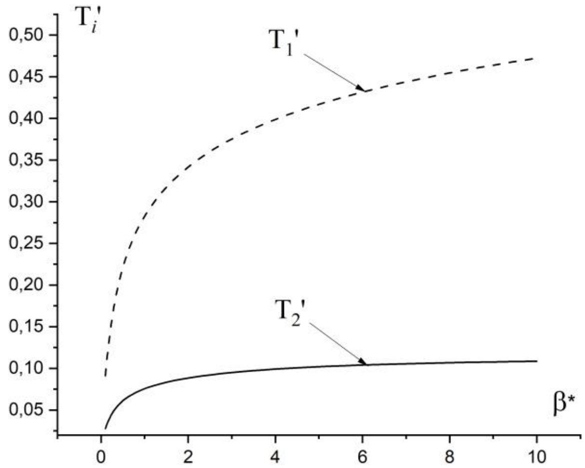

Fig. 10 Temperature characteristics T'_i at different values of the coefficient β^* ($h_2/h_1 = 1$, Bi $= 1$)

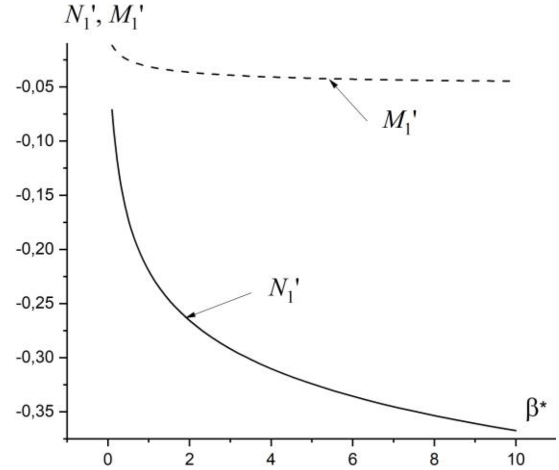

Fig. 11 Normal forces $N_1{}'$ and bending moments $M_1{}'$ at different values of the coefficient β^* ($h_2/h_1 = 1$, Bi $= 1$)

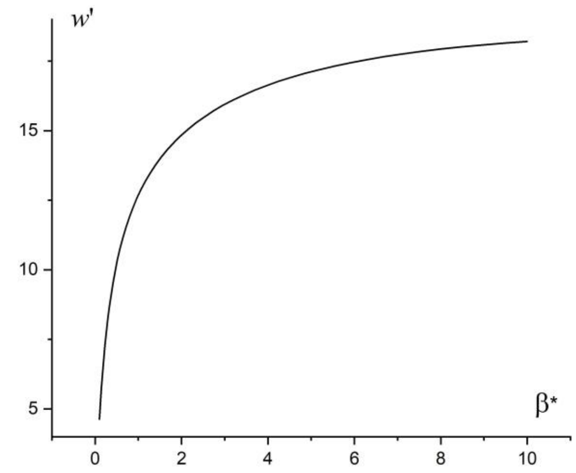

Fig. 12 Radial deflections w' at different values of the coefficient β^* ($h_2/h_1 = 1$, Bi $= 1$)

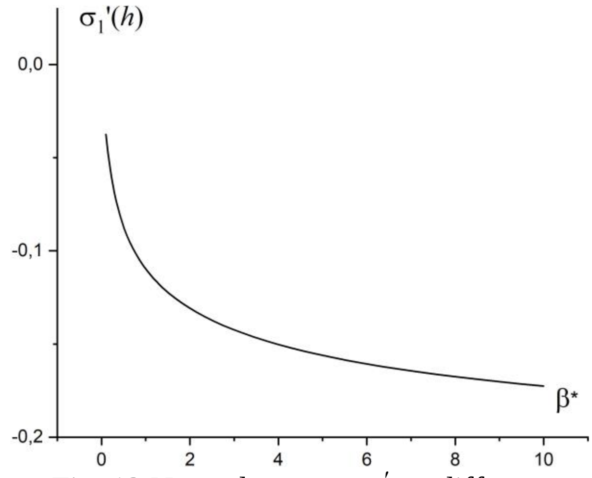

Fig. 13 Normal stresses $\sigma_1^{'}$ at different values of the coefficient β^* ($h_2/h_1 = 1$, Bi $= 1$)

5 Method of prediction of sandwich panel operability

To evaluate the operability of the sandwich panel, after finding the components of the displacement vector from the system of equations Eq. 10 using relations Eq. 8, we write down the expressions of the components $\boldsymbol{\sigma_{11}}$, $\boldsymbol{\sigma_{22}}$, $\boldsymbol{\sigma_{12}}$, $\boldsymbol{\sigma_{13}}$, $\boldsymbol{\sigma_{23}}$ of the stress tensor $\hat{\boldsymbol{\sigma}}$. Based on the found components of the stress tensor in the sandwich panel, we determine the stress intensities $\boldsymbol{\sigma_i}$ in it. These stress intensities in the sandwich panel are calculated by the formula [29]:

$$\boldsymbol{\sigma_i = \sqrt{\frac{3I_2(\hat{\sigma}) - I_1^2(\hat{\sigma})}{2}}}.$$

Here $I_j(\hat{\sigma})$ is the j-th invariant of the stress tensor. After that, according to the known Huber-Mises condition for an isotropic body, we compare the value of the stress intensity in the sandwich panel σ_i with its ultimate strength σ_M. If the condition $\sigma_i \leq \sigma_M$ [26] is met, then the considered layered sandwich panel retains its operability as a whole structural element. Note that the strength limit σ_M of the entire sandwich panel can be chosen as the smallest of the values of the strength limits σ_M^n of the materials of its n-th component layers, i.e. $\sigma_M = min\,\{\sigma_M^n\}$. This means that a sandwich panel loses its operability if at least one of its component layers fails according to the condition $\sigma_i^n \geq \sigma_M^n$. The values of σ_i^n and σ_M^n are determined experimentally for the material of the n-th component layer of the sandwich panel.

CONCLUSION

A methodology for solving the problems of thermal conductivity and quasi-static thermoelasticity for a three-layer sandwich panel has been developed. The temperature and parameters of the stress-strain state of a three-layer sandwich panel of the ceramic-metal-ceramic structure are investigated depending on the values of the dimensionless Biot criterion, the ratio of thicknesses of its outer and inner layers, and the dimensionless coefficient characterizing the time for the studied values to reach the steady-state mode of convective heating of the sandwich panel by the external environment. Based on the numerical analysis, it was found that by changing the ratio of the thickness of the sandwich panel's constituent layers and the conditions of convective heat transfer, it is possible to influence the temperature and parameters that determine the stress-strain state of the three-layer sandwich panels of the considered structure. A method for predicting the operability of a sandwich panel as a structural element is proposed. The dependences of the studied values obtained as a result of numerical analysis can serve as a scientific basis for predicting the reliability of the operation of three-layer sandwich panels of the ceramic/metal/ceramic (C/M/C) type as elements of various engineering structures.

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