

FLUTTER MEASUREMENT OF A LINEAR TURBINE BLADE CASCADE WITH AN ANGULAR POSITION DEVIATION OF ONE BLADE

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Abstract: Experimental flutter testing of high-aspect ratio rotor blades is a mainstay of turbomachinery research. However, rotor blades are never identical, and geometrical errors between actual and nominal geometries exist due to limited machining precision or assembly imperfection. The paper presents the initial phase of the controlled flutter research of a linear turbine blade cascade with a geometric deviation in one blade position. A subsonic wind tunnel with four flexibly mounted blades in an otherwise rigid blade cascade is employed at one angle of incidence and three low reduced frequencies. Measurements are performed with an angular position deviation ($\pm 1.5^\circ$) of one blade in pure bending and torsion modes. A tangible effect of one blade's slight incidence angle offset on the vibrating blade cascade aerodynamic stability is demonstrated, and this research effort opens the door to a more extensive testing campaign.

KEYWORDS: linear blade cascade; turbine rotor blade tip; controlled flutter measurement; position deviation

1 Introduction

The quality of production and assembly of bladed discs is essential for the performance and safe operation of turbomachinery. However, geometric errors are unavoidable [1–4] and manufacturing and assembly imperfections may play a role in dynamic unbalance, mistuning or vibration of the bladed discs. There are two main mechanisms of blade row vibration: forced response due to blade row interaction phenomena or inflow distortion and self-excited oscillation known as flutter. The most severe turbine and compressor blade failures are due to flutter, which occurs at minor reduced frequencies and affects high-aspect ratio rotor blades. In flutter occurrence, the unsteady aerodynamics feeds energy into the blade leading to a rapid increase in oscillation amplitude. Flutter testing in real turbomachinery is impossible for safety and expense reasons, and understanding of the phenomenon comes through simplified experimental rigs [5,6] and theoretical and numerical studies [7–11].

Many factors, such as reduced frequency, mode shape or unsteady blade loading, affect blade flutter. A blade mode shape is one of the most significant factors in determining blade cascade aerodynamic stability. Rotor blades are never identical, and their mode shapes may be affected by geometric variations due to unintended manufacturing and assembly errors or even intentional design modifications [12, 13]. In general, experimental methods for performance or stability analysis of coupled fluid-structure systems require sophisticated approaches and advanced system identification and data processing techniques [14–18].

Oscillating linear blade cascades are frequently used in experiments [19–23] and two methods are known for controlled flutter testing: a Travelling Wave Mode (TMW) and an

Influence Coefficient Method (ICM). Flutter in uniform turbomachine blade rows can be realistically represented using the TWM approach, where all blades oscillate in the same mode with the same amplitude and frequency but at various inter-blade phase angles. In the ICM testing, only one blade oscillates, and the response is measured on all the cascade blades. Both techniques theoretically lead to the same results by assuming a linear superposition of the unsteady pressures in the cascade for low amplitude vibration; however, the level of an agreement depends on the specific situation [24, 25].

In this study, a technique of controlled flutter testing proposed by Tsymbalyuk [26] is used to estimate the unsteady aerodynamic forces and moments in a linear blade cascade vibrating in a pure bending and a pure torsion mode using the TWM approach. A subsonic wind tunnel with four flexibly mounted blades in an otherwise rigid blade cascade is employed at one angle of incidence and three low reduced frequencies. Measurements are performed with an angular position deviation ($\pm 1.5^\circ$) of the second downstream flexibly mounted blade. The experimental setup is often used to produce test data for validation of numerical methods; see examples in [27–29]. This paper, similarly to the work [30], reports only the initial results of the ongoing research and the findings are already important as no flutter investigation of a linear turbine blade cascade with blade position deviations is presented in the literature.

2 Experimental system

The experimental setup is an in-draft wind tunnel with an atmospheric inlet and a Roots blower installed at the end of the system. The sucked airflow enters the system through filters and turbulence-reducing mesh before accelerating in a convergent nozzle. The working medium then discharges in a vacuum drum chamber (Fig. 1), where the linear blade cascade is placed and runs through the outlet duct toward the blower. Flow velocity is set by a variable speed drive and control valves. For this study, the experimental setup was operated up to an inlet isentropic Mach number equal to 0.5 approximately.



Fig. 1. Vacuum drum chamber with a test section: the air flow direction is from right to left

Fig. 2 shows the model of the test section with four flexibly mounted blades in the middle of a linear cascade consisting of eight blades in total and installed pressure probes. Rigid blades are fixed in grooves on upper and lower discs, and each flexibly mounted blade is a part of an electromagnetic shaker connected to a supporting frame via a designed elastic element. The oscillating blades 1 and 3 are attached to the supporting frame top. In order to eliminate the effect of aerostatic loading acting on flexibly mounted blades, the position of the supporting frame can be adjusted by a linear stepper motor to restore the initial position of the flexibly mounted blades. The left side wall of the test section (from an upstream view) can be solid (applied here) or perforated to suppress acoustic resonance. A flush-mounted static port and a Pitot probe are installed upstream of the blade cascade in the left side wall to monitor the dynamic pressure at the inlet. A traversing mechanism with a Pitot-static probe operates downstream of the blade cascade to measure the total and static pressure distribution and is used when the blade cascade does not oscillate. The pressure measurements are performed using calibrated differential transducers BHV 5355.

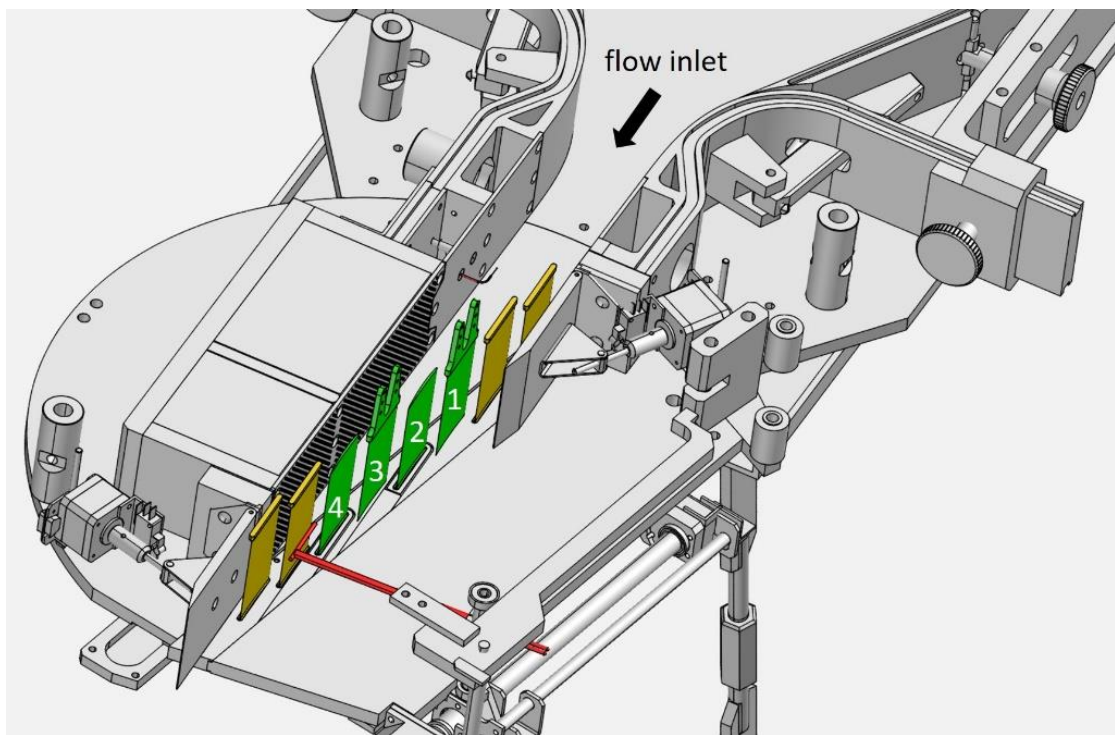


Fig. 2. Test section model: flexibly mounted blades 1-4 (green), rigid blades (yellow), pressure probes (red)

Fig. 3 depicts the model of the electromagnetic shaker with a flexibly mounted blade. Each flexibly mounted blade (12) has two degrees of freedom and can be operated in two fundamental modes: pure bending and pure torsion. Elements (1), (14) and a crossbar (10) form a flexible framework, and a pair of small moving coils (5) is attached to the crossbar. These coils are partially inserted into permanent magnets (6, 8), and a phase delay between the electric currents running through two moving coils defines the required oscillation mode. An additional mass (13) can be mounted at both ends of the crossbar for system frequency tuning. Finally, two precisely manufactured elastic elements (2) are used to connect the main body of an electromagnetic shaker (3) to a base (4). A sealing cover (11) is fixed at the blade root to reduce the leakage through the wall slots. Two non-contact eddy current displacement sensors Schenck IN-085 (9) are attached to the main body (3). The measured displacement signals y_1 , y_2 are used to define a bending motion y and a torsion motion α in the blade mid-chord using Eq. 1, where h is the distance between the displacement sensors.

$$\begin{aligned} y &= (y_1 + y_2)/2 \\ \alpha &= (y_2 - y_1)/h \end{aligned} \quad (1)$$

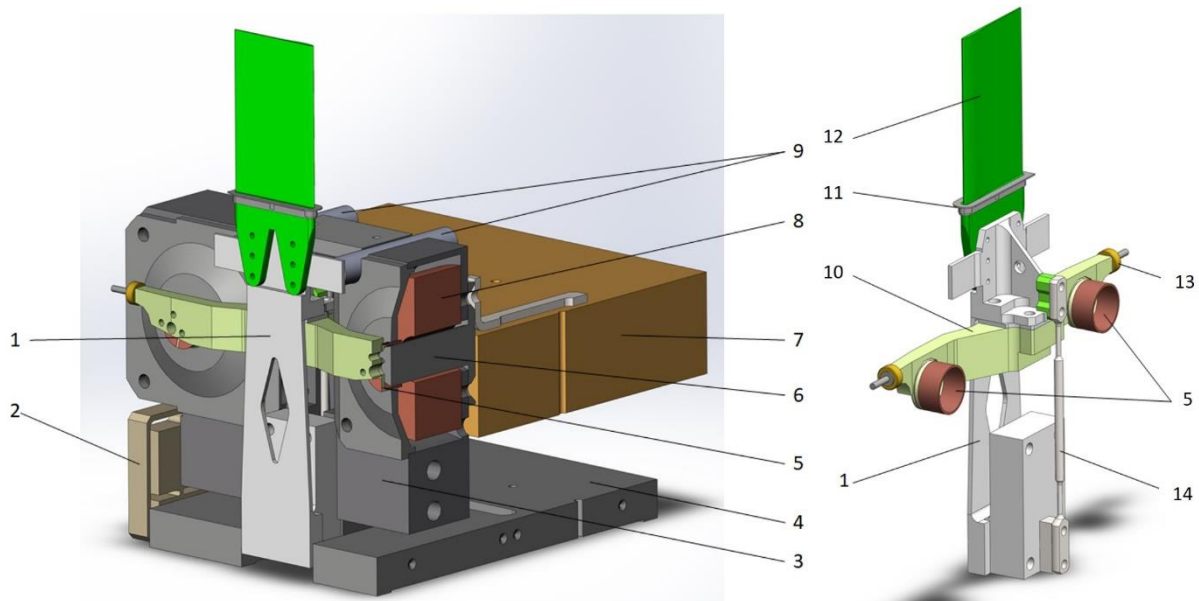


Fig. 3. Schematic of the electromagnetic shaker with a flexibly mounted blade, taken from [31]: 1 - main elastic element; 2 - flexible element of base; 3 - main body; 4 - base; 5 - moving coil; 6 - magnetic core; 7 - mass of electromagnetic shaker; 8 - permanent magnet solenoid; 9 - a pair of eddy current displacement sensors Schenck IN-085, 10 - crossbar; 11 - sealing cover; 12 - blade profile; 13 - additional mass for tuning; 14 - auxiliary elastic element

Fig. 4 shows the oscillation modes of the flexible suspension with a blade profile as analysed using FEM. In a pure torsion mode, the torsion axis is located in the mid-chord of the airfoil and motion amplitudes are the same along the blade height as expected. However, in a pure bending mode, the blade is susceptible to deflection due to its low natural frequency (the first natural frequency is approximately three times higher than the excitation frequency), and the maximum deformation can be found at the blade tip; this deflection behaviour was also verified experimentally [27]. Flow can further deflect the blade, and the deformation can be estimated once the aerodynamic loading is known. A correction technique [32] was used to obtain the aerodynamic unsteady loading as if the oscillating blades were rigid.

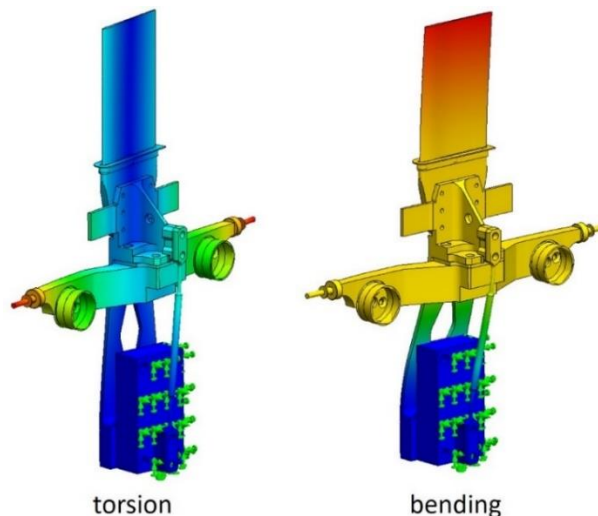


Fig. 4. Oscillation modes of the flexible suspension with a blade profile using FEM: blue - zero deformation, red - maximum deformation

The tested blade is made of stainless steel, and its profile shape is constant along the blade height representing a reduced section of the tip profile of the last stage blade of a steam turbine rotor. The flexibly mounted blade cascade schematic is shown in Fig. 5, where only angular position deviations of blade 2 are investigated. This blade was primarily chosen due to the simplicity of setting the blade cascade geometric error. The blade profile has a height of 79.5 mm, the blade chord b is 50 mm, the blade pitch T is 47 mm, and the cascade stagger angle β is -77° . The test section height is 80 mm, and the blade tip clearance in the cascade is 0.5 mm. The experiments were performed for three values of reduced frequency $K = 0.225$, 0.174 and 0.142, which correspond to the inlet isentropic Mach number $Ma = 0.263$, 0.388 and 0.478. Because the excitation frequency is fixed at $f_{exc} = 73$ Hz, different reduced frequencies in tests are achieved by varying the inlet flow velocity (hence, the Mach number and the reduced frequency are changed simultaneously). The measurements were performed at a low incidence $i = -2^\circ$, where minimal aerostatic loading was observed. This angle is between the blade chord line and the inlet flow direction, see Fig. 5, and can be easily set in the experimental setup. The angular position deviations of blade 2 are adjusted to $\pm 1.5^\circ$, which are the maximum possible values due to narrow wall slots. In future studies, more work will be done covering a wider range of incidence angles of the blade cascade and incidence angle offsets of individual blades.

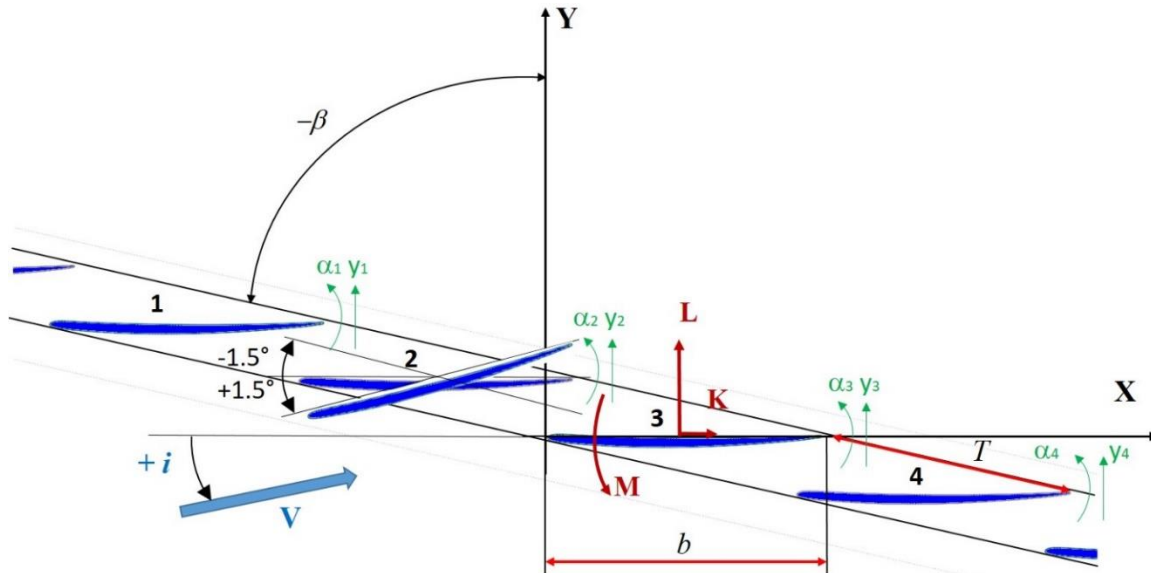


Fig. 5. Schematic of the vibrating blade cascade, reproduced from [31]

The experimental methodology employs high accuracy control of blade cascade motion with and without fluid flow. Therefore, each blade motion control uses a two-channel analogue controller with negative feedback. The controller subtracts the signals of the non-contact displacement sensors from reference harmonic signals. The signal differences generate electric currents in the moving coils via power amplifiers, and the associated forces reduce the signal differences. The reference signals control the required blade motion (frequency, amplitude and phase shift) and are generated using a simultaneously updating analogue output module NI 9264 programmed in LabVIEW. The displacement signals and the electric currents of the vibrating blade cascade are acquired by three PXI 4472b modules in a NI PXI 1042 chassis. Eq. 2 describes the bending and torsion motion of the n^{th} blade ($n = 1 \dots 4$), see Fig. 5,

$$\begin{aligned} y_n &= Y \sin(\omega t + (n - 1)IBPA) \\ \alpha_n &= A \sin(\omega t + (n - 1)IBPA) \end{aligned} \quad (2)$$

where Y is the bending motion amplitude, A is the torsion motion amplitude, ω is the angular frequency at the excitation, t is the time, IBPA means the inter blade phase angle. The amplitude of bending motion was set to 0.7 mm, and the torsion motion amplitude was 0.5 deg with the rotation axis in the blade mid-chord. The TWM approach was tested in all measurements with an IBPA step of 30 deg.

3 Mathematical background

3.1 Steady aerodynamic loading

Aerostatic forces and moments acting on each flexibly mounted blade in flow are measured using steady displacements and stiffness coefficients obtained using system calibration. The nondimensional coefficients of aerostatic force L_s and moment M_s are defined by Eq. 3,

$$C_{L_s} = 2 \frac{L_s}{qbH}, C_{M_s} = 2 \frac{M_s}{qb^2H} \quad (3)$$

where q is the inlet aerodynamic pressure, b is the blade chord and H is the blade height.

3.2 Unsteady aerodynamic loading

The equation of excited motion of a single blade of a unit height with two degrees of freedom under the influence of unsteady aerodynamic force L and moment M can be expressed by Eq. 4 (see also Fig. 5, where aerodynamic loading in x direction is ignored [26]),

$$\begin{aligned} m\ddot{y} + g_y\dot{y} + k_y y + mx_m\ddot{\alpha} &= L + F_{exc} \\ I\ddot{\alpha} + g_\alpha\dot{\alpha} + k_\alpha \alpha + mx_m\ddot{y} &= M + M_{exc} \end{aligned} \quad (4)$$

where m is the oscillating mass, I is the moment of inertia, g_y is the coefficient of structural damping for the motion in y direction, g_α is the coefficient of structural damping for the torsion in α direction, k_y is the coefficient of structural stiffness for the motion in y direction, k_α is the coefficient of structural stiffness for the torsion in α direction, x_m is the distance between the gravity axis and the axis of rotation and F_{exc} is the excitation force and M_{exc} is the excitation moment. It is valid for the experimental setup that x_m is zero; hence the equations of motion in Eq. 4 are mechanically uncoupled, and the fundamental modes are a pure bending motion and a pure torsion.

The excitation force F_{exc} and moment M_{exc} are a function of the electromagnetic forces F_1 , F_2 generated in a pair of small moving coils as formulated in Eq. 5,

$$\begin{aligned} F_{exc} &= F_1 + F_2 \\ M_{exc} &= h_c(F_1 - F_2) \end{aligned} \quad (5)$$

where h_c is the distance between the axes of moving coils. It is expected that the aerodynamic loading is small; hence the sensitivity of the measuring device is increased by operating the system in resonance under the condition that the natural frequencies of bending and torsion modes are the same and equal to the excitation frequency.

As mentioned, the experimental technique employs high accuracy control of blade cascade motion with and without fluid flow. The right-hand side of Eq. 4 can be written as F_0 and M_0 for the case of no flow ($L = 0$, $M = 0$). By assuming the system motion is the same with and without flow, the left-hand side of Eq. 4 is identical in both situations and can be subtracted from each other, yielding the expression for the aerodynamic force L and the aerodynamic moment M in Eq. 6,

$$\begin{aligned}
L &= F_0 - F_{exc} = \sum_{j=1}^2 \zeta_j (i_{0j} - i_{excj}) \\
M &= M_0 - M_{exc} = \sum_{j=1}^2 h_c \zeta_j (i_{0j} - i_{excj})
\end{aligned} \tag{6}$$

where ζ is the complex electromagnetic force coefficient (experimentally obtained by system dynamic calibration [26]), i represents the harmonic electrical current running through the moving coil and the subscript j denotes the number of moving coils.

The unsteady aerodynamic work W_{AC} acting on a reference blade per motion cycle is defined in Eq. 7,

$$W_{AC} = \pi[Y|L| \sin(\Phi_{Lb}) + A|M| \sin(\Phi_{Mt})] = \pi[Y\Im(L) + A\Im(M)] \tag{7}$$

where Φ_{Lb} is the phase delay between the aerodynamic force L and the bending motion in y direction and Φ_{Mt} is the phase delay between the aerodynamic moment M and the torsion motion in α direction. The system stability is affected only by the imaginary parts of the aerodynamic force $\Im(L)$ and the aerodynamic moment $\Im(M)$. A positive value of the aerodynamic work W_{AC} means aerodynamic instability. Eq. 7 can be reduced to a non-dimensional coefficient AW using Eq. 8, where $C_L = L/(qb)$, $C_M = M/(qb^2)$ are the complex aerodynamic coefficients. The coefficient of qHb^2 corresponds to the kinetic energy of the air volume upstream of the blade cascade.

$$AW = \frac{W_{AC}}{qHb^2} = \pi \left[\frac{Y}{b} \Im(C_L) + A \Im(C_M) \right] \tag{8}$$

4 Results

4.1 Flow periodicity and aerostatic forces and moments

A spatial periodicity of the flow through the blade cascade is evaluated in two ways: (i) by the measurement of the total pressure distribution downstream of the nonvibrating blade cascade and (ii) by the nondimensional coefficients C_{Ls} and C_{Ms} (Eq. 3). Fig. 6 shows the total pressure distribution normalized by the inlet total pressure at $K = 0.174$ (equivalent results were obtained for other values of reduced frequency). The blade 2 incidence angle offset of ± 1.5 deg primarily affects the wake width and the wake symmetry of the blade itself and, to a lesser degree, of the directly neighbouring blades 1 and 3. Only the pressure distribution around blade 4 is not influenced by the imposed position changes of blade 2.

A similar conclusion can be drawn from the values of nondimensional coefficients C_{Ls} and C_{Ms} in Fig. 7, where the aerostatic loading on blade 2 changes considerably with the incidence angle offset, while almost negligible change in C_{Ls} and C_{Ms} is found for blade 4. Moreover, the trends of C_{Ls} and C_{Ms} for incidence angle offset of ± 1.5 deg tend to be symmetric around the value for zero incidence angle offset, however, the discrepancy might be caused by experimental errors in measuring the aerostatic loading, typically of the order of magnitude 0 for the force and -2 for the moment.

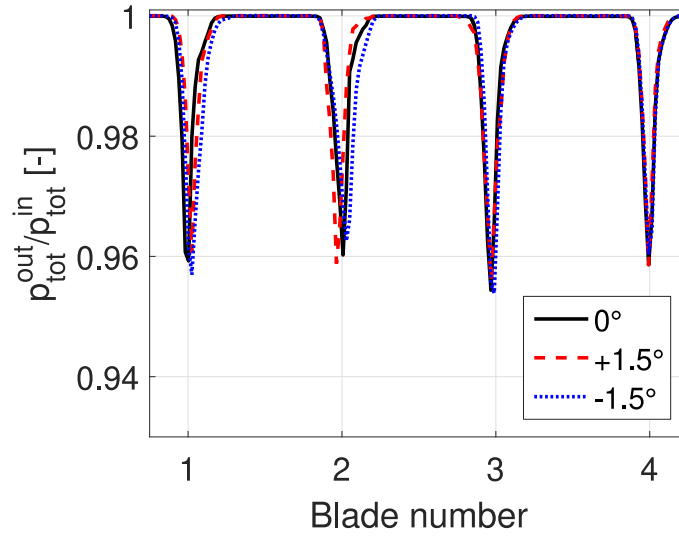


Fig. 6. Nondimensional total pressure distribution downstream of the blade cascade at $K = 0.174$

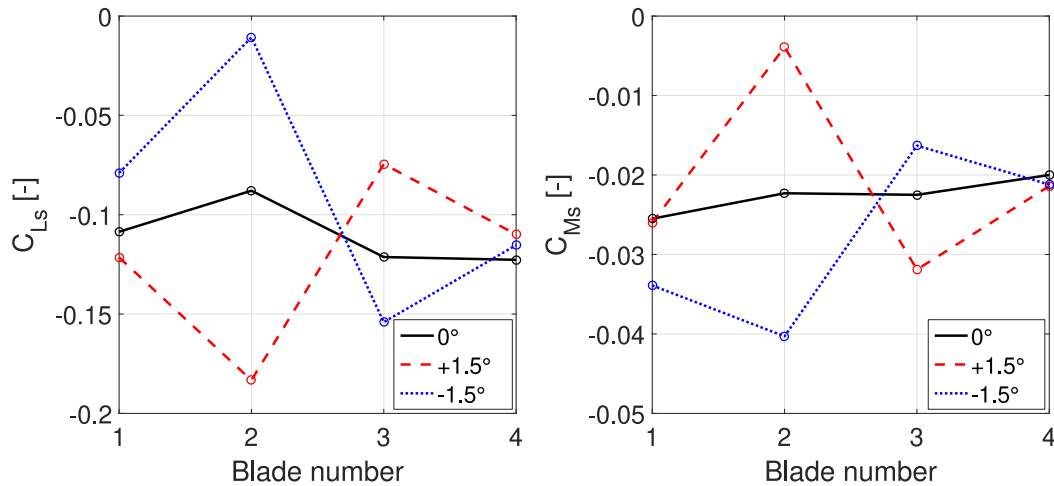


Fig. 7. Nondimensional coefficients of aerostatic force C_{Ls} and moment C_{Ms} at $K = 0.174$

4.2 Unsteady aerodynamic forces and moments

4.2.1 Pure torsion

Fig. 8 shows the aerodynamic work coefficients AW of the vibrating blades at various low reduced frequencies for pure torsion. All four blades are aerodynamically unstable at lower values of IBPA, and the maximum AW of each blade can be found at a value of IBPA, which increases with the blade position; see Table 1. The aerodynamic stability of the blades does not change with the reduced frequency, which goes well in line with the previous findings [31] and the theory [9,11] for torsion mode. The behaviours of blades 2 and 3 are alike and are influenced primarily by the adjacent blades (therefore, a characteristic S-shape is obvious). On the other hand, blades 1 and 4 are affected by the direct and distant vibrating blades and the nonvibrating blades. An indistinctive mean offset in each AW trend indicates no strong effect of the blade motion on itself.

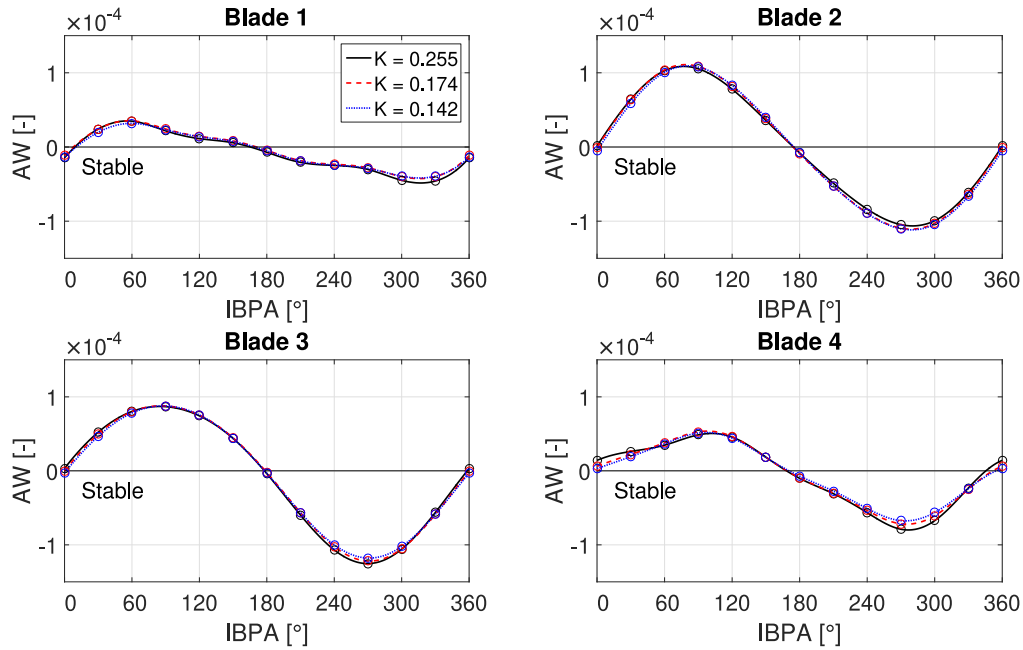


Fig. 8. Aerodynamic work coefficients of the vibrating blades at various reduced frequencies for pure torsion

Table 1. Overview of the IBPAs at the maximum aerodynamic work coefficient AW_{max} for pure torsion

K [-]	Blade 1 [°]	Blade 2 [°]	Blade 3 [°]	Blade 4 [°]
0.255	56	77	85	102
0.174	57	79	86	98
0.142	59	81	88	97

The aerodynamic work coefficients AW of the vibrating blades with blade 2 angle position deviations of ± 1.5 deg for pure torsion at $K = 0.255$ are shown in Fig. 9, similar trends were observed for the other reduced frequencies. The plots of AW demonstrate that the influence of this deviation on the aerodynamic stability of other blades is measurable, and surprisingly the effect of the displaced blade on the self-stability is not the most obvious as expected. For each blade, the IBPA range for the flutter region remains nearly unchanged by the considered geometry modification, and some differences can be found in the maximum/minimum values of AW .

Fig. 10 shows the differences between the aerodynamic work coefficient for the blade 2 angular position deviations of ± 1.5 deg and no position deviation at all reduced frequencies tested. The vibrating blade cascade has two different characteristics of stabilizing/destabilizing behaviour, and blades 3 and 4 have a visual symmetry in this feature. At lower IBPA values, the angular position deviation of blade 2 by $+1.5$ deg leads to a stabilizing effect of blades 3 and 4; at higher IBPA values, there is a destabilizing effect, and vice versa. On the other hand, blades 1 and 2 have no symmetry in the stabilizing/destabilizing effect. The results are reasonably consistent in all three cases of reduced frequencies tested.

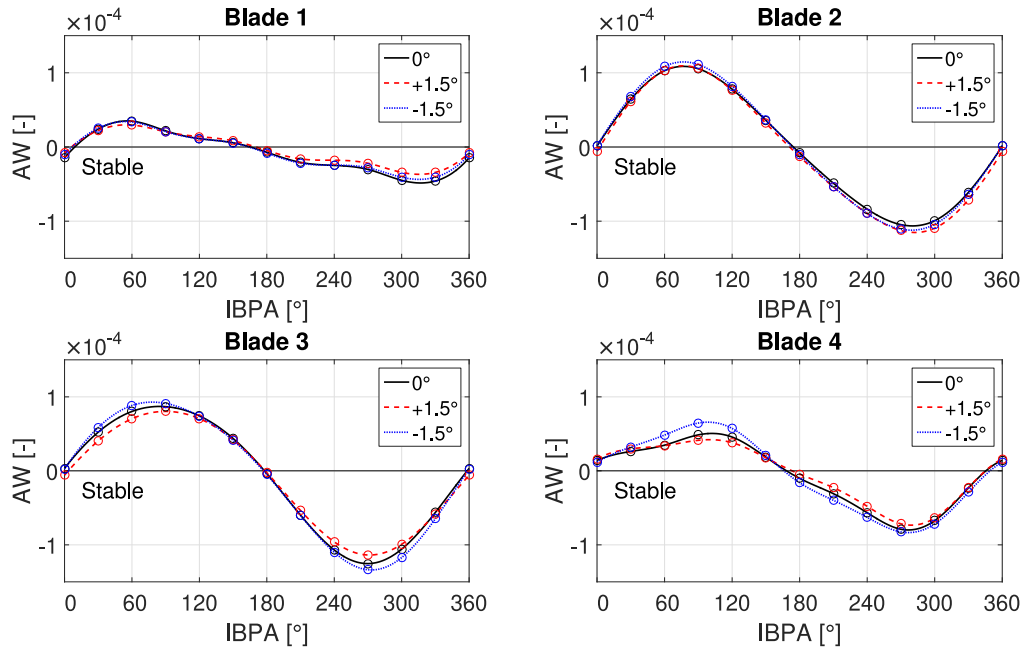


Fig. 9. Aerodynamic work coefficients of the vibrating blades with blade 2 angle position deviations for pure torsion at $K = 0.255$

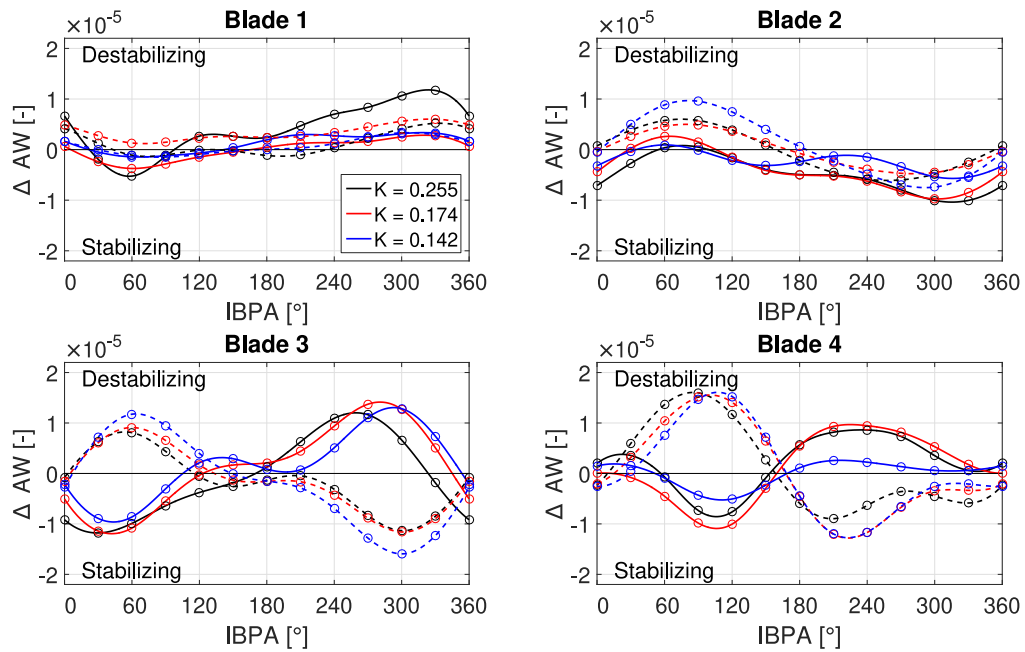


Fig. 10. Difference of aerodynamic work coefficient of the vibrating blade cascade for pure torsion: solid lines ($+1.5^\circ$), dashed lines (-1.5°)

Table 2. Overview of the ratios $AW_{max}/AW_{max,0^\circ}$ for pure torsion

K [-]	angle deviation [$^\circ$]	Blade 1 [-]	Blade 2 [-]	Blade 3 [-]	Blade 4 [-]
0.255	+1.5	0.85	1.01	0.92	0.83
	-1.5	0.98	1.06	1.07	1.30
0.174	+1.5	0.90	1.02	0.94	0.80
	-1.5	1.04	1.04	1.08	1.29
0.142	+1.5	0.96	1.00	0.97	0.91
	-1.5	0.96	1.09	1.12	1.30

Table 2 summarizes the ratios of maximum coefficient AW_{max} at ± 1.5 deg to maximum coefficient AW_{max} at 0 deg for pure torsion to quantify the sensitivity of angular position deviation of blade 2 on the cascade stability. If this ratio is larger than 1, the blade aerodynamic stability (locally) deteriorates. The most sensitive is blade 4; its stability improved with the blade 2 incidence offset of -1.5 deg and significantly deteriorated for the blade 2 incidence offset of $+1.5$ deg, consistently for all reduced frequencies under investigation. In addition, only minor differences in IBPA at AW_{max} were observed, except for blade 3, see Table 3.

Table 3. Overview of the differences $IBPA(AW_{max}) - IBPA(AW_{max,0^\circ})$ for pure torsion

K [-]	angle deviation [$^\circ$]	Blade 1 [$^\circ$]	Blade 2 [$^\circ$]	Blade 3 [$^\circ$]	Blade 4 [$^\circ$]
0.255	+1.5	-1	0	5	-2
	-1.5	-2	0	-7	-4
0.174	+1.5	0	-2	9	-3
	-1.5	-1	0	-5	0
0.142	+1.5	0	-1	8	-3
	-1.5	0	0	-5	2

4.2.2 Pure bending

Fig. 11 shows the aerodynamic work coefficients AW of the vibrating blades at various low reduced frequencies for pure bending. All four blades are aerodynamically stable in the full range of IBPA, and for the lowest reduced frequency, blades 1 and 2 are the closest to the stability threshold at IBPA specified in Table 4. The aerodynamic stability of the blades changes with the reduced frequency, which agrees with the previous experimental results [31] and the theory [9, 11] for bending mode. Each blade's stability is strongly affected by its motion, which has a stabilizing effect, and by direct and distant vibrating blades.

Table 4. Overview of the IBPAs at the maximum aerodynamic work coefficient AW_{max} for pure bending

K [-]	Blade 1 [$^\circ$]	Blade 2 [$^\circ$]	Blade 3 [$^\circ$]	Blade 4 [$^\circ$]
0.255	80	69	355	344
0.174	77	70	17	351
0.142	77	72	113	352

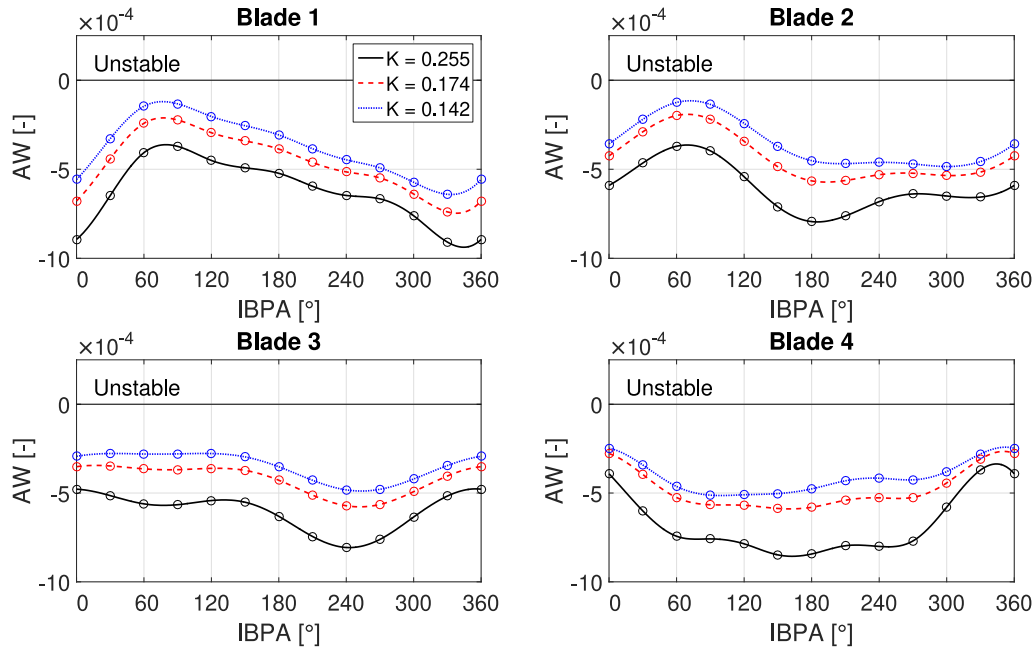


Fig. 11. Aerodynamic work coefficient of the vibrating blade cascade at various reduced frequencies for pure bending

The aerodynamic work coefficients of the vibrating blades with blade 2 angular position deviations of ± 1.5 deg for pure bending at $K = 0.255$ are shown in Fig. 12. In contrast to torsion mode, the effect of position deviations is more distinct at each blade stability. Moreover, the incidence offset of -1.5 deg destabilizes the blade 2 itself over the full IBPA range, especially for $\text{IBPA} \geq 180$ deg, which was also observed for other reduced frequencies, as detailed in Fig. 13. Here no evident symmetry in stabilizing/destabilizing behaviour is observed; however, some indications could be found again for blades 3 and 4.

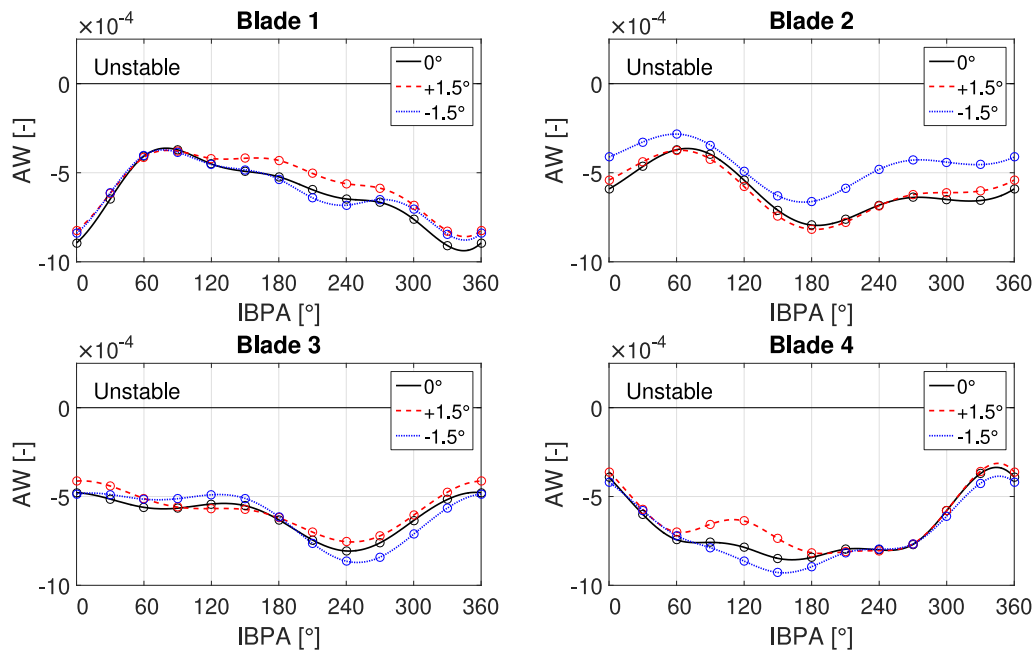


Fig. 12. Aerodynamic work coefficient of the vibrating blade cascade at various reduced frequencies for pure bending at $K = 0.255$

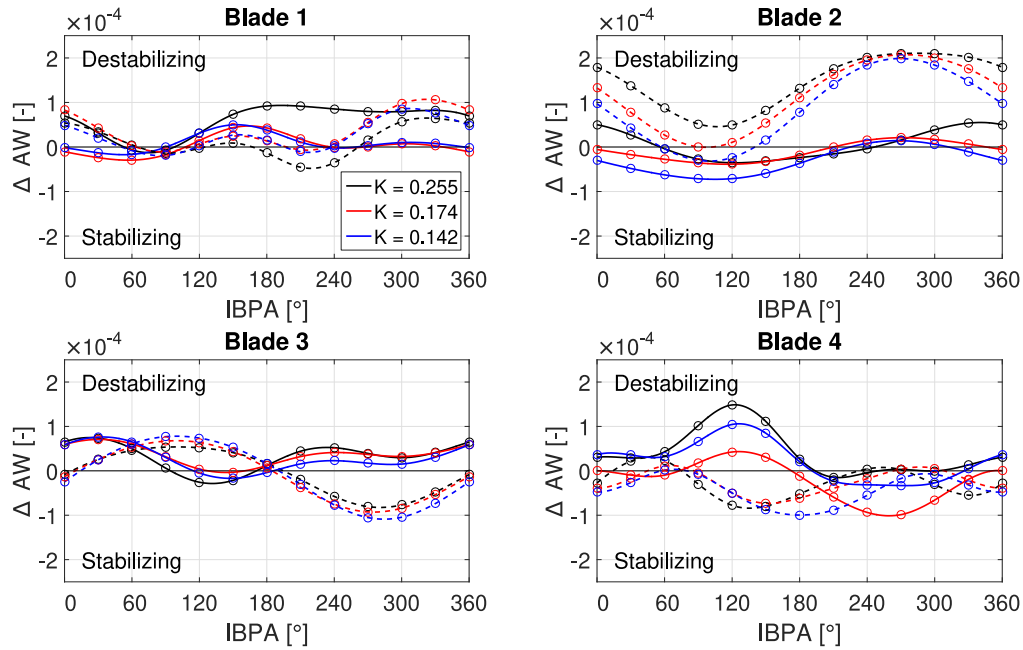


Fig. 13. Difference of aerodynamic work of the vibrating blade cascade for pure bending: solid lines (+1.5°), dashed lines (−1.5°)

Table 5 summarizes the ratios of maximum coefficient AW_{max} at ± 1.5 deg to maximum coefficient AW_{max} at 0 deg for pure bending. There are several cases, where the blade aerodynamic stability considerably deteriorates ($AW_{max}/AW_{max, 0^\circ} > 1$) due to angular position deviations of blade 2: blade 3 ($K = 0.142$, ± 1.5 deg) or blade 2 ($K = 0.255$, -1.5 deg) and, on the contrary, where the blade aerodynamic stability considerably improves: blade 2 ($K = 0.142$, $+1.5$ deg). The stability of blade 3 is mostly deteriorated by the angular position deviations of blade 2 in the investigated pure bending motion. Finally, huge differences in IBPA at AW_{max} were observed, especially on blades 3 and 2; see Table 6 for details and the findings also prove that bending mode is more sensitive to angular position deviations of blade 2 than torsion mode.

Table 5. Overview of the ratios $AW_{max}/AW_{max, 0^\circ}$ for pure bending

K [-]	angle deviation [°]	Blade 1 [-]	Blade 2 [-]	Blade 3 [-]	Blade 4 [-]
0.255	+1.5	0.97	0.97	1.16	1.07
	-1.5	0.97	1.29	0.99	0.87
0.174	+1.5	0.90	0.86	1.25	1.00
	-1.5	0.96	1.12	1.16	0.88
0.142	+1.5	0.93	0.63	1.38	1.15
	-1.5	0.87	0.90	1.39	0.84

Table 6. Overview of the differences $IBPA(AW_{max}) - IBPA(AW_{max,0^\circ})$ for pure bending

$K [-]$	angle deviation $[^\circ]$	Blade 1 $[^\circ]$	Blade 2 $[^\circ]$	Blade 3 $[^\circ]$	Blade 4 $[^\circ]$
0.255	+1.5	1	-6	10	2
	-1.5	-3	-10	18	3
0.174	+1.5	4	-2	5	1
	-1.5	-3	-10	96	-3
0.142	+1.5	4	-3	-79	4
	-1.5	-3	-9	-8	-2

CONCLUSION

The experimental investigation of controlled flutter in a linear turbine blade cascade with an angular position deviation of one blade was performed for the first time. The TWM approach with an IBPA step of 30 deg was applied to four vibrating blades. A series of measurements was carried out with an angular position deviation ($\pm 1.5^\circ$) of the second downstream blade in pure bending and torsion modes in subsonic flow at one angle of incidence and three low reduced frequencies $K = 0.225, 0.174$ and 0.142 .

For pure torsion mode, all four vibrating blades were aerodynamically unstable at lower values of IBPA, and no strong effect of the blade motion on itself was noticed. However, the blade 2 angular position deviations of ± 1.5 deg had a major impact on the aerodynamic stability of blade 4, and also, an apparent symmetry of stabilizing/destabilizing effect due to blade 2 angular position deviations was observed on blades 3 and 4.

For pure bending mode, all four vibrating blades were aerodynamically stable in the full range of IBPA, and each blade's stability was strongly affected by its motion, which had a stabilizing effect. The stability of blade 3 was mostly deteriorated by the angular position deviations of blade 2. No evident symmetry in stabilizing/destabilizing effect due to blade 2 angular position deviations was observed; however, some indications could be found again for blades 3 and 4.

Overall, a tangible effect of one blade's slight incidence angle offset on the vibrating blade cascade aerodynamic stability was demonstrated; the findings also showed that bending mode is more sensitive to angular position deviations of blade 2 than torsion mode. This initial research effort opened the door to a more extensive testing campaign. The intended types of position deviations will be realized according to the actual geometric errors, emphasizing the effect on the blade throat size and position [1–3]. In such investigations, a non-dimensional or otherwise generalizable form of the deviations and their effects will be preferred to provide information on the relative importance.

Finally, although linear blade cascades do not closely simulate flow conditions in a turbomachine (e.g. a radial static pressure gradient field cannot be modelled, and a truly periodic character of the flow is infeasible), they offer several attractive features [33], and experimental datasets are valuable for numerical code validation or asymptotic approach theory confirmation, as shown in this study.

Author Contributions

Conceptualization, P.E. and V.T.; methodology, V.T.; software, P.E. (LabVIEW, MATLAB) and V.T. (LabVIEW) and M.E. (FEM); validation, all; formal analysis, all; investigation, all; resources, P.E.; data curation, all; writing—original draft preparation, P.E.; writing—review

and editing, all; visualization, all; supervision, P.E.; project administration, P.E.; funding acquisition, P.E. All authors have read and agreed to the published version of the manuscript.

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Conflict of Interest

The authors declare no conflict of interest.

Nomenclature

Symbols

α	torsion displacement [rad]
β	cascade stagger angle [deg]
$\Im$	imaginary part
ω	excitation angular frequency [rad/s]
Φ	phase delay [rad]
ζ	electromagnetic force coefficient [N/A]
A	torsion motion amplitude [rad]
aoi	angle of incidence [deg]
AW	dimensionless unsteady aerodynamic work per motion cycle [-]
b	blade chord [m]
C_L, C_M	aerodynamic coefficients [-]
F	aerodynamic force [N]
f	frequency [Hz]
g	coefficient of structural damping [Ns/m]
H	blade height [m]
h	distance between displacement sensors [m]
h_c	distance between axes of moving coils [m]
I	moment of inertia [kgm ²]
i	harmonic electrical current [A]
K	reduced frequency $K = 2\pi f_{exc} b/V$ [-]
k	coefficient of structural stiffness [N/m]
M	aerodynamic moment [Nm]
m	oscillating mass [kg]
Ma	Mach number - inlet [-]
q	aerodynamic pressure [Pa]
T	blade pitch [m]
t	time [s]
V	flow velocity [m/s]
W_{AC}	unsteady aerodynamic work per motion cycle [J]
x_m	distance between gravity axis and torsion axis [m]
Y	bending motion amplitude [m]
y	bending displacement [m]

Subscripts

0	no flow condition
1,2	indexes for a pair
α	angular direction
cr	critical

<i>exc</i>	excitation
<i>j</i>	number of moving coil
<i>Lb</i>	related to aerodynamic force L and bending motion
<i>Mt</i>	related to aerodynamic moment M and torsion motion
<i>n</i>	number of vibrating blade
<i>s</i>	aerostatic
<i>y</i>	bending direction

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