

Optimizing the Geometry of an L-Shaped Building to Enhance Energy Efficiency and Sustainability

Ewa ROKITA-MAGDZIARZ¹, Barbara GRONOSTAJSKA², Marcin MAGDZIARZ^{3*}

¹*Faculty of Architecture, Wrocław University of Science and Technology, Wyspińskiego 27, Wrocław, 50-370, Poland*

²*Department of Architectural and Construction Design, Faculty of Architecture, Wrocław University of Science and Technology, Wyspińskiego 27, Wrocław, 50-370, Poland*

³*Hugo Steinhaus Center, Department of Applied Mathematics, Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology, Wyspińskiego 27, Wrocław, Poland*

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Abstract – The geometric form of a building strongly influences its material use, heat losses, and energy efficiency. This paper presents an analytical optimization of L-shaped residential buildings aimed at minimizing the external surface area for a prescribed volume. Both symmetric and asymmetric configurations are examined under realistic design constraints, including fixed or bounded wing aspect ratios and fixed building height. Using explicit optimization methods and Karush-Kuhn-Tucker conditions, closed-form expressions for the optimal geometric parameters and the minimal envelope area are derived. The results show that unconstrained optimization leads to degenerate cuboid shapes, highlighting the importance of geometric constraints to preserve the L-shaped form. The results obtained provide practical design guidelines for architects and engineers, supporting informed early-stage decisions that balance functional requirements, regulatory constraints, architectural intent, and energy performance. The proposed approach should be understood as an early-stage geometric optimization method based on envelope-area minimization, rather than as a complete dynamic energy-performance assessment. Case studies of existing houses demonstrate that the proposed approach can reduce external surface area or confirm near-optimality of practical designs, supporting energy-efficient early-stage architectural decisions.

Keywords – Building optimization; mathematical modelling; architectural design; L-shaped house; sustainable architecture; energy efficiency.

1. INTRODUCTION

The geometry of the building is one of the most influential determinants of its structural behavior, material consumption, and energy requirements. Within the broad spectrum of residential typologies, the L-shaped house stands out as a particularly significant example of modern architecture. L-shaped houses represent a commonly used residential layout in both traditional and contemporary architecture. Their popularity results from functional flexibility,

* Corresponding author.

E-mail address: marcin.magdziarz@pwr.edu.pl

Ewa ROKITA-MAGDZIARZ  <https://orcid.org/0009-0009-4378-6756>

Barbara GRONOSTAJSKA  <https://orcid.org/0000-0002-8425-220X>

Marcin MAGDZIARZ  <https://orcid.org/0000-0002-2239-7155>

the possibility of separating different zones of the dwelling, the creation of sheltered outdoor spaces, and the adaptability of the plan to irregular plots and site-specific constraints. Despite these practical advantages, the geometry of such buildings is often selected primarily on the basis of functional, aesthetic, or regulatory considerations, while explicit analytical optimization of the building envelope is rarely used at the early design stage. Composed of two orthogonally intersecting wings, this form provides both practical and aesthetic benefits in traditional as well as contemporary designs. By disrupting continuous wind flow around the building, the L-shaped geometry improves wind resistance and structural stability compared to more linear building forms. Despite this, their design is frequently driven by tradition, stylistic norms, or regulatory constraints rather than by analytical optimization. Consequently, many realized examples diverge considerably from theoretically optimal solutions, resulting in unnecessary increases in material demand, construction costs, and operational energy use.

The relationship between building form and energy performance has been extensively documented in the literature. Depecker *et al.* [1] demonstrated the impact of the shape of the building on heating demand, showing that compact envelopes reduce heat losses. This principle was further emphasized by Hegger *et al.* [2] in their guide on sustainable architectural design, which underscores the importance of geometry in reducing operational energy consumption. In a comprehensive lifecycle energy assessment, Ramesh *et al.* [3] showed that the form-related parameters can represent a significant share of the total use of energy use. Numerous other studies have sought to model and quantify this relationship, including those of Catalina *et al.* [4], Ourghi *et al.* [5], Schlueter and Thesseling [6], and Jedrzejuk [7], [8], all of which confirmed the strong link between compactness and performance. Steemers [9] further argued that decisions made at the morphological level of architectural design are among the most effective levers to reduce energy demand.

More recently, D'Amico and Pomponi [10] proposed a dimensionless compactness index that enables a scale-independent assessment of building geometries, addressing the shortcomings of conventional surface-to-volume ratios. Their findings underscore that geometry is as critical to sustainability as material selection or building systems.

Studies at the urban and typological scales corroborate these conclusions. Steadman's investigations into design evolution and building typologies [11], [12] illustrate the persistent influence of form on efficiency, while Julia *et al.* [13] and Hargreaves [14] incorporated typological and density-related variables into urban energy models. Angel's global analysis of urban expansion [15], together with UN reports [16] and policy instruments such as the London Strategy [17] and California's energy regulations [18], emphasize that residential design choices have lasting consequences for sustainable urban development.

An additional foundation for this research is provided by theoretical work on architectural form generation. Early studies [19]–[21] introduced shape grammars, demonstrating that design processes can be formalized through rule-based systems. Mitchell [22] strengthened this perspective by integrating formal logic into computational design, while Knight [23] developed these ideas into a comprehensive framework for systematic shape exploration. Synthesized by Oxman [24], these contributions form the basis of much contemporary computational design theory, which conceives architecture as the outcome of generative rules informed by performance criteria. Within this paradigm, geometry becomes an explicit and controllable design variable, subject to optimization alongside other constraints.

Optimization has consequently emerged as a central component of sustainable architectural design. Convex optimization, in particular, provides a rigorous mathematical framework for addressing constrained design problems and identifying global optima in form-finding applications [25]. Okeil [26], Caruso *et al.* [27], Jin and Jeong [28], and Vartholomaos [29]

employed such techniques in architectural contexts, spanning parametric design studies and performance-driven optimization. Hachem [30], [31] extended these methods to the neighbourhood scale, optimizing residential morphologies to improve solar access and energy efficiency. Collectively, these works demonstrate that optimization techniques constitute practical and effective tools to advance sustainability in the built environment.

Recent studies also confirm the importance of energy efficiency and sustainability in residential buildings. Rieksta *et al.* [32] analysed heat demand in residential buildings, Brakovska and Blumberga [33] examined household decisions related to energy efficiency, and Argalis *et al.* [34] investigated bio-based building materials from manufacturing waste.

Building on this body of knowledge, this paper introduces a rigorous mathematical framework for optimizing L-shaped geometry houses. In this study, the external surface area is used as a geometric proxy related to material demand and potential heat exchange through the building envelope. A smaller envelope area may contribute to reduced material consumption and lower transmission heat losses, provided that other parameters, such as thermal transmittance, glazing ratio, airtightness, solar gains, ventilation losses, and climate conditions, remain comparable. Therefore, the results presented in this paper should not be interpreted as a complete energy simulation or full sustainability assessment. Instead, the proposed analytical framework is intended to support early-stage design decisions by identifying compact L-shaped configurations within prescribed architectural and practical constraints.

In what follows, we develop and apply a rigorous optimization framework for the geometry of L-shaped houses. Section 2 presents an analytical formulation of the building envelope area and derives optimal geometric configurations for symmetric and asymmetric L-shaped layouts under a range of realistic design scenarios, including fixed volume, fixed or bounded wing aspect ratios, and prescribed building height. Closed-form solutions are obtained using classical optimization techniques and Karush-Kuhn-Tucker conditions, providing clear design rules for minimizing the external surface area. Section 3 illustrates the practical applicability of the theoretical results through case studies of existing residential designs, demonstrating how the proposed framework can inform early-stage architectural decisions aimed at improving energy efficiency and sustainability. Section 4 concludes the article by summarizing the key results, discussing their implications for sustainable housing design, and suggesting directions for future research.

2. OPTIMIZATION OF GEOMETRIC DIMENSIONS

The following section formulates the optimization problem for L-shaped residential buildings. The analysis focuses on the relationship between building volume, plan proportions, height, and external surface area. Separate models are developed for symmetric and asymmetric L-shaped configurations. In each case, the objective is to minimize the external envelope area while preserving the L-shaped topology through appropriate geometric constraints.

2.1. Symmetric L-Shaped House

An illustrative sketch of a symmetric L-shaped house is shown in Fig. 1. The building footprint is composed of two perpendicular rectangular wings that intersect orthogonally to form a corner typical of L-plans. Both wings share the same geometric parameters: plan length L (measured along the longitudinal axis of each wing), uniform wing width B (measured across the short axis), and constant height H (vertical dimension).

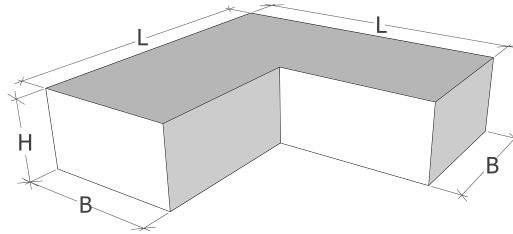


Fig. 1. View of a typical symmetric L-shaped house.

The volume of the house is given by

$$V = H (2LB - B^2). \quad (1)$$

The external surface area of the L-shaped house, also called the building envelope), is equal to

$$S = 4LH + 2LB - B^2. \quad (2)$$

Note that the first component represents the area of the walls. The remaining part is equal to the area of the roof. It should be noted that the area of the building in contact with the ground is not included in S .

The main objective of this section is to identify the optimal L-shaped house geometry that minimizes the external surface area S . We will examine different scenarios.

2.1.1. First Case: Fixed Volume V

In this scenario, the building volume V is assumed to be fixed (for example, as specified by the investor). The objective is to identify the values of L , B , and H that minimize the building's external surface area of the building S .

However, some standard calculations show that in this setting the minimal area of S is obtained for $L = B$. Then, the house takes on a cuboid shape, which means that the L-form of the building is lost. Therefore, this is not a satisfactory solution.

A similar situation occurs when we assume that the floor area $F = 2LB - B^2$ and height H of the building are fixed. Then the minimal envelope area is also obtained for $L = B$. Again, this solution yields a cuboid-shaped configuration, which is not desirable in this context.

It should be emphasized that the unconstrained minimization of the external surface area naturally favours compact conventional forms, such as square or rectangular cuboid buildings. This is consistent with the general compactness principle: for a given volume, more compact forms usually have a smaller envelope area and therefore a lower potential for transmission heat losses, assuming comparable envelope properties. In the present analysis, this tendency is reflected by the fact that, when no additional geometric constraints are imposed, the optimal solution degenerates into a cuboid and the characteristic L-shaped topology is lost. Consequently, the aim of this study is not to demonstrate that an L-shaped building is globally more compact than a conventional cuboid. Rather, the objective is to identify the most compact configuration within the admissible class of L-shaped buildings, where the L-shaped form is required by functional, architectural, regulatory, or site-related constraints. From this perspective, the proposed formulas provide a quantitative tool for reducing the envelope area of L-shaped designs relative to non-optimized L-shaped alternatives, while preserving the intended building topology.

2.1.2. Second Case: Fixed Wing Aspect Ratio $r = L/B$

At the preliminary stage of the design process, it is justifiable for an investor or designer to treat the wing aspect ratio $r = L/B$ as a prescribed parameter, since this ratio encapsulates functional, regulatory, and contextual constraints that typically precede detailed geometric optimization.

From a regulatory and site-planning point of view, zoning regulations, setback requirements, and plot geometry may impose preferred or restrictive proportions on building wings.

Finally, considerations of architectural coherence and design intent may also warrant fixing the wing aspect ratio in order to preserve visual balance, façade rhythm, and compatibility with the surrounding built environment.

Assuming that V and $r = L/B$ are fixed, we obtain from Eq. (1)–(2) that

$$H = \frac{V}{2LB - B^2}$$

and

$$S = S(B) = \frac{4Vr}{B(2r-1)} + B^2(2r-1). \quad (3)$$

Therefore, the derivative of S with respect to B equals

$$\frac{dS}{dB} = 2B(2r-1) - \frac{4Vr}{B^2(2r-1)}.$$

Solving the equation $\frac{dS}{dB} = 0$ we obtain that the minimum of $S(B)$ is obtained for

$$B_{\min} = \left(\frac{2Vr}{4r^2 - 4r + 1} \right)^{1/3}. \quad (4)$$

Consequently, the remaining optimal parameters are

$$L_{\min} = rB_{\min} = r \left(\frac{2Vr}{4r^2 - 4r + 1} \right)^{1/3}. \quad (5)$$

$$H_{\min} = \frac{V}{2L_{\min}B_{\min} - B_{\min}^2} = \frac{V}{(2r-1) \left(\frac{2Vr}{(2r-1)^2} \right)^{2/3}} = \frac{[(2r-1)V]^{1/3}}{(2r)^{2/3}} \quad (6)$$

The minimal envelope is equal to

$$S_{\min} = S(B_{\min}) = 3 \left(\frac{4V^2r^2}{2r-1} \right)^{1/3}, \quad (7)$$

see Eq. (3).

The expressions given in Eqs. (4)–(7) offer a practical basis for the design of an L-shaped house that attains the minimum external surface area $S_{\min}$ for a prescribed volume V and ratio r . Using these relations enables designers to optimize the building geometry, thereby reducing material consumption and construction costs. In addition, minimizing the external surface area enhances energy efficiency by decreasing heat losses through the building envelope, which in turn improves overall sustainability and lowers long-term operational costs.

Let us analyse the following particular example: take $V = 300 \text{ m}^3$ and $r = 2$. Then, from Eqs. (4)–(7) we obtain the following optimal parameters: $B_{\min} = 5.11 \text{ m}$, $L_{\min} = 10.22 \text{ m}$, $H_{\min} = 3.83 \text{ m}$, $S_{\min} = 234.89 \text{ m}^2$.

It should be noted that, from a practical perspective, the parameters derived above are well suited to real-world residential construction. They provide viable design options while ensuring structural efficiency, economic feasibility, and improved energy performance. Consequently, these results offer a reliable basis for architects and engineers aiming to implement optimized house designs in practice.

Fig. 2 illustrates the external surface area S of a symmetric L-shaped house as a function of the wing width B and the wing aspect ratio $r = L/B$ for a fixed building volume $V = 300 \text{ m}^3$. The surface is presented as a three-dimensional plot, where each curve corresponding to a fixed value of r exhibits a clear minimum. These minima, marked by red dots, identify the optimal wing width $B_{\min}$ that minimizes the external surface area S for a given aspect ratio. The figure visually demonstrates that, although S increases for very small or very large values of B , an intermediate optimal configuration exists for each r , thereby confirming the analytical results derived from Eq. (3) and highlighting the dependence of geometric optimization on the prescribed aspect ratio.

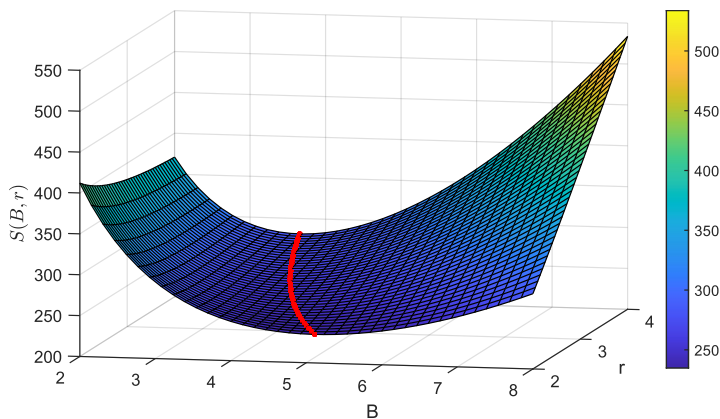


Fig. 2. External surface $S = S(B, r)$ as a function of parameters B and r , see Eq. (3). Here $v = 300 \text{ m}^3$. For each fixed r , there is a minimum of S attained at $B_{\min}$ and denoted here by a red dot.

Compactness measure. A common method for characterizing the relationship between a building's external surface area S and its volume V is the surface-to-volume ratio [2], [3]. Lower values of this ratio are typically associated with greater geometric compactness. However, an important drawback of the S/V ratio is its dependence on scale: geometrically similar buildings of different sizes (for example, cubes with different edge lengths) yield different S/V values.

To overcome this limitation, a recently introduced compactness metric has been proposed, which evaluates building compactness purely on the basis of shape, independent of absolute

size [10]. This measure is defined as $S/S_{\min}$, see [10]. Here, S denotes the exterior surface area of the building and $S_{\min}$ is the minimum external surface area required to enclose a given volume V . This metric is particularly useful in the early design phase, as it quantitatively indicates how closely a given form approaches optimal compactness, thereby supporting informed design decisions and the exploration of more efficient geometric alternatives.

We now evaluate this compactness measure for the L-shaped house with fixed V and r . Substituting equations (3) and (7) yields

$$\frac{S}{S_{\min}} = \frac{\frac{4Vr}{B(2r-1)} + B^2(2r-1)}{3 \left(\frac{4V^2r^2}{2r-1} \right)^{1/3}} = \frac{4Vr + B^3(2r-1)^2}{3B[2Vr(2r-1)]^{2/3}}. \quad (8)$$

By construction, $S/S_{\min} \geq 1$, with equality attained for the optimal values Eqs. (4)–(6). Consequently, this compactness measure provides a quantitative indicator of the geometric efficiency of the house design. It serves as a valuable tool for assessing shape optimality, guiding design refinement, and identifying opportunities to reduce both energy demand and construction costs.

2.1.3. Third Case: Wing Aspect Ratio r from a Fixed Range

In the early design stage, it is reasonable for an investor or architect to assume that the wing aspect ratio $r = L/B$ belongs to a prescribed range, $r \in [a, b]$, rather than being completely free. The main reasons are practical and come from constraints that typically precede any geometric optimization of the envelope. The ratio r strongly influences the usability of each wing: room proportions, corridor efficiency, furniture layouts, and adjacency requirements

In this scenario, we assume that the ratio r is constrained to a prescribed interval, $r \in [a, b]$. As before, volume V is given. Under these assumptions, our objective is to determine the optimal house geometry that minimizes the external surface area S , as defined in Eq. (3).

To this end, we employ the method of Lagrange multipliers combined with the Karush-Kuhn-Tucker (KKT) conditions [26]. Specifically, we seek the optimal parameters (B^*, r^*) that minimize the external surface area, in accordance with Eq. (3).

More precisely, we seek to find the minimum of the function

$$S(B, r) = \frac{4Vr}{B(2r-1)} + B^2(2r-1),$$

with constraints $a \leq r \leq b$ and $B > 0$. The constants V , a , and b are fixed, with $V > 0$, $a > 1$, $b > 1$ and $a < b$. The constraint functions are:

$$g_1(r) = a - r \leq 0 \text{ (i.e., } r \geq a),$$

$$g_2(r) = r - b \leq 0 \text{ (i.e., } r \leq b).$$

The corresponding Lagrangian $L = L(B, r, \lambda_1, \lambda_2)$ is defined as

$$\mathcal{L} = \frac{4Vr}{B(2r-1)} + B^2(2r-1) + \lambda_1(a - r) + \lambda_2(r - b).$$

Here λ_1 and λ_2 are the Lagrange multipliers.

The necessary conditions for a local minimum at (B^*, r^*) are the KKT conditions:

I. Stationarity conditions $\frac{\partial \mathcal{L}}{\partial B} = 0$ and $\frac{\partial \mathcal{L}}{\partial r} = 0$.

The first condition yields

$$B = \sqrt[3]{\frac{2Vr}{(2r-1)^2}},$$

the second one gives

$$-\frac{4V}{B(2r-1)^2} + 2B^2 - \lambda_1 + \lambda_2 = 0.$$

II. Primal Feasibility: The constraints must be satisfied

$$a - r \leq 0, \quad r - b \leq 0.$$

III. Dual Feasibility: The Lagrange multipliers must be non-negative

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0.$$

IV. Complementary Slackness:

$$\lambda_1(a - r) = 0, \quad \lambda_2(r - b) = 0.$$

Solving the above KKT system, we obtain that the unique points (B^*, r^*) corresponding to the minimum of S with given constraints, are equal to

$$R^* = a \tag{9}$$

and

$$B^* = \left(\frac{2Va}{(2a-1)^2} \right)^{1/3}. \tag{10}$$

The corresponding minimal value $S^* = S(B^*, r^*)$ is therefore equal to

$$S^* = 3 \left(\frac{4V^2 a^2}{2a-1} \right)^{\frac{1}{3}}. \tag{11}$$

The remaining optimal parameters are

$$L^* = aB^* = a \left(\frac{2Va}{4a^2 - 4a + 1} \right)^{1/3}, \tag{12}$$

$$H^* = \frac{V}{2L^*B^* - (B^*)^2} = \frac{[(2a-1)V]^{1/3}}{(2a)^{2/3}}. \tag{13}$$

The above results Eqs. (9)–(13) give a practical basis for the design of an L-shaped house that attains the minimum external surface area for a prescribed volume and ratio r from a

predefined interval $[a, b]$. They also show that the optimal value of the ratio $r \in [a, b]$ is equal to a , i.e. the left end of the interval $[a, b]$.

Let us consider the following example. Take $V = 200 \text{ m}^3$, $a = 3$ and $b = 4$. Then, from Eq. (9)–(13) we find that the optimal parameters are: $r^* = 3$, $B^* = 3.63 \text{ m}$, $L^* = 10.90 \text{ m}$, $H^* = 3.03 \text{ m}$ and $S^* = 198.12 \text{ m}^2$ and $S^* = 198.12 \text{ m}^2$. Fig. 3 presents the corresponding external surface area $S(B, r)$. The surface is shown as a three-dimensional graph in the admissible range of r constrained by $a = 3$ and $b = 4$. The visualization clearly illustrates how the external surface varies within this domain and highlights the influence of both geometric parameters on the building envelope. The global minimum of S corresponding to the optimal solution obtained analytically from the constrained optimization problem, is marked by a red dot. This minimum occurs at the lower bound of the admissible aspect-ratio interval, confirming that the optimal configuration is attained for $r^* = a$. The figure thus provides a graphical validation of the KKT-based analysis and illustrates the effect of imposing bounds on the wing aspect ratio during the design process.

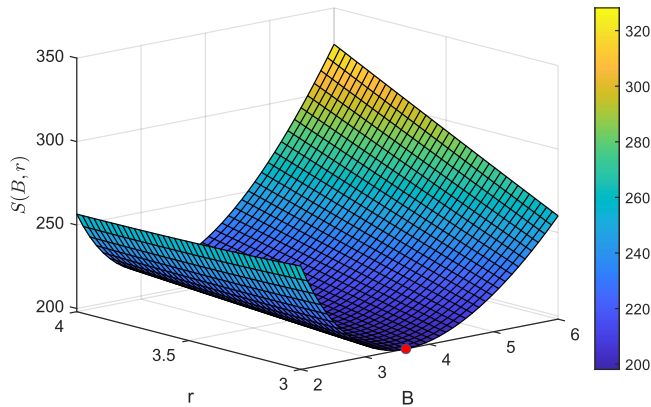


Fig. 3. Plot of the external surface $S(B, r)$ with $V = 200 \text{ m}^3$. For the constraints $a = 3$, $b = 4$ the global minimal surface derived from Eq. (11) is equal to 198.12 m^2 and is denoted by the red dot.

2.2. Asymmetric L-Shaped House

In this section, we derive the optimality condition for an asymmetric L-shaped house. An illustrative sketch of the asymmetric L-shaped house is shown in Fig. 4.

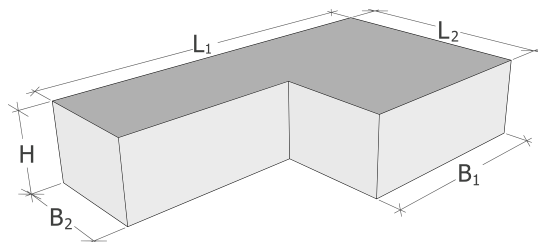


Fig. 4. View of an asymmetric L-shaped house.

It is composed of two perpendicular wings of unequal dimensions. Unlike the symmetric configuration, the wings differ in length and/or width, resulting in a non-uniform footprint

that departs from mirror symmetry. The intersection of the two rectangular wings forms a characteristic L-shaped plan, creating a sheltered inner corner that can be used as a courtyard, terrace, or a protected outdoor space.

Variation in wing proportions allows the building to respond more effectively to site-specific constraints, such as plot geometry, orientation, access points, or solar exposure. The asymmetric L-shaped configuration offers greater design freedom than its symmetric counterpart, making it particularly suitable for irregular plots and context-driven residential designs, while retaining the climatic and spatial advantages associated with L-shaped forms.

Let us introduce the wing aspect ratios

$$r_1 = \frac{B_1}{L_1} \text{ and } r_2 = \frac{B_2}{L_2} .$$

Then, the volume of the asymmetric house from Fig. 4 is equal to

$$V = H \left[L_1 L_2 - L_1 (1 - r_1) L_2 (1 - r_2) \right] = H L_1 L_2 (r_1 + r_2 - r_1 r_2) . \quad (14)$$

Therefore, height H can be expressed as

$$H = \frac{V}{L_1 L_2 (r_1 + r_2 - r_1 r_2)} . \quad (15)$$

The external surface area of the asymmetric L-shaped house is given by

$$S = L_1 L_2 (r_1 + r_2 - r_1 r_2) + 2(L_1 + L_2)H = L_1 L_2 (r_1 + r_2 - r_1 r_2) + \frac{2V(L_1 + L_2)}{L_1 L_2 (r_1 + r_2 - r_1 r_2)} . \quad (16)$$

The first component in the formula above represents the area of the roof. The remaining part is equal to the area of the walls. In addition, in this case the area of the building in contact with the ground is not included in S .

The main goal of this section is to find the optimal L-shaped house geometry that minimizes the external surface area S . We will consider different scenarios.

2.2.1. First Case: Fixed Volume V

In this scenario, the building volume V is assumed to be fixed. However, similarly to the symmetric case, this formulation yields a minimum of the external surface area S when $L_1 = L_2 = B_1 = B_2$. In this situation, the building degenerates into a cuboid, and the characteristic L-shaped configuration is no longer preserved. Consequently, this solution is not acceptable within the context of L-shaped house design.

An analogous situation arises when the floor area F and the building height H are prescribed. In this case, the external envelope area is likewise minimized for $L_1 = L_2 = B_1 = B_2$, leading once again to a cuboid configuration. As a result, the defining L-shaped geometry is lost, rendering this solution unsuitable for the present design context.

2.2.2. Second Case: Fixed Wing Aspect Ratios r_1 and r_2

At the early stage of the design process, it is reasonable to treat the wing aspect ratios $r_1 = B_1/L_1$ and $r_2 = B_2/L_2$ and as fixed parameters. In an asymmetric L-shaped house, the two wings typically serve different functional purposes, imposing predefined requirements on

room proportions, circulation, and minimum widths. These functional constraints are naturally expressed through prescribed aspect ratios rather than fully free geometric variables.

Additionally, site conditions, zoning regulations, and setback requirements often restrict the proportions of admissible wings before detailed geometric optimization is undertaken. Structural and economic considerations, such as preferred spans, standardized components, and roof geometry, also favour specific length-to-width ratios. Fixing r_1 and r_2 therefore ensures that the optimization remains consistent with practical, regulatory, and constructional constraints, while preserving the intended asymmetric L-shaped character of the building.

In what follows, we will find the optimal shape (minimal envelope S) of the asymmetric L-shaped house, assuming that r_1 and r_2 are fixed parameters. As before, we assume that V is also fixed.

Using Eq. (16), we obtain that the equations $\frac{\partial S}{\partial L_1} = 0$ and $\frac{\partial S}{\partial L_2} = 0$ take the form:

$$\begin{aligned} L_2(r_1 + r_2 - r_1 r_2) + \frac{2V}{L_1 L_2 (r_1 + r_2 - r_1 r_2)} - \frac{2V(L_1 + L_2)}{L_1^2 L_2 (r_1 + r_2 - r_1 r_2)} &= 0, \\ L_1(r_1 + r_2 - r_1 r_2) + \frac{2V}{L_1 L_2 (r_1 + r_2 - r_1 r_2)} - \frac{2V(L_1 + L_2)}{L_1 L_2^2 (r_1 + r_2 - r_1 r_2)} &= 0. \end{aligned}$$

After some tedious calculations, we find that the solution of the above system of equations, which defines the minimum of S , has the form

$$L_1^{\min} = L_2^{\min} = \left(\frac{2V}{(r_1 + r_2 - r_1 r_2)^2} \right)^{1/3}. \quad (17)$$

Consequently,

$$B_1^{\min} = r_1 L_1^{\min} = r_1 \left(\frac{2V}{(r_1 + r_2 - r_1 r_2)^2} \right)^{1/3}, \quad (18)$$

$$B_2^{\min} = r_2 L_2^{\min} = r_2 \left(\frac{2V}{(r_1 + r_2 - r_1 r_2)^2} \right)^{1/3}, \quad (19)$$

$$H^{\min} = \frac{V}{L_1^{\min} L_2^{\min} (r_1 + r_2 - r_1 r_2)} = \left(\frac{(r_1 + r_2 - r_1 r_2)V}{4} \right)^{1/3}, \quad (20)$$

$$S^{\min} = L_1^{\min} L_2^{\min} (r_1 + r_2 - r_1 r_2) + \frac{2V(L_1^{\min} + L_2^{\min})}{L_1^{\min} L_2^{\min} (r_1 + r_2 - r_1 r_2)} = 3 \left(\frac{4V^2}{r_1 + r_2 - r_1 r_2} \right)^{1/3}. \quad (21)$$

The expressions given in Eqs. (17)–(21) provide a practical framework for the design of an asymmetric L-shaped house that achieves the minimum external surface area $S_{\min}$ for a prescribed building volume V and fixed wing aspect ratios r_1 and r_2 . Employing these relations

allows designers to optimize the building geometry in a systematic manner, leading to a reduction in material usage and construction costs.

As a representative example, consider a building volume of $V = 300 \text{ m}^3$ and aspect ratios $r_1 = 0.4$, $r_2 = 0.6$. Substituting these values into (17)–(21) yields the optimal parameters $L_1^{\min} = L_2^{\min} = 10.13 \text{ m}$, $B_1^{\min} = 4.05 \text{ m}$, $B_2^{\min} = 6.08 \text{ m}$, $H^{\min} = 3.85 \text{ m}$ and $S^{\min} = 233.86 \text{ m}^2$.

From a practical point of view, the dimensions obtained are well aligned with typical residential construction standards. They represent feasible design solutions that balance structural efficiency, economic viability, and enhanced energy performance. Fig.5 depicts the corresponding external surface area S as a function of the wing lengths L_1 and L_2 . The global minimum of S , corresponding to the optimal solution obtained analytically in Eq. (21), is indicated by a red marker. Therefore, the figure provides a graphical validation of the analysis.

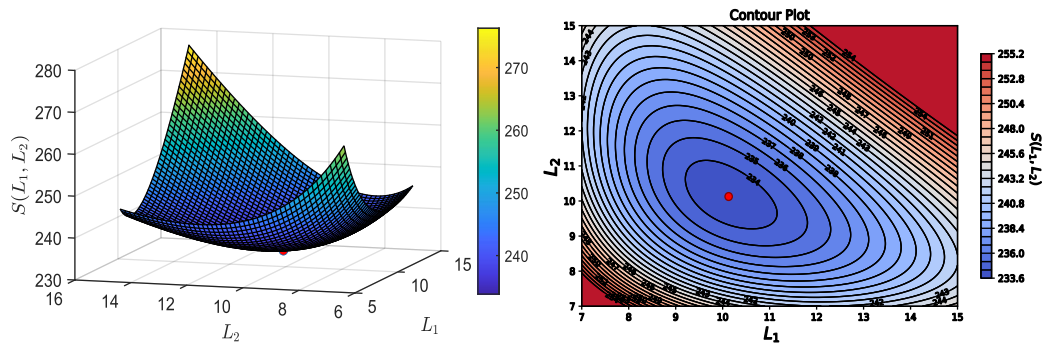


Fig. 5. Left panel: plot of the external surface $S(L_1, L_2)$. Right panel: corresponding contour plot. The red dot indicates the location of the minimal surface obtained using formula Eq. (21). Parameters: $V = 300 \text{ m}^3$, $r_1 = 0.4$, $r_2 = 0.6$.

2.2.3. Third Case: Wing Aspect Ratios r_1 and r_2 from Fixed Ranges

At the early stage of the design process, it is reasonable to assume that the wing aspect ratios r_1 and r_2 lie within prescribed intervals $r_1 \in [a_1, b_1]$ and $r_2 \in [a_2, b_2]$, rather than being completely unrestricted. This assumption is primarily motivated by practical considerations arising from constraints that typically precede any detailed geometric optimization of the building envelope. The aspect ratios have a significant impact on the functional quality of each wing. In addition, site-specific factors, including plot geometry, setback regulations, easements, minimum boundary distances, and local planning rules, often further constrain the range of admissible building proportions.

For these reasons, we assume that the ratios are constrained to prescribed intervals: $r_1 \in [a_1, b_1]$ and $r_2 \in [a_2, b_2]$. As before, the volume V is fixed. Under these assumptions, our objective is to determine the optimal house geometry that minimizes the external surface area S , as defined in Eq. (16).

To this end, we employ the method of Lagrange multipliers with KKT conditions, see the analysis in Sec. 2.1.3 and [26]. More precisely, we will find the optimal parameters L_1^* , L_2^* , r_1^* and r_2^* that minimize the external surface area S , in accordance with Eq. (16). The constraints are $a_1 \leq r_1 \leq b_1$, $a_2 \leq r_2 \leq b_2$, $L_1 > 0$, $L_2 > 0$. Here $0 < a_1 < b_1 < 1$, $0 < a_2 < b_2 < 1$ and $V > 0$ are fixed constants. The corresponding Lagrangian is

$$\mathcal{L} = \frac{2V(L_1 + L_2)}{L_1 L_2 (r_1 + r_2 - r_1 r_2)} + L_1 L_2 (r_1 + r_2 - r_1 r_2) + \lambda_1 (a_1 - r_1) + \lambda_2 (r_1 - b_1) + \lambda_3 (a_2 - r_2) + \lambda_4 (r_2 - b_2).$$

To find the optimal values of L_1 , L_2 , r_1 and r_2 we apply the KKT conditions, cf. Sec. 2.1.3. Solving the system of KKT conditions, after some tedious calculations, we find that the optimal parameters minimizing S are the following:

$$r_1^* = b_1, \quad r_2^* = b_2. \quad (22)$$

$$L_1^* = L_2^* = \left(\frac{2V}{(b_1 + b_2 - b_1 b_2)^2} \right)^{1/3}, \quad (23)$$

and therefore

$$S^* = 3 \cdot \left(\frac{4V^2}{b_1 + b_2 - b_1 b_2} \right)^{1/3}. \quad (24)$$

The remaining optimal parameters are

$$B_1^* = b_1 L_1^* = b_1 \left(\frac{2V}{(b_1 + b_2 - b_1 b_2)^2} \right)^{1/3}, \quad (25)$$

$$B_2^* = b_2 L_2^* = b_2 \left(\frac{2V}{(b_1 + b_2 - b_1 b_2)^2} \right)^{1/3}, \quad (26)$$

$$H^* = \frac{V}{L_1^* L_2^* (b_1 + b_2 - b_1 b_2)} = \left(\frac{(b_1 + b_2 - b_1 b_2)V}{4} \right)^{1/3}. \quad (27)$$

The results summarized in Eqs. (22)–(27) provide a practical foundation for the design of an L-shaped house that minimizes the external surface area for a prescribed building volume and wing aspect ratios constrained to predefined intervals $r_1 \in [a_1, b_1]$ and $r_2 \in [a_2, b_2]$. In particular, they demonstrate that the optimal values of the aspect ratios within these intervals are attained at their upper bounds, that is, $r_1 = b_1$ and $r_2 = b_2$.

As an illustrative example, consider a building volume of $V = 200 \text{ m}^3$ with the admissible aspect-ratio ranges defined by $a_1 = 0.3$, $b_1 = 0.5$, $a_2 = 0.2$ and $b_2 = 0.8$. Substituting these values into Eqs. (22)–(27) yields the optimal parameters $r_1^* = 0.5$, $r_2^* = 0.8$, $L_1^* = L_2^* = 7.90 \text{ m}$, $B_1^* = 3.95 \text{ m}$, $B_2^* = 6.32 \text{ m}$, $H^* = 3.56 \text{ m}$ and $S^* = 168.69 \text{ m}^2$.

From a practical standpoint, these dimensions are well aligned with typical residential construction standards. They represent feasible design solutions that balance structural efficiency, economic viability, and improved energy performance.

Fig. 6 illustrates the graph of the minimal surface calculated for each $r_1 \in [a_1, b_1]$ and $r_2 \in [a_2, b_2]$ using Eq. (21). The global minimum of S is indicated by a red marker. This minimum is attained at the upper bounds of the admissible aspect-ratio intervals $r_1 = b_1$ and $r_2 = b_2$, confirming the analytical results Eqs. (22)–(27).

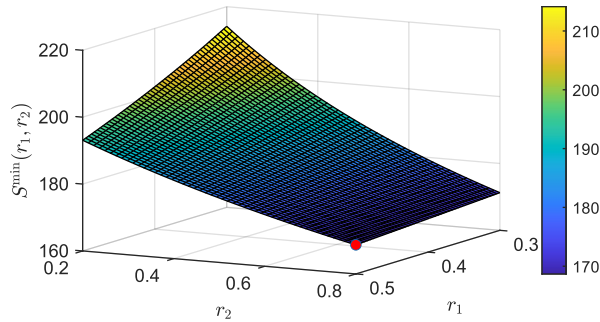


Fig. 6. Plot of the minimal surface calculated for each $r_1 \in [a_1, b_1]$ and $r_2 \in [a_2, b_2]$ using Eq. (21). The red dot indicates the global minimum of S . Clearly, the global minimum is obtained for $r_1 = b_1$ and $r_2 = b_2$, in accordance with the results (22)–(27). Parameters: $V = 200 \text{ m}^3$, $a_1 = 0.3$, $b_1 = 0.5$, $a_2 = 0.2$ and $b_2 = 0.8$. The surface reaches its highest values (about 215) in the back-left corner where both r_1 and r_2 are small, and decreases smoothly toward the front-right corner, where the values are lowest (about 170–175). A red dot marks a point near $r_1 \approx 0.5$, $r_2 \approx 0.8$, close to the minimum of the surface. A vertical color bar on the right maps colors from blue (approximately 170) through green to yellow (approximately 210).

2.2.4 Fourth Case: Fixed Height H and Wing Aspect Ratios r_1 and r_2

At the early design stage, it is reasonable to treat the building height as a predefined parameter, since it is typically constrained by factors that precede geometric optimization. Local zoning and planning regulations often impose strict limits on allowable building height, number of storeys, and roof geometry. In addition, functional requirements such as desired ceiling heights, storey-to-storey dimensions, and the inclusion of attic or technical spaces are usually specified by the investor at the outset.

We now assume that H , r_1 , r_2 and V are fixed. In this setting, the building envelope area is equal to

$$S = \frac{V}{H} + 2H \left(L_1 + \frac{V}{L_1 H (r_1 + r_2 - r_1 r_2)} \right).$$

The minimum of this function is obtained for

$$L_1^{\min} = \sqrt{\frac{V}{H(r_1 + r_2 - r_1 r_2)}}. \quad (28)$$

The remaining optimal parameters are

$$L_2^{\min} = L_1^{\min}, \quad B_1^{\min} = r_1 L_1^{\min}, \quad B_2^{\min} = r_2 L_2^{\min}, \quad S^{\min} = \frac{V}{H} + 4H \sqrt{\frac{V}{H(r_1 + r_2 - r_1 r_2)}}. \quad (29)$$

The expressions given in Eqs. (28)–(29) establish a practical basis for the design of an asymmetric L-shaped house that attains the minimum external surface area $S_{\min}$ for prescribed H , V , r_1 and r_2 . Making use of these relations enables a systematic optimization of the building geometry, which in turn contributes to reduced material consumption and lower construction costs.

3. CASE STUDIES

To emphasize the practical significance of the results of theoretical optimality derived in the previous section, we consider an application in the form of two case studies. In particular, we analyse two L-shaped house designs currently offered by architectural firms, denoted by A and B (see Fig. 7). These buildings were intentionally selected to represent a wide range of geometric shapes, proportions, and sizes. The objective is to illustrate how the derived formulas can be applied to the analysis of real-world structures with varying dimensions and configurations. For each house, both the visual characteristics and the key geometric parameters were extracted using data available on the project websites. This methodology ensures that the case studies are not only illustrative, but also reproducible, as it can be readily extended to other architectural examples.

For each case study, the geometric parameters were extracted from publicly available architectural project information, including dimensioned plans, project descriptions, and visual documentation available on the project websites. The extracted dimensions were then simplified to match the analytical L-shaped model used in this study. In particular, the building footprints were approximated by two perpendicular rectangular wings, and the corresponding lengths, widths, and heights were used to compute the volume and external surface area. The consistency of the extracted parameters was checked by comparing the resulting floor areas and volumes with the values reported in the project descriptions, where available. The case studies should therefore be understood as illustrative applications of the proposed analytical framework rather than as certified architectural surveys or detailed energy audits.



Fig. 7. Images of two L-shaped houses analysed as case studies using the results obtained in previous sections. Design and image sources: www.hausbaudirekt.de/haus/solution-110-v4-2-2 (panel A); www.maramani.com/products/l-shaped-4-bedroom-house-1d-14313 (panel B).

Analysis of House A. Given parameters of House A are as follows: $L_1 = 13.7$ m, $L_2 = 14.9$ m, $B_1 = 8.7$ m, $B_2 = 4.6$ m, $H = 3.6$ m. This gives $r_1 = 0.63$, $r_2 = 0.31$, $V = 549.5$ m³ and $S = 358.5$ m².

Using the optimality findings from Equations (17)–(21), House A's ideal parameters are as follows: $L_1^{\min} = L_2^{\min} = 12.5$ m, $B_1^{\min} = 7.9$ m, $B_2^{\min} = 3.9$ m, $H^{\min} = 4.6$ m and $S^{\min} = 351.9$ m². The above optimization procedure would reduce the external surface by 6.6 m².

The application of the optimality results from Eq. (28) and Eq. (29) gives the external surface almost identical to the original one. From this point of view, the original design is very close to optimal. This result is expected due to the very limited degrees of freedom in Eqs. (28)–(29).

Analysis of House B. Given parameters of House B are the following: $L_1 = 22$ m, $L_2 = 19.5$ m, $B_1 = 8.8$ m, $B_2 = 8.5$ m, $H = 4.3$ m. This gives $r_1 = 0.4$, $r_2 = 0.43$, $V = 1220.3$ m³ and $S = 640.7$ m².

Using the optimality findings from Equations (17)–(21), House B's ideal parameters are as follows: $L_1^{\min} = L_2^{\min} = 17.7$ m, $B_1^{\min} = 7.1$ m, $B_2^{\min} = 7.7$ m, $H^{\min} = 5.8$ m and $S^{\min} = 624.1$ m².

The above optimization procedure would reduce the external surface by 16.6 m². It would result in reasonable savings in construction costs and energy consumption of the house.

Note that the application of the optimality results from Eq. (28) and Eq. (29) gives the external surface smaller than the original one by about 1 m². From this point of view, the original design is close to optimal. This result is expected due to the very limited degrees of freedom in Eqs. (28)–(29).

The two case studies confirm the practical usefulness of the proposed analytical framework. For both houses, the derived formulas make it possible to compare the existing design with an optimized geometric configuration and to assess whether the external surface area can be reduced.

In House A, optimization with fixed wing aspect ratios indicates a possible reduction in the external envelope area, showing that some geometric improvement is possible. However, when the building height is treated as fixed, the optimized surface area is almost identical to the original one. This suggests that the existing design is already close to optimal under realistic design constraints.

For House B, the optimization also leads to a reduction in the external surface area. This indicates potential savings in construction materials and a possible decrease in heat losses through the envelope. Nevertheless, these results should be interpreted as geometric improvements rather than as complete energy-performance predictions, since factors such as glazing, orientation, insulation, ventilation, and climate are not included in the model.

Overall, the case studies show that the proposed method can support early-stage architectural decisions by providing a simple quantitative measure of geometric efficiency. It helps designers identify whether a given L-shaped layout is already close to optimal or whether modest dimensional adjustments could improve compactness while preserving the intended architectural form.

4. CONCLUSIONS

This paper presents a systematic analytical study of geometric optimization of L-shaped residential buildings with the objective of minimizing the external surface area for a prescribed building volume. Since the external surface area is directly related to material consumption, heat losses, and operational energy demand, the results obtained provide a clear geometric basis for improving both energy efficiency and sustainability at the early design stage. In this study, we have employed the external surface area as a geometric proxy for material demand and potential envelope heat exchange. A minimized envelop area has shown it can reduce material consumption and transmission heat losses, assuming factors like thermal transmittance, glazing ratios, and climate conditions have remained constant. Consequently, these findings have not been presented as a complete energy simulation or full sustainability assessment. Instead, this analytical framework has been developed to guide early-stage design by identifying compact L-shaped configurations within practical constraints.

For symmetric L-shaped houses, several optimization scenarios were investigated. It was shown that when only the volume or floor area is fixed, the minimum external surface area is attained for a degenerate cuboid configuration, leading to the loss of the L-shaped form. This observation highlights the necessity of introducing additional geometric constraints to preserve the intended building topology. By fixing the aspect ratio, explicit formulas for

optimal wing dimensions, height, and minimal envelope area were derived. These closed-form expressions allow designers to directly determine optimal proportions that reduce the envelope area while maintaining functional and architectural requirements. When the wing aspect ratio was constrained to a prescribed interval, the constrained optimization analysis based on KKT conditions demonstrated that the optimal solution is attained at the boundary of the admissible range, providing a clear design guideline for selecting feasible proportions. An analogous optimization framework was developed for asymmetric L-shaped houses, which offer greater flexibility and are particularly relevant for irregular plots and context-driven designs. For fixed wing aspect ratios, the optimal configuration was shown to correspond to equal wing lengths, despite differing widths, and explicit expressions for the optimal geometry and minimal external surface area were obtained. When the aspect ratios were constrained to admissible intervals, the analysis revealed that the minimum envelope area is achieved at the upper bounds of these intervals. Additional scenarios with fixed building height further illustrate how regulatory and functional constraints can be incorporated into the optimization process without sacrificing analytical tractability.

The practical relevance of the theoretical results was demonstrated through real-world case studies of existing L-shaped house designs. In both cases, the application of the derived formulas led to a reduction of the external surface area or confirmed that the original designs were already close to optimal. These examples illustrate that even modest geometric adjustments, guided by analytical optimization, can lead to tangible savings in construction materials and energy demand.

In general, the results presented show that geometric optimization of L-shaped buildings can be effectively addressed using relatively simple analytical tools, yielding explicit design formulas that are directly applicable in architectural practice. The proposed approach supports informed early-stage decision-making, enabling designers to balance functional requirements, regulatory constraints, architectural intent, and energy efficiency. Future research may extend this framework to include additional factors such as thermal zoning, façade orientation, window-to-wall ratios, or multi-storey configurations, further strengthening the link between geometric form and sustainable building performance.

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