The home-field paradox: These go to eleven

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Abstract

Two baseball teams plan a best-of-seven series, beginning with three games at the home field of one team and followed by up to four at the home field of the other, stopping when one team reaches four wins. The teams are evenly matched, but the home team wins with a fixed probability $p > 1/2$. Which team is expected to win more often? Which team is expected to win the series? Are these the same? Does it depend on p? Peter Winkler recently described a "home-field paradox" that arises in this problem. We describe this problem and its analysis, and explore a number of computational investigations into it, varying the series length, the value of p, and the sequence of home and away games. Can we design series schedules and p values that exacerbate the home-field paradox? Or ameliorate it? Or make it even more confounding? Can we push this problem to the utmost level, so that we can truly make the home-field paradox go to eleven?

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1 Introduction

Suppose two rival baseball teams from the Empire State, the Albany AlleyCats and the Binghamton Bumblebees, meet annually for a series of games, where the winner is the first team to reach four wins. By tradition, the first three games are played at Albany, followed by up to four at Binghamton. The teams are evenly matched, but the home team has an edge: the hosting team wins 54% of the time. This is a grudge match, with city pride on the line, so the two teams always bring their best effort to every game, regardless of past wins and losses. Which team would you expect to come out on top?

Surely the Bumblebees have the edge when the series stretches to seven games. But many times the series is over in fewer games, and it seems plausible that on average the

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series would be decided in five or six. If that's the case, then we expect that more games are played at Albany, so the AlleyCats ought to have an edge here.

On the other hand, the teams bring their best effort to every game, so we should treat them as independent events. In this case, there is no harm in assuming that Albany and Binghamton play all seven games, even if one team reaches four wins at an earlier point. So from that point of view, the Bumblebees should come out on top.

Peter Winkler described this conundrum in 2021 [Win21], as well as its resolution. Winkler's fictional teams were the Appleton Aardvarks and the Brockville Bandicoots, but we've changed the setting here to the Empire State, after an early version of this work was presented at the AMS Fall Eastern Sectional Meeting in Albany in 2024.[1] Winkler showed that *both* of the heuristic analyses above are right, in some sense: Albany wins more *games* on average, but Binghamton wins more *series*. This phenomenon was dubbed the *home-field paradox* in his article.

We investigate and expand on this paradox further here. We'll refer to the two teams as simply A and B. Suppose then that A and B are evenly matched, except the home team wins with probability p, and these teams meet in a series of odd length s, according to a particular *home-site schedule*, which is given by a sequence H_s of length s over the set $\{A, B\}$. In Winkler's case, we have $p = 0.54$, $s = 7$, and $H_7 = AAABBBB$. We say H_s *exhibits the home-field paradox* if, for some $p > 1/2$, team A is expected to win more games per series, but B is expected to win more series overall. How often does the home-field paradox arise, if we vary the home-site schedule H_s, for a given series length s? How does this change with s? What value of p produces the strongest effect in the paradox, for a given sequence H_s? Are there even stranger examples waiting to be discovered, e.g., for a given sequence H_s, is the set of probability values p where one team enjoys a game advantage always a connected set in the real interval $[1/2, 1]$? Can we find more bizarre examples, and truly make the home-field paradox go to eleven?[2]

We investigate these questions and more here. We start with an empirical overview of home-field advantage in professional baseball in Section 2. We then analyze Winkler's series pattern $AAABBBB$, along with all other seven-game home-site patterns in Section 3, and then to all series of length $s \leqslant 9$ in Section 4. These analyses suggest a path to automating a procedure for characterizing the game bias, which we develop in Section 5, and we use this to explore the strange ways that the home-field paradox can present itself in successively longer series in Sections 6 and 7. We turn to some simulations in Section 8, and close with some potential problems for future investigations in Section 9.

Let's start with a historical view of the home-field paradox in Major League Baseball.

[1] The Albany AlleyCats and Binghamton Bumblebees don't actually exist, but the Tri-City ValleyCats of the Frontier League represent Albany, Schenectady, and Troy, and were once affiliated with the Houston Astros, and the Binghamton Rumble Ponies are a Double-A club associated with the New York Mets. Support your local Minor League Baseball teams!

[2] An expression recognized even in the august OED, meaning to an extreme or intense degree, or surpassing the maximum level, after the 1984 Rob Reiner mockumentary *This is Spinal Tap*.

2 Empirical home-field advantage

What advantage have home teams enjoyed during the past fifty years? While there are several ways to interpret this question, perhaps the simplest interpretation is to ask how often home teams win. Considering all games over the fifty seasons between 1976 and 2025, the home team has won 53.8% of the time. The season that exhibited the greatest home winning percentage was 1978 (57.4%) and the lowest percentage was realized in the strike-shortened 1994 season (51.6%). The percentages have not varied greatly, however, with nearly two-thirds of these fifty seasons seeing home-field winning percentages between 53% and 55%. There is interesting consistency in other ways as well. If we look at the months of April through September separately, we find that the monthly home-team winning percentages all lie between 53.3% and 54.6%. Further, the days of the week exhibit the same steadiness: daily home team winning percentages all fall between 53.5% and 54.2%. Winkler's adoption of $p = 0.54$ certainly lands in the heart of the strike zone!

Some readers may be interested to know how the home winning percentage varies by club. If we simply consider percentage of home games won, the New York Yankees come out on top over these fifty years with a home win rate of 61.0%. The Tampa Bay (Devil) Rays sit at the other end of the dugout with a home win rate of 45.0% during those years. Another way to interpret a team's home-field advantage might be to compare their win rate at home against their win rate when they play on the road. Thinking this way, the club with the largest disparity in these two values is the Colorado Rockies, and the club with the smallest difference here is the Cincinnati Reds.

The percentages discussed so far were based on all games over the regular and postseason combined. Since this article considers series that most closely resemble playoff match-ups, it is also natural to investigate how home teams have managed in the postseason alone over the last fifty years. Home winning percentages vary a bit more in this case: 80% of the postseason games were won by the home team in 1982, which was the highest percentage, but only 36.6% were won by the home team in 2023, which was lowest. Though the variance was larger, the mean is on par with the full season number: over all postseason games during these fifty years, 54.8% were won at home.

One further bit of investigation may be of interest. Over the last fifty years there have been 37 postseason series (League Championship and World Series) that have gone to a seventh game. How have home teams fared in these series? Over all of these series, the average number of games won by the home team is 3.92. Four times the home team won every game in the series, and twice the home team never won at all, including the 2023 World Series. Also, in 19 of these 37 seven game series (51.4%), the series winner has been the team with four home games.

The seven-game playoff series in this data set all employed the home-site schedule $BBAAABB$, rather than Winkler's $AAABBBB$. We discuss both of these orderings, and many more, over the next few sections. First, let's review the analysis of Winkler's series, and quantify the effect of the home-field paradox there.

3 Seven-game series

Let's first calculate the expected length of the series in the original problem, where $H_7 = AAABBBB$. For this, let $\Pr_T(H_s; k, p)$ denote the probability that team $T \in \{A, B\}$ wins the series in exactly k games, when the home-site schedule is H_s, assuming the home team wins with probability p. Clearly our expected series length is then

$$L(H_s; p) = \sum_{k=\frac{s+1}{2}}^{s} k\big(\Pr_A(H_s; k, p) + \Pr_B(H_s; k, p)\big)$$

We'll suppress the H_s in $\Pr_T(H_s; k, p)$ when the context is clear.

We can write down these probabilities for Winkler's series. A series is decided in four games if either A sweeps the first four games, which occurs with probability $\Pr_A(4, p) = p^3(1-p)$, or B does, which happens with probability $\Pr_B(4, p) = p(1-p)^3$. If A wins in five games, then it either lost its first away game, or one of its games at home, so these contribute $p^4(1-p)$ and $3p^2(1-p)^3$, respectively, and the analysis is similar when B wins in five, so

$$\Pr_A(5, p) = p^4(1-p) + 3p^2(1-p)^3, \;\; \Pr_B(5, p) = p(1-p)^4 + 3p^3(1-p)^2$$

In general, we note that $\Pr_A(k, p) = \Pr_B(k, 1-p)$. Using a similar strategy, we find

$$\begin{aligned}\Pr_A(6, p) &= p^5(1-p) + 3p(1-p)^5 + 6p^3(1-p)^3\\ \Pr_B(6, p) &= p(1-p)^5 + 3p^5(1-p) + 6p^3(1-p)^3\end{aligned}$$

We could now use that $\Pr_A(7, p) + \Pr_B(7, p)$ is simply the complement of the sum of all the prior values, but it is convenient to determine each value separately:

$$\begin{aligned}\Pr_A(7, p) &= p^6(1-p) + 9p^4(1-p)^3 + 9p^2(1-p)^5 + (1-p)^7\\ \Pr_B(7, p) &= p(1-p)^6 + 9p^3(1-p)^4 + 9p^5(1-p)^2 + p^7\end{aligned}$$

This produces

$$L(AAABBBB; p) = 20p^6 - 60p^5 + 82p^4 - 64p^3 + 31p^2 - 9p + 7$$

so $L(AAABBBB; 0.54) = 5.8153$. This comports with our prior intuition that a typical series probably resolves in five or six games. This value is just slightly higher than the expected length if there were no home-field advantage: $L(AAABBBB; 1/2) = 5.8125$.

Next, we can compute the probability that B wins the series, which we denote by $S_B(H_s; p)$. Certainly $S_B(H_s; p) = \sum_{k=\frac{s+1}{2}}^{s} \Pr_B(H_s; k, p)$, but it is helpful to employ Winkler's observation that if B hosts the last possible game, then series ending in fewer than s games produce no net advantage for A or B; an advantage for B arises only in the last game. For convenience, let $\Pr(H_s; k, p)$ denote the probability that the series ends at the kth game, so

$$\Pr(H_s; k, p) = \Pr_A(H_s; k, p) + \Pr_B(H_s; k, p)$$

If H_s ends with B, then

$$\begin{aligned} S_B(H_s;p) &= \frac{1}{2}(1-\Pr(H_s;s,p)) + p\Pr(H_s;s,p) \\ &= \frac{1}{2} + \left(p-\frac{1}{2}\right)\Pr(H_s;s,p) \end{aligned}$$

Either way, we obtain

$$S_B(AAABBBB;p) = 20p^7 - 70p^6 + 108p^5 - 95p^4 + 52p^3 - 18p^2 + 4p \tag{1}$$

Thus, we can quantify B's advantage in this series:

$$S_B(AAABBBB;0.54) = 0.512548$$

so indeed B wins more series on average.

We can compute the expected advantage in the number of games per series won by a team in a similar way. We define the *game bias for* A, relative to a given home-site sequence H_s, as the expected value of the difference in the number of games won by A over those won by B when the home-field advantage is p. We denote this by $\beta(H_s;p)$. If A wins in k games, then this game differential is $s+1-k$; if B wins in k games, then it is $k-s-1$, so

$$\beta(H_s;p) = \sum_{k=\frac{s+1}{2}}^{s} (s+1-k)\big(\Pr_A(H_s;k,p) - \Pr_B(H_s;k,p)\big)$$

Note that by construction $\beta(H_s;p) = -\beta(H_s;1-p)$, and in particular $\beta(H_s;1/2) = 0$. In this way, we compute

$$\beta(AAABBBB;p) = (1-2p)\left(1-9p+31p^2-64p^3+82p^4-60p^5+20p^6\right) \tag{2}$$

We can now verify that indeed A expects to win more games using Winkler's parameters: $\beta(AAABBBB;0.54) = 0.01477$, that is, A is expected to win 0.01477 more games than B per series played when the home-field advantage is $p = 0.54$.

This verifies the home-field paradox for $H_7 = AAABBBB$ when $p = 0.54$: B has the advantage in the series, but A wins more games on average. Moreover, we can determine all values of p where the home-field paradox arises for this pattern of home games. Certainly, if $p = 1$ then B wins every series, four games to three, so the game bias is back in B's favor at that point. Since the game bias is continuous in p, there must exist some value $p_0 \in (1/2, 1)$ where $\beta(AAABBBB;p_0) = 0$. This value is in fact unique: to four decimal places, $p_0 = 0.7787$. Figure 1 displays the graph of $\beta(AAABBBB;p)$ over $1/2 \leqslant p \leqslant 1$.

We also see that we can exacerbate the home-field paradox with another choice of p: the game bias for A is maximized when $p = 0.6712$: here the value is more than three times larger than that witnessed at 0.54, as $\beta(AAABBBB;0.6712) = 0.04441$.

We might next ask if the home-field paradox appears for other patterns of home games for seven-game series. There are $\binom{7}{3} = 35$ different home-site schedules for a seven-game

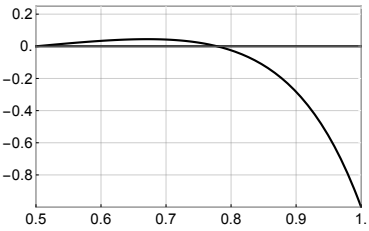


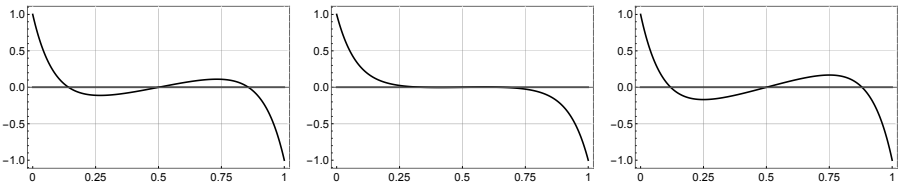
Figure 1: The game bias (2) for Winkler's seven-game series.

series where there are three home games at A and four at B, and we could repeat the analysis above for each of them. However, it is helpful to note first that, for a series of s games, these expected value calculations are immune to the ordering of the first $(s+1)/2$ positions in the sequence, since no series can end in fewer than $(s+1)/2$ games. We can therefore group the home-site sequences into equivalence classes, based on their last $(s-1)/2$ positions only. For example, the class BBB consists of Winkler's site pattern $AAABBBB$, together with $AABABBB$, $ABAABBB$, and $BAAABBB$. We might also note that the class ABB includes the sequence $BBAAABB$, which is currently used in Major League Baseball for seven-game championship series, and BAB covers the pattern $BBAABAB$, which often arises in other sports. We denote such an equivalence class of home-site sequences by $\overline{H}_s$, and denote it by its final $(s-1)/2$ entries. In the future we use $\overline{H}_s$ to index the game bias polynomials and other functions in a simpler way.

There are therefore only eight cases to consider when $s = 7$. In each of them, team B has the advantage in the series, with B having four home games. In fact, the probability that B wins the series is independent of the home-site sequence, and is given by (1). For the game bias in A's direction, similar analyses produce the following expressions:

$$\begin{aligned}
\beta(AAA;p) &= 4(1-2p)\left(1-3p+9p^2-17p^3+21p^4-15p^5+5p^6\right)\\
\beta(AAB;p) &= (1-2p)\left(1-3p+17p^2-48p^3+74p^4-60p^5+20p^6\right)\\
\beta(ABA;p) &= (1-2p)\left(2-5p+21p^2-52p^3+76p^4-60p^5+20p^6\right)\\
\beta(ABB;p) &= (1-2p)\left(1-2p+2p^2\right)\left(1-6p+16p^2-20p^3+10p^4\right)\\
\beta(BAA;p) &= (1-2p)\left(3-9p+33p^2-68p^3+84p^4-60p^5+20p^6\right)\\
\beta(BAB;p) &= (1-2p)\left(1-2p+2p^2\right)\left(1-2p+12p^2-20p^3+10p^4\right)\\
\beta(BBA;p) &= 2(1-2p)\left(1-3p+11p^2-26p^3+38p^4-30p^5+10p^6\right)
\end{aligned}$$

Some straightforward analysis reveals that the equivalence class of Winkler's home-site sequence $\overline{H}_7 = BBB$ is the only one among all of the seven-game series patterns that exhibits the home-field paradox. Does the home-field paradox arise for series of other lengths? We investigate this question next.



(a) $\beta(ABBB;p)$ (b) $\beta(BABB;p)$ (c) $\beta(BBBB;p)$

Figure 2: Game bias $\beta(\overline{H}_9;p)$ for nine-game home-site sequence classes $\overline{H}_9$ that exhibit the home-field paradox.

4 Series with length 3, 5, or 9

Major League Baseball's division-league playoff series are best-of-five affairs, and the postseason wild card playoffs are best-of-three (though the latter are all played at one site). We can compute the game bias polynomial $\beta(\overline{H}_s;p)$ for all home-site schedules of lengths $s=3$ and $s=5$ in the same way as our analysis for the $s=7$ cases, and determine if any home-site patterns produce the paradox. The polynomials for the $2^{(s-1)/2}$ different cases of length s are shown below for these cases. Each of these polynomials has a single root in $[0,1]$ at $p=1/2$, so the home-field paradox first arises at series length $s=7$.

$$\begin{aligned}
\beta(A;p) &= 2(1-2p)(1-p+p^2)\\
\beta(B;p) &= (1-2p)(1-2p+2p^2)\\
\beta(AA;p) &= 3(1-2p)(1-2p+4p^2-4p^3+2p^4)\\
\beta(AB;p) &= (1-2p)(1-3p+9p^2-12p^3+6p^4)\\
\beta(BA;p) &= 2(1-2p)(1-2p+5p^2-6p^3+3p^4)\\
\beta(BB;p) &= (1-2p)(1-2p+2p^2)(1-3p+3p^2)
\end{aligned}$$

What about longer series? There are sixteen different equivalence classes $\overline{H}_9$, and it turns out that exactly three exhibit the home-field paradox. One of these is the analogue of Winkler's series at $s=7$ ($\overline{H}_9=BBBB$), but the paradox arises in two additional cases as well: $ABBB$ and $BABB$. Their plots over $[0,1]$ are displayed in Figure 2. The maximal bias toward A over $1/2\leqslant p\leqslant 1$ for $\overline{H}_9=ABBB$ in part (a) is 0.1112 games per series, achieved at $p=0.7317$, for $\overline{H}_9=BABB$ in part (b) it is 0.002826 occurring at $p=0.5898$, and for $BBBB$ in part (c) it is 0.1683, at $p=0.7504$. This last example shows a bias for A that is 3.8 times larger than the maximal game bias for A exhibited by the case $\overline{H}_7=BBB$.

One might wonder if longer series might demonstrate even greater game bias toward A. For example, does there exist a series length s, home-site sequence class $\overline{H}_s$, and value $p_1\in(1/2,1)$ where the game bias toward A exceeds the bias toward B when $p=1$? That is, if we suppose that the sequence ends with B (so that $\beta(\overline{H}_s,1)=-1$), can we find qualifying $\overline{H}_s$ and p_1 so that $\beta(\overline{H}_s,p_1)>1$? It may seem counter-intuitive to think that such a situation might exist. Could it be that A, on average, wins more than one full game more than B who, on average, wins the series?

To answer these questions, and enable further investigation of the home-field paradox, we need to automate our method for computing the game bias polynomials and counting their roots in $[0, 1]$. We develop this in the next section.

5 Automation

We need to develop two procedures to automate our study of the home-field paradox: computing the game bias polynomial for a given home-site schedule, and determining the number of roots of a polynomial in a given real interval. We discuss each of these in turn here.

5.1 Game bias polynomials

We can develop an algorithm for computing the game bias polynomial $\beta(\overline{H}_s; p)$, given a series length s and an equivalence class for the home-site sequence $\overline{H}_s$. We assume s is odd, and that the sequence has $(s-1)/2$ games at A and $(s+1)/2$ games at B. There are $2^{(s-1)/2}$ different equivalence classes $\overline{H}_s$, and we can represent a particular class as a nonnegative integer $h_s < 2^{(s-1)/2}$: the binary representation of h_s (padding on the left with zeros so that the length is $(s-1)/2$) determines the sites of the last $(s-1)/2$ games, using $A \leftrightarrow 0$ and $B \leftrightarrow 1$. We call h_s the *index* associated with $\overline{H}_s$. Given a value for h_s, we can count the number of 1's in its binary representation, and then from this and the value of s we can determine the number of games at A and at B in the first $(s+1)/2$ games. Call these two values n_A and n_B.

Suppose a team sweeps the series, that is, they win the first $(s+1)/2$ games. The probability of this happening is $p^{n_A}(1-p)^{n_B}$ if the winner is A, and $p^{n_B}(1-p)^{n_A}$ if the winner is B. In the first case, A earns a game advantage of $(s+1)/2$, in the latter case, the award is $-(s+1)/2$. Suppose instead a team clinches the series in game number g, with $(s+3)/2 \leqslant g \leqslant s$, that is, a team reaches $(s+1)/2$ wins in game g. Let's update the values of n_A and n_B to count the number of home games played at A and at B respectively over the first $g-1$ games. We know the series winner won game g, so suppose that among the first $g-1$ games they won w_A games at A, and w_B games at B. There are $\binom{n_A}{w_A}$ ways to select w_A games from the n_A contests held at A over the first $g-1$ games, and similarly there are $\binom{n_B}{w_B}$ ways to select the games won at B. If game g was played at A's home field, then the probability that A wins in game g is

$$\binom{n_A}{w_A}\binom{n_B}{w_B} p^{w_A+1+(n_B-w_B)}(1-p)^{w_B+(n_A-w_A)}$$

That is, we count a factor of p for each win at A and each loss at B, and a factor of $1-p$ for each win at B or loss at A. The award to A in this case is the game advantage for A over B: in this case, A won $(s+1)/2$ games and B won $g-(s+1)/2$, so the game advantage for A here is the difference, $s+1-g$. The other cases, so A clinching the series at B, or B winning the series, either at A or at B, are similar.

Our algorithm needs to loop over the possible values of g, and for each of these values, it needs to consider all possible values of w_A and w_B that are consistent with the context. That is, we require that $0 \leqslant w_A \leqslant n_A$, $0 \leqslant w_B \leqslant n_B$, and $w_A + w_B = (s-1)/2$.

Each term that we compute has the form $cp^a(1-p)^b$, where c is an integer and $a+b = g$. We store this in a compact manner by recording the coefficient c in a table indexed by the game number g, and the exponent a on p. We have thus developed the following algorithm.

Algorithm 1: Compute the game bias polynomial for a home-site schedule.

Input : an odd positive integer s, and a nonnegative integer $h_s < 2^{(s-1)/2}$.
Output: A finite list (c_i, a_i, b_i) so that $\beta(\overline{H}_s; p) = \sum_i c_i p^{a_i}(1-p)^{b_i}$, where $\overline{H}_s$ is the home-site sequence class corresponding to the binary representation of length $(s-1)/2$ of h_s.

1 **foreach** (g, j) *with* $\frac{s+1}{2} \leqslant g \leqslant s$ *and* $0 \leqslant j \leqslant g$ **do**
2 $c[g, j] \leftarrow 0$
3 $n_A \leftarrow$ # games at A in first $\frac{s+1}{2}$ games of a series determined by h_s
4 $n_B \leftarrow \frac{s+1}{2} - n_A$
5 $c[\frac{s+1}{2}, n_A] \leftarrow \frac{s+1}{2}$; $c[\frac{s+1}{2}, n_B] \leftarrow -\frac{s+1}{2}$
6 **for** $g \leftarrow \frac{s+3}{2}$ **to** s **do**
7 **if** *game* g *is at* A **then**
8 $g_A \leftarrow 1$; $g_B \leftarrow 0$
9 **else**
10 $g_A \leftarrow 0$; $g_B \leftarrow 1$
11 **for** $w_A \leftarrow \max(0, \frac{s-1}{2} - n_B)$ **to** $\min(\frac{s-1}{2}, n_A)$ **do**
12 $w_B \leftarrow \frac{s-1}{2} - w_A$
13 Add $(s+1-g)\binom{n_A}{w_A}\binom{n_B}{w_B}$ to $c[g, w_A + g_A + n_B - w_B]$
14 Subtract $(s+1-g)\binom{n_A}{w_A}\binom{n_B}{w_B}$ from $c[g, w_B + g_B + n_A - w_A]$
15 $n_A \leftarrow n_A + g_A$; $n_B \leftarrow n_B + g_B$
16 **foreach** (g, j) *with* $\frac{s+1}{2} \leqslant g \leqslant s$ *and* $0 \leqslant j \leqslant g$ **do**
17 **if** $c[g, j] \neq 0$ **then**
18 print $(c[g, j], j, g-j)$

We implemented Algorithm 1 in C++, using the Boost library [Sch14] for big-integer arithmetic when computing binomial coefficients and values for the table c. Our code uses a 64-bit integer for index h_s (allowing one to obtain the 1-bit count in h_s needed for line 3 in a single popcount instruction), so we can handle series lengths $s \leqslant 129$. The implementation is quite speedy, for example at $s = 51$ we compute more than 5500 game bias polynomials per second on a single core running at 3.6 GHz.

5.2 Counting real roots

We also require a method for determining the number of real roots in $[0, 1]$ for a polynomial with integer coefficients. For this we employ Sturm's method, which uses exact arithmetic to find the number of real roots of an integer polynomial in a given real interval. This method involves running Euclid's GCD algorithm starting with the given polynomial and its derivative, and counting sign changes in the resulting sequence of polynomials evaluated at each given endpoint: for details see [Bar95, pp. 179–182]. This is implemented in SageMath [The22] as the method `number_of_roots_in_interval(0,1)` for the ring of polynomials with rational coefficients. We use SageMath to read each sequence of triples (c_i, a_i, b_i) produced by Algorithm 1, which together encode a game bias polynomial, construct the polynomial, and count its number of real roots in $[0, 1]$. When $s = 51$, this procedure processes about 2.63 polynomials per second per core on a 32-core computer running at 2.5 GHz.

We note that Sturm's method does not detect the multiplicity of roots in the given interval, so these calculations do not rule out the possibility of roots of multiplicity 2 or higher. However, for $s \leqslant 43$ we checked the root counts of all game bias polynomials using the `CountRoots` function in Mathematica, which does account for multiplicities, and we obtained identical results. (One could of course check for multiple roots by computing the gcd of the polynomial with its derivative.)

6 Complete searches

Using the tools of Section 5, we can now conduct broad experiments on the home-field advantage. For example, we can survey *all* possible home-site sequences $\overline{H}_s$ for a fixed value of s, at least as long as s is not too large, and determine how frequently the home-field paradox appears. We can seek to answer the question we raised near the end of Section 1: does there exist an $\overline{H}_s$ ending in B and a value $p_1 \in (1/2, 1)$ for which $\beta(\overline{H}_s; p_1) > 1$?

It does not take long to find a positive answer. The smallest s where this occurs is $s = 21$, where exactly six classes $\overline{H}_{21}$ have this property:[3]

$$\begin{gathered} ABABBBBBBB,\ BBABBBBBBB,\ AABBBBBBBB, \\ BABBBBBBBB,\ ABBBBBBBBB,\ BBBBBBBBBB \end{gathered} \tag{3}$$

More succinctly, these are the classes whose indices are 383, 895, 255, 767, 511, and 1023 respectively, when we convert to a binary value using $A \leftrightarrow 0$ and $B \leftrightarrow 1$. Using this shorthand, the game bias polynomial for the last example in (3) satisfies $\beta(1023; 0.8805) = 1.1670$. So here is a case where A's game bias exceeds one full game!

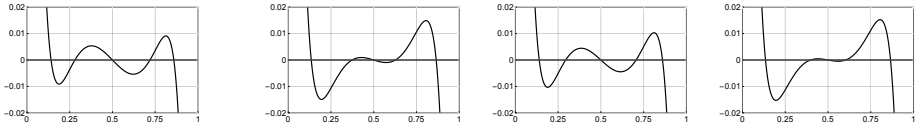
Our survey however turns up another interesting, and indeed unexpected, twist in the home-field paradox: there exist home-site sequences for which the corresponding game bias changes sign more than once over the range $1/2 < p < 1$! These interesting hits first occur

[3]Perhaps we can tip our hat to Pirates legend #21 Roberto Clemente for numerical inspiration.

for series of length 19:[4] there are exactly four equivalence classes of home site sequences for which $\beta(\overline{H}_{19}; p)$ has exactly five roots in $[0, 1]$. Here the game bias changes sign *twice* over $1/2 < p < 1$. These four classes are

$$AAAAABABB,\ ABAAABABB,\ BAAAABABB,\ BBAAABABB$$

or, more succinctly, indices 11, 139, 267, and 395. Plots of their game bias polynomials are displayed in Figure 3.



(a) $\beta(11; p)$ (b) $\beta(139; p)$ (c) $\beta(267; p)$ (d) $\beta(395; p)$

Figure 3: Game bias polynomials at $s = 19$ with five crossings in $[0, 1]$.

This presents a natural question: do there exist home site sequences whose game bias polynomials possess even more sign changes over the range $0 \leqslant p \leqslant 1$? Can we find examples with *seven* crossings? If so, what is the smallest series length where this occurs?

We can use our algorithms from Section 5 to investigate this, and extend our reach by splitting our generation and analysis of polynomials across multiple computational cores. We expanded to 64 cores, each running at 3.6 GHz, beginning at $s = 33$, where checking all 2^{16} polynomials required just two minutes of elapsed time. That case uncovered 584 polynomials with five crossings in $[0, 1]$, but none with seven or more. We can continue the search from here, but we face an exponential climb in run times, given a fixed computational resource. No polynomials with seven crossings arise for several additional steps, but finally at $s = 45$ we find exactly 132 classes where the game bias polynomial has seven crossings in $[0, 1]$. That calculation required just under four hours. So a big save[5] by number 45! One of these 132 examples, where the corresponding index for the sequence is 4193611, is displayed in Figure 4. We completed these exhaustive searches through $s = 51$.

7 Targeted searches

Can we find even stranger instances of the home-field paradox if we allow longer series? The number of polynomials to compute and analyze doubles each time we move to the next series length, and the polynomials themselves become more complicated as we raise s, so our ability to complete exhaustive searches is certainly limited. Perhaps now is the time to plan for some more targeted searches.

We can hope to learn something by studying the first examples we found with a particular number of crossings. From (3), we see that all of the series at $s = 19$ where the game

[4] Quite appropriate indeed! #19 Tony "Mr. Padre" Gwynn had plenty of interesting hits over his twenty-year MLB career!

[5] Yankees enthusiasts might credit the save to #45 Gerrit Cole; Red Sox devotees may of course substitute Hall of Famer #45 Pedro Martinez.

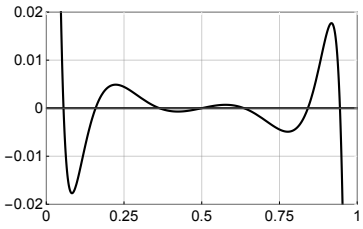


Figure 4: The game bias polynomial for $\overline{H}_{45} = BBBBBBBBBBBABABAABABB$ (index 4193611), with seven crossings. Its roots in $(1/2, 1)$ occur near $p = 0.6361$, 0.8415, and 0.9446.

bias has five roots in $[0, 1]$ end with the pattern $AAABABB$, so the index of each one is 11 mod 2^7. This pattern persists in the six sequences at $s = 21$ whose bias polynomials have five roots in $[0, 1]$: the indices here are 139, 267, 395, 651, 779, 907. All are 11 mod 2^7.

The home-site schedules found at $s = 45$ with $\rho = 7$ show an even stronger pattern: all 132 of them end with $BABAABABB$, so all of the corresponding indices are 331 mod 2^9. The same suffix is universal for all 508 of the sequences with $\rho = 7$ at $s = 47$, and all 1280 of them at $s = 49$. We might therefore consider fixing the tail of the home-site schedule to a particular pattern in searches of larger series lengths for even stranger game bias polynomials.

We can also look at the prefix of these home-site schedules. At $s = 45$, we find that all 64 sequences over $\{A, B\}$ of length 6 occur when we look at the first six positions of the 132 sequences with $\rho = 7$. At $s = 47$, all 256 possible sequences of length 8 occur in the first eight positions among the 508 with this feature, and all 1024 possibilities occur in the first ten positions among the 1280 with $\rho = 7$ at $s = 49$. The first few positions appear to have little effect on the value of ρ, so perhaps we should simply choose a fixed prefix.

Might a particular prefix be an attractive choice? Using our 132 game bias polynomials at $s = 45$, we can compute the value of the first local maximum of each one, and determine which sequence produces the largest value here. This is the sequence $\overline{H}_{45} = BBBBBBBBBBBABABAABABB$, or index 4193611. Its game bias polynomial is displayed in Figure 4. At $s = 47$, the best sequence is $\overline{H}_{47} = BBBBBBBBBBBBABABAABABB$, or index 8387915, and at $s = 49$, it is $\overline{H}_{49} = BBBBBBBBBBBBBABABAABABB$, index 16776523. This therefore suggests employing a fixed prefix consisting of a string of games at B.

We pursued a targeted search inspired by these observations, fixing both a prefix and a suffix for the home-site schedules we consider. In a search at length s, we use a string of games at B of length $\lfloor s/6 \rfloor$ for the prefix, and we set up to six positions for the suffix, using $AABABB$ as our model. This vastly reduces the number of polynomials we need to consider in our targeted search.

We used Algorithm 1 to produce the game bias polynomials in this search, and employed SageMath as before to determine the number of real roots in $[0, 1]$ for each one. We ran this

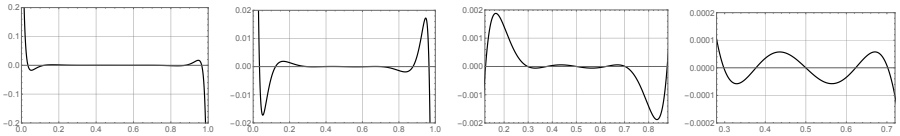


Figure 5: The game bias polynomial $\beta(\overline{H}_{73}; p)$ for the sequence class with index 68703253323, at four levels of magnification. Each successive plot zooms in by a factor of 10. Its roots in $(1/2, 1)$ lie near $p = 0.6229$, 0.7021, 0.8771, and 0.9652.

at successive odd values for s, with $s = 53$ as our starting value,[6] searching for a sequence where the game bias polynomial has $\rho = 9$ roots in $[0, 1]$. We started with a suffix of size four ($BABB$), and raised that to five ($ABABB$) and then six ($AABABB$) as s grew larger. Translating to binary, each of these is decimal 11, in a field of the given size. Each search produced many polynomials with $\rho = 5$ or $\rho = 7$, and but no examples with nine crossings appeared until $s = 73$, where there was exactly one in our targeted search space, to our great relief.[7] This sequence has index 68703253323, so its home-site sequence class is

$$BBBBBBBBBBBBAAAABAAAABBBAABBABAABABB$$

Its plot is displayed in Figure 5, at four successive levels of magnification. The scale of the vertical axis stretches by a factor of 10 in each successive graph here, in order to see all nine roots in $[0, 1]$.

Can we use this method to find still more complicated instances of the home-field paradox? In the spirit of *Spinal Tap*'s Nigel Tufnel, can we make these go to eleven? For this, let's first collect some more examples of home-site schedules with nine crossings. We can do this by mining the vein we just found, searching at $s = 75$, 77, and 79. Here we narrow our focus by fixing nine suffix bits, and looking only at indices that are 331 mod 2^9. This congruence class was already indicated by our examples with seven crossings, and fits our lone example at $s = 73$. These three searches produce 4, 9, and 11 sequences respectively that exhibit nine crossings. All of these are exactly the same not just in their last nine positions, but moreover in their last 19 sites. They all end with

$$AAAABBBAABBABAABABB \tag{4}$$

that is, their respective indices are all congruent to 29515 mod 2^{19}. We can use this to refine the search for our next foray. We start at $s = 85$, and jump by six after each run, so that we can home in on promising sequences more quickly. We continue using a fixed prefix of size $\lfloor s/6 \rfloor$ consisting of a string of games at B throughout this search. For the suffix, we use the last 14 positions of (4) at $s = 85$, and increment this by 1 after each jump by six, until reaching length 18 positions. This is enough to outfit searches up to $s = 109$, with 2^{18} polynomials to consider at that level. Each polynomial requires about 2.3 seconds to analyze at that stage, so that amounts to just over five hours of elapsed time, running 32 threads in parallel. But all the searches through $s = 109$ came up empty!

[6] Much like the Dodgers, who for years enjoyed having #53 Don Drysdale as a starting pitcher.

[7] Perhaps we can thank career left-handed reliever #73 Ricardo Rincón, from the *Moneyball*-era Athletics.

We ran $s = 115$ using the same suffix length 18, and we needed 28 hours to check all 2^{20} polynomials in our targeted set. This finds more than 185000 sequences with nine crossings, but none with eleven. Our computations are becoming unwieldy at this point, even in the targeted search, but we can trim the space a little more. We have still not taken advantage of the full nineteen elements of (4) for our suffix, so let's employ this full complement at $s = 121$. Also, our prefix at this point is a string of 20 B's, but we can perform an initial search with a prefix of length 21, using 20 B's followed by a single A. (If that fails, we could always try a prefix of 21 B's.) This would keep our first effort at $s = 121$ to 2^{20} polynomials, which ought to be manageable. We could check this sample in about 29 hours, again running in parallel on 32 cores.

Success! We find 35 of the game bias polynomials constructed at $s = 11^2$ have 11 roots in $[0, 1]$! Indeed these do go to eleven! All 35 of these end with the same sequence of 29 games, ten more than we fixed in the search:

$$BBBBBABBABAAAABBBAABBABAABABB \tag{5}$$

We can use some of this to refine searches at the values we had skipped over, $s = 117$ and $s = 119$. We use the last 24 positions of (5) for searches at these values, and employ the same strategy for the prefix, with $\lfloor s/6 \rfloor = 19$ consecutive B's. This makes these searches much more tractable, with 2^{15} polynomials to check at $s = 117$, which required about 45 minutes, and 2^{16} at $s = 119$, which needed about 100 minutes. Both of these searches succeeded: we found 47 examples with eleven crossings at $s = 119$, and three at $s = 117$. Length 117 is the smallest value where we detected the home-field paradox with eleven roots in the expected game bias polynomial over $0 \leqslant p \leqslant 1$.[8]

The three winning indices at $s = 117$ are

$$288230057777328971,\ 288230195216282443,\ 288230329970881355 \tag{6}$$

and the home-site schedule $\overline{H}_{117}$ corresponding to the first of these is

$$BBBBBBBBBBBBBBBBBABBABABBBABBBBBABBABAAAABBBAABBABAABABB \tag{7}$$

Among the three examples (6), the game bias polynomial for (7) has the largest peak value at its first extremum over $p > 1/2$, so we chose this one to display. Its plot is shown in Figure 6, at six successive levels of magnification. We see all eleven roots if we zoom in far enough!

8 Simulations

Our game bias polynomials tell us the predicted margins for game differentials for any series pattern we choose. Can we check some of these with empirical data? It is probably

[8] As much as the authors were hoping we would see an example in time for (respectively) a Braves #99 Spencer Strider or Yankees #99 Aaron Judge reference, our value here is well past the maximum allowable uniform number. It is also significantly higher than the fastest pitch (in miles per hour) ever recorded, notwithstanding the singular feat of Sidd Finch [Pli]. We will therefore be content with remarking that no Major League Baseball team has ever won 117 or more games in a regular season, and this is the smallest integer with this property. The 1906 Cubs and the 2001 Mariners both came within one!

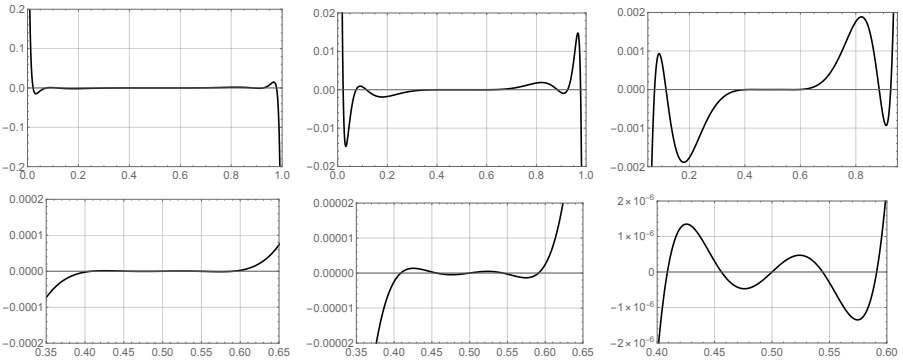


Figure 6: The game bias polynomial $\beta(\overline{H}_{117}; p)$ for the sequence class with index 288230057777328971, at six levels of magnification. Each successive plot zooms in by a factor of 10. The five roots with $p > 1/2$ are approximately 0.54404, 0.59109, 0.88430, 0.92570, and 0.97785.

too much to ask the hard-working players of the AlleyCats and Bumblebees (or the stalwart fans of Albany and Binghamton) to schedule even one 117-game series. But we can certainly run simulations to see if our margins are on the money.

It is quite simple to implement a program that takes as input an odd integer s, a home-site sequence class $\overline{H}_s$, a probability p, and a number of iterations M, and run a simulation of M series with home-site schedule in $\overline{H}_s$, where the home team wins with probability p in each game. The program can track how many times each team wins a series, as well as the game differential in each series when one team reaches $(s+1)/2$ wins. We can then report empirical values for the probability of winning a series, and the average game bias per series for team A.

For example, we can run $M = 10^7$ iterations of Winkler's pattern with $s = 7$: $\overline{H}_7 = BBB$. With $p = 0.54$, we found Binghamton won 5126110 series to Albany's 4873890, but Albany won 29152656 games in all, versus Binghamton racking up just 29005657, so the bias for Albany is 146999 games over 10^7 series. This matches the expected values from Section 3 reasonably well: our predictions were that Binghamton would win the series with probability 0.5125, and that Albany's game bias per series would be 0.01477 games per series.

We can try this with our series of size 117 given by (7), but we will need to ramp up the iteration count substantially. We pick five values for p to test in turn, corresponding to the local extrema of the game bias polynomial $\beta(\overline{H}_{117}; p)$, whose coefficients are given in Section A, over $[1/2, 1]$. The approximate values here are $p = 0.5240$, 0.5745, 0.8204, 0.9094, and 0.9671, and the corresponding predicted game bias values per series are in turn $\beta(\overline{H}_s; p) = 4.711 \cdot 10^{-7}$, $-1.349 \cdot 10^{-6}$, 0.001885, -0.0009304, and 0.01475. Let's put the Law of Large Numbers to work: can we run enough iterations to see these trends emerge? Using the methods described earlier, along with the home-site sequence (7), we computed the expected game bias $\beta(\overline{H}_{117}; p)$, the expected series length $L(\overline{H}_{117}; p)$, and the expected probability for B to win a series $S_B(\overline{H}_{117}; p)$, for each of these five values of p. Then we

can compare these results to the values that we computed empirically over M trials. We label these quantities $\beta^*(\overline{H}_{117}; p, M)$, $L^*(\overline{H}_{117}; p, M)$, and $\beta^*(\overline{H}_{117}; p, M)$ respectively. For the three largest peaks, we use $M = 5 \cdot 2^{39} \approx 2.75 \cdot 10^{12}$. The first two peaks are much smaller, so we increase the iteration count to $M = 2^{44} \approx 1.76 \cdot 10^{13}$ for these cases. Our simulations tallied results for about $4.8 \cdot 10^{15}$ games overall (which would indeed be rather taxing to measure for actual ball games).

Table 1 summarizes our results. We see extremely good agreement for the game bias values in the three largest peaks, fairly close agreement at the second peak, and not particularly close agreement for the first peak, where the empirical value is an order of magnitude larger than the expected one. However, this predicted edge is extremely small here: we clearly need many more iterations to see better convergence at this value. We do at least see a qualitative change in the sign of the bias here, as we expected. The other values we measured—length of series and probability of a series win for B—are in excellent agreement with their theoretical values, over all five probability values p.

	Peak 1	Peak 2	Peak 3	Peak 4	Peak 5
p	0.524011	0.574458	0.820426	0.909374	0.967093
M	2^{44}	2^{44}	$5 \cdot 2^{39}$	$5 \cdot 2^{39}$	$5 \cdot 2^{39}$
$\beta(\overline{H}_{117}; p)$	$4.71114 \cdot 10^{-7}$	$-1.34875 \cdot 10^{-6}$	.00188510	$-.000930389$	.0147532
$\beta^*(\overline{H}_{117}; p, M)$	$4.94017 \cdot 10^{-6}$	$-3.68381 \cdot 10^{-6}$	.00188891	$-.000937678$	.0147520
$L(\overline{H}_{117}; p)$	109.3599106	109.4363334	111.1558019	112.7030648	114.4853687
$L^*(\overline{H}_{117}; p, M)$	109.3599111	109.4363353	111.1558034	112.7030679	114.4853686
$S_B(\overline{H}_{117}; p)$	0.5017770	0.5055667	0.5309503	0.5531934	0.6002402
$S_B^*(\overline{H}_{117}; p, M)$	0.5017772	0.5055667	0.5309499	0.5531935	0.6002401

Table 1: Expected values versus empirical results over M trials using $\overline{H}_{117}$ given by (7): game bias for A per series β, series length L, and probability of B winning the series S_B. The empirical results are denoted by β^*, L^*, and S_B^*.

9 Further explorations

A number of questions arise for potential further investigations. We'll send out our closer with a few pitches for future batters to take a swing at.[9]

- Our complete searches show that the proportion of home-site equivalence classes that do not exhibit the home-field paradox, that is, where the game bias per series favors B for all $p \in (1/2, 1]$, gradually diminishes as s increases. Our exhaustive checks show this proportion decreasing through $s = 51$, where it is approximately 0.70306. Figure 7 shows this proportion for $11 \leqslant s \leqslant 51$. Does this proportion have a limiting value as $s \to \infty$? If so, is the limit positive, or do almost all home-site sequences

[9]Given our last question, we have to give the nod to Astros, Braves, Mets, Phillies, and Red Sox Hall of Famer #13 Billy Wagner.

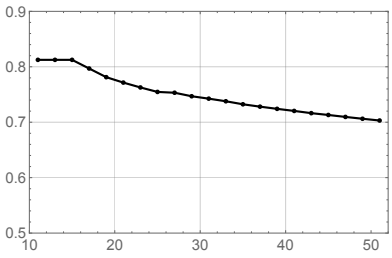


Figure 7: Proportion of home-site sequence classes $\overline{H}_s$ which do not exhibit the home-field paradox, for odd s with $11 \leqslant s \leqslant 51$.

exhibit the home-field paradox? Do at least a positive proportion of the home-site sequence classes as $s \to \infty$ possess no home-field paradox?

- Is the number of roots ρ in $[0, 1]$ of the game bias polynomial $\beta(\overline{H}_s; p)$ unbounded as $s \to \infty$? Can you determine a lower bound on s for a sequence with a given number of real roots ρ?

- Is $s = 73$ the smallest series length where there is a home-site sequence having $\rho = 9$ crossings in $[0, 1]$ in its game bias polynomial? How about $s = 117$ for $\rho = 11$?

- Can you find an odd integer s and a sequence $\overline{H}_s$ where the game bias polynomial has at least 13 roots in $[0, 1]$?

Readers wishing to study these and other questions on this topic may be interested in the polynomials found here, or in detailed results of the searches. These are available in the appendices.

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A Polynomials

A number of game bias polynomials are described and plotted in the body of the article, but are often not given explicitly there. We give full details for each of these polynomials here. In each case, we indicate the corresponding figure in the body of the text, the series length (and polynomial degree) s, the number of roots ρ of the polynomial in $[0, 1]$, the corresponding index h_s, and the full sequence of coefficients, beginning with the constant term.

Figure 2

$s = 9$, $\rho = 3$.

- $h_9 = 7$: 1, −15, 89, −321, 795, −1375, 1645, −1310, 630, −140.
- $h_9 = 11$: 1, −13, 80, −305, 780, −1369, 1645, −1310, 630, −140.
- $h_9 = 15$: 1, −16, 94, −331, 805, −1379, 1645, −1310, 630, −140.

Figure 3

$s = 19$, $\rho = 5$.

- $h_{19} = 11$:

  1, −28, 385, −3555, 24502, −131318, 559059, −1917992, 5356963, −12261869, 23077230, −35706196, 45244485, −46564540, 38379770, −24772748, 12082070, −4192760, 923780, −97240.

- $h_{19} = 139$:

  1, −28, 385, −3560, 24546, −131497, 559500, −1918706, 5357734, −12262407, 23077450, −35706236, 45244485, −46564540, 38379770, −24772748, 12082070, −4192760, 923780, −97240.

- $h_{19} = 267$:

  1, −28, 385, −3556, 24510, −131345, 559108, −1918042, 5356990, −12261875, 23077230, −35706196, 45244485, −46564540, 38379770, −24772748, 12082070, −4192760, 923780, −97240.

- $h_{19} = 395$:

  1, −28, 385, −3560, 24545, −131491, 559486, −1918690, 5357725, −12262405, 23077450, −35706236, 45244485, −46564540, 38379770, −24772748, 12082070, −4192760, 923780, −97240.

Figure 4

$s = 45$, $\rho = 7$, $h_{45} = 4193611$:

1, −67, 2231, −51215, 908487, −13097363, 157929009, −1624036154, 14448080422, −112437588674, 772201825006, −4714057608896, 25733500169292, −126246597576220, 558976531155300, −2241667410763582, 8166917852021796, −27098553957489296, 82060840107340412, −227172841927167280, 575675488037166368, −1336663105479933424, 2845573595846061584, −5556032680625032666, 9949797540040958150, −16337583138180366022, 24581748355784235090, −33858380249755415410, 42634709392062938790, −48993856460211024690, 51267169299909283230, −48715477423590710940, 41895731535907479030, −32477154875561257290, 22580823150849009090, −13996845799333528290, 7677574577824905210, −3692448817508615370, 1539021395625045150, −547639506346397340, 163084282283606580, −39541012436985540, 7497956657216220, −1043094828130680, 94684453367400, −4208197927440.

Figure 5

$s = 73$, $\rho = 9$, $h_{73} = 68703253323$.

1, −109, 5920, −222225, 6491652, −155888760, 3172076105, −55767151467, 858973142848, −11718621819230, 142876238635698, −1568650602150519, 15610271005436721, −141602324457691194, 1176645903063678813, −8995085227745063722, 6350166044975954743 5, −4153570870585511432 46, 252452260080981233 5209, −1429465575241954588 2932, 7557737578342020477 1164, −3738593967807078151010 88, 1733403864270356542505120, −7544950397560099006632115, 3087392595132122056558 9280, −1189190829688922232861651 97, 4316377874746491744825446 08, −1477828137419730994873985435, 4776859830386537239949862050, −1458833928219576912290207878 4, 4212124387605758570995328 4315, −1150468934437276790586274305 15, 2973957926149982188094917892 04, −7278669835098859824192023508 01, 1687182901718835617033026828500, −3704815389352456144058966320085, 7707843925845689005017665401335, −1519482226961065813536325447943 9, 2838285190485635409774204824 6931, −5023219648761891836596246671 2396, 8421866155883549083234702154 0187, −1337316918145127052523428415305 54, 2010594272606212037359823255756 81, −2860921653128527714916512577010 84, 3850946989828306912138072163608 29, −4900686814626174625168489534678 37, 5892213975010297815221672641917 78, −6687902662871632011104090498428 77, 7159762810345139122743847279398 68, −7221994976570396175949014494524 60, 6855799422274382875434824480852 42, −6116845859657059331170192241227 33, 5121742198684535184067071815320 55, −4017889026847141728347606727126 50, 2947431325835915928613068663661 109, −2017546081275089317017563420882 75, 1285531009278450702451024589217 20, −7603669111547090924275845744113 4, 4161785836335187223745769384577 1, −2100328168537000467909158456704 5, 9732876267377091969607087412050, −4121413416282038924451374344185, 1585791916324762564704225316769, −5507217923228307091816121906 06, 1712433783715094696817316160 86, −4721019038825622231106642142 2, 1140011643022053844403023790 4, −2374092550528650408992974166, 4177777471392448939307677 84, −6041070246546689640605901 4, 6892354071081871254491902, −5817918041862237018078 28, 3230341544020908488189 2, −88502508055367355840 8.

Figure 6

$s = 117$, $\rho = 11$, $h_{117} = 288230057777328971$:

1, −175, 15281, −922859, 43514286, −1696097727, 56419038379, −1634236880853, 41805854538422, −954619944053673, 19626641483980714, −365976631585238765, 6228782503314202761, −97300901153217419784, 1401981633612542107251, −1871512359433882563 3972, 2323681584257971496374 18, −2692919371642303417276701, 2922163503822872169461 4079, −2977508233458415951385585 36, 2856139542631833881548115299, −2585156704553514788221636047 1, 2212500066666788711925054000 718, −1793876806305963097037836468732, 1380270902886430246650617121 1298, −1009443063981200112322615373517 89, 7026994613800501784479896278085 10, −4662294200778547132475637698292010, 2951867744387490973212884078700 1055, −1785434473803467770596629123956077 16, 1032726410527025931435198646337230060, −5717828385754664333590918498547496479, 3032915480407344978581918214370270 9651, −1542493484681279245203066643876525359 25, 7527395674632487878706602760768024223 85, −3527161106677350004612728438036824228637, 1587975911825981708511657315843528454 0265, −6873205703900679753257901377385275922 8205, 2861618169194506890127549831374784760619 93, −1146631034982588897639148844965565512834889, 4423865894915643529421233888627368662043581, −1644136731270013446477923659394461717636670 1, 5888557135175745200824898900598250178391 8845, −2033193555185639257575984764096217696 78390917, 6770155816327216613195921170636309098232 54861, −2174743047686885547238927934165819287728416033, 6741156333993511971881626182302130218786956231, −2016948949472268492424736354268782475118161 5415, 5826341096634233885041554645890167371120859 0225, −1625300866629166693046250497035889663533328467939, 4379203597533903276176821114605752692940056678 57, −1139875059404618487428616523018894243721883905591, 2866739268536034806603222681150635906237814559177, −6967021152653862440008160390978499468853465957163, 1636381184825899411345187685991923907185763710 8645, −3714856877921679821541475471103734618884875602 6495, 8151796859827962006035928190889818068636166984 6997, −1729192315759042222688252978316943141212483823982 51, 3545917846757992508888912056912582367710549968553 05, −7029394972595390165336311797808842404348629044494 15, 1347135740120093652627942306602113388787480778021105, −2495748773236186257416958356694947694408815070200635, 4469614990612207807346768985434241144904404984647 53, −7737387654874921389178807457848403297426878995394150, 1294614137016632795191900643459249546606174672819 6702, −2093467414693858170065667365827900290739055650097 6506, 3271306708071836262488112699084161888120680539270 4122, −4939077663636917196931492218475669792708221782625797 8, 7203956588518201887110272708212387678426634507460344, −1014889072385818302296491113002706350695363349453018 26, 1380702596942660515614529026509913765880859194690436 94, −1813499569517724247902162822471952729213675518838467 26, 2299120101932370697862412005036635419633334217868772 40, −2812631957144559153986147245479309827336773683230817430, 3319249441336559713474373425159188315022369208878164 04, −3777454806745206488597114906017352712701240350804786 14, 4144146447219210378521826658677377026418200912522048 26, −4381048335023200074832552080641674617379711245330031 64, 4461128558556213501700218232727395562864285366865714 26,

$-437357762617233441792690804857339770864338897635957392$, $412610339753534863772578332091160086425280192508994108$,
$-374389218989774863980855160375624023113951139168757822$, $326540573686111323386567417199534781831888048007462046$,
$-273597856059883581491644375915484676654622201227667944$, $220069613257560063008026276224644835681950764514078958$,
$-169811368735661983687593230998052047906400133397434810$, $125601718002300800187609289252246809437754856028522832$,
$-889779960942940533673209612801719131188538781523244 56$, $603163108603384432318329447729953214026033510285047 44$,
$-390865127303251601915732691679981803541372334022482 90$, $241879352309425572383722243335818428153748560496561 16$,
$-142774702172808595310585190963351046093240913979096 12$, $802864283397074746063734859328175037912055738257937 4$,
$-429517921024055549057548136959124549769863037147915 6$, $218285524052333190418542770181164623830232977973399 2$,
$-105212897276288729934092417748569034415087183616983 2$, $480110424327737354309117756577669458866990220548092$,
$-207010125404814129365976364568153155416821928064908$, $84155368296825234212277819478548936788754969726372$,
$-321788974264066911635741224680272137215607874772 36$, $115425120778988311511064485198986632362994884566 56$,
$-387232127123689683341658264442209018229883250738 0$, $121093974746226161902169932943266361366561949963 6$,
$-351638743928549659947871468157548464512371191204$, $944063663173719501019040457287295152045405094 96$,
$-233160920074270403676779558983645920294704785 32$, $526650996027858792413823794024284828753137932 4$,
$-108047731104693183889462621122332102113598362 8$, $199695581113677803796411967616791203731730972$,
$-329195844971053446596478154684908597867 98972$, $478096292804426714867954146146465578139666 0$,
$-602210721021683264015099475212138339185080$, $644517141108315686622502086334672354227 12$,
$-569867439597572107796686366873483555504 8$, $399642907255141931271783270905475210360$,
$-208417491637898919520814581260308371 68$, $718522641375811264743238894759788240$,
$-122824383141164318759528016198254 40$.

B Search summaries

Two searches are described in the body of the article: an exhaustive search over all home-site schedules for series lengths $s \leqslant 51$, and targeted explorations for lengths $53 \leqslant s \leqslant 117$ of home-site schedules subject to particular constraints. Table 2 summarizes the results from the first search. This shows the number of classes of home-site schedules for series of each odd length s where the game bias polynomial has ρ roots in $[0, 1]$, for each value of ρ that occurred. Table 3 displays the results of the second search, recording the length α of the fixed prefix and that of the suffix ω, the integer representation of the suffix ι, the number of home-site schedules tested N, and the number of game bias polynomials having ρ roots in $[0, 1]$, for each value of ρ that occurred.

s	$\rho = 1$	$\rho = 3$	$\rho = 5$	$\rho = 7$
1	1			
3	2			
5	4			
7	7	1		
9	13	3		
11	26	6		
13	52	12		
15	104	24		
17	204	52		
19	400	108	4	
21	790	228	6	
23	1562	476	10	
25	3091	989	16	
27	6170	1986	36	
29	12236	4052	96	
31	24330	8194	244	
33	48352	16600	584	
35	95988	33756	1328	
37	190856	68336	2952	
39	379643	138337	6308	
41	755344	279550	13682	
43	1502278	565366	29508	
45	2990776	1142190	61206	132
47	5952078	2306626	129396	508
49	11848476	4653074	274386	1280
51	23590868	9385088	575468	3008

Table 2: Number of home-site schedule equivalence classes $\overline{H}_s$ for series of length s for which the game bias polynomial $\beta(\overline{H}_s; p)$ has ρ roots in $[0, 1]$. These are complete searches: the sum of the values in row s is $2^{(s-1)/2}$.

s	α	ω	ι	N	$\rho=1$	$\rho=3$	$\rho=5$	$\rho=7$	$\rho=9$	$\rho=11$
53	8	4	11	2^{14}	2281	12435	1644	24		
55	9	4	11	2^{14}	2337	12373	1644	30		
57	9	4	11	2^{15}	4664	24628	3410	66		
59	9	4	11	2^{16}	9546	49031	6812	147		
61	10	4	11	2^{16}	9643	48840	6892	161		
63	10	4	11	2^{17}	19415	97318	14036	303		
65	10	5	11	2^{17}	38706	71000	20808	558		
67	11	5	11	2^{17}	38606	70741	21142	583		
69	11	5	11	2^{18}	77025	140992	42881	1246		
71	11	6	11	2^{18}	153789	73579	32336	2440		
73	12	6	11	2^{18}	153432	73800	32399	2512	1	
75	12	9	331	2^{16}	2048	24089	34470	4925	4	
77	12	9	331	2^{17}	4288	48052	68516	10207	9	
79	13	9	331	2^{17}	5264	48027	67252	10518	11	
85	14	14	13131	2^{14}		7331	4982	3992	79	
91	15	15	29515	2^{15}		28838	1245	2233	452	
97	16	16	29515	2^{16}		51699	5302	6277	2258	
103	17	17	29515	2^{17}		80696	22819	17040	10517	
109	18	18	29515	2^{18}		85671	86635	42951	46887	
115	19	18	29515	2^{20}		369367	370518	123535	185156	
117	19	24	6845259	2^{15}				465	32300	3
119	19	24	6845259	2^{16}				1007	64482	47
121	21	19	29515	2^{20}			693337	176411	178793	35

Table 3: Targeted search results. Here, $\alpha = \lfloor s/6 \rfloor$ is the size of the fixed prefix $BB\cdots B$ (except at $s = 121$, where it is 20 B's followed by a single A), ω is the size of the fixed suffix, ι is the integer representation of the suffix, and N is the number of home-site schedules tested.

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