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BEHAVIOUR OF RECTANGULAR REINFORCED CONCRETE COLUMNS WITH DISPROPORTIONATE CROSS-SECTION DIMENSIONS SUBJECTED TO COMPRESSION AND BIAXIAL BENDING - CASE STUDY

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Rezumat

Scopul prezentei lucrări este de a atrage atenția asupra comportamentului stâlpilor din beton armat cu secțiune dreptunghiulară cu dimensiuni disproporționate solicitați la compresiune și încovoiere biaxială, având în vedere faptul că în procesul de proiectare curent, calculul stâlpilor este realizat în raport cu fiecare direcție principală fără a considera efectul lor combinat.

Lucrarea este bazată pe un studiu de caz privind o clădire ce este în faza de proiectare, în care, pentru a obține o comportare seismică adecvată, au fost folosiți stâlpi cu dimensiuni disproporționate ale secțiunii.

Atât dimensionarea clădirii cât și stâlpilor este realizată urmând prevederile codurilor de proiectare seismică din România ("P100-1/2013 - Cod de proiectare seismică - Partea 1 - Prevederi de proiectare pentru clădiri" și "SR-EN 1998/2004 - Proiectarea structurilor pentru rezistența la cutremur").

Comparațiile rezultatelor și concluziile sunt bazate pe curbele de interacțiune și suprafețele de interacțiune calculate cu expresii din literatura de specialitate și programe de analiză secțională.

Cuvinte cheie: Structuri de beton armat, Secțiuni disproporționate, Compresiune, Încovoiere biaxială

Abstract

The purpose of the current paper is to raise awareness on the behaviour of rectangular reinforced concrete columns with disproportionate cross-section dimensions subjected to compression and biaxial bending, considering the fact that in the current structural design process, the design of columns is usually done with respect to each of the two principal directions without considering their combined effect.

The paper is based on a case study regarding a building that is currently in the design phase, in which to obtain a correct seismic behaviour, rectangular columns with disproportionate cross-section dimensions were used.

The design of both the building and the columns follows the Romanian seismic design codes (“P100-1/2013 - Seismic design code - Part 1 - Design provisions for buildings” and “SR-EN 1998/2004 - Design of structures for earthquake resistance”).

Results were compared and conclusions were drawn based on the interaction curves and interaction surfaces computed with expressions from specialized literature and sectional analysis software.

Keywords: Reinforced concrete structures, Disproportionate sections, Compression, Biaxial bending

1. INTRODUCTION

In current practice, due to the ever-growing request of new constructions corresponding to a decrease in the time allotted to design them, some shortcuts are used to finish the project faster.

One of these shortcuts consists of only computing columns on their principal directions and omitting their combined effect. This omission is in compliance with the design codes only when the relative eccentricities applied to the axial force due to the bending moment on each direction are small and when the columns are not slender.

In usual, regular, low to mid-rise structures with square columns or rectangular columns with similar cross-section dimensions, this omission may not be that important, but in some cases, as is the case-study shown in this paper, the importance is significant.

The purpose of the current paper is to raise awareness about this effect and to show the differences between results computed with the code specified expressions and results from sectional analysis software.

2. INTERACTION SURFACE

Considering a column with a rectangular cross-section, biaxial bending can be represented by eccentricities applied to the axial force (Figure 1), in which:

$$e_y = \frac{M_{Edz}}{N_{Ed}}; e_z = \frac{M_{Edy}}{N_{Ed}} \quad (1)$$

where: e_y, e_z = eccentricities along y-axis and z-axis of the section;

M_{Edy}, M_{Edz} = design bending moments about y-axis and z-axis;

N_{Ed} = design value of the axial force.

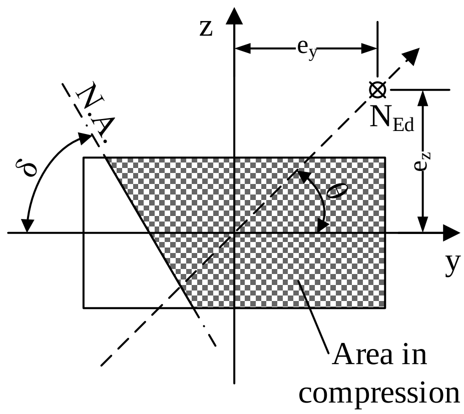

Figure 1. Eccentricities

The failure surface for a reinforced concrete column subjected to both axial force and biaxial bending can be three-dimensionally plotted as in Figure 2, in which 1 and 2 are uniaxial interaction curves N-M_y and N-M_z, 3 is the load contour M_y-M_z at a constant axial load and 4 is the interaction curve N-M_θ corresponding to biaxial bending (the axial load is acting on a direction rotated with angle θ with respect to y).

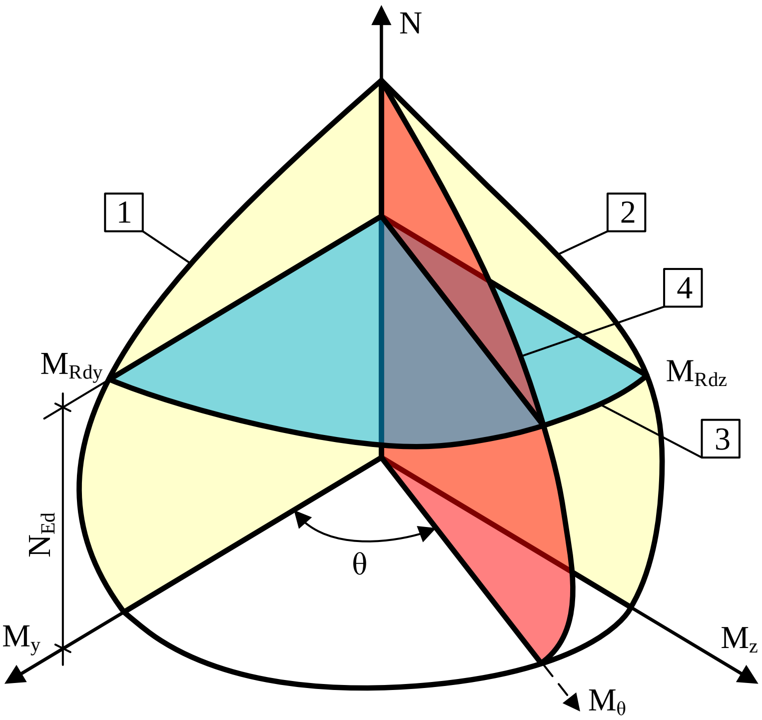

Figure 2. Failure surface

If the point with coordinates N_{Ed}, M_{Edy} and M_{Edz} is situated inside the domain delimited by the failure surface then the section capacity is not reached.

In specialized literature two distinct methods for determining the failure surface are presented: the approximate method and the exact method.

The approximate method was studied in-depth, especially before the use of computers was common. It consists of the analytical determination of the failure surface through empirical expressions by comparison to results from biaxial bending tests on rectangular columns.

The first analytic expressions for the failure surface were introduced by Bresler [1] in 1960. Later, Pannel [2], Ramamurthy [3], Furlong [4], Meek [5] and others added to Bresler's findings through either new test and comparisons or slight modifications to the expressions.

In recent years, due to technological advancements, it is generally preferred to determine the exact failure surface. Usually, it is computed with the aid of sectional analysis software due to the increased difficulty in considering static, geometric and physical conditions expressed with respect to the inclined neutral axis (Figure 1). [6]

Most of the specialized programs use the "fibre decomposition method" where the cross-section is decomposed into fibres and the sectional response is computed by "adding" the uniaxial behaviour of each fibre through numerical integration of stress resultants. [7]

Recent studies on this subject were made for various concrete and steel-concrete composites with varying sections (rectangular, L and T shapes, polygonal, circular etc.) by Fafitis [8], Chen, Teng and Chan [9], Sfakianakis [10], Chiorean [7] and others.

2.1. Analytical method

The second equation that was introduced by Bresler [1] was found to supply the most accurate results in comparison with performed tests, it is also known as the "ellipse" method and it approximates the failure surface through a family of load contours:

$$\left(\frac{M_x}{M_{xo}}\right)^{\alpha} + \left(\frac{M_y}{M_{yo}}\right)^{\beta} = 1 \qquad (2)$$

where: M_x, M_y = components of the bending moment about x and y axes;
M_{xo}, M_{yo} = bending moment resistance with respect to axes x and y;

α, β = exponents depending on column dimensions, amount and distribution of reinforcement, stress-strain relationships of concrete and reinforcement, amount of concrete cover, size and distribution of stirrups.

Bresler's load contour equation is still being used today in code provisions. For example, the following expression is given in Eurocode 2 as the criterion for checking columns in biaxial bending with or without axial force:

$$\left(\frac{M_{Edz}}{M_{Rdz}}\right)^a + \left(\frac{M_{Edy}}{M_{Rdy}}\right)^a \leq 1 \qquad (3)$$

where: M_{Edy}, M_{Edz} = design bending moments about y-axis and z-axis, including a second-order moment;

M_{Rdy}, M_{Rdz} = bending moment resistance with respect to axes x and y;

a = exponent that is chosen as follows:

- for circular and elliptical cross-sections, $a = 2$;
- for rectangular cross-sections, linear interpolation between values from Table 1.

Table 1. Exponent values

N_{Ed}/N_{Rd}	0.1	0.7	1.0
a	1.0	1.5	2

3. CASE-STUDY

The case-study involves a building displayed in Figure 3 that is part of a residential complex currently in the design phase, with the following characteristics:

- location: Brașov, Romania;
- height regime: partial basement + seven storeys, with a setback at the seventh storey, with storey height = 3m;
- structural system: three-dimensional reinforced concrete frames (columns, beams and slabs);
- foundations: raft foundation under the partial basement, continuous foundations under the ground floor.

Figure 3. 3D rendering of the building

3.1. Structural modelling

The analysis was performed at the Technical University of Civil Engineering Bucharest, Structural Mechanics Department and the software used was ETABS for structural analysis, XTRACT for sectional analysis and MATHCAD for the creation of 3D plots.

The final structural model is displayed in Figure 4. The parameters used for structural analysis are the following:

- structural parameters:
  - structural material - concrete C30/37 with $E = 33GPa$, $G = 13.75GPa$, $\gamma \approx 25kN/m^3$ and $\nu = 0.2$;
  - structural layout - 3 spans of 6.1m, 6.0m and 5.8m, 6 bays of 6.4m;
  - columns - 1D bars with cross-section 30x110cm;
  - beams - 1D bars with cross-section 30x55cm;
  - slab - 2D shell, height = 15cm, mesh elements ≤ 1.00m^2, rigid diaphragm;

  - elevator shaft - 30cm - shell, mesh elements ≤ 1.00m^2;
- loads:
  - permanent loads - element weights + 2.50kN/m^2;
  - variable loads:
    - live load = 2.00 kN/m^2;
    - snow load = 1.60 kN/m^2, in accordance with CR 1-1-3;
  - seismic loads, modelled using response spectra with the following parameters:
    - peak ground design acceleration - $a_g = 0.2g$ for MRI = 225 years, in accordance with P100-1/2013;
    - corner periods: $T_B = 0.14s, T_C = 0.7s, T_D = 3s$;
    - importance-exposure factor: $\gamma_{I,e} = 1.00$;
    - behaviour factor: $q = 6.75$.
- load combinations: in accordance with CR 0/2012 and P100-1.

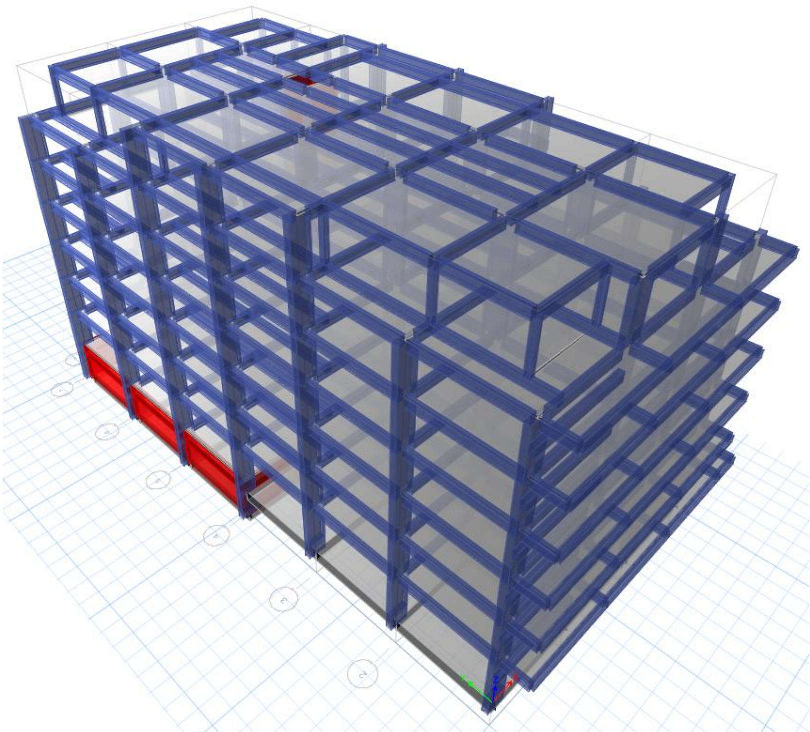

Figure 4. Structural model

As per P100-1 provisions, the beams and columns were considered cracked by halving their elasticity (Young) modulus.

The combination of modal responses was done by “Complete Quadratic Combination” and the combination of effects of the components of seismic action on the horizontal was done by “Square Root of the Sum of Squares” of the value of the action effect due to each horizontal component.

The vertical component of the seismic action was not considered as the structure satisfies the code criteria.

The number of vibration modes was considered such that the sum of the effective modal masses amounts to at least 90%.

3.2. Analysis results

All checks specified within the codes were performed for the structure with satisfactory results, but they will not be detailed here since they exceed the purpose of the current paper.

The column reinforcement (Figure 5) was dimensioned following all code provisions considering initially only the interaction curves with respect to the principal axes of the section (Figure 6 and Figure 7).

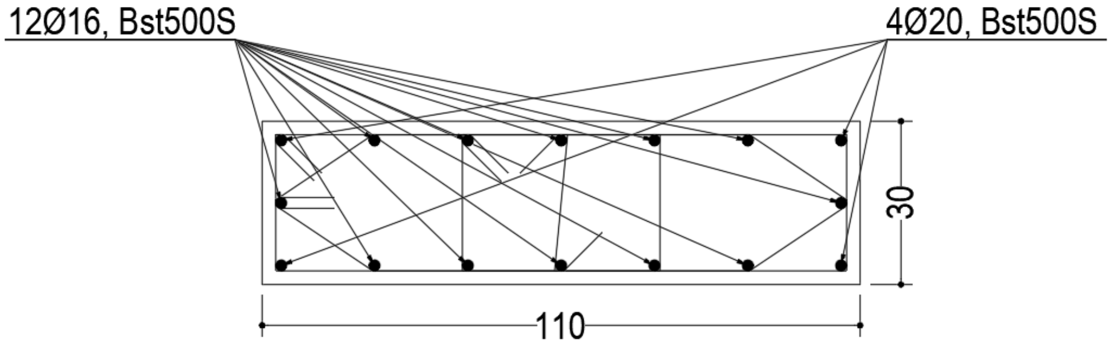

Figure 5. Column section reinforcement

The interaction curves were computed using Xtract, considering the constitutive models affected by safety factors and imposing a limiting strain of 0.2% for concrete and 7.5% for the reinforcement.

The dots in the two graphs are the pairs of design axial force - design bending moment about the corresponding principal axis, for all columns at all storeys, exported from ETABS, affected with correction factors in accordance with P100-1/2013.

The two graphs show that the column section does not reach its capacity under the action of the individual moments about each of the two principal axes.

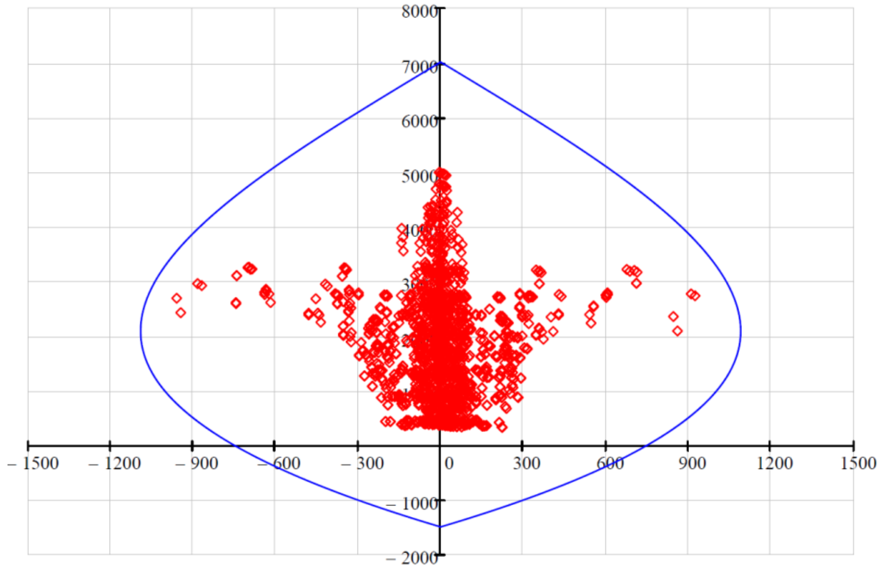

Figure 6. Interaction curve N-M_y (major axis)

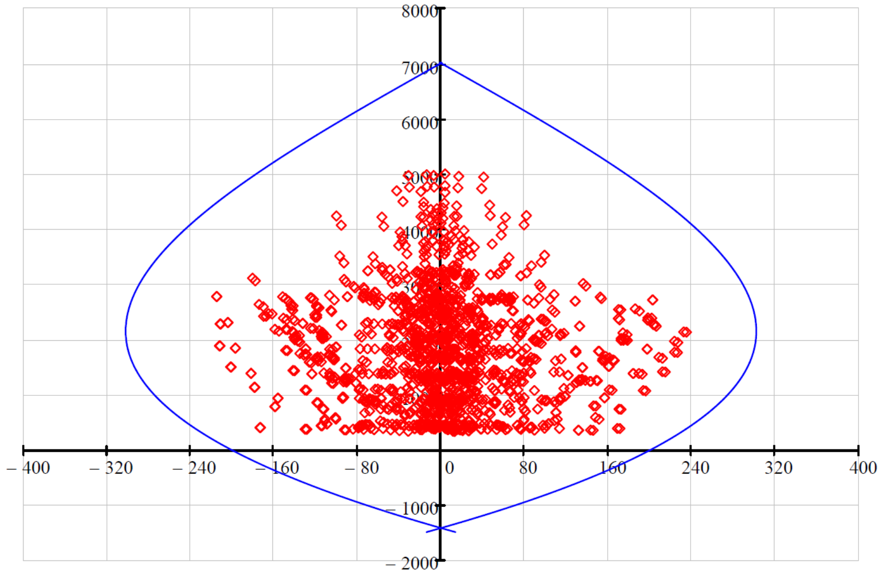

Figure 7. Interaction curve N-M_z (minor axis)

Since in some cases the relative eccentricities exceed the limit recommended in Eurocode 2, further analysis was necessary.

The interaction surfaces displayed in Figure 8 and Figure 9 were generated in Mathcad from results computed with Xtract and with the expression given in Eurocode 2 - equation (4), by using several load contours for intervals of 100kN axial force and rotating the angle of loading with 22.5 degrees.

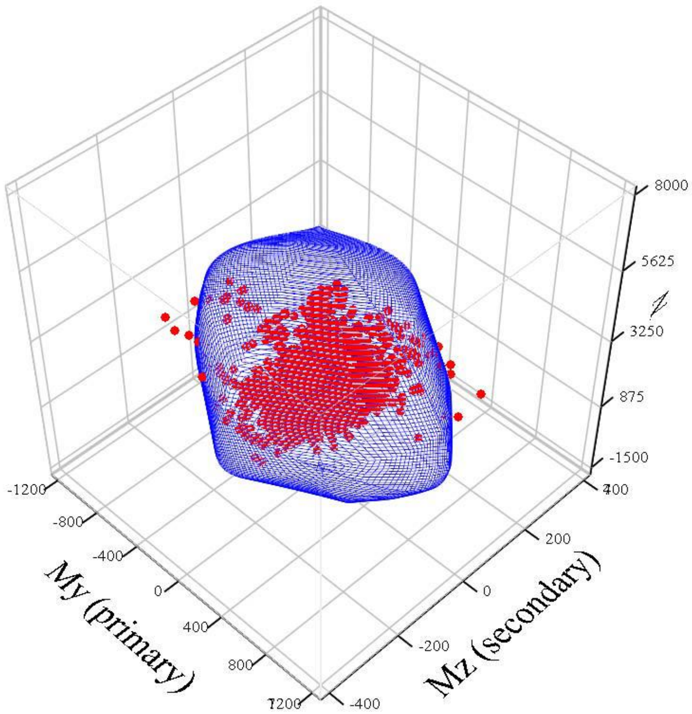

Figure 8. Failure surface computed with Xtract results

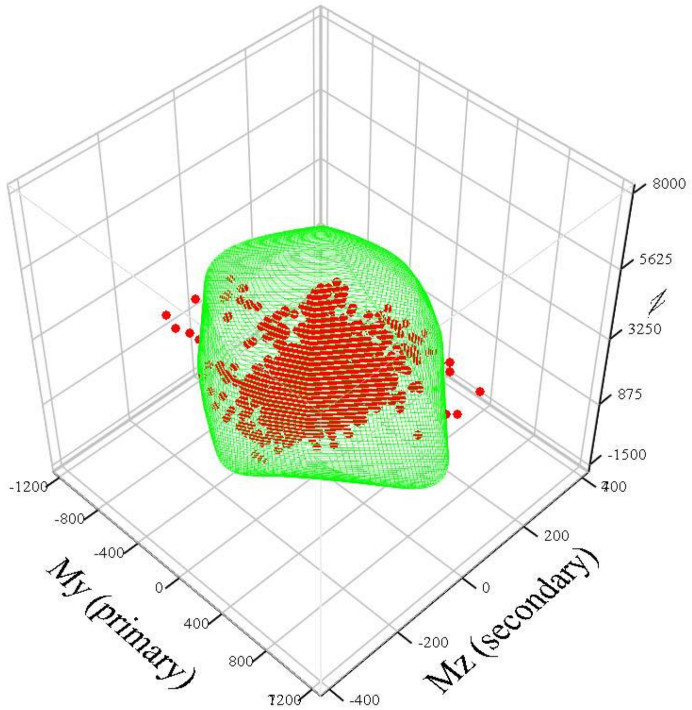

Figure 9. Failure surface computed with EC2 expression

In both graphs there are a few cases in which the capacity of the column is exceeded, causing a need for reinforcement and/or section dimensions increase.

Before any conclusions are drawn, it must be said that the cross-section dimensions of the columns studied in this paper are close to those of a shear wall. The definition for a shear wall is that the length of its section is at least four times the width.

So, in this case, the solution regarding an increase in cross-section dimensions, especially their length, requires a full redesign of the structure with different types of elements (shells instead of bars) that change the behaviour of the structure completely.

4. CONCLUSIONS

An important fact about the N-M_y-M_z points that are outside the domain delimited by the failure surface is that they all derive from the load combination that takes into account the simultaneous effect of the horizontal components of seismic action (in this case, modelled by a square root of the sum of squares).

This shows that even if the columns are positioned along the two principal directions of the structure, they can still be subjected to important biaxial bending effects, due to the combined effect of the horizontal seismic components.

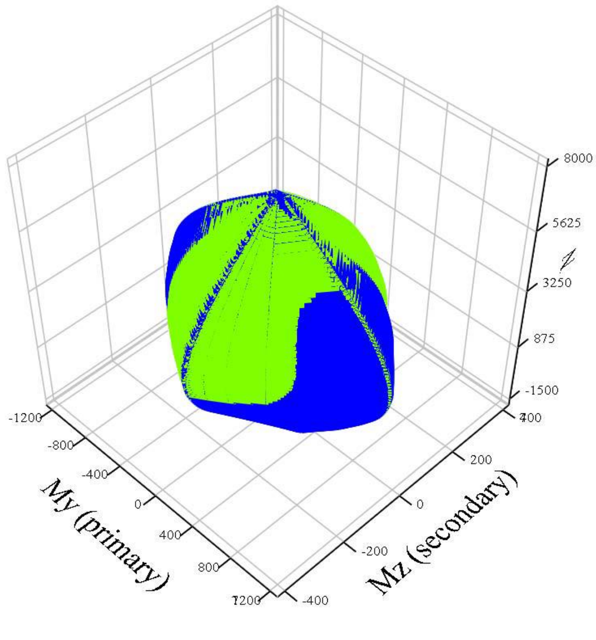

Figure 10. Failure surface comparison

From the comparative analysis of the results determined from sectional analysis software versus the results computed with the Eurocode 2 expression (Figure 10), the following conclusions can be drawn:

- at low values of the axial force - less than 20% of the axial force capacity of the column, the load contours computed with the Eurocode 2 expression are almost linear and underestimating the ones determined from Xtract (on the safe-side) - Figure 11;

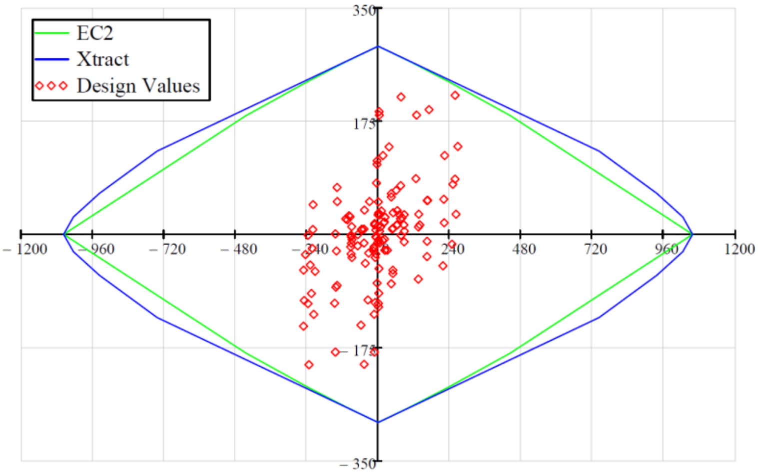

Figure 11. Load contour comparison for 20% of axial force capacity

- at high values of the axial force - more than 70% of the axial force capacity of the column, the load contours computed with the Eurocode 2 expression are overestimating the ones determined from Xtract (not on the safe-side) - Figure 12;

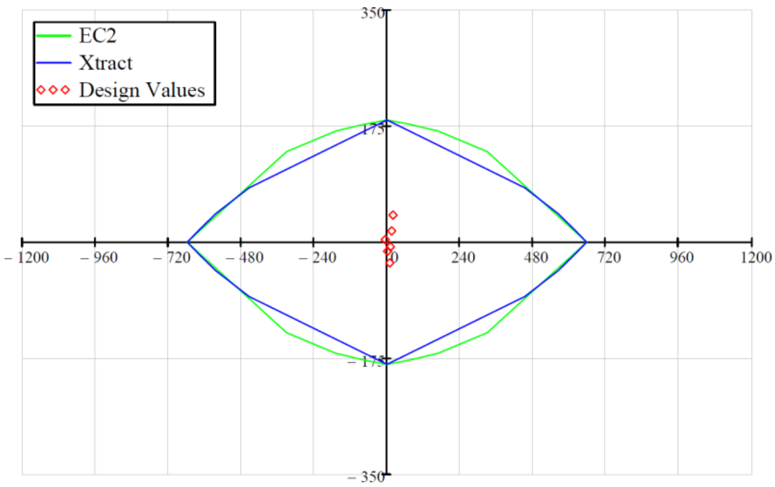

Figure 12. Load contour comparison for 70% of axial force capacity

- between the two limits specified above, the load contours computed with the Eurocode 2 expression are overestimating the values close to the bending moment about the minor axis and underestimating the values close to the bending moment about the major axis - Figure 13.

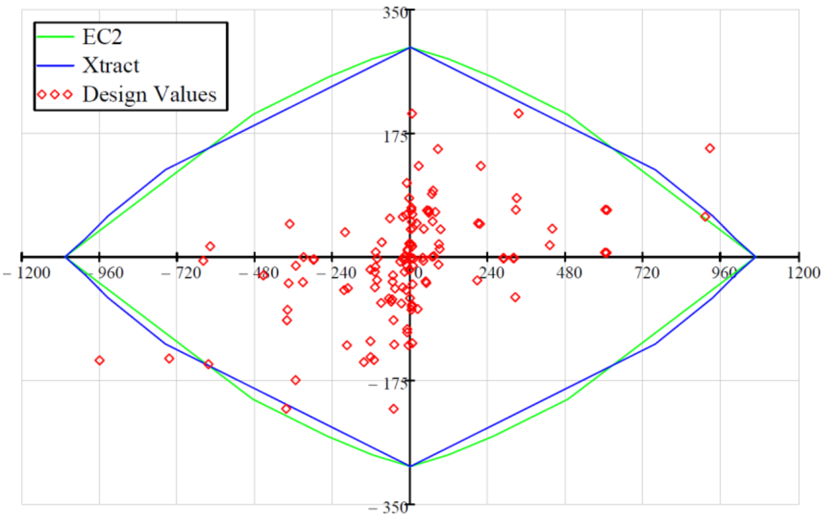

Figure 13. Load contour comparison for 50% of axial force capacity

The most important conclusion stems directly from the analysis results: a biaxial bending check should always be performed, at least in cases when the columns have disproportionate sections.

Also, considering the fact that the determination of the actual interaction surface with the help of sectional analysis software is tedious (having to determine several interaction curves for several angles and connect them through load contours), the simple analytical expression supplied by Eurocode 2 gives results that are close to the real ones within a reasonable margin of error.

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