

PROBLEM OF GAS PHASE FLOW IN A WATER-SATURATED POROUS MEDIUM

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The aim of this study is to develop efficient numerical methods for the solution of the fluid–fluid displacement problem in porous media, encountered in the development and exploitation of oil and gas reservoirs and in the construction and operation of underground gas storage in aquifers. At present, hydro-gas-dynamic simulations of underground gas storage facilities are performed using numerical methods that do not fully account for the actual physical properties of the porous reservoir, the gas, and the water, nor for the specific features of the displacement process. The same limitations apply to problems arising during the operation of gas reservoirs in contact with active edge-water aquifers. In this regard, there arises a need to develop efficient numerical methods for hydro-gas-dynamic simulations that take these factors into account, thus enabling a more accurate representation of the process. The numerical implementation of the algorithm is based on a finite difference scheme on a moving grid. The proposed difference–iterative method with moving grids can be employed to identify a rational reservoir stimulation strategy.

Keywords: gas volume, underground gas storage facility, water-drive regime, aquifer, mass flow rate, moving boundary.

INTRODUCTION

The present study extends the results reported in Gasimov *et al.* (2025) and focuses on the analysis of nonstationary gas-volume filtration in a bounded aquifer. The problem includes applications associated with the construction and operation of underground gas storage facilities and the development of gas reservoirs under water-drive conditions.

The possibility of gas storage in horizontal water-bearing formations with a sufficiently tight clay caprock was theoretically substantiated for the first time by Charny (1962). Later, a number of approximate approaches were developed, which were used to carry out hydrogas-dynamic calculations of gas-water displacement in an unbounded aquifer (Coats *et al.*, 1967).

The work by Charny *et al.* (1969) further advances this topic by developing a numerical method for hydro-gas-dynamic simulations of gas storage in horizontal aquifers with an impermeable caprock, considering a battery of regu-

lating wells, as well as bounded reservoirs with impermeable contours. These studies are based on a modified piston displacement model (MPDM).

Different hydrodynamic models are used to model the non-stationary filtration of gas and water in porous media. As shown in Charny *et al.* (1969), the results of investigations conducted within the framework of the MPDM exhibit good agreement with known experimental and field observations.

It has been established by Charny *et al.* (1968) that, during cyclic operation of underground gas storage, relative phase permeabilities, capillary forces, and the real properties of gas and water have a substantial impact on the gas-water and water-gas displacement processes. Therefore, it is of interest to assess the agreement between the results of the MPDM and the relative permeability model (RPM), and to compare them with known field observations.

Under cyclic operation of underground gas storage, changes in gas saturation and phase pressures, outside the near-

wellbore region, exhibit relatively smooth behaviour. However, even in this case, a difference-iterative method implemented on a fixed grid fails to accurately track the time-dependent position of the gas volume.

It is well established that underground gas storage facilities are mainly used to balance seasonal and short-term variations in gas demand. Consequently, their operation is cyclic, and each cycle includes three periods: gas injection into the aquifer, a storage shut-in period, and production (withdrawal) of the injected gas.

A previous study (Solovieva *et al.*, 1974) was devoted to the numerical simulation of the first period, i.e. gas injection into the aquifer. The cited work presents guidelines for selecting appropriate models when simulating processes associated with the cyclic operation of underground gas storage in horizontal aquifers. It was demonstrated that the hydrodynamic effect caused by pressure buildup in the gas zone, in the presence of a gas volume in the aquifer, is independent of the displacement model employed.

A later study (Telkov *et al.*, 2008) addressed complex three-dimensional problems of filtration theory associated with applied issues in the design and development of oil, oil-and-gas, and gas fields, as well as hydrocarbon production. The work presented analytical solutions for unsteady and steady-state spatial filtration of compressible and incompressible fluids and gases, which were based on the classical contributions to subsurface hydromechanics by S.Y. Leibenzon, V. N. Shelkachev, I. A. Charny, M. Musket, and other prominent researchers.

PROBLEM STATEMENT

Let us formulate the problem of gas-volume migration in a water-saturated porous medium within the framework of the MPDM (Charny 1962; Coats *et al.*, 1967).

Let, at time $t = 0$, the reservoir contain a gas volume with mean gas saturation S_1 and a closed outer boundary $r = \xi(0)$, while outside the gas-bearing part of the reservoir there is a water-bearing region with saturation $S = 1$ and a closed outer boundary $r = R$.

Since the gas viscosity is extremely small compared with the water viscosity, as shown in Charny *et al.* (1969), the pressure in the gas-saturated region may be assumed spatially uniform at each time instant and equal to the volume-averaged (mean) pressure over the gas-saturated volume. This pressure is determined from the material-balance equation (Zakirov *et al.*, 1974):

$$p(t) = Z(p) \frac{U_{nl}}{U_{cm}} \frac{p_{am}}{Z(p_{am})} \frac{W^*(t)}{mS_1 \pi H \xi^2(t)} \equiv F(\xi(t), t, p),$$

where $W^*(t)$ denotes the amount of gas contained in the reservoir at time t , reduced to atmospheric pressure p_{at} and the standard temperature T_{st} (in Kelvin).

Depending on the specification of the function $W^*(t)$, one obtains either a problem of gas reservoir development or a problem describing the filtration process during the operation of an underground gas storage facility.

If, to determine the pressure in the water-bearing part of the reservoir, one employs the differential equation of elastic-regime filtration (Pirmamedov, 1968), then, in dimensionless variables, the following problem is obtained: find the functions $p(r, t)$ and $\xi(t)$ from the governing equations and conditions:

$$a(r, p) \frac{\partial p}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (rb(r, p) \frac{\partial p}{\partial r}), \quad (1)$$

$$(r, t) \in G_T^{(2)} = \{\xi(t) < r < R, 0 < t \leq T\},$$

$$p(r, 0) = p^0(r), \xi(0) = \xi^0, (r, t) \in G_0^{(2)}, 0 < \xi^0 < R, \quad (2)$$

$$p(r, t) = F(r, t, p), r = \xi(t), 0 < t \leq T, \quad (3)$$

$$p(r, t) = \psi(t), r = R, 0 < t \leq T, \quad (4)$$

$$\gamma(r, p) \frac{d\xi}{dt} = \frac{\partial p}{\partial t}, r = \xi(t), 0 < t \leq T \quad (5)$$

where

$$a(r, p) = \beta^* H \geq c_1 = \text{const} > 0,$$

$$b(r, p) = \frac{kH}{\mu_2(p)} \geq c_2 = \text{const} > 0,$$

$$\gamma(r, p) = -\frac{m(p)\mu_2(p)\bar{S}}{k} < 0,$$

$\beta^* = m\beta_2 + \beta_s$, β_2 and β_s are the coefficients of volumetric compressibility of the aquifer, the water, and the porous matrix, respectively; $p^0(r)$ and $\psi(t)$ are prescribed continuous functions satisfying the interface (matching) conditions.

$$p^0(r) = F(\xi(0), 0, p^0(\xi(0))), p^0(R) = \psi(0), \quad (6)$$

and $\bar{S}$ is the overall (effective) displacement coefficient, normalised by the total formation thickness H .

Note that, unlike the classical one-phase Stefan problem with a moving boundary, the sought function is not a prescribed constant but depends on $\xi(t)$. For this reason, the problem under consideration will be referred to as a one-phase Stefan-type problem.

SOLUTION ALGORITHM

A distinctive feature of problem (1) to (5) is the presence of a moving boundary with an unknown law of time evolution, which renders the problem non-linear and creates additional difficulties both in proving the existence and uniqueness of the solution and in constructing algorithms for obtaining an approximate solution.

Due to the difficulties associated with obtaining exact analytical solutions of Stefan-type problems, the need arose to

seek approximate solutions (Douglas *et al.*, 1959). Research on the numerical solution of such problems has developed along two main directions (Solovieva *et al.*, 1974). In studies belonging to the first direction, Stefan-type problems are treated as free-boundary problems in which the moving boundaries are explicitly identified. In contrast to this approach, the second direction is characterised by the construction of a generalised solution to the problem (Solovieva *et al.*, 1974).

Let us proceed to the numerical solution of problem (1) to (5). Here, an approach is proposed in which the free boundary is explicitly isolated by means of a change of variables. This transformation makes it possible, through a special choice of the computational grid, to approximate the equations governing the motion of the free boundary with high accuracy and to effectively track regions with large pressure gradients. Note that, in the original variables (r, t), the proposed nonuniform grid of nodes is moving.

After the change of independent variables

$$\sigma = \frac{r - \xi(t)}{R - \xi(t)} + 2, \quad t = t,$$

the problem (1) to (5) takes the following form: find the functions $p(\sigma, t)$ and $\xi(t)$ satisfying the governing equations and conditions:

$$\frac{\partial p}{\partial t} = K_2 \frac{\partial^2 p}{\partial \sigma^2} + K_1 \frac{\partial p}{\partial \sigma}, \quad 2 < \sigma < 3, \quad 0 < t \leq T \quad (7)$$

$$p(\sigma, 0) = p^0(\sigma), \quad 2 \leq \sigma \leq 3, \quad \sigma = \frac{r - \xi(0)}{R - \xi(0)} + 2, \quad (8)$$

$$\xi(0) = \xi^0, \quad (9)$$

$$p(\sigma, t) = F(\xi(t), t, p), \quad \sigma = 2, \quad 0 < t \leq T, \quad (10)$$

$$p(\sigma, t) = \psi(t), \quad \sigma = 3, \quad 0 < t \leq T, \quad (11)$$

$$\gamma(\sigma, p) \frac{d\xi}{dt} = \frac{\partial \xi}{\partial r} \frac{\partial p}{\partial \sigma}, \quad \sigma = 2, \quad 0 < t \leq T, \quad (12)$$

where

$$K_2(\sigma, p, \xi) = \frac{b(\sigma, p)}{a(\sigma, p)} \left(\frac{\partial \sigma}{\partial r} \right)^2, \quad \frac{\partial \sigma}{\partial r} = (R - \xi(t))^{-1},$$

$$K_1(\sigma, p, \xi) = \left[-a(\sigma, p) \frac{\partial \sigma}{\partial t} + \left(\frac{\partial \sigma}{\partial r} \right)^2 \frac{\partial b}{\partial \sigma} + \frac{1}{r} b(\sigma, p) \frac{\partial \sigma}{\partial r} \right],$$

$$\frac{\partial \sigma}{\partial t} = [(\sigma - 3)\xi'] (R - \xi(t))^{-1}, \quad r = (R - \xi(t))(\sigma - 2) + \xi(t).$$

The system of equations (7) to (12) is nonlinear, since the coefficients K_1 , K_2 , and F , as well as $\frac{\partial \sigma}{\partial r}$, involve the unknown functions $p(\sigma, t)$ and $\xi(t)$. Let us apply the method of successive (Picard) iterations. For each fixed iterate $\xi^{(m)}(t)$, $m \geq 0$, at the p -iteration we obtain the following problem: find $p^{(l+1)}(\sigma, t)$ (treating $p^{(l)}(\sigma, t)$, $\xi^{(m)}(t)$ as known) $l \geq 0$, satisfying the governing equations and conditions:

$$\frac{\partial p^{(l+1)}}{\partial t} = K_1^{(l,m)} \frac{\partial p^{(l+1)}}{\partial \sigma} + K_2^{(l,m)} \frac{\partial^2 p^{(l+1)}}{\partial \sigma^2}, \quad (13)$$

$$p(\sigma, 0) = p^0(\sigma), \quad 2 \leq \sigma \leq 3, \quad (14)$$

$$p^{(l+1)}(\sigma, t) = F(\xi^{(m)}(t), t, p^{(l)}), \quad \sigma = 2, \quad 0 < t \leq T, \quad (15)$$

$$p^{(l+1)}(\sigma, t) = \psi(t), \quad \sigma = 3, \quad 0 < t \leq T, \quad (16)$$

where $K_a^{(l,m)} = K_a(\sigma, p^{(l)}, \xi^{(m)})$, l – is the iteration number on p , m is the iteration number on ξ .

For the numerical solution of problem (13) – (16), we employ a monotone implicit difference-iterative scheme (Samarski, 1983; Samarski *et al.*, 2003) designed for the case of a rapidly varying coefficient multiplying the first derivative.

We discretise the domain $G_T = \{2 \leq \sigma \leq 3, 0 \leq t \leq T\}$ by a space–time grid of nodes:

$$\widehat{\omega}_{h_i \tau_n}^* = \{(\sigma_i, t_n); \sigma_i = \sigma_{i-1} + h_i, i = \overline{1, M_1 - 1}; \sigma_0 = 2, \sigma_{M_1} = 3; t_n = t_{n-1} + \tau_n, n = \overline{1, N}, t_0 = 0, t_N = T\}$$

We replace the differential equation (7) by a two-level implicit monotone finite-difference scheme:

$$\prod \widehat{Y}^{(l+1)} = \widehat{Y}_t^{(l+1)} - \widetilde{\chi}^{(l,m)} \widetilde{K}_2^{(l,m)} \widehat{Y}_{\sigma\sigma}^{(l+1)} - \widetilde{K}_1^{(l,m)} \widehat{Y}_\sigma^{(l+1)} -$$

$$\widetilde{K}_1^{-(l,m)} \widehat{Y}_\sigma^{(l+1)} = 0,$$

$$\widetilde{\chi} = (1 + \widetilde{V})^{-1}, \quad \widetilde{V}_i = \frac{1}{2\widetilde{K}_{2,i}} [h_{i+1} \widetilde{K}_{1,i}^+ - h_i \widetilde{K}_{1,i}^-],$$

$$\widetilde{K}_{a,i} = K_a(\sigma_i, \hat{\eta}, Y_i), \quad \hat{\eta} = \xi(t_{n+1}), \quad K_1^\pm = \frac{1}{2} [K_1 \pm |K_1|] \geq 0 (\leq 0).$$

The initial and boundary conditions are approximated as follows:

$$Y_i^{(0)} = p^0(\sigma_i), \quad i = \overline{0, M_1 - 1}, \quad (18)$$

$$\eta^{(0)} = \xi(0) = \xi^0, \quad (19)$$

$$\widehat{Y}_0^{(l+1)} = F(\hat{\eta}^{(m)}, \hat{t}, \widehat{Y}_0^{(l)}), \quad n = \overline{0, N}, \quad (20)$$

$$\widehat{Y}_{M_1}^{(l+1)} = \hat{\psi}, \quad n = \overline{0, N}. \quad (21)$$

The iterative procedure is organised as follows. Suppose that, on the time layer $i = t_n$ ($n \geq 1$), the grid functions Y_i and η have been computed at all nodes of the grid $\widehat{\omega}_{h_i \tau_n}^*$; these values are taken as the initial (zero) approximations for $\widehat{Y}_i^{(0)} = Y_i$ and $\hat{\eta}^{(0)} = \eta$ on the $(n+1)$ -st time layer. Assuming that, on the grid $\widehat{\omega}_{h_i \tau_{n+1}}^*$, the l -th iterate of $\widehat{Y}_i^{(l)}$ is known for a fixed $\hat{\eta}^{(m)}$ from relations (17), (20), and (21) we obtain a system of algebraic equations with a tridiagonal matrix for computing $\widehat{Y}_i$, $i = \overline{0, M_1}$:

$$\begin{cases} -c_0^l \widehat{Y}_0^{(l+1)} + b_0^{(l)} \widehat{Y}_1^{(l+1)} = -d_0, & i = 0, \\ a_i \widehat{Y}_{i-1}^{(l+1)} - c_i \widehat{Y}_i^{(l+1)} + b_i \widehat{Y}_{i+1}^{(l+1)} = -d_i, & 1 \leq i \leq M_1 - 1, \\ a_{M_1} \widehat{Y}_{M_1-1}^{(l+1)} - c_{M_1} \widehat{Y}_{M_1}^{(l+1)} = -d_{M_1}, & i = M_1 \end{cases} \quad (22)$$

$$c_0 = 1, b_0 = 0, d_0 = F(\hat{\eta}^{(m)}, \hat{t}, \widehat{Y}_0^{(l,m)}), a_{M_1} = 0,$$

$$\begin{aligned}
c_{M_1} &= 1, d_{M_1} = \hat{\psi}, \\
a_i &= h_i^{-1} h_i^{-1} \left[(\tilde{\chi}^{(l,m)} \tilde{K}_2^{(l,m)})_i - h_i \tilde{K}_{1,i}^{(l,m)} \right], \\
b_i &= h_{i+1}^{-1} h_i^{-1} \left[(\tilde{\chi}^{(l,m)} \tilde{K}_2^{(l,m)})_i + h_i \tilde{K}_{1,i}^{(l,m)} \right], \\
c_i &= a_i + b_i + \tau_{n+1}^{-1}, d_i = \tau_{n+1}^{-1} Y_i, i = \overline{1, M_1 - 1}.
\end{aligned}$$

System (22) is solved by the standard Thomas algorithm (the tridiagonal sweep method) (Aziz *et al.*, 1982; Samarski, 1983). The stability condition for the sweep method

$$a_i > 0, b_i > 0, c_i \geq a_i + b_i, 0 \leq \chi_\alpha < 1, a = 1, 2.$$

is satisfied trivially, and therefore the system of equations (22) is solvable by the sweep method for arbitrary step sizes of the grid $\tilde{\omega}_{h_i \tau_{n+1}}^*$ and for arbitrary coefficients $\tilde{K}_{a,i}$, $a = 1, 2$, i.e., the implicit monotone scheme (17) is absolutely stable.

Using the sweep method and the known values of $\hat{Y}_i^{(l)}, \hat{\eta}_i^{(l)}$, $i = \overline{0, M_1}$, we compute $\hat{Y}_i^{(l+1)}$ at all nodes of the grid $\tilde{\omega}_{h_i \tau_{n+1}}^*$. For each fixed $\hat{\eta}^{(m)}$ ($m \geq 0$), the iterations with respect to Y are continued up to an index $l = l_m$ such that

$$\max_i |\hat{Y}_i^{(l_m+1)} - \hat{Y}_i^{(l_m)}| \leq \varepsilon, (\varepsilon > 0).$$

Using the computed $\hat{Y}_i^{(l_m)}$ for a fixed $\hat{\eta}^{(m)}$ we then determine $\hat{\eta}^{(m+1)}$ from condition

$$\gamma(\sigma_0, \hat{Y}_0^{(l_m)}) \frac{\hat{\eta}^{(m+1)} - \eta}{\tau_{n+1}} = \frac{1}{R - \hat{\eta}^{(m)}} \hat{Y}_{\sigma,0}^{(l_m)}, \sigma_0 = 2 \quad (23)$$

which is a finite-difference analogue of condition (12). For this updated $\hat{\eta}^{(m+1)}$, the entire computational procedure is repeated until an index $l = l_{m+1}$ is reached such that

$$\max_i |\hat{Y}_i^{(l_{m+1})} - \hat{Y}_i^{(l_m)}| \leq \varepsilon_1,$$

where $(\varepsilon_1 > 0)$ is a sufficiently small number, and so on.

The iteration loop on the $(n+1)$ -st time layer is terminated at $m + m_0$ such that

$$|\hat{\eta}^{(m_0)} - \hat{\eta}^{(m_0-1)}| \leq \varepsilon_2,$$

where $\varepsilon_2 > 0$ is a sufficiently small number; the corresponding $\hat{\eta}^{(m_0)}$ and $\hat{Y}_i^{(l_{m_0})}$ are then taken as the solution of the finite-difference problem on the $(n+1)$ -st time layer.

REMARK

Assuming that, at each fixed iteration with respect to η and Y , the problem is linear, one can estimate the discretisation (finite-difference) approximation error for the scheme (17) to (21), (23).

Substituting $Z = Y - P$ into (17), we obtain for the grid function z the relation

$$\prod \hat{z} + \hat{\psi} = 0,$$

where $\hat{\psi} = \hat{p}_\tau - \{\tilde{\chi} \tilde{K}_2 \hat{p}_\sigma + \tilde{K}_1^+ \hat{p}_\sigma + \tilde{K}_1^- \tilde{p}_\sigma\}$ denotes the truncation (discretisation) error of the monotone scheme evaluated on the exact solution of equation (7).

RESULTS OF NUMERICAL SIMULATIONS

For the problem of constructing and operating underground gas storage facilities in aquifers, the following data were used:

$$K = 3.74 \text{ Darsi}, H = 8 \text{ m}, R = R_0 = 10000 \text{ m}, \\ p^0(r) = p_0 = 34 \text{ kg / sm}^2$$

$$\begin{aligned}
\mu_1 &= 0.000124 \text{ puaz}, \rho_1^0 = 0.023 \text{ q / sm}^3, U_{st} = 293 \text{ K}, \\
U_{pl} &= 330 \text{ K}, \rho_2^0 = 1 \text{ q / sm}^3, m = 0.2, r_s = 10 \text{ sm}, \\
\mu_2 &= 0.01 \text{ puaz}.
\end{aligned}$$

Table 1 presents the distributions of pressure and gas saturation within the reservoir at the end of different operational periods (injection, shut-in, and withdrawal). By the end of the withdrawal period, the maximum gas saturation over the entire gas-bearing region reaches 0.4899. A similar pattern is observed in the second cycle as well.

The rate of pressure decline during prolonged shut-in is of significant importance under cyclic operation of the storage facility. The calculations showed that, after gas injection into the storage is terminated, pressure redistribution begins within the reservoir water-drive system, both in the gas-bearing and water-bearing zones. In the gas-bearing part of the reservoir, the pressure decreases (Table 2), whereas in the water-bearing part it increases (Table 3); simultaneously, gas saturation is redistributed. It should be noted that at the beginning of the shut-in period the pressure decline rate in the storage is the highest and then gradually decreases. Table 3 shows that the gas pressure at the bottom-hole of the injection well decreases by 11.3 atm during the first month of shut-in, and by 2.89, 1.82, and 1.21 atm in the subsequent months, respectively.

Figure 1 shows the pressure-time curves in the gas region. The solid line corresponds to calculations based on the MPDM, while the dashed line represents the results obtained using the RPM. The results demonstrate good agreement not only in a qualitative but also in a quantitative sense. In particular, the maximum pressure values practically coincide; moreover, they are attained at the same time instant ($t = 0.12 \times 10^6$). Thus, the resulting hydro-gas-dynamic effect does not depend on the displacement model employed. From a physical viewpoint, this can be explained by the fact that at a high gas injection rate, i.e. during an increase in the *in-situ* gas volume, the expansion of the reservoir gas volume does not have sufficient time to compensate for the pressure buildup when the reservoir volume increases relatively slowly. As time progresses, water displacement, i.e. the growth of the gas volume, compensates for the pressure increase, and the pressure tends toward a certain constant value corresponding to a self-similar solution.

Such a phenomenon is observed not only during the initial stage of gas storage filling, but also at later stages under cyclic gas injection.

It should be noted that this hydrodynamic effect must be taken into account in the design and analysis of under-

Table 1. Distribution of pressure and gas saturation

Distribution of gas pressure						
	I			II		
t , day r, m	95	130	274	427	468	637
1	53.69	43.11	35.73	52.68	46.37	32.47
8	53.58	43.11	35.82	52.52	46.37	32.03
20	53.29	43.11	35.84	52.43	46.37	33.29
36	53.12	43.11	35.91	52.35	46.37	33.45
68	52.86	43.11	35.96	52.23	46.37	33.65
132	52.36	43.11	36.04	52.04	46.37	33.90
260	51.59	43.10	36.15	51.73	46.36	34.24
500	48.48	43.06	36.32	51.24	46.35	34.71
1000	45.76	42.68	36.84	50.30	46.25	35.64
Distribution of gas saturation						
1	0.7270	0.6584	0.4899	0.7274	0.6694	0.3820
8	0.6395	0.6298	0.4801	0.6619	0.6524	0.3803
20	0.5401	0.5327	0.4296	0.5656	0.5598	0.3757
36	0.4795	0.4706	0.3943	0.5080	0.5041	0.3662
68	0.3959	0.3919	0.3671	0.4397	0.4360	0.3432
132	0.3147	0.3141	0.2958	0.3604	0.3593	0.3060
260	0.2705	0.2704	0.2562	0.2976	0.2971	0.2642
500	0.1890	0.2004	0.1806	0.2353	0.2361	0.2140
1000	0.0446	0.0664	0.0545	0.1556	0.1639	0.1189

ground gas storage facilities, since otherwise hydraulic fracturing of the formation may occur.

As noted above, the investigation of the problem of constructing and operating a gas storage facility in a horizontal bounded aquifer with impermeable caprock, base, and lateral boundaries is of direct practical interest.

In this connection, hydro-gas-dynamic simulations were carried out for the construction and operation of such underground gas storage facilities. It is evident that in storage systems of this type, the advance of the gas volume is possi-

ble only due to the compressibility of water; since water has low compressibility, the gas volume propagates over a relatively small distance. At the same time, the gas saturation coefficient turns out to be higher than in the analogous case where a system of relief (pressure-control) wells is present.

A comparison of the obtained results showed that, for identical technological processes and equal radii of the external boundary, the results differ. In particular, Fig. 2 presents the pressure variation curves at the well ($r = r_s$) during the injection of 100 million m^3 of gas at a constant mass flow rate

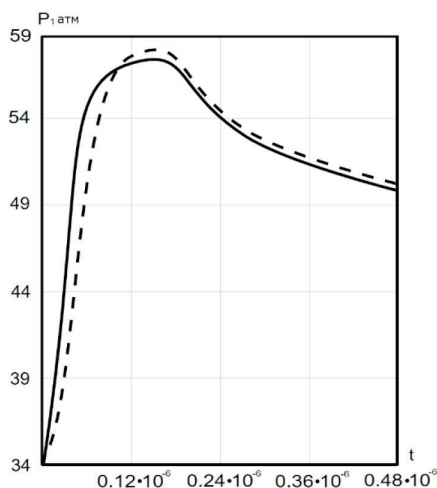


Fig. 1. Curves of pressure changes.

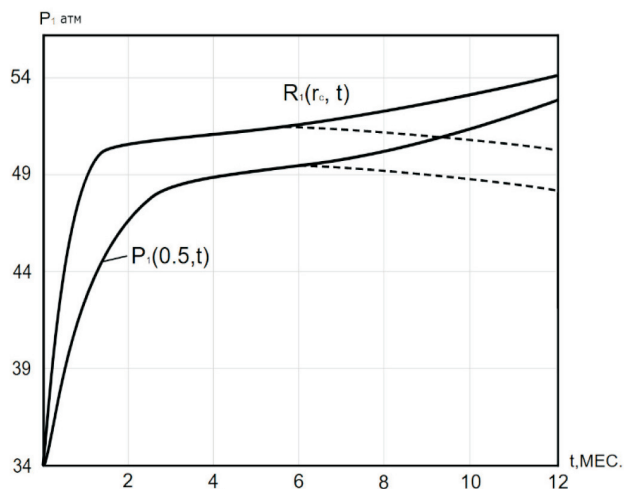


Fig. 2. Change of pressure at the well.

Table 2. Distribution of pressure (in atm) and gas saturation

Distribution of gas pressure (in atm)					
t, month	4	5	6	7	8
r, m					
1	53.35	44.05	41.16	39.34	38.13
2	53.29	44.05	41.16	39.34	38.13
4	53.22	44.05	41.16	39.34	38.13
6	53.18	44.05	41.16	39.34	38.13
12	53.10	44.05	41.16	39.34	38.13
20	53.02	44.05	41.16	39.34	38.13
36	52.92	44.05	41.16	39.34	38.13
68	52.74	44.05	41.16	39.34	38.13
132	52.45	44.05	41.16	39.34	38.13
260	51.96	44.03	41.15	39.33	38.12
500	51.12	43.98	41.11	39.31	38.10
1000	48.62	43.50	40.86	39.15	37.98
Distribution of gas saturation					
1	0.7353	0.6805	0.6617	0.6517	0.6469
2	0.7203	0.6805	0.6617	0.6517	0.6469
4	0.7024	0.6739	0.6587	0.6505	0.6431
6	0.6765	0.6633	0.6535	0.6446	0.6348
12	0.6077	0.6055	0.6015	0.5961	0.5899
20	0.5533	0.5490	0.5452	0.5412	0.5373
36	0.4966	0.4918	0.4873	0.4827	0.4783
68	0.4108	0.4082	0.4060	0.4039	0.4020
132	0.3280	0.3277	0.3275	0.3273	0.3272
260	0.2776	0.2777	0.2777	0.2778	0.2779
500	0.2055	0.2122	0.2141	0.2151	0.2156
1000	0.0690	0.0930	0.1041	0.1111	0.1158

over a period of four months (the solid line corresponds to an impermeable boundary, while the dashed line corresponds to the presence of a relief well battery). It can be seen that, in the initial period, the influence of the impermeable boundary is negligible; however, at later times its effect becomes significant, and, due to the low compressibility of water, the pressure in the gas-bearing region begins to increase linearly.

Additional calculations were performed to investigate the effect of boundary impermeability on the process of nonstationary gas filtration in an aquifer under different injection rates. Over the course of one year, gas was injected into the aquifer at a constant mass flow rate, with total injected volumes of 70, 80, 90, 100, and 110 million m³, respectively.

Figures 3 and 4 show the time evolution of the average pressure and the average gas saturation. As expected, at higher injection rates the effect of boundary impermeability on the displacement process manifests itself much more rapidly than at lower rates. For example, when injecting 110

million m³ of gas, this effect becomes noticeable after about five months, whereas for an injection volume of 60 million m³ the influence is practically not observed over the course of a year.

An analysis of the obtained results indicates that the effect of boundary impermeability, as in the case of the modified piston displacement model (MPDM) (Charny, 1969), becomes significant when the gas volume reaches approximately 8% of the total pore volume of the aquifer.

CONCLUSIONS

The conducted studies made it possible to identify several specific features of the displacement process during the cyclic operation of underground gas storage facilities, which must be taken into account when performing corresponding hydro-gas-dynamic investigations. Recommendations are provided for the application of appropriate models in the simulation of processes associated with the cyclic operation of underground gas storage facilities in horizontal aquifers. It was shown that the hydrodynamic effect associated with

Table 3. Distribution of water pressure (in atm)

Distribution of water pressure (in atm)						
r, m	t, month	4	5	6	7	8
	5000	36.50	37.61	38.18	38.38	38.43
	5125	36.31	37.39	37.97	38.21	38.29
	5250	36.13	37.18	37.79	38.05	38.15
	5375	35.97	36.98	37.59	37.89	38.02
	5500	35.81	36.79	37.41	37.73	37.89
	5625	35.66	36.60	37.24	37.58	37.76
	5750	35.53	36.44	37.07	37.44	37.64
	5875	35.40	36.28	36.91	37.28	37.52
	6000	35.29	36.12	36.76	37.16	37.40
	6125	35.17	35.98	36.61	37.03	37.29
	6250	35.07	35.85	36.47	36.90	37.18
	6375	34.98	35.72	36.34	36.78	37.08
	6500	34.89	35.60	36.21	36.67	36.98
	6625	34.81	35.49	36.09	36.55	36.88
	6750	34.74	35.38	35.98	36.45	36.79
	6875	34.67	35.29	35.87	36.35	36.70
	7000	34.61	35.19	35.77	36.25	36.61
	7125	34.55	35.10	35.67	36.15	36.53
	7250	34.49	35.03	35.58	36.06	36.44
	7375	34.44	34.96	35.48	35.98	36.37

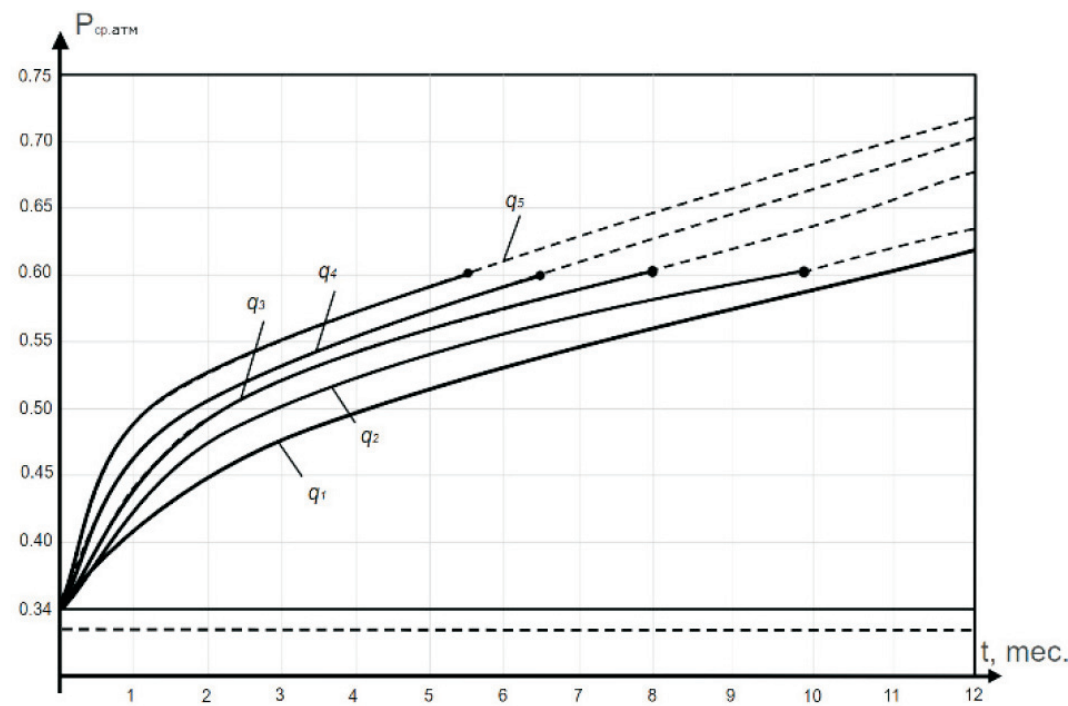


Fig. 3. Change of P_{avg} over time.

pressure buildup in the gas region in the presence of a gas volume within the aquifer is independent of the displacement model chosen. Also, the results obtained using the MPDM and the RPM are in good agreement not only quali-

tatively but also quantitatively, and it was shown that the resulting hydro-gas-dynamic effect does not depend on the model employed.

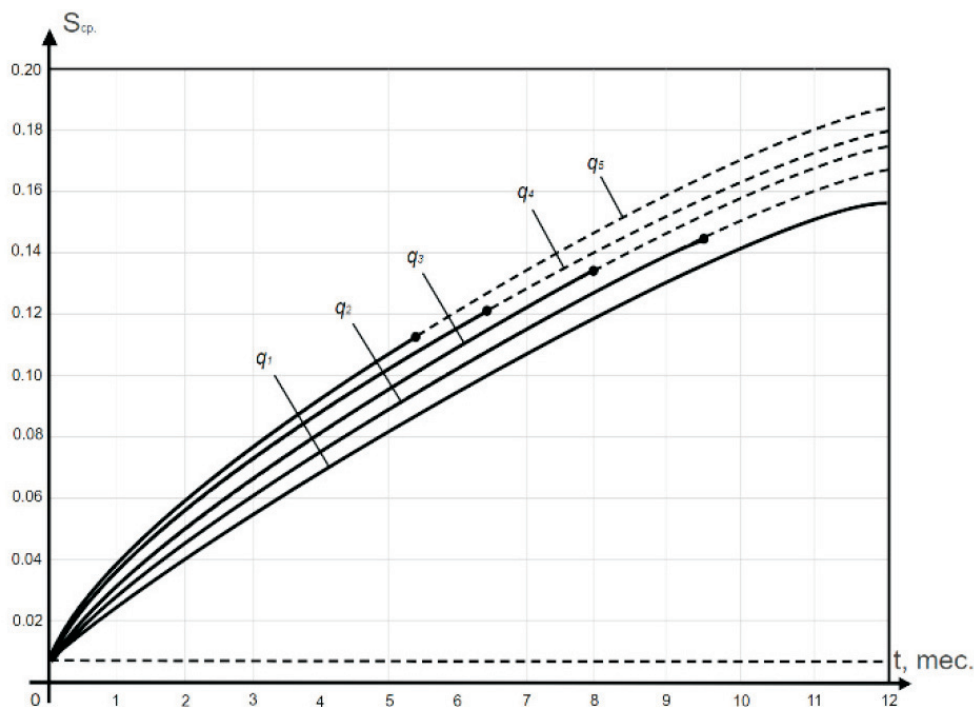


Fig. 4. Change in quantity over time.

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GĀZES PLŪSMAS NORISE AR ŪDENI PIESĀTINĀTĀ PORAINĀ VIDĒ

Šī pētījuma mērķis ir izstrādāt efektīvas skaitliskās metodes šķidrums-šķidrums izspiešanas norises risināšanai porainās vidēs, kas rodas naftas un gāzes rezervuāru izveidošanā un izmantošanā, kā arī pazemes gāzes uzglabāšanas cisternu būvniecībā un ekspluatācijā. Piedāvāto metodi var izmantot, lai noteiktu racionālu rezervuāru izveidošanas stratēģiju.