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Original Paper

ON A NUMERICAL METHOD FOR SOLVING THE PROBLEM OF WATER DISPLACEMENT BY GAS

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The objective of the work is to develop efficient numerical methods for solving the problem of displacement of one fluid by another in a porous medium, arising during the development and operation of oil and gas fields, and creation and operation of underground gas storage facilities in aquifers. At present, when conducting hydrodynamic calculations of underground gas storage facilities, methods are used that do not fully take into account the real properties of the gas and water reservoir, as well as the specifics of the process. In this regard, there is a need to develop efficient numerical methods for conducting hydro gas dynamic calculations that allow to take into account the specified factors and thereby to model the process more accurately. All of the above also apply to problems arising during the operation of gas fields bordering active edge waters. The theoretical justification for the possibility of storing gas in horizontal aquifers with a sufficiently sealed clay roof was first given by Charny et al. *(1969). Subsequently, various approximate methods were proposed, and hydrodynamic calculations of water displacement by gas were carried out on their basis. Numerical implementation of the algorithm was carried out by the finite difference method on a moving grid. The proposed difference-iteration method in moving grids can be used to determine a rational method of influencing the reservoir.*

Keywords: *Rapoport–Leas model, displacement front, slippage effect, oscillating solutions, super compressibility coefficient, matrix sweep method, hypothetical reservoir.*

INTRODUCTION

The problem of multiple filtration is associated with important practical tasks in oil and gas production, hydraulic engineering, etc. This problem is most relevant for the theory and practice of oil and gas production. In fact, important issues of development and operation of oil, gas, gas condensate fields, problems of creation and operation of underground gas storage facilities in horizontal water-saturated formations, etc. are connected with the processes of multiple filtration. Studies of the processes of joint filtration of oil, gas and water in a porous medium, which take into account real geological and production data, the properties of the filtered liquids and gas, relative phase permeabilities and capillary forces, have important theoretical and practical interest. Comparison of experimental and production data with the results of hydrodynamic studies conducted with significant schematisation of real processes shows a significant discrepancy between theoretical and practical results. Therefore, to obtain an adequate description of the processes, it is necessary to simultaneously take into account the influence of most factors on the filtration process, which in mathematical terms leads to the solution of systems of nonlinear differential equations in frequent derivatives, the complexity of which does not allow them to be studied in sufficient depth analytically.

Numerical modelling of the process of water displacement by gas was conducted within the framework of the Rapoport–Leas model (Coats *et al*., 1967). In it, the authors use the Douglas, Peaceman, and Rechford method (Douglas *et al*., 1959) for the numerical solution of the problem of water displacement by gas (the process of gas injection into an aquifer is considered). From the numerical calculations presented in this work, it is evident that, compared to known ideas about the process, a more intensive spreading of the gas volume occurs. This fact is explained by the error of the numerical method used. In fact, the difference scheme considered in the work poorly takes into account the presence of large saturation gradients in the filtration area.

In some studies (Babaev *et al*., 1977; Gasymov *et al*., 2017) the problems of gas displacement by water have been investigated taking into account some properties of fluids, as well as relative phase permeabilities and capillary forces, based on the difference-iteration method in moving grids. To study the influence of the above-mentioned factors on the filtration process, a computational algorithm has been developed that has the property of adaptability to the characteristics of the problems and is distinguished by high accuracy.

There have been experimental studies (Latypov *et al*., 2011) of issues related to the processes of water displacement by gas from models of aquifer reservoirs through horizontal wellbores in relation to solving problems of underground gas storage (UGS). The aim of the work was to study and compare the features of the processes of water displacement by gas through horizontal and vertical wellbores in micro-heterogeneous porous formations when forming a gas-saturated volume in them, intended for UGS.

The work (Telkov *et al*., 2008; Yushkov *et al*., 2013) presents the basic provisions on the development and operation of oil and gas fields, on the methods of exploiting oil and gas wells and the system of collecting oil and gas at the fields. Basic methods of increasing oil recovery from formations are shown. Characteristics of oil, gas, formation fluids, and economic indicators of oil deposit development are given.

STATEMENT OF THE PROBLEM

Let us consider a flat-radial movement of a gas volume in a water-saturated porous medium, taking into account relative phase permeabilities and capillary forces.

Let a horizontal aquifer (gas-bearing) circular formation of constant thickness H and length R be covered by an impermeable roof and base. In the centre of the formation there is a hydrodynamically perfect injection (operational) well of radius $r = r_s$, and on the outer contour, which is a circle, $(r = R)$, a battery of control wells is located. Let us assume that at the initial moment of time t = 0 there is trapped gas (residual water) in the formation.

The system of equations of isometric two-phase filtration, describing the specified process in an isotropic deformable porous medium, taking into account the relative phase permeabilities and capillary forces, can be represented in the following form (Aziz *et al.,* 1982; Gasymov *et al*., 2017):

$$\frac{1}{r}\left\{\frac{\partial}{\partial r}\left[(r\lambda_1)\frac{\partial p}{\partial r}\right]+\frac{\partial}{\partial r}\left[(r\lambda_1)\frac{\partial p_k}{\partial r}\right]\right\}=\frac{\partial}{\partial t}(m\rho_1(1-S)), \quad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[(r\lambda_2)\frac{\partial p}{\partial r}\right]=\frac{\partial}{\partial t}(m\rho_2 S), \quad (2)$$

$$(r,t)\in G_T=\{(r,t):\ r_s<r<R,\ 0<T\le T\},$$

$$\lambda_\alpha = K(r)\frac{f_\alpha(S)\rho_\alpha(p_\alpha)}{\mu_\alpha(p_\alpha)}>0, \alpha=1,2,$$

$$S_1+S_2=1, p_1-p_2=p_k(S),\ S_2\equiv S, p_2=p.$$

where the index $\alpha = 1$ corresponds to the gas phase, $\alpha = 2$ the water phase, S is the water saturation of the steam volume in fractions of a unit, $p_k(S)$ — interfacial capillary pressure.

Let us formulate the boundary conditions. As is known, underground gas storage facilities are intended mainly to cover the unevenness of demand for gas; accordingly, their operation is cyclical, each cycle consists of three periods: gas injection into the aquifer, storage downtime, and extraction of the injected gas.

Let us assume that the first cycle corresponds to the time interval $(0, T_1] \subset 0, T]$. Let us divide the period of time $(0, T_1]$ into three parts: $(0, t_1], (t_1, t_2), (t_2, T_1]$ corresponding to the three periods mentioned above.

Let's assume that in the layer at $t \in (0, t_1]$ gas is pumped through the central well at a given (arbitrary) mass flow rate $Q_1(t)$. Then $r = r_s$ we have:

$$\begin{cases} \lambda_1\dfrac{\partial p_k}{\partial r}=q_1(t), \\ \dfrac{\partial p}{\partial r}=0, r=r_s, 0<t\le T \end{cases} \quad (3)$$

where $q_1(t)=\dfrac{Q_1(t)}{2\pi r_s H}$. Obviously, during storage downtime (i.e.at $Q_1(t) = 0$) $q_1(t) \equiv 0$.

On the outer circuit, during the entire cycle, the absence of flow is set for the gas phase, and for the water phase, either water pressure (or flow):

$$\begin{cases} \lambda_1\left(\dfrac{\partial p}{\partial r}+\dfrac{\partial p_k}{\partial r}\right)=0, \\ p(r.t)=\varphi(t), r=R, 0<t\le T \end{cases} \quad (4)$$

where $\varphi(t)$ is a given function.

Thus, it is required to find the functions p and p_k satisfying the system of nonlinear differential equations (1)-(4) and the initial conditions:

$$p(r,0)=p^0(r), p_k(r,0)=p_k^0(r). \quad (5)$$

Note. As numerical calculations have shown, in poorly permeable deposits it is necessary to take into account the effect of gas slippage along the pore walls (Gasymov *et al*., 2017). In this case, the gas mobility λ_1 has the form:

$$\lambda_1(r,p_1,S,B)=K(r)\frac{f_1(S)(p_1+B)}{\mu_1(p_1)Z(p_1)}$$

where B is a constant, which is a characteristic of the gas and porous medium, $Z(p_1)$ coefficient of gas super compressibility.

NUMERICAL ALGORITHM OF THE PROBLEM

Analysis of the conducted studies showed that significant difficulties in the numerical solution of problem (1) – (5) arise due to the fact that, firstly, nonlinear differential equations (1), (2) describe both convective (transfer of values of the displacing phase from point to point) and "diffusion" (action of capillary forces) processes occurring in the reservoir. Therefore, difference analogues of differential equations (1), (2) should take into account both processes well. Secondly, in numerical modelling of multiphase filtration processes, it is necessary to approximate the boundary conditions with high accuracy, since near the boundaries of the filtration region, a sharp change in phase pressures occurs during the entire process.

The above features are typical for most multiphase filtration problems. In many studies, only some of the listed features are taken into account when solving these problems numerically.

In previous works (Babaev *et al.*, 1977; Gasymov *et al.*, 2017) it was shown that in numerical modelling of the processes of gas (oil) displacement by water it is necessary to use the difference-iteration method in moving grids. Its idea is that a moving (pseudo Lagrangian) grid is introduced into the spatial variable along with the Euler grid, which moves together with the displacement "front". Grid thickening occurs in the vicinity of the "front" $r = r^*(t)$, the selection criterion of which is the maximum value of the water saturation gradient by r.

For the numerical solution of problem (1)–(5), the existence and uniqueness of which is assumed, we introduce in the region $\overline{G_T} = \{r_s \le r \le R, 0 \le t \le T\}$ an arbitrary movable non-uniform spatio-temporal grid of nodes (Babaev *et al.*, 1977):

$$\hat{\Omega}_{h_{i,n}\tau_n} = \{(r_{i,n}t_n): r_{i,n} = r_{i-1,n} + h_{i,n},\ i = \overline{1, M-1};\ r_{0,n} = r_s,$$
$$r_{M,n} = R,\ t_n = t_{n-1} + \tau_n,\ n = \overline{1, N-1},\ t_0 = 0, t_N = T\}$$

The moving space-time grid is constructed on the basis of *a priori* information about the properties of the solution, namely, it is taken into account that in problems of multiphase filtration in a porous medium during the entire process there are large gradients of phase pressures near the boundary of the filtration region $[r_s, R]$ and in these areas the steps are chosen to be quite small.

Using an implicit conservative scheme on the grid, $\widehat{\Omega}_{h_{i,n}\tau_n}$ we approximate the differential problem (1)–(5) by the following difference problem:

$$\left\{(r\hat{\lambda}_1)^{(-)}\hat{Y}_{1,\bar{r}}\right\}_{\hat{r}} + \left\{(r\hat{\lambda}_1)^{(-)}\hat{Y}_{2,\bar{r}}\right\}_{\bar{r}} =$$
$$= r\{(1-S)(m\rho_1)'Y_{1,t} + [(1-S)(m\rho_1)' - m\rho_1 S']Y_{2,t}\}, \qquad (6)$$

$$\left\{(r\hat{\lambda}_2)^{(-)}\hat{Y}_{1,\bar{r}}\right\}_{\hat{r}} = r\{S(m\rho_2)'Y_{1,t} + (m\rho_2 S')Y_{2,t}\}, \qquad (7)$$

$$1 \le i \le M-1, 1 \le n \le N-1.$$

$$Y_{1,i}^0 = p(r_i, 0), Y_{2,i}^0 = p_k(r_i, 0), n = 0, 0 \le i \le M. \qquad (8)$$

$$\begin{cases} [\hat{\lambda}_1 \hat{Y}_{2,r}]_0 = \hat{q}_1, \\ [\hat{Y}_{1,r}]_0 = 0, i = 0, 0 \le n \le N-1. \end{cases} \qquad (9)$$

$$\begin{cases} [\hat{\lambda}_1 (\hat{Y}_{1,\bar{r}} + \hat{Y}_{2,\bar{r}}]_M = 0, \\ [\hat{Y}_{1,\bar{r}}]_M = \hat{\varphi}, i = M,\ 0 \le n \le N-1. \end{cases} \qquad (10)$$

Here,

$$(r\lambda_\alpha)_i^{(-)} = r_{i-\frac{1}{2}}\lambda_{\alpha, i-\frac{1}{2}}.$$

$$((r\lambda_\alpha)_i^{(-)} Y_{\bar{r}})_{\hat{r},i} = \hbar_i^{-1}\left[(r\lambda_\alpha)_{i+\frac{1}{2}} Y_{r,i} - (r\lambda_\alpha)_{i-\frac{1}{2}} Y_{\bar{r},i}\right],$$
$$\hat{Y}_{\alpha,i} = Y_{\alpha,i}^{n+1}, Y_{\alpha,i} = Y_{\alpha,i}^n, Y_{\alpha,r,i} = Y_{\alpha,\bar{r},i+1},$$
$$Y_{\alpha,\bar{r},i} = h_i^{-1}(Y_{\alpha,i} - Y_{\alpha,i-1}),$$

$$Y_{\alpha,t,i} = \tau_{n+1}^{-1}(\hat{Y}_{\alpha,i} - Y_{\alpha,i}),\ \hbar_i = \frac{1}{2}(h_1 + h_{i+1}),\ \alpha = 1, 2.$$

To solve the obtained nonlinear difference problem (6)–(10), iterative methods are used: the simple iteration method and the quasi linearisation method. The essence of the simple iteration method is that the values of the nonlinear coefficients of the problem (6)–(10) at any iteration cycle are taken from the previous iteration, i.e. they are considered known.

Thus, at each iteration, a linear system of algebraic equations with a tridiagonal matrix is obtained, which is solved by the matrix sweep method (Babaev *et al.*, 1977; Samarski *et al.*, 2003):

$$\begin{cases} -C_0^{(l)}\overline{Y}_0^{(l+1)} + B_0^{(l)}\overline{Y}_1^{(l+1)} = -F_0^{(l)}, i = 0, \\ A_i^{(l)}\overline{Y}_{i-1}^{(l+1)} - C_i^{(l)}\overline{Y}_i^{(l+1)} + B_i^{(l)}\overline{Y}_{i+1}^{(l+1)} = -F_i^{(l)}, 1 \le i \le M-1, \\ A_M^{(l)}\overline{Y}_M^{(l+1)} - C_M^{(l)}\overline{Y}_M^{(l+1)} = -F_M^{(l)}, i = M \end{cases} \qquad (11)$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix},$$

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}, F = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

The elements of the matrices and the components of the vector F have the form (using the simple iteration method):

$$c_{11,0} = h_1^{-1}, c_{12,0} = c_{21,0} = 0, c_{22,0} = h_1^{-1}(\lambda_1)_0^{(l)},$$

$$b_{11,0} = c_{11,0}, b_{12,0} = b_{21,0} = 0, b_{22,0} = h_1^{-1}(\lambda_1)_0^{(l)},$$

$$F_{1,0} = q, F_{2,0} = 0,$$

$$a_{11,i} = \hbar_i^{-1}a_1^{(-)}, a_{12,i} = a_{11,i}, a_{21,i} = \hbar_i^{-1}a_2^{(-)}, a_{22,i} = 0,$$

$$b_{11,i} = \hbar_i^{-1}a_2^{(+)}, b_{12,i} = b_{11,i}, b_{21,i} = \hbar_i^{-1}a_2^{(+)}, b_{22,i} = 0,$$

$$c_{11,i} = a_{11,i} + b_{11,i} + r_i \tau_{n+1}^{-1} [(m\rho_1)'(1-s)]_i^l ,$$

$$c_{12,i} = a_{12,i} + b_{12,i} + r_i \tau_{n+1}^{-1} [(m\rho_1)'(1-s) - m\rho_1 s']_i^{(l_1} ,$$

$$c_{21,i} = a_{21,i} + b_{21,i} + r_i \tau_{n+1}^{-1} [(m\rho_2)' s]_i^{(l_1} ,$$

$$c_{22,i} = a_{22,i} + b_{22,i} + r_i \tau_{n+1}^{-1} [(m\rho_2)' s]_i^{(l_1} , 1 \le i \le M-1,$$

$$a_{11,M} = h_M^{-1} (\lambda_1)_M^{(l_1)} , a_{12,M} = a_{11,M} , a_{21,M} = h_M^{-1} (\lambda_2)_M^{(l_1)} , \quad a_{22,M} = 0,$$

$$c_{11,M} = a_{11,M} , c_{12,M} = a_{11,M} , c_{21,M} = 1, c_{22,M} = 0, \quad F_{1,M} = 0, F_{2,M} = \varphi,$$

$$a_\alpha^{(-)} = h_i^{-1} (r\lambda_\alpha)_{i-\frac{1}{2}}^{(l)} , a_\alpha^{(+)} = h_{i+1}^{-1} (r\lambda_\alpha)_{i-\frac{1}{2}}^{(l)} , \alpha = 1{,}2,$$

The iteration process is as follows. According to the values of the grid functions $l - th$ iteration for a given time layer, the matrix sweep method from (16) determines $(l + 1)$ the - *th* iteration of the desired grid vectors. The iteration cycle on each time layer continues until $l = l_0$, at which

$$\max_i \left| Y_{\alpha,i}^{(l_0+1)} - Y_{\alpha,i}^{(l_0)} \right| \le \varepsilon_\alpha , \alpha = 1{,}2, \tag{12}$$

where $\varepsilon_1 , \varepsilon_2$ - given sufficiently small positive numbers.

RESULTS OF NUMERICAL CALCULATIONS

To compare and evaluate the influence of capillary forces and relative phase permeabilities on the displacement process, numerical calculations were carried out for the problem of exploitation of gas fields bordering active marginal waters; the following data were used:

$$R = R_0 = 100\, m; H = 10\, m; r_s = 0.1\, m; m = 0.2, \quad k = 3{,}02\, Darsi;$$

μ_1 = 0.00013 Poise; μ_2 = 0.01 Poise; μ_0 = 1 Poise; $k_0 = 10^{-12}\ m^2$;

$$\rho_1 = \rho_1^0 = 0.063\, {}^{g}\!/_{sm^3};\ \rho_2 = \rho_2^0 = 1\, {}^{g}\!/_{sm^3};\ P^0(r) = P_0 = 100\ atm;$$

where $R_0 , P_0 , \mu_0 , k_0 , \rho_1^0 , \rho_2^0$ are characteristic quantities used to reduce the original problem to a dimensionless form.

The curves of the dependence of relative phase permeabilities, capillary forces on water saturation and the coefficient of super compressibility on pressure, for the displacement of gas by water and water by gas, are shown in Figures 1–2 (Zakirov *et al.*, 1974).

In Rapoport-Leas model, the concept of "front" should be introduced. Usually, the point at which there is a saturation jump is taken as the "front" of displacement. Here, in the analysis of numerical calculations, the "front" was understood to be the first point at which water saturation differed from the saturation of "bound water" S = 0.23 by 0.0001.

Due to the fact that the "front" in the numerical solution of problems, taking into account capillary forces, can be located between the grid nodes, then when determining its position in this model, an error of the order of a step is allowed, which generally speaking depends on the problem under consideration and on time.

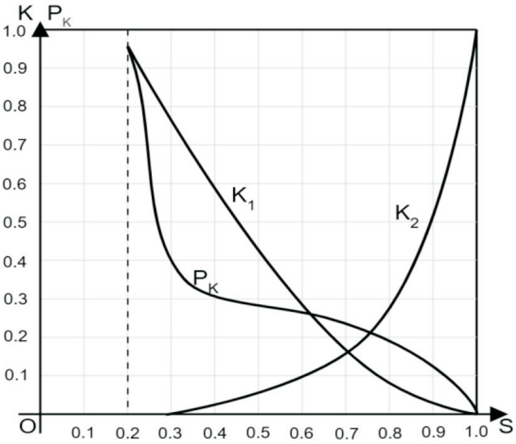

Fig. 1. Curves showing the dependence of K_1, K_2, and P_k on S during gas displacement by water.

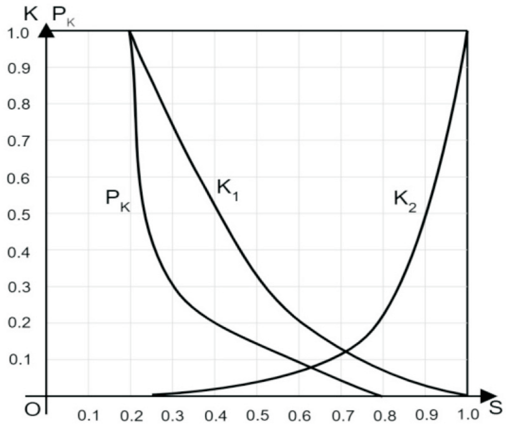

Fig. 2. Curves showing the dependence of K_1, K_2, and P_k on S during water displacement by gas.

The results of numerical calculation of one model problem using different grids for the spatial variable are shown in Figures 3 and 4. Here are presented the curves of gas saturation distribution at different moments of time: the dots correspond to the calculation results obtained by the difference-iteration method in moving grids, and the solid lines — without a moving grid. It is evident from the figure that near the injection well (where the steps in r are small) the results for both methods coincide, but in the area of a sharp change in gas saturation they differ. Moreover, this difference increases over time. Thus, after 3- x months of gas injection into the aquifer with k = 0.5 Darcy and R = 1 km, the spread of the gas volume, when calculated without a moving grid, occurs more intensively and outpaces the spread of the gas volume, calculated with the introduction of a moving grid by approximately 50 m, and after four months of injection — by approximately 70 m.

Figure 4 shows graphs of changes in water saturation in the formation for different moments in time, with the dotted lines corresponding to the solution obtained by the difference-iteration method without a moving grid.

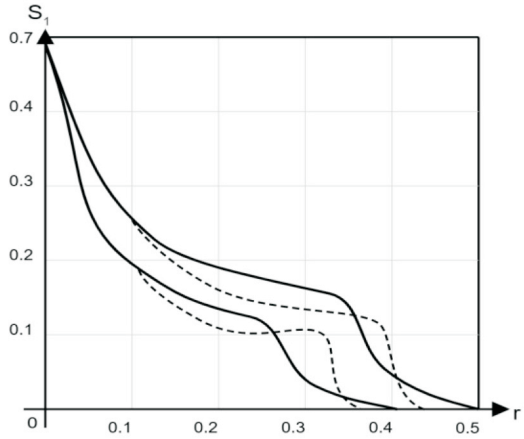

Fig. 3. Gas saturation distributions at different points in time.

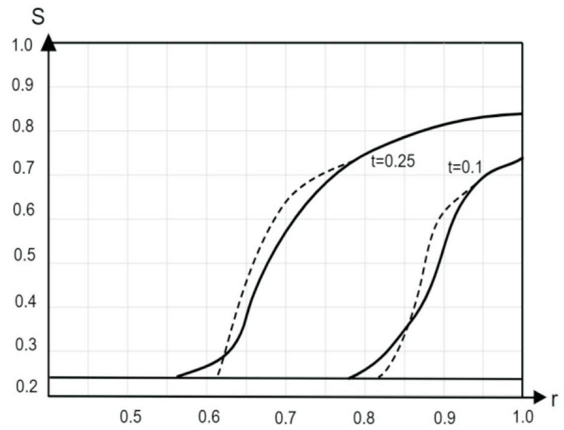

Fig. 4. Gas saturation distributions at different points in time.

The analysis of the obtained results showed that, firstly, at low flow rates, calculations even with very large time steps are carried out without any complications, however, as can be seen from Figure 5 (R = 100 m), the displacement "front" obtained by the difference-iteration method without a moving grid is ahead of the displacement "front" obtained with a moving grid by approximately 14 m. And secondly, at high flow rates, the difference-iteration method without a moving grid gives oscillating solutions (Fig. 5).

To determine the degree of influence of the formation heterogeneity in permeability on the filtration process, calculations were carried out for different values of permeability K. The results of the numerical calculations are presented in Tables 1–2. It is evident from the tables that higher formation permeability leads to the gas moving along the formation over a greater distance; however, the gas saturation coefficient (displacement completeness) is lower. For example, if k = 0.5 D the gas saturation coefficient is 0.195 and the gas moves along the formation over a distance of 875 m, while at k = 0.5 D this coefficient will be equal to 0.174, and the gas displacement is approximately 1000 m.

To assess the heterogeneity of a formation, the following dimensionless parameter is usually adopted (Levykin, 1973):

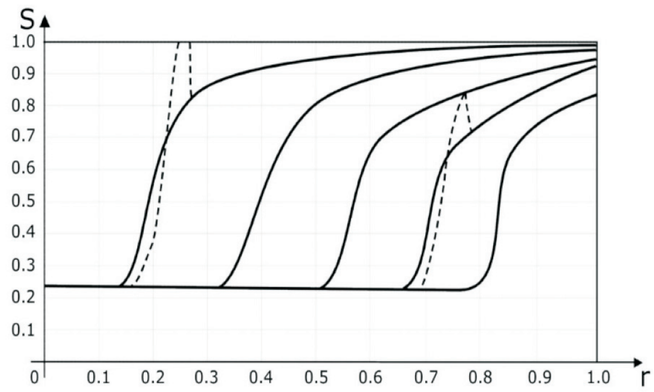

Fig. 5. Changes in water saturation at high flow rates.

Table 1. Distribution of gas pressure (in atm.), after four months of injection

K, D / r, m	0.5	1.0	1.5	2.0
1	88.63	71.11	63.27	58.47
20	87.49	70.41	62.73	58.04
68	86.36	69.74	62.23	57.64
132	85.14	69.04	61.73	57.24
250	82.99	67.83	60.86	56.56
375	80.75	66.53	59.92	55.11
500	78.65	65.28	59.00	54.44
625	76.68	64.13	58.16	53.82
750	73.99	63.00	57.36	53.14
875	69.41	61.18	56.40	51.94
1000	65.89	58.73	54.70	50.68

Table 2. Distribution of gas saturation after four months of injection

K / r, m	0.5	1.0	1.5	2.0
1	0.77818	0.7631	0.7566	0.7511
20	0.5413	0.5490	0.5522	0.5536
68	0.3867	0.3993	0.4059	0.4098
132	0.3090	0.3214	0.3278	0.3318
250	0.2338	0.2433	0.2487	0.2524
75	0.1992	0.2049	0.2083	0.2107
500	0.1811	0.1841	0.1859	0.1874
625	0.1644	0.1722	0.1733	0.1738
750	0.0994	0.1528	0.1639	0.1643
875	0.0176	0.0745	0.1174	0.1376
1000	0.0052	0.0133	0.0306	0.0528

$$K_{no} = \frac{1}{K_{sp}} \sqrt{\frac{1}{S} \sum_{i=1}^{n} S_i (K_i = K_{sp})^2} \qquad (13)$$

Here $S = \pi(R^2 - r_s^2), K_{no} \geq 0$ - the heterogeneity parameter of the formation, K_{sp} – the average weighted permeability value of the formation over the area of the formation, K_i – the permeability of different zones of the formation with an area of $S_i, i = \overline{1,n}$. Since the thickness of the hypothetical layer under consideration is constant, then K_{sp} is defined as follows:

$$K_{sp} = \frac{2}{R^2 - r_0^2} \sum_{i=1}^{n} \int_{r_i}^{r_{i+1}} r K_i(r) dr \tag{14}$$

where $r_1 = r_0$, $r_n R$.

Calculations have shown that with increasing value K_{no} the degree of heterogeneity also increases and the influence of heterogeneity, generally speaking, depends on the nature of the change in permeability. Table 3 shows the results of calculations after the injection period for the piecewise constant function and and for the average value K_{sp}. It can be seen from the table that in the third variant the influence of heterogeneity is significant.

Thus, for lower values of permeability (homogeneous layer, Table 1), and for higher values of the parameter K_{no} (heterogeneous formation, Table 3), the gas pressure in the formation increases significantly, and with an excessive increase in pressure, various harmful consequences are possible: opening of existing or formation of new cracks in the roof of the storage facility, underground gas losses, explosions and fires, which indicates the need to take into account all of the above factors.

The question of the influence of the real properties of gas and water on the displacement process was also studied. The densities of gas and water at the current reservoir pressure were determined by the formulas:

$$\rho_1 = A_g P_1 / Z(P_1), \tag{15}$$

$$\rho_2 = A_g P_2 + B_w. \tag{16}$$

In the numerical calculations it was assumed: A_g = 0,023, A_w = 0,01033, B_w = 0,99989. The following cases were considered: the density of water is constant and equal to ρ_2^0, the density of gas is determined by formula (16) for $Z(P_1) \equiv 1$; the density of water and gas is determined by formulas (15) and (16), where $Z(P_1) \equiv 1$; the compressibility of water and the super compressibility of gas are taken into account ($Z(P_1) \equiv 1$; determined according to Fig. 6). The calculation results are presented in Table 4. From the analysis of the presented results it follows that taking into account the real properties of gas and water has an impact on the displacement process, and therefore, in all calculations the real properties of the filtered fluids were taken into account.

CONCLUSIONS

A computational algorithm was developed on an adaptive grid taking into account relative phase permeabilities and capillary forces to solve the problem, allowing for the consideration of the features and some properties of the solution. The results showed that at low flow rates, calculations even with very large time steps are carried out without complications, however, the displacement "front", obtained without a moving grid is ahead of the displacement "front"

Table 3. The influence of formation heterogeneity on the filtration process

K =	$\begin{cases} 1{,}5, r_0 \le r < 2, \\ 2.0, 2 \le r < 6, \\ 2.5, 6 \le r \le 10 \end{cases}$		$\begin{cases} 3.75, r_0 \le r < 2, \\ 1.00, 2 \le r < 5, \\ 4.00, 5 \le r \le 10 \end{cases}$		$\begin{cases} 2.0, r_0 \le r < 2.5, \\ 1.0, 2.5 \le r < 5, \\ 0.25, 7.5 \le r \le 10 \end{cases}$	
K_{no} / K_{sr}	0.123/2.3		0.36/3.36		0.789/0.58	
r,m	*KRKRK*	K_{sr}	*K*	K_{sr}	*K*	K_{sr}
1	54.85	56.15	56.30	51.96	110.75	82.94
2	54.78	56.07	56.26	51.91	110.45	82.53
3	54.72	56.02	56.24	51.87	110.26	81.74
4	54.67	55.97	56.21	51.84	110.12	81.60
5	54.64	55.93	56.19	51.82	110.00	81.49
6	54.60	55.90	56.18	51.81	109.90	81.40
8	54.54	55.84	56.16	51.78	109.41	81.24
12	54.43	55.74	56.12	51.73	108.94	80.98
20	54.25	55.58	56.06	51.66	108.27	80.53
36	53.92	55.30	55.98	51.56	107.07	79.72
68	53.37	54.81	55.85	51.40	105.05	78.36
132	52.44	54.04	55.64	51.13	101.69	76.06
260	49.02	52.13	55.27	50.69	95.14	67.94
500	43.69	46.18	54.68	49.92	80.45	53.40

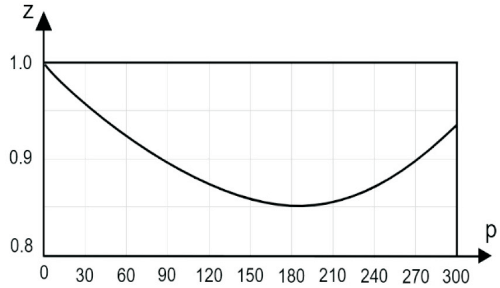

Fig. 6. Dependence of the overcompression coefficient on pressure.

Table 4. Distribution of pressure (in atm.) and gas saturation in the formation

Distribution of pressure (in atm.) and gas saturation in the formation						
Var: r, m	$\rho_1 = \rho_1(P_1)$, $Z(P_1) = 1$, $\rho_2 = \rho_2^0$		$\rho_1 = \rho_1(P_1)$, $Z(P_1) = 1$, $\rho_2 = \rho_2(P)$		$\rho_1 = \rho_1(P_1)$, $Z = Z(P_1)$, $\rho_2 = \rho_2(P)$	
1	52.76	0.7316	51.55	0.7333	50.69	0.7298
2	52.70	0.7165	51.49	0.7179	50.64	0.7151
3	52.67	0.7022	51.46	0.7084	50,61	0.7061
4	52.64	0.7007	51.43	0.7017	50.58	0.6988
5	52,62	0.6868	51.41	0.6895	50.56	0.6844
6	52.61	0.6748	51.39	0.6772	50.55	0.6728
8	52.58	0.6573	51.36	0,6590	50,52	0.6557
12	52,54	0.6102	51.32	0.6126	50.48	0.6082
20	52.47	0.5551	51.25	0.5573	50.41	0.5535
36	52.38	0.4994	51.16	0.5016	50.32	0.4973
68	52.22	0.4156	51,00	0.4180	50.17	0.4137
132	51.97	0.3313	50.75	0.3335	49.93	0.3303
260	51,56	0.2794	50.33	0.2806	49.52	0.2785
500	50.84	0.2109	49.61	0.2125	48.82	0.2087
1000	48.85	0,0802	47.66	0.0839	46.85	0,0760
1500	45.60	0,0127	44.78	0,0135	44.14	0,0124

of the one obtained with a moving grid. Using the developed method, the influence of various factors (wettability and permeability of rocks, viscosity of gas, water, etc.) on the process of two-phase filtration was investigated using computational experiments. The obtained results can be recommended for use in the creation, operation and forecasting of changes in the main indicators of underground gas storage facilities.

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SKAITLISKĀ METODE JAUTĀJUMA RISINĀŠANAI PAR ŪDENS IZSPIEŠANU AR GĀZI

Darba mērķis bija izstrādāt efektīvas skaitliskās metodes, risinot problēmu, saistītu ar viena šķidruma aizvietošanu ar citu porainā vidē, kas notiek naftas un gāzes ieguves laikā, kā arī pazemes gāzes uzglabāšanas laikā. Piedāvāta iterācijas metode, lai noteiktu racionālu veidu, kā pārvaldīt rezervuāru.