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Using the GUGiK grid model to determine the geometric center of Poland

Abstract. Determining the geometric center of a given geographical area – such as a country, province, or region – is a key component in cartography visualization, spatial analysis, territorial planning, and the symbolic marking of central points. This article presents a comprehensive methodology for calculating the geometric center of Poland, utilizing a regular grid of spatial data models defined by GUGiK (Główny Urząd Geodezji i Kartografii, Head Office of Geodesy and Cartography), with cells measuring $0.01^\circ \times 0.01^\circ$, in accordance with national cartographic and geodetic standards. The area of Poland was subdivided into over 400,000 sub-areas (trapezoids), for which centroids were calculated based on the coordinates of their vertices. The overall geometric center of the country was then derived as a weighted average of these centroids, using area of the small sub-areas as the weighting factor.

Several computational variants were analyzed, involving different coordinate reference systems (PL-LAEA, PL-1992, and local systems), as well as various projection methods from ellipsoidal to planar surfaces. The study identified the most accurate approach, capable of determining the geometric center of Poland with a precision of a few centimeters. Special attention was given to the impact of boundary definitions – whether including or excluding territorial waters – demonstrating that the location of the center can shift by several kilometers depending on the adopted boundary variant.

Final coordinates of Poland's geometric center were determined for both variants: without territorial waters (variant A) and including them (variant B), and compared with previous attempts to define the country's center. The paper highlights the importance of consistent boundary definitions and appropriate reference models in geodetic calculations and shows the potential to apply the proposed methodology to smaller administrative units such as voivodeships, counties, and municipalities. The findings are relevant for geodetic authorities, spatial planners, and for promotional or symbolic purposes related to the designation of national or regional “geographical centers”.

Keywords: cartographic projections, geometric center, centroid, trapezoidal grid, spatial analysis

1. Introduction

Determining the location of the center point of a given area (country, region, etc.) is an important element of geographical orientation and may also have economic, social, and even symbolic significance. The geometric center of a country is sometimes used in spatial analyses, e.g., to assess the spatial relationships of other objects relative to the center, which has wide

application in geographic information systems (Burrough et al., 2015; Longley et al., 2015). A properly marked point in the field – the center of a country, part of a country, or a given city – can become not only a reference point, but also a tourist attraction and an element of local identity.

Determining the centroid also plays an important role in cartographic visualization, where it is a key element in map design and interpre-

tation. Indicating the center of an area allows for proper centering of the cartographic composition, which affects the perception of spatial relationships and facilitates the viewer's orientation in the terrain (Dent et al., 2008). The use of a computational centroid is particularly useful in map generalization and the creation of small-scale thematic representations, where simplification of shapes must go hand in hand with maintaining a clear reference to the location of the area (Brewer, 2016). Furthermore, in multimedia cartography and GIS systems, the designated centroid often is the basis for automatic map scaling and cropping (Slocum et al., 2009). Research also emphasizes the importance of clearly defining the centroids in transport network analyses and public service location planning, which, in thematic visualizations, allows for a clear presentation of spatial accessibility (Jabłoński et al., 2024; Kimerling et al., 2016). Contemporary map design standards recommend the conscious and methodical use of the concept of centroid to ensure the consistency and comparability of cartographic studies (Peterson, 2021).

For these reasons, methods are being developed to determine such points in an unambiguous and precise manner. The geometric center of an area is most often identified with the centroid of an irregular polygon, whose boundaries are defined by coordinates in the adopted reference system (Hofmann-Wellenhof & Moritz, 2005). Classic examples include: the determination of the center of France by Adolphe Joanne (Institut national de l'information géographique et forestière [IGN], 2024), the determination of the center of the United States using a cardboard model (Rogerson, 2015), and the determination of the center of Australia based on the division of the area into equal parts (Aitken & Betts, 2008; Royal Geographical Society of Queensland, 2018). The position of the center depends on many factors: primarily on the shape and extent of the analyzed area, which may change over time (Prescott & Triggs, 2008), and on the calculation method used. Contemporary approaches are often based on dividing the analyzed territory into smaller units (e.g., trapezoidal grids, Voronoi polygons, etc.) and averaging the coordinates of local centroids using weights (most often area) (Bláha & Dolejš, 2017; Okabe et al., 2000; Pędzich, 2025). This methodology is used, among others,

by the French National Geographic Institute (Hangouet, 2019), as well as by American authors analyzing the location of the centers of individual US states (Rogerson, 2015).

Such calculations are usually performed in projected coordinate systems (e.g., PL-LAEA, PL-1992), which transform the surface of the ellipsoid onto a plane. The choice of projection affects the accuracy of measurements, especially in the case of surface area calculations (Bugayevskiy & Snyder, 1995). In GIS environments, such as QGIS, functions are available that automate the process of determining the centroid, center of mass, or so-called pole of inaccessibility (Deakin et al., 2002; Pędzich, 2025; QGIS Development Team, 2025). However, the choice of the appropriate function should be conscious and mathematically justified (Kraak & Ormeling, 2020).

All these studies emphasize the importance of clearly defining the boundaries of the analyzed area and selecting the appropriate calculation model, as changing any of these parameters can lead to a shift in the determined center by up to several kilometers (Nielsen & Hansen, 2015). Unfortunately, many publications lack algorithmic details and source data, which makes it difficult to compare results. Also in Poland, points marked as the "center of the country" are often the result of historical administrative decisions, not precise geodetic calculations.

In the 1960s, the Head Office of Geodesy and Cartography (Główny Urząd Geodezji i Kartografii, GUGiK) designated the geometric center of Poland in the town of Piątek, but the lack of documentation makes it impossible to verify this location today (Cisak & Nycek, 2001). In 2018, new initiatives to determine the center of Poland emerged (Meus, 2019; Róžański, 2018) based on dividing the country into sub-areas and weighted average centroids. Tests have shown that the boundary used to define the area (with or without territorial waters) significantly affects the final result of the calculations.

In technical and scientific literature, terms such as "centroid", "geometrical center" and "center of mass" differ depending on the spatial context in which they are used (Encyclopedia Britannica, 2025; Polskie Towarzystwo Informacji Przestrzennej, 2025; Weisstein, 2025). Therefore, this study adopts terminology in which "centroid" refers to the center of a local sub-area, and "geometric center" refers to the

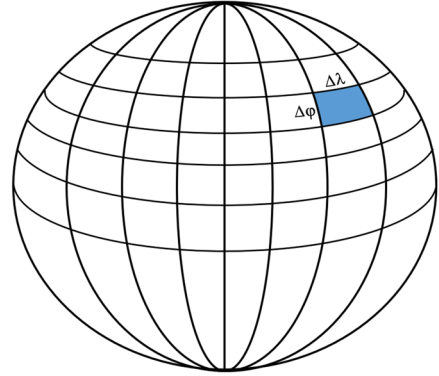
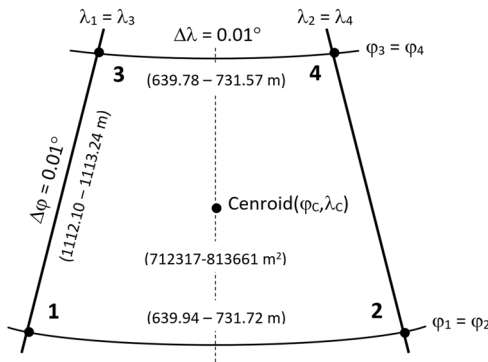


Figure 1. Example of a sub-area – meridian-parallel trapezoid; on the left: position of the trapezoid on the ellipsoid; on the right: dimensions of one of the trapezoids (within the range: $\varphi_{\min} = 49^\circ$, $\varphi_{\max} = 55^\circ$, $\lambda_{\min} = 14^\circ$, $\lambda_{\max} = 24^\circ$)

center of the entire study area. This distinction is necessary in analyses conducted using regular geospatial grids, such as those based on GUGiK models (2024).

2. Algorithm for calculating the coordinates of the geometric center of Poland

The assumptions adopted to calculate the coordinates of the geometric center for the area of Poland are presented below.

1. The boundary of the area is a polygon on a geodetic reference surface (ellipsoid) whose vertices have coordinates φ , λ , $h = 0$ in the GRS80h geocentric geodetic coordinate system and the PL-ETRF2000 reference system (Obwieszczenie, 2024). The vertices are the points of the Polish border taken from the National Register of Boundaries (GUGiK, 2025).

2. The area of the above-mentioned polygon is divided into sub-areas (trapezoids) forming a meridian-parallel grid on an ellipsoid (Figure 1, left), with dimensions $\Delta\varphi = \Delta\lambda = 0.01^\circ$. In Poland, such a sub-area has an average size of 1113 m $\times$ 700 m and an average area of 76 ha (Figure 1, right). The ellipsoidal coordinates of the nodes of such a grid are consistent with the description of the analogous meridian-parallel grid used to define the spatial data models of the GUGiK (2024). The nominal range of coordinates of the nodes of this grid is: from $\varphi_{\min} = 49.00^\circ\text{N}$ to $\varphi_{\max} = 55.00^\circ\text{N}$ and from $\lambda_{\min} = 14.00^\circ\text{E}$ to $\lambda_{\max} = 24.20^\circ\text{E}$ (GUGiK, 2024). Only those sub-areas that do not extend beyond the accepted boundaries of Poland will

be used to calculate the coordinates of the geometric center.

3. Due to the irregular shape of Poland in the above-mentioned meridian-parallel grid, there will be complete sub-areas located entirely within Poland and incomplete sub-areas – border sub-areas located near its border (Figure 2). Examples of both complete (CS) and border (BS) sub-areas are shown in Figure 3. Complete sub-areas have four vertices at the intersections of the grid's meridians and parallels, while marginal areas may have at least three vertices at the intersections of meridians, parallels, and the area boundary line (Figure 3).

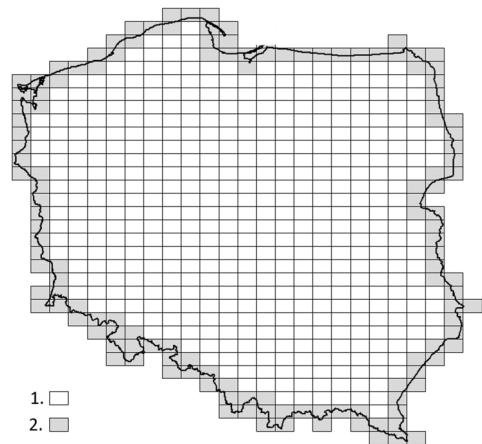


Figure 2. Diagram showing the division of Poland into sub-areas – meridian-parallel trapezoids with a size of $\Delta\varphi = \Delta\lambda = 0.01^\circ$ (1 – complete sub-areas; 2 – incomplete sub-areas – border areas)

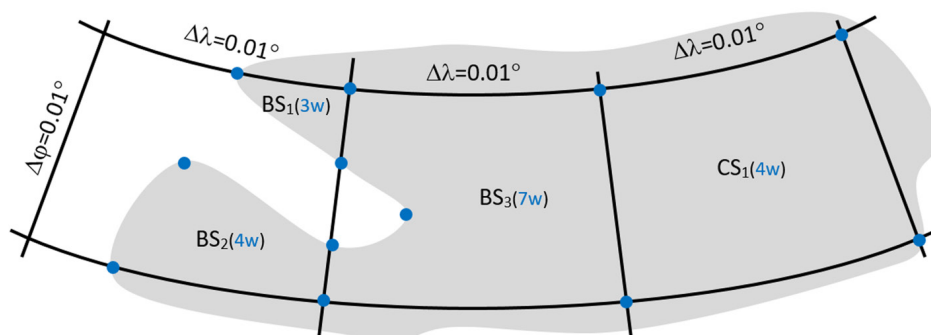


Figure 3. Examples of a complete sub-area CS_1 with four vertices and boundary sub-areas BS_1 with three vertices, BS_2 with four vertices, BS_3 with seven vertices

4. The coordinates of the geometric center of the area will be calculated in two stages. In the first stage, the coordinates of the sub-areas' centroids will be calculated, and in the second stage, the average coordinates will be calculated, which will also be the coordinates of the geometric center of the entire area. The average coordinates will be calculated using weights, i.e., values characterizing the sub-areas. It was assumed that the weight value will be equal to the area of a given sub-area.

5. The calculation of the coordinates of the sub-area centroids and the coordinates of the geometric center of the entire area can be performed directly on the ellipsoid (in the geodetic coordinate system φ, λ) or on a plane, in a selected projection of the ellipsoid onto a plane (in the X,Y coordinate system), and then converted to the geodetic system φ, λ .

6. The accuracy of calculating the coordinates of the geometric center should not be less than 10 centimeters, because the point that is the geometric center of Poland may be marked in the field with a sign in the form of an obelisk (monument).¹

The above assumptions indicate that the basis for the algorithm for calculating the coordinates of the geometric center of the area is the calculation of the coordinates of the centroids of the sub-areas and the area of each sub-area.

3. Calculation of the coordinates of the sub-area centroid and its area (stage I)

3.1. Calculations for complete sub-areas

A trapezoidal sub-area on an ellipsoid with vertices 1, 2, 3, 4 (Figure 1, right) has an axis of symmetry lying in the plane of the meridian $\lambda_m = (\lambda_1 + \lambda_2)/2$. Therefore, the center of the sub-area will lie on this meridian and, consequently, the ellipsoidal longitude of the centroid of this trapezoid will be equal to $\lambda_C = \lambda_m$.

Calculating the ellipsoidal latitude of the sub-area centroid, certain simplifications of shape can be assumed, resulting from its small size ($\Delta\varphi = \Delta\lambda = 0.01^\circ$). For this purpose, the shape of sub-areas located in five places (A, B, C, D, E) in the area covering Poland was analyzed (Figure 4). The difference in the length of the northern and southern parallels for any such trapezoid is a maximum of 16 cm. Therefore, such a trapezoid could be considered approximately as a rectangle whose two sides (meridian arcs) are identical and whose two bases (parallel arcs) differ by a maximum of 16 cm. The coordinates of the geometric center of such a rectangle could be calculated as the average values of the latitude and ellipsoidal longitude. However, the impact of this simplification on the calculated latitude of the centroid would need to be analyzed.

The legs and bases of the meridian-parallel trapezoid are arcs of the meridians and parallels of the ellipsoid. For a small trapezoid, the lengths of these arcs differ minimally from the

¹ The value refers to the accuracy of the location of the first group of terrain details, which include, among others, monuments (Obwieszczenie, 2022).

lengths of the chords. The difference in length Δd between an arc d with radius r and the corresponding chord can be estimated from equation (1):

$$\Delta d = \frac{d^3}{6r^2} \quad (1)$$

The lengths of a short meridian arcs d_p and parallel d_r of an ellipsoid (e.g., several kilometers long) can be calculated with an accuracy of 1 mm as the lengths of arcs on a sphere with a radius of, for example, $R = 6371$ km, according to the relationship (2):

$$\begin{cases} d_p = R\Delta\varphi \\ d_r = R\cos\varphi\Delta\lambda \end{cases} \quad (2)$$

In a trapezoid with dimensions $\Delta\varphi = \Delta\lambda = 0.01^\circ$ (approximately $1100 \text{ m} \times 700 \text{ m}$), the differences in arc length and chord length for the meridian arc Δd_p and the parallel Δd_r calculated using equation (1) based on the values of d calculated using equation (2) are of the order of a thousandth of a millimeter. Therefore, it can be assumed that a meridian-parallelogram trapezoid with dimensions $\Delta\varphi = \Delta\lambda = 0.01^\circ$ can be considered a flat trapezoid with a southern base $d_{rS} = d_{r12}$ and a northern base $d_{rN} = d_{r34}$ and a height $d_p = d_{p13} = d_{p24}$ (Figure 1, right). The lengths of the bases d_r and heights d_p of a given trapezoid can be expressed as the lengths of the segments between the vertices of the trapezoid and calculated using the X, Y, Z coordinates of the vertices. The X, Y, Z coordinates of a point on the ellipsoid are calculated on the basis of the geodetic coordinates φ, λ, h (Szpunar, 1982).

To determine the latitude φ_C of the center of mass for such a trapezoid, one could use the formula used for flat trapezoids (3) (Harris & Stocker, 1998):

$$d = \frac{h(a+2b)}{3(a+b)} \quad (3)$$

where: a, b – the larger (lower) and smaller (upper) bases of a flat trapezoid, respectively,

h – the height of the trapezoid,

d – the distance between the center of mass and base a .

In the meridian-parallel trapezoid (Figure 1, right), the equivalents of bases a and b in formula (3) are the lengths of the southern and northern arcs of the parallel, respectively, and the equivalent of height h is the arc length of the meridian corresponding to the latitude difference $\varphi_3 - \varphi_1$. The distance d corresponds to the latitude difference $\varphi_C - \varphi_1$. However, given the small length difference between the southern and northern bases of the trapezoid (maximum 16 cm), one may wonder whether such a trapezoid can be treated as a rectangle, for which the latitude φ_C of the centroid can be calculated as the average value of the latitudes φ_S and φ_N . Therefore, it would be necessary to estimate the extent to which the above-mentioned difference between the lengths of the bases of the trapezoid affects the calculation of the centroid coordinate as the average of the widths of the southern and northern bases of the trapezoid. For this purpose, formula (3) can be used, which expresses the relationship between the length d (corresponding to the difference $\varphi_C - \varphi_1$) and the lengths of the bases of the trapezoid. The length error m_d , depending on the base length error (e.g., m_a), will be expressed by the relationship (4a):

$$m_d = \frac{\partial d}{\partial a} = \frac{\partial}{\partial a} \left(\frac{h(a+2b)}{3(a+b)} \right) \cdot m_a = \frac{hb}{(a+b)^2} \cdot m_a \quad (4a)$$

where the equivalent of the error m_a will be the difference in length $m_a = a - b$. In the case of a small trapezoid, when the difference in length between bases a and b is minimal, a simplification $a \approx b$ can be applied in the denominator of formula (4a). Under these assumptions, formula (4a) simplifies to (4b):

$$m_d \approx \frac{h}{12b} \cdot m_a \quad (4b)$$

Assuming average trapezoid dimensions of $0.01^\circ \times 0.01^\circ$ for the area of Poland ($h \approx 1113 \text{ m}$, $m_a \approx 0.16 \text{ m}$, $b \approx 700 \text{ m}$) we obtain $m_d = 0.02 \text{ m}$.

This value represents the inaccuracy of the centroid position on the meridian, i.e., the inaccuracy of the calculated ellipsoidal latitude φ_C , resulting from simplifying the trapezoid to a rectangle and calculating φ_C as the average of φ_1 and φ_3 . The inaccuracy of the φ_C coordinate of

a few centimeters will not reduce the accuracy of the coordinate of the geometric center of the entire area, calculated from many sub-areas (trapezoids) and assumed to be at the decimeter level.

Therefore, the coordinates of the centroid φ_C, λ_C of the complete sub-areas, which are trapezoidal in shape along the meridian-parallel on the ellipsoid and with dimensions $\Delta\varphi = \Delta\lambda = 0.01^\circ$ can be calculated (with centimeter accuracy) as the average values of the width and ellipsoidal length of the vertices of this trapezoid (Figure 1, right), according to equation (5).

$$\begin{cases} \varphi_C = \frac{\varphi_1 + \varphi_2}{2} \\ \lambda_C = \frac{\lambda_1 + \lambda_2}{2} \end{cases} \quad (5)$$

The coordinates of the centroid φ_C, λ_C of the complete sub-areas calculated on the ellipsoid using formula (5) will be used later in this study as a reference in tests, in relation to coordinates calculated using other methods (see Section 5).

The area P_E of the meridian-parallel trapezoid in the range from φ_1 to φ_2 and from λ_1 to λ_2 (complete sub-area) can be calculated directly on the ellipsoid using formula (6) (Szpunar, 1982):

$$P_{E(\varphi_1\varphi_2\lambda_1\lambda_2)} = a^2(1-e^2)(\lambda_2 - \lambda_1) \left[\begin{aligned} &(\sin \varphi_2 - \sin \varphi_1) + \frac{2}{3}e^2(\sin^3 \varphi_2 - \sin^3 \varphi_1) + \\ &\frac{3}{5}e^4(\sin^5 \varphi_2 - \sin^5 \varphi_1) + \\ &\frac{4}{7}e^6(\sin^7 \varphi_2 - \sin^7 \varphi_1) + \frac{5}{9}e^8(\sin^9 \varphi_2 - \sin^9 \varphi_1) + \dots \end{aligned} \right] \quad (6)$$

where: a, e – the major axis and the first eccentricity of the ellipsoid, respectively.

3.2. Calculation for boundary sub-areas

Boundary sub-areas are irregular polygons on the surface of the ellipsoid, with an area smaller than the area of the complete sub-area (Figure 3). For such a border sub-area, which is a polygon with vertices $i = 1, 2, 3, \dots, n$, it is convenient to calculate the centroid coordi-

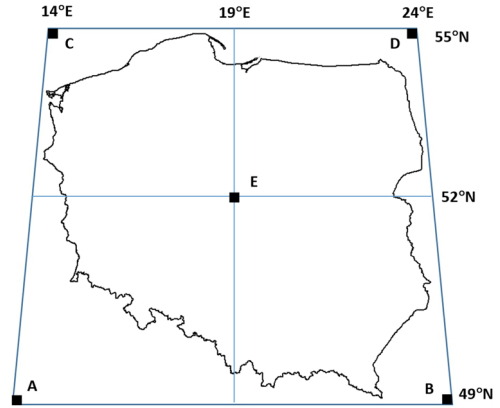


Figure 4. Diagram showing the location of the five test sub-areas A-E (trapezoids $0.01^\circ \times 0.01^\circ$) in Poland, approximated by the parallels 49°N and 55°N and the meridians 14°E and 24°E

ates using formula (7) applied to a plane for flat polygons (Bourke, 1988):

$$\begin{cases} x_C = \frac{1}{6P_P} \left| \sum_{i=1}^n (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \right| \\ y_C = \frac{1}{6P_P} \left| \sum_{i=1}^n (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \right| \end{cases} \quad (7)$$

where: x_i, y_i – reduced coordinates of the vertices of the polygon, e.g., using the

average coordinates x, y , in the Cartesian coordinate system XY ,
 P_P – area of the polygon on the plane

In this case, the surface of the ellipsoid should be mapped onto a plane using a selected map projection, obtaining the x, y coordinates of the vertices in the Cartesian system, based on the ellipsoidal coordinates φ, λ . A single projection can be used for the entire area or a local projection, i.e., only for a given sub-area. The P_P area

of such a polygon on a plane can be calculated using formula (8) (Harris & Stocker, 1998):

$$P_P = \frac{1}{2} \left| \sum_{i=1}^n x_{i+1} (y_{i+2} - y_i) \right| \quad (8)$$

4. Calculation of the coordinates of the geometric center of the entire area (stage II)

In accordance with the assumptions (Section 2, point 4), the coordinates a_{SG} , b_{SG} of the geometric center of the area will be calculated as weighted averages with weight S , based on the coordinates a_C , b_C of the centroids of all $j = 1, 2, 3, \dots, k$ sub-areas, according to formula (9):

$$\begin{cases} a_{SG} = \frac{1}{\sum_{j=1}^k S_j} \sum_{j=1}^k a_{C,j} \cdot S_j \\ b_{SG} = \frac{1}{\sum_{j=1}^k S_j} \sum_{j=1}^k b_{C,j} \cdot S_j \end{cases} \quad (9)$$

The coordinates a_C , b_C can be Cartesian coordinates (x_C , y_C) on a plane or coordinate φ_C , λ_C , on an ellipsoid. The weight S can be, for example, the area of a subregion (P) on an ellipsoid (P_E) or a plane (P_P). It is possible to use another value representative of the sub-area (e.g., population, etc.) as the weight.

4.1. Calculation variant tests

In order to optimize the algorithm for calculating the coordinates of the sub-area centroid or the geometric center of the area, tests were performed for several calculation variants in different coordinate systems. The tests were performed for five complete sub-areas (A-E) located in Poland (Figure 4). The calculations were performed on an ellipsoid and on the planes of national and local coordinate systems.

4.1.1. Variant I (reference)

– calculation of the coordinates φ_C , λ_C of the sub-area centroids as average values based on the coordinates φ_W , λ_W of the sub-area vertices using formula (5),

– calculation of the coordinates φ_{SG} , λ_{SG} (9) of the geometric center of the area as a weighted average of φ_C , λ_C with a weight equal to the area of the sub-area P_E on the ellipsoid (6).

The coordinates of the centroids of the sub-areas and the geometric center of the area (as well as the lengths of the sides and areas of the sub-areas) obtained in this variant on the ellipsoid can be considered the most probable. Therefore, the results from this variant were taken as a reference for the results obtained in other variants.

4.1.2. Variant IIa

– calculation of the plane coordinates x_W , y_W of the vertices of the sub-areas based on φ_W , λ_W , in the projection used in a given plane coordinate system,

– calculation of the coordinates x_C , y_C of the centroids of the sub-areas using formula (7),

– calculating (eq. 9) the coordinate X_{SG} , Y_{SG} of the geometric center of the area as a weighted average of x_C , y_C with a weight equal to the area P_P sub-area calculated on the plane using formula (8),

– convert X_{SG} , Y_{SG} to ellipsoidal coordinates φ_{SG} , λ_{SG} .

4.1.3. Variant IIb

Calculation steps a), b) and d) are as in option IIa, in step c) the area P_E calculated using formula (6) on the ellipsoid is used as the weight.

4.1.4. Variant IIIa

Calculation steps a) and b) are as in variant IIa, – conversion of x_C , y_C to ellipsoidal coordinates φ_C , λ_C ,

– calculation of the coordinates φ_{SG} , λ_{SG} of the geometric center of the area using formula (9) as a weighted average of φ_C , λ_C with a weight equal to the P_P area calculated using formula (8) on the plane.

4.1.5. Variant IIIb

Steps a) and b) are as in variant IIIa, in step c) the area P_E calculated using formula (6) on the ellipsoid is used.

Calculations in variant I were performed on the current reference surface, the GRS80 ellipsoid (in the PL-ETRF2000 system, Obwieszczenie, 2024). Calculations in variants II

and III are performed using the X, Y coordinate system on a plane. For this purpose, two national plane coordinate systems applicable in Poland (Obwieszczenie, 2024) and two local plane systems used for a given sub-area were selected. Due to the projection of the ellipsoid surface onto a plane used in the above-mentioned systems, we can expect distortions in length, angle, or surface area. Below are the characteristics of the above systems:

- PL-1992 national system (with equiangular projection, Obwieszczenie, 2024),
- PL-LAEA system (with equiangular azimuthal projection, Obwieszczenie, 2024),
- the “Local-GK” system – limited to a given sub-area, a rectangular XY system on a plane in Gaussian-Kruger projection system tangent to the ellipsoid, with a central meridian whose ellipsoidal length was calculated on the basis of the minimum $\lambda_{\min}$ and maximum $\lambda_{\max}$ values of the ellipsoidal longitude of the vertices of the sub-area, using formula (10):

$$\lambda_0 = \frac{\lambda_{\min} + \lambda_{\max}}{2} \quad (10)$$

- the “Local-CTP” system – limited to a given sub-area, a rectangular XOY layout on a quasi-plane tangent at a selected point of the sub-area, in which the x, y coordinates of the i -th vertex of the sub-area are calculated using formulas (11):

$$\left\{ \begin{array}{l} \varphi_0 = \frac{\varphi_{\min} + \varphi_{\max}}{2} \\ \lambda_0 = \frac{\lambda_{\min} + \lambda_{\max}}{2} \\ M_0 = \frac{a(1-e^2)}{(1-e^2 \sin^2 \varphi_0)^{3/2}} \\ N_0 = \frac{a}{(1-e^2 \sin^2 \varphi_0)^{1/2}} \\ x_C = M_0 \cdot (\varphi_i - \varphi_0) \\ y_C = N_0 \cdot \cos \varphi_0 \cdot (\lambda_i - \lambda_0) \end{array} \right. \quad (11)$$

The first part of the test consisted in comparing the shape of sub-areas that were implemented on the surfaces of the above-mentioned planar systems with the shape of a sub-area on an ellipsoid. Therefore, the lengths of the sides $d_{12}, d_{34}, d_{13}, d_{24}$ (Figure 1, right) of sub-areas A-E (Figure 4), their surface areas (P),

and the coordinates of the sub-area centroids ($\Delta\varphi_C, \Delta\lambda_C$) were calculated. The lengths of the sides on the plane were calculated as the lengths of segments based on the x_W, y_W coordinates of the vertices of the sub-area, the surface area using formula (8), and the coordinates of the sub-area centroids using formula (7). Table 1 shows the above values on the ellipsoid (second column) and the differences with the lengths, areas, and coordinates of the centroids calculated on the plane of a given flat system (columns 2–5). In sub-areas A and B on the ellipsoid, the lengths and areas are equal, as in sub-areas C and D. In the case of flat coordinate systems, due to projection distortions, the lengths of the sides and the area values in these sub-areas may differ.

In the case of the length of the sub-area sides, the greatest discrepancies compared to the corresponding sides on the ellipsoid were obtained in the PL-LAEA system (2.92 m – sub-area D) and in the PL-1992 system (1.05 m – sub-area B). In the case of surface area, the largest differences were obtained in the PL-1992 system (over 1500 m² – sub-areas A and B). In the PL-LAEA system, the differences in area are significantly smaller (maximum 6 m² – sub-area D). The discrepancy in the area for the equidistant projection used in the PL-LAEA system results from the method of calculating the area. The area of the sub-area on the ellipsoid is preserved on the projection plane of the equidistant projection, but within the curved boundaries of the meridian and parallel images. In contrast, the area on the plane of this system was calculated on the basis of the flat coordinates x, y (using formula 8), i.e. for a sub-area whose sides are straight line segments.

High consistency in both the length of the sides and the area of the sub-areas was achieved when using both local systems. This consistency results from the adjacency of the sub-area plane to the ellipsoid surface, obtained after applying the Gauss-Krüger projection with the central meridian running through the center of the sub-area. The distortions in length and area occurring in such a projection and in such a small area are close to zero. In the case of the PL-1992 system, the central meridian is the same for all sub-areas ($\lambda_0 = 19^\circ$), hence the projection distortions may be large in some cases. Additional distortions (e.g., in sub-areas

A, B, and E) result from the scale factor used in this projection ($m_0 = 0.9993$). A similar situation occurs in the case of the PL-LAEA system, where there is one point of contact between the plane and the ellipsoid for the entire area of Poland ($\varphi_0 = 52^\circ$, $\lambda_0 = 10^\circ$). In order to achieve consistency between the shape of the sub-area and the ellipsoid in its image on the mapping plane, it would be necessary to use a projection that ensures the greatest possible contact of the projection plane to the surface of the ellipsoid across the entire area.

Table 1 shows that the discrepancies in the coordinates of the sub-area centroids in each of the analyzed systems do not exceed 1–2 cm. This means that the coordinates of the sub-area centroid (a meridional-parallel trapezoid with a size of $\Delta\varphi = \Delta\lambda = 0.01^\circ$) can be calculated in each of the above-mentioned systems, and they will correspond to the coordinates of the centroids of these sub-areas on the ellipsoid. Therefore, the position of the geometric center of an area composed of multiple sub-areas will be influenced, for example, by the surface area used as a weight in the calculation of the mean coordinates.

The second stage of the test was to calculate the ellipsoidal coordinates φ_{SC} , λ_{SC} of the geometric center of the entire area, consisting of sub-areas A, B, C, D, E (Figure 4). These coordinates were calculated in all variants I–III. Table 2 shows the differences in coordinates: $\Delta\varphi = \varphi_{SG} - \varphi_{SG_E}$ and $\Delta\lambda = \lambda_{SG} - \lambda_{SG_E}$, obtained in relation to the reference coordinates of the geometric center on the ellipsoid in variant I and amounting to: $\varphi_{SG_E} = 51^\circ 50' 44.362''$ i $\lambda_{SG_E} = 19^\circ 00' 18.000''$.

It can be concluded that good agreement with both reference coordinates at a level not worse than 25 cm was obtained for the PL-LAEA system in variant IIIa (−0.23 m and +0.07 m). Better consistency at a level below 10 cm was achieved in the PL-1992 and PL-LAEA systems in variant IIIb (−0.01 m and 0.00 m). Significantly greater discrepancies with the reference coordinates were obtained in variant IIa, in both state systems. It is worth noting that in variants IIa and IIb, the coordinates of the geometric center of the area are calculated on the basis of the planar coordinates x_C , y_C of the sub-area centroids, and in variants IIIa and IIIb on the basis of the ellipsoidal coordinates φ_C , λ_C of these centroids (Table 2). Very good agree-

ment of the coordinates of the geometric center of the area, regardless of the variant, was obtained in the “Local-GK” and “Local-CTP” systems (the maximum discrepancy occurred in the “Local-CTP” system and amounted to 0.02 m).

5. Calculation of the coordinates of the geometric center of Poland

The results of the test carried out on five complete sub-areas (Tables 1 and 2) indicate that the coordinates of the geometric center of Poland can be calculated with centimeter accuracy in the PL-1992 (variant IIIb), PL-LAEA (variant IIIb) and Local-GK (variant IIIa or IIIb). Calculation of these coordinates with slightly lower, decimeter accuracy is also possible in variant IIIa, in the PL-LAEA system. However, variant IIIb, in which more accurate coordinates can be obtained, is inconvenient because it requires the surface area on the ellipsoid. This is difficult to calculate for irregular, border sub-areas. It is much easier to calculate the area of border sub-areas on a plane (the area calculated in this way was used in variants IIa and IIIa). Therefore, the calculation of the coordinates of the geometric center of Poland was performed in variant IIIa, in the Local-GK system and, for comparison, in the PL-LAEA system. The data used were the coordinates φ_W , λ_W of the vertices of the sub-areas in the form of meridian-parallel trapezoids with a size of $\Delta\varphi = \Delta\lambda = 0.01^\circ$, forming a grid consistent with the data model grid (GUGiK, 2024). Only those sub-areas of this grid that covered the territory of Poland within its administrative boundaries were used. The sets of points for this border were downloaded from the National Register of Boundaries (GUGiK, 2025) in two versions: version A – covers the border without territorial waters, version B – covers the border including territorial waters (Figure 5).

Below are the characteristics of the sub-areas comprising the territory of Poland, given for the two above-mentioned versions of the boundaries (A) and (B):

- Total number of sub-areas in Poland territory: 414820 (A) and 427079 (B)

- Total area of all sub-areas in [m²]:

- in the Local-GK system: 313932947354 (A) and 322720452475 (B),

- in the PL-LAEA system: 313932947690 (A) and 322720452823 (B),

Table 1. Comparison of side lengths, surface areas, and centroid coordinates for sub-areas on an ellipsoid and on planes of different coordinate systems

sub-area	$d_{12}[m] / d_{34}[m]$ $d_{13}[m] / d_{24}[m]$ $P[m^2]$ φ_C / λ_C	$\Delta d_{12}[m] / \Delta d_{34}[m]$ $\Delta d_{13}[m] / \Delta d_{24}[m]$ $\Delta P[m^2]$ $\Delta \varphi_C / \Delta \lambda_C [m]$			
	ellipsoid	PL-1992	PL-LAEA	Local-GK	Local-CTP
A	731.72 / 731.57 1,112.10 / 1,112.10 813,661 49°00'18" / 14°00'18"	+0.69 / +0.69 +1.05 / +1.04 +1,528 0.01 / 0.00	+0.04 / +0.04 -0.07 / -0.07 -6 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.02 / 0.00
B	731.72 / 731.57 1,112.10 / 1,112.10 813,661 49°00'18" / 24°00'18"	+0.69 / +0.69 +1.05 / +1.05 +1,539 +0.01 / 0.00	-1.71 / -1.72 +2.62 / +2.63 0 -0.01 / -0.01	0.00 / 0.00 0.00 / 0.00 0 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.02 / 0.00
C	639.94 / 639.78 1,113.24 / 1,113.24 712,317 55°00'18" / 14°00'18"	+0.35 / +0.35 +0.62 / +0.61 785 0.01 / 0.00	+0.10 / +0.09 -0.17 / -0.16 -1 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0.0 -0.02 / 0.00
D	639.94 / 639.78 1,113.24 / 1,113.24 712,317 55°00'18" / 24°00'18"	+0.36 / +0.36 +0.62 / +0.62 +792 0.01 / 0.00	-1.67 / -1.68 +2.91 / +2.92 -6 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.02 / 0.00
E	686.78 / 686.23 1,112.67 / 1,112.67 764,078 52°00'18" / 19°00'18"	-0.48 / -0.48 -0.78 / -0.78 -1,069 0.01 / 0.00	0.80 / 0.81 -1.31 / -1.32 4 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.01 / 0.00	0.00 / 0.00 0.00 / 0.00 0 -0.02 / 0.00

Table 2. Comparison of the coordinates of the geometric center of the area calculated in different variants (I-III)²

Difference from reference coordinates	PL-1992		PL-LAE		Local-GK	Local-CTP
	„IIa” „IIb”	„IIIa” „IIIb”	„IIa” „IIb”	„IIIa” „IIIb”	„IIIa” „IIIb”	„IIIa” „IIIb”
$\Delta \phi [m]$	+9,099.75 +9,204.57	-110.29 -0.01	+9,260.43 +9,260.66	-0.23 -0.01	0.00 -0.01	0.02 0.02
$\Delta \lambda [m]$	+1.74 +0.03	+1.69 0.00	-717.90 -718.04	+0.07 0.00	0.00 0.00	0.00 0.00

– Number of complete sub-areas: 409833 (A) and 421980 (B)

– Number of border sub-areas: 4987 (A) and 5099 (B)

– Minimum/maximum number of vertices of complete areas: 4/4

– Minimum/maximum number of vertices of edge areas: 3/439

– Minimum/maximum area of a complete sub-area in $[m^2]$:

715481.79 / 813499.33 (A)

711965.16 / 813499.33 (B)

– Minimum/maximum area of a border sub-area in $[m^2]$:

0.38 / 810857.15 (A)

0.02 / 810857.15 (B)

² In the case of both local systems, due to their heterogeneity across the entire area, only variants IIIa and IIIb can be implemented.

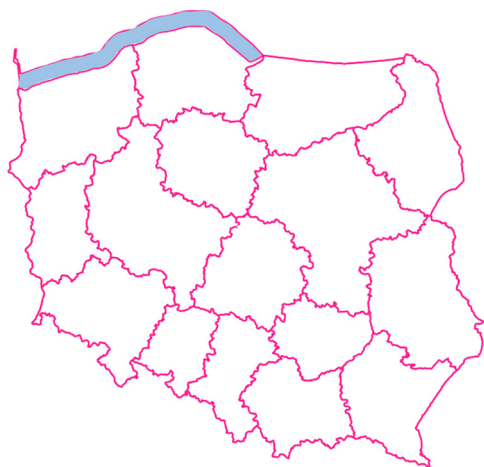


Figure 5. Poland's borders as recorded in the National Register of Boundaries (the territorial waters of the Baltic Sea are marked in blue, GUGiK, 2025)

The ellipsoidal coordinates of the geometric center of the above-defined area of Poland obtained as a result of applying variant IIIa in the Local-GK and PL-LAEA systems are the same with an accuracy of 1 cm and are as follows in the PL-ETRF2000 reference system (Figure 6):

– in variant (A): $\varphi_{SG(A)} = 52^{\circ}05'49.6170''$,
 $\lambda_{SG(A)} = 19^{\circ}24'20.8183''$

Łódź Voivodeship, Łęczyca County, PIĄTEK Commune, BALKÓW District, Plot ID 100406_5.0001.641

– in variant (B): $\varphi_{SG(B)} = 52^{\circ}09'50.5140''$,
 $\lambda_{SG(B)} = 19^{\circ}20'13.1125''$.

Łódź Voivodeship, Kutno County, Commune KUTNO, District STRZEGOCIN, Plot ID 100206_2.0027.1/3

Therefore, for the adopted sub-area grid $\Delta\varphi = \Delta\lambda = 0.01^{\circ}$, already used in Poland in spatial reference models (GUGiK, 2024), it is possible (with the same effect) to use both the PL-LAEA system and the Local-GK system. The coordinates of the geometric center of Poland may change if a differently defined area of Poland is used for their calculation, i.e., according to a different course of Poland's borders. For example, if the coordinates of the geometric center of Poland were calculated only on the basis of complete sub-areas (without border sub-areas), we would obtain the following values for the coordinates of the center:

– in variant (A): $\varphi'_{SG(A)} = 52^{\circ}05'59.4172''$,
 $\lambda'_{SG(A)} = 19^{\circ}24'22.3151''$

– in variant (B): $\varphi'_{SG(B)} = 52^{\circ}09'59.9325''$,
 $\lambda'_{SG(B)} = 19^{\circ}20'15.3928''$.

The point designated in this way would be located on average (variants A and B): 291 m north and 36 m east of the geometric center of Poland, determined on the basis of all sub-areas (Figure 6).

6. Discussion and conclusions

The method for determining the geometric center of Poland proposed in the article involves dividing the country into smaller sub-areas (complete and border), calculating the coordinates of the centroids of these sub-areas, and calculating the average coordinates of the centroids, using their surface area as a weight. A meridian-parallel grid with a mesh size of $\Delta\varphi = \Delta\lambda = 0.01^{\circ}$ was used to divide the area of Poland into sub-areas. This grid is used to create geospatial models in reference systems applicable in Poland. In Poland, it consists mainly of complete sub-areas (almost 99%) and a small number of border sub-areas. An analysis performed for a complete sub-area showed that its centroid coordinates can be calculated with an accuracy of single centimeters as the average ellipsoidal coordinates of the sub-area vertices. The results of the tests showed that the shape and position of the centroids of such a sub-area on the PL-1992, PL-LAEA, and local Local-GK and Local-CTP coordinate systems differ minimally from their counterparts in the original sub-area on the ellipsoid. Particularly noteworthy in this regard are the two local systems, in which both the lengths of the sides and the areas of the sub-areas, as well as the coordinates of the centroids, are almost identical to those obtained for sub-areas on the ellipsoid. In the case of the PL-LAEA state system, the differences are greater (for side lengths of approximately 3 m, for areas of approximately 6 m²). However, these discrepancies did not affect the calculated centroid coordinates, which for this system (and the above-mentioned local systems) differ from the centroids of the sub-areas on the ellipsoid by a maximum of 2 cm.

Subsequent numerical tests involved calculating the coordinates of the geometric center of the entire area consisting of five sub-areas. The tests were performed in several variants, differing in the type of coordinates used (x, y or

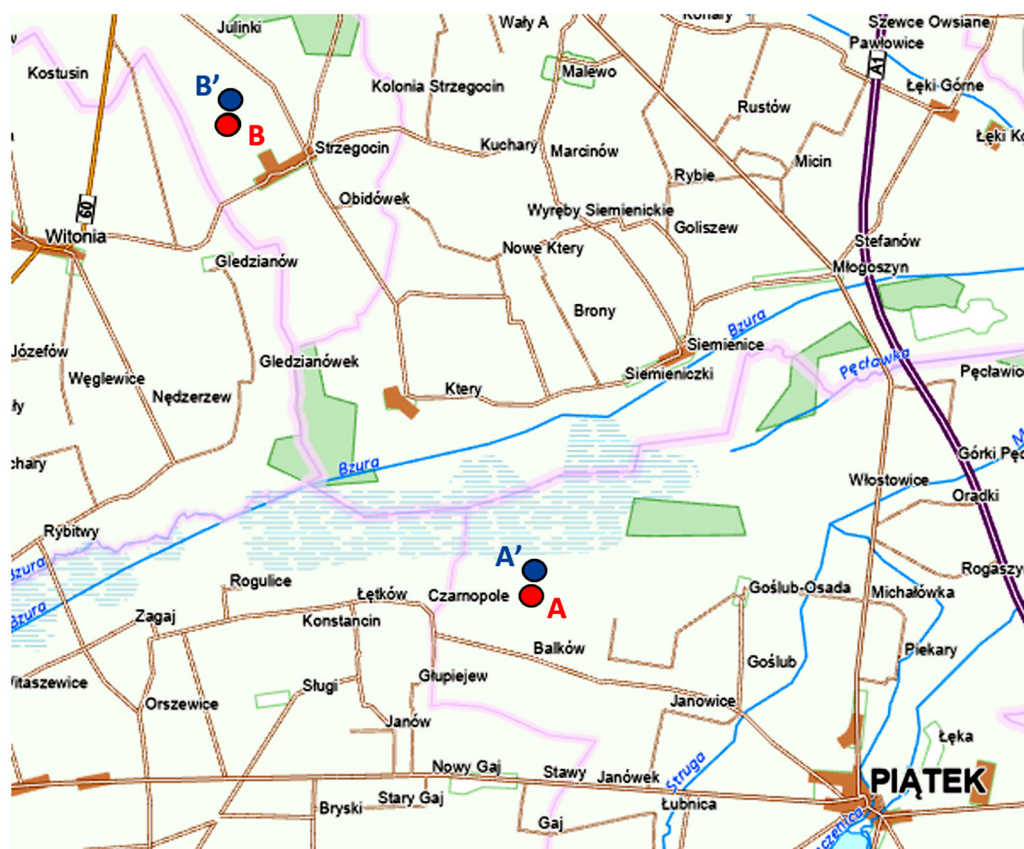


Figure 6. Location of the geometric center of Poland determined in the following variants: for complete and border sub-areas: A – without territorial waters; B – with territorial waters, only for complete sub-areas; A' – without territorial waters; B' – with territorial waters

φ, λ) and the type of weight (area on a plane or area on an ellipsoid). The coordinates of the centroids together with the weights were used to determine the position of the geometric center of the area. The test results indicate that it is advantageous to calculate the coordinates of this center in the PL-LAEA or Local-GK (or Local-CTP) systems, in variant IIIa. This variant consists in calculating the x, y coordinates of the centroids and the area of the sub-areas on the plane of a given system, converting the x, y coordinates to ellipsoidal φ, λ and then calculating the average values of φ, λ using weights in the form of the above-mentioned areas of the sub-areas on the plane. The differences in the coordinates of the geometric center of the area calculated in these

systems in variant IIIa differed from the reference coordinates on the ellipsoid by a maximum of 1–2 cm. Therefore, it was decided that the calculation of the coordinates of the geometric center of Poland would be carried out in the PL-LAEA and Local-GK systems, in variant IIIa. The strategy of using the projection plane of flat systems resulted from the difficulty of calculating the coordinates of the centroid of irregular polygons (border sub-areas) directly on the ellipsoid. While calculating the coordinates of the centroids of complete sub-areas is not a problem, calculating the coordinates of the centroid of a boundary (irregular polygon) on an ellipsoid is not easy. The use of a plane allows the calculation of the coordinates of the centroids of any sub-areas (polygons) as

a function of the Cartesian coordinates x , y . This can apply to both homogeneous systems (e.g., PL-LAEA) and heterogeneous local systems (e.g., Local-GK).

The calculated coordinates of the geometric center of Poland in variant IIIa, in both the PL-LAEA and Local-GK systems, proved to be consistent with an accuracy greater than 1 cm. Therefore, other factors may determine the choice of system, e.g., the ease of generating x , y coordinates in a given system based on φ , λ grid vertices, or the number of numerical operations. Alternatively, it would also be possible to calculate the coordinates of the centroids of the border sub-areas in the above-mentioned planar systems (variant IIIa), and the coordinates of the centroids of the complete sub-areas as the average ellipsoidal coordinates of the vertices of the sub-areas. On the other hand, omitting the border sub-areas results in a significant, almost 300 m shift of the determined geometric center. This shift is greater in the meridional direction. This is due to the fact that the eastern and western boundaries are similar to the meridional line and are approximately parallel to each other. Thus, the western and eastern border sub-areas affect the position of the geometric center to a similar extent. Therefore, their absence will not significantly change the position in the W-E direction. The situation is different with the southern and northern borders, which are highly diversified. The northern border is approximately parallel, while the southern border mostly runs in a S-E direction and in the eastern part in a N-E direction.

The calculated coordinates of the geometric center depend largely on the adopted border of Poland. The calculations were based on the area of Poland without territorial waters (variant A) and with territorial waters included (variant B). The enlargement of the area by the territorial waters shifted the geometric center 7.4 km north and 4.7 km west.

The coordinates of the geometric center of Poland calculated using the proposed method differ from the coordinates given in other publications (Meus, 2019; Róžański, 2018). This may be due to the different area of Poland

used in these publications. Another reason may be the density of centroid points placement. For the grid used in this study ($\Delta\varphi = \Delta\lambda = 0.01^\circ$) already used in Poland in spatial references (GUGiK, 2024), the calculation of the coordinates of the geometric center is performed for over 400,000 centroids (the number of all sub-areas). However, it is possible to use a different, e.g., denser grid with a smaller size $\Delta\varphi$, $\Delta\lambda$. This raises the question: how dense should the grid be to calculate the geometric center of Poland? It makes sense to use an already-made grid with a mesh size of $\Delta\varphi = \Delta\lambda = 0.01^\circ$ which is used in Poland in spatial reference models (GUGiK, 2024). Tests show that for such a grid, the coordinates of the geometric center of Poland will be determined with an accuracy of single centimeters. In addition, the determined geometric center of Poland would result from the already used basis for modelling spatial references and would be consistent with them. The ellipsoidal coordinates φ , λ of the nodes of this grid are fixed. Due to the small size of the grid's mesh (approximately 1,100 m $\times$ 700 m), this grid can also be used to determine the geometric center of smaller administrative units (provinces, counties, and municipalities).

However, the basis for determining the location of the geometric center of Poland (or any other area) using any method is to assume a specific course of Poland's borders. In the calculations above, two variants of Poland's border were assumed. If the so-called "center of Poland" were to be a single point, it would be necessary to first decide which border would define the area of Poland and then calculate the location of its geometric center for that area.

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