Polish Cartographical Review
Vol. 57, 2025, pp. 1–20
DOI: 10.2478/pcr-2025-0001
Received: 14.01.2025
Accepted: 29.04.2025

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Methods of centroid determination in QGIS – proposal of an algorithm using the "Model Designer" tool

Abstract. The article talks about determination of centroids. The introduction outlines a number of definitions of centroids. Then, various methods of their determination and example applications are discussed. The paper also describes algorithms of calculation of centroid coordinates implemented in QGIS. Then, with the use of the model designer tool, an algorithm was developed to determine centroids on a reference surface, followed by calculations in QGIS. We calculated the coordinates of selected polygons on a surface in the systems PL-1992, PL-LAEA and PL-LCC, as well as on the GRS80 ellipsoid, Krasowski ellipsoid and on a sphere. The test area was Poland in its administrative borders, including territorial waters. The paper is concluded with an analysis and comparison of the obtained results.

Keywords: centroid, centre of gravity, pole of inaccessibility, QGIS, model designer

1. Introduction

This paper was inspired by the work of a geodetic undertaking "Honorowy Południk Krakowski" (in English "Honourable Krakow Meridian") led by the surveyor Mariusz Meus, who worked on determination of centroids of Poland and of a few cities in Poland. Discussions of the author of this article with the initiator of the undertaking were the main motivation to make independent calculations using simple tools which do not need software.

Many sources give various definitions of centroids and centres of areas, which may cause some confusion.

According to Webster's Third New International Dictionary, a centroid is the centre of mass or centre of a figure (Merriam Company, 1961).

In the physicist's dictionary, the centre of mass, also referred to as the centre of inertia, is "a point characterising distribution of mass in a body or a system of bodies or material points. For a system made of n material points of mass m_n each, the coordinates of the centre of mass are calculated as the weighted average of the coordinates of points with masses m_n determined in these points" (Cach et al., 2017, p. 315).

The dictionary also includes an explanation of the term centre of gravity: "Centre of gravity – concept related to a perfectly rigid body referring to the point of application of the force that is the resultant of the gravitational forces acting on all the particles of that body. To find the centre of gravity, the body is freely suspended at least twice, each time using a different suspension point, and vertical lines are drawn from these points. The intersection point of these lines determines the centre of gravity of the body. This point may lie outside the physical boundaries of the body" (Cach et al., 2017, p. 315).

The mathematician's encyclopaedic dictionary (Filist et al., 1998) provides a definition of the centre of mass for a finite system of material points as the weighted average of the coordinates of these points with weights equal to masses placed in these points.

Another mathematical dictionary (Siwek, 2008, p. 35, 430) includes a definition of barycentre as the centre of mass, defined in the following way: "if $\{P_0. P_1,\dots,P_n\}$ is a set of n+1 points in the Euclidean (finite) space, and $q_0, q_1, \dots, q_n$ is a sequence of n+1 of natural numbers the sum of which is equal to one, then point P, whose position vector relative to a fixed point O is expressed with the equality

$$\overrightarrow{OP} = q_0\overrightarrow{OP_0} + q_1\overrightarrow{OP_1} + \cdots + q_n\overrightarrow{OP_n}$$

does not depend on point O and is called the centre of mass of the system of points $P_0. P_1,\dots,P_n$ with respective weights $q_0, q_1, \dots, q_n$."

Obviously, physics and mathematics use the terms of centre of mass, centre of gravity or barycentre. For a uniform body this overlaps with the geometric centre. Thus we have a widely accepted meaning of centroid as the centre of mass. In GIS we typically analyse polygons on the projection plane and then a centroid is the geometric centre, the location of which is calculated with mathematical formulas from Cartesian coordinates (Deakin et al., 2002).

According to Open Geospatial Consortium "a centroid is allocated to the centre of an area, region or polygon. In case of irregular polygons, the centre of mass is determined mathematically and weighted to obtain something like an approximated "centre of mass". Centroids are important in GIS since discrete locations X-Y are often used to index or refer to a polygon in which it is located. Sometimes information about attributes is "added", "suspended" or "attached" to the location of the centre of mass" (Open Geospatial Consortium, 2018).

According to the geomatic lexicon of the Polish Association for Spatial Information, "a centroid is a point inside a polygon representing an object; the point is allocated some information about the object and its location within the polygon may be defined as, e.g., centre of gravity or selected by the user" (Gaździcki, 2003, p. 11).

The geomatic dictionary of State Forests provides the following definition of centroid: "a point defining the geometric centre of an object or a system of objects. The location of the centroid is generally important when creating labels and in some cases it is an element connecting the object with its description" (Lasy Państwowe, n.d.).

GIS dictionary by ESRI provides two definitions (ESRI, n.d. a):

− "In spatial analyses, a centroid is defined as the geometric centre or an average location of a spatial object. In case of linear, polygon or 3D objects, it is the centre of mass (or centre of gravity) and may be located inside the object. In case of multipoints, polylines or polygons with many parts, it is calculated on the basis of a weighted average of the centre of all parts of the element.

− In cartography it is the centre of an area, balance point or centre of mass."

In the book Longley et al. (2006, p. 351), centroids are defined as 2D equivalents of averages. A centroid described also as the centre point of a set of points on a plane may be calculated as the weighted average of x and y coordinates. "It is a point that minimises the sum of squared distances and is the equilibrium point of the plane. A centroid is a summative index representing the location of a set of points". It is a point useful especially to present changes of the location of the set of points in time. The publication also provides a definition of the point of minimum aggregate travel (MAT), i.e., a point for which the sum of distances from the other points of the set, measured in a straight line, is the smallest. MAT is an equivalent of median. This point is often mistaken with a centroid (Longley et al., 2006, p. 353–354).

According to DeMers (2009), a centroid for geographical objects is the exact geometrical centre. He also lists three types of centroids: a simple centroid, also called a geographical centroid, calculated by dividing the polygon into trapeziums (the method is described in the next chapter), centre of mass (calculated as the average of coordinates) and centre calculated as the weighted average of coordinates of a set of points.

A centroid in geoinformation sciences may thus be the geometric centre of an area, the centre of mass of a set of points or a subjectively determined point within an area. If it is to be a point to which labels or information about an area is attached, it can be placed anywhere within the area. A centroid, as the geometric centre, is determined for various purposes, very often, for example, as the centre of a country.

A centre of mass of a set of points is used, for instance, in spatial analyses when studying the spatial distribution of various phenomena, e.g. population.

2. Examples of centroids determination

Literature describes many examples of centroid determination for various areas, e.g. for countries, using various methods.

The location of the centre of Italy was analysed by Aebischer (2014). He emphasised the difficulties defining the area, especially the borders; in the case of Italy, these may be the borders of the peninsula, administrative borders with the main islands or without islands. He made the calculations using various methods and tools, also with QGIS.

In Italy there are four candidates for the geographical centre: Foligno, Monteluco, Reti and Narni. In the two latter places, there are monuments and plaques stating that this is the centre of Italy. Already in the ancient times, Rieti was mentioned to be the centre of the peninsula (Aebisher, 2014).

Aebischer also showed a few central points of the European Union (EU) determined in different years. In 2004, the French National Institute of Geographic and Forest Information (IGN) determined the centre of the EU for 25 countries in Kleinmaisched in Germany. Earlier, the centre of the EU, comprising 12 member states was Saint-Andre-le-Coq in central France. For 15 member states, the centre was in Viroinval in Belgium. In some places, monuments were erected (Aebischer, 2014). In 2020, when Great Britain left the EU, IGN informed that the new centre of the EU is in Gadheim in Germany (IGN, 2021).

The article by Deakin et al. (2002) describes a few possible definitions and methods of centroid determination. It compares the results of calculations with results obtained with GIS programmes. Centroids of Victoria (a state of Australia) and Melbourne were also determined.

Barmore (1994) determined the centres of USA and Wisconsin in various ways: on Clarke's ellipsoid, in 3D, on the plane in the Lambert azimuthal equal-area projection and in the Albers equal-area conic projection. He compared these results against each other and with the Deetz method used a few dozen years earlier to determine the centre of mass of the USA. Then a map of the USA was made in equal-area projection on thin cardboard, cut out around its borders and the centre of mass was determined manually. Barmore stated that for real areas on the Earth, one should calculate average locations on the surface of a sphere or an ellipsoid. And he decided this method was the best. The area of the USA was divided into 30x60 minutes' ellipsoidal quadrilaterals. For quadrilaterals inside the borders and quadrilaterals for which more than half of the area was located within the area, the surface area was calculated using the ellipsoid geometry.

To calculate the geographical centre of Pennsylvania, Boscoe (2001) used his own method of dividing this state into 5x5 minutes' spherical quadrilaterals. The coordinates of the quadrilateral were determined as the weighted average of the coordinates of the centres of the quadrilaterals, and the weights were their surface areas.

Kalyuzhin et al. (2020) determined the centre of the Baikal Lake with various methods and tools, e.g. in MapInfo, Geomedia and ArcInfo and obtained different results. Using the determined point as a tourist attraction when sailing on the Baikal Lake was mentioned as an interesting application.

A geometric centre was also determined for Poland. In 1966, the Main Office of Geodesy and Cartography determined the geometric centre of Poland, which, according to local traditions, was located 3.4 km north west of the centre of Piątek. Due to a lack of documentation on the calculations made then, it is impossible to locate the point physically. It was not mapped physically. In 1976, on the Piątek market square, in the place of Polish Tourist and Sightseeing Society (in Polish: Polskie Towarzystwo Turystyczno-Krajoznawcze, PTTK) meetings, a monument was erected of the *geometric centre of Poland* (Cisak, 2001; Meus, 2019). A new centre of Poland was determined by Meus (2019), taking into account not only administrative units but also territorial waters. The newly determined point is located in Nowa Wieś near Kutno. In 2018, the point was marked with a granite memorial post (Meus, 2019). The calculations were based on an algorithm and computer programme developed by Różański (2018).

When calculating the centre of an area, e.g. a country, a number of problems need to be taken into account. The area may consist of a few parts, e.g., islands, so there arises the question if the centre should be determined for the whole area or only the main area, e.g. the largest, for example in case of Italy it could be the area of the peninsula. Another problem are changing borders. Each change entails a change of the centre of the area. For instance, EU borders changed many times, so when new countries joined it, the location of the centre changed. Another problem is the curvature of the Earth's surface. For an ellipsoid, a sphere and a plane, we will get different centres' coordinates. The results will also differ slightly when we use different ellipsoids. We can also calculate the centre of an area on a plane. How can we then use cartographic projection?

An interesting example of the use of the centroid determined for a set of points are demographic analyses on populations and their changes over time. Results of such analyses were presented on the portal of the United States Census Bureau (USGS). Their maps present three centres of population, i.e., the geographical centre of the area and points representing the average and the median. They also present the changes in the location of these points against time from 1790 to 2020 (US Census Bureau, 2021).

An interesting application of the centre of mass for spatial analyses of the spread of COVID-19 in Europe was described in the article (Yavorska & Bun, 2022). On the basis of weekly data on the illness, trajectories of centres of mass for new infections and fatalities were calculated for selected European countries. Graphs presented in the article show a dominant role of some countries and regions at different stages of the pandemic.

The centre of mass is also a useful tool for spatial analyses in the economy. Kandogan (2014) analysed trajectories of changes of centres of mass in the economy of 150 countries, regional economic blocks and the whole world in the period from 1970 to 2009. The results showed, for example, the growth of the importance of Asia and, in the case of economic blocks, the shift of the centre of mass towards geometric centres of the blocks.

You can find many more examples of the applications of centroids. This article concentrates mainly on centroids as the geometric centres of polygons representing selected geographical objects.

3. Methods of centroid determination

There are many methods of determination of the centroid of an area. The simplest method is determination of the centre of a spherical or ellipsoidal quadrilateral limited with meridians and parallels at the borders of the area. Literature also describes many other interesting methods. The most often used ones described in the article (Deakin et al., 2002) include:

− Centroid calculated on the basis of moments – the Moment Centroid method. The algorithm described in the article divides the polygon into triangles and then, for each triangle, determines moments, i.e., products of the surface areas of the triangles and the average of the planar Cartesian coordinates. The sum of moments divided by the surface area is the coordinates of the centroid.

− Centroid calculated on the basis of dividing the polygon into equal parts – the Area Centroid method. The polygon is divided into two equal parts along a line. If we find a second line dividing the polygon into equal parts, then the intersection of the lines is the location of the centroid. Unfortunately, for complicated polygons, different pairs of lines give different centroid locations. If a figure is not symmetrical, then we will have the location of centroids in another point than for the Moment Centroid.

− Centroid calculated on the basis of dividing the polygon into equal parts on a curved surface – Volume Centroid, 3D area method. A method similar to the previous one but it works in 3D. For example, when we have a centroid on a sphere, we divide it into two equal parts along two lines of the great circle. The intersection point is the centroid.

− Centroids calculated on the basis of averages – the Averages method. The coordinates of the centroid are the average values of the coordinates of all points of the figure. For calculations, we may use the average, median and mode. The average may be calculated as the arithmetic average, square average, harmonic and geometric average.

− Centroid as the centre of the smallest rectangle bounding the area – the Minimum Bounding Rectangle Centroid method. In this method, we determine the extreme values of the x and y coordinates; parallel lines to the coordinate axes determine a rectangle, the centre of which is the centroid.

− Centroid determined from the minimum condition of its distance from the vertices – the Minimum Distance Centroid method. The minimum of the sum of the distance to the vertices and to the centroid determines its location.

− Centroid determined on the basis of buffers – the Negative Buffer Centroid method. Inside the polygon, buffers (polygons) are created the sides of which are parallel to the sides of the outer polygon and equally distanced from the sides of the outer polygon. Subsequent iterations lead to the creation of a triangle the geometric centre of which is the centroid.

− Centroid as the centre of a circle – the Circle centroid method. Centroid is defined in this method as the centre of a circle inscribed in the polygon.

Deakin et al. (2002) also discusses the advantages and disadvantages of the above mentioned methods:

− Methods based on the average are easy to calculate but depend on the number of vertices and their location and they are not sensitive to shape. They are good for determination of the centroid of a set of points but not for polygons.

− The minimum bonding rectangle method is also very easy, but not sensitive to the shape of the whole polygon.

− The surface area method yields various results for different balance lines.

− Negative buffer and circle methods are not appropriate for elongated polygons of complicated shapes and are difficult to perform with a large number of sides.

− The methods of moments and minimum distance are the best, while the moments method is much easier.

Rogerson (2015) proposed a method of determination of the *geographical centre*. The method minimises the sum of squares of sections of large circles on a sphere between all points in a given area to the centre point. It projects all vertices of the polygon on a plane in the azimuthal equidistant projection. Then the centre of the polygon is calculated and its planar Cartesian coordinates are recalculated back into geographical coordinates. The method was used by the author to calculate the centre of the USA (its area without Alaska or islands) and the states.

DeMers (2009) described the methods of calculation of a so-called simple or geographical centroid. He uses the so-called trapezium rule, so a given polygon is divided into a number of overlapping trapeziums. For each trapezium, the centre is determined as the average of its vertices and then the centroid is calculated for the whole polygon as the weighted average of the centroids with weights equal to surface areas of the trapeziums.

Aboufadel and Austin (2006) criticised the method used by the US Census Bureau of determining centroids by projecting points from the sphere onto Sanson projection (sinusoidal projection), since the use of various projections leads to determination of various locations of centroids. The authors proposed a method which does not employ cartographic projection. They proposed to divide the area into smaller areas and for each centre of a small area determine a vector in the 3D system attached to the centre of the sphere. Then centroid coordinates are calculated in 3D as weighted average of the vector coordinates. The authors stated that even if a point is located below the sphere, it is a good representative for a given set of points.

Różański (2018) proposed his method and used it for the area of Poland. The area was divided into a million rectangles. For such small areas, their shape and area were very similar in shape and area to the corresponding areas of the reference ellipsoid. The rectangles were projected on a plane with Albers projection. Then their centroids were determined; centroid coordinates were determined in geocentric 3D Cartesian system, each of them was allocated a weight equal to the surface area of the rectangle. Then the coordinates of a point which was the centre of mass of the set of centroids of the small rectangles were calculated.

To calculate centroids, GIS programmes can be employed. The two most popular programmes for that are ArcGIS and QGIS.

ArcGIS is an advanced geospatial platform developed by Esri that enables the integration and analysis of geographical data. It allows users to create maps, manage spatial data, and share it efficiently. The platform is used across various domains – from urban management and environmental protection to business process optimisation and support for strategic decision-making. ArcGIS is commercial software that offers a wide range of advanced tools for the creation, analysis, and visualization of spatial data (Esri, n.d. b). A tool for determination of centroids in ArcGIS is *find centroids*, which creates point objects representing geometric centres (centroids) of multipoint, linear and polygon objects. The parameter *centroid location* determines their location. Using the value *real geometric centroid* places the centroid in the so-called real centroid, even if it is located beyond the borders of the entry object. Using the value *within each entry object* yields the location of the centroid within the object.

The article presents results of calculations obtained in the programme QGIS, which has many various functions to create centroids and the algorithms it uses were described in the programme documentation. QGIS is a cross-platform, free, and open-source GIS software. It is a highly functional programme designed for creating, managing, analysing, and visualising spatial data. Its capabilities can be extended through the use of plug-ins – small programs that enable the execution of various geoinformation-related tasks. Plug-ins are available in the QGIS repository and must be installed before use. Examples include QuickOSM, a plug-in for downloading data from the OpenStreetMap database; QGIS Cloud, for publishing maps online; and MMQGIS, for processing vector data (Open Source Geospatial Foundation, 2025).

A useful tool when performing many operations in QGIS used by the author in his calculations is the *Model Designer.* With this tool, we can combine many operations that use various QGIS functions and plug-ins into one operation.

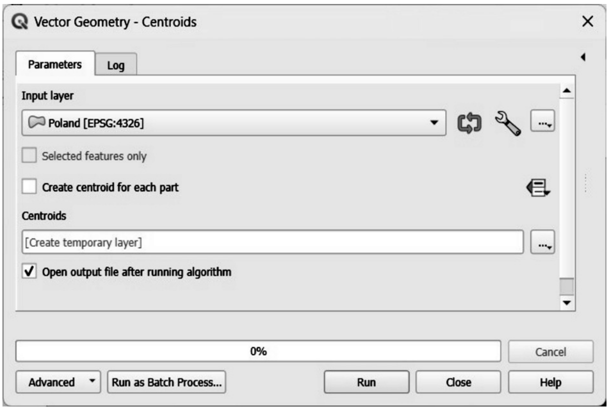

Figure 1. Parameters of the function *Centroids*

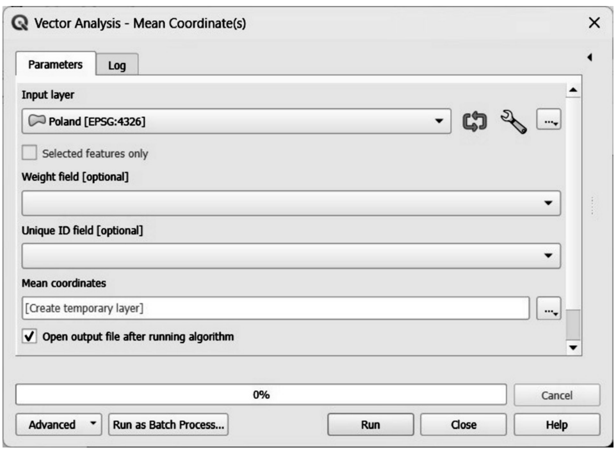

Figure 2. Parameters of the function *mean coordinate(s)*

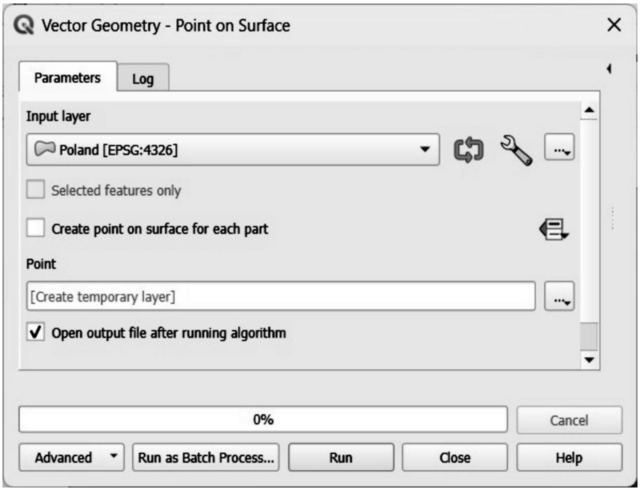

Figure 3. Parameters of the function *point on surface*

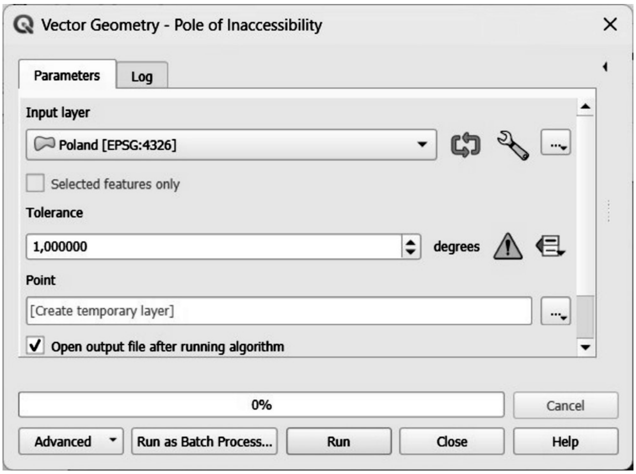

Figure 4. Parameters of the function *pole of inaccessibility*

QGIS has four functions to determine the centroid of an area:

- centroids,
- mean coordinate(s),
- point on surface,
- pole of inaccessibility.

The plug-in *realcentroid* can also be used for that.

The *centroids* function calculates the geometric centre of a polygon as the average of its vertices. In case of many objects, it is the centre of all its components or it can be a set of points located in its particular constituents, depending on the option we choose (Figure 1; QGIS project, 2025a).

Spatial analyses often calculate the centre of mass (*mean coordinate(s)* in QGIS), which is the weighted average of the points located on a layer (QGIS project, 2025b). We can give the name of weight fields (Figure 2).

The *point on surface* functions yields a point definitely located on a surface of a given geometry (QGIS project, 2025c). One can select the option of determining these points for the whole set of objects or for each part of the object (Figure 3).

Pole of inaccessibility employs the algorithm determining for the polygon layer the farthest internal point from the border of the polygon (QGIS project, 2025d).

The algorithm, referred to as “polylabel” in literature (Agafonkin, 2016), is an iterative approach guaranteeing determination of a real pole of inaccessibility within a given tolerance (in layer units). More precise tolerances require a larger number of iterations and need more calculation time. All algorithms work on objects located on a plane in cartographic projection, so the results will vary depending on the used projection or reference surface. The tolerance is given in layer system units (Figure 4).

According to its description, the *realcentroid* plug-in creates an internal point of a polygon. After computing the average of the planar coordinates of the polygon's vertices, a horizontal line passing through the point with the computed coordinates is determined. The longest segment of the intersection between this line and the polygon is then identified. The middle of the section is the centroid (Siki, 2014). A simple interface of the plug-in is presented in Figure 5.

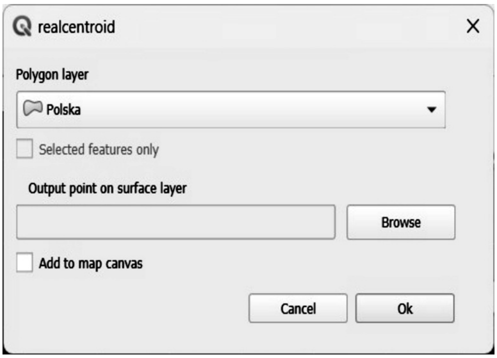

Figure 5. Interface of the plug-in *realcentroid*

4. Assumptions adopted in calculations, methodology and source data

In the further parts of the article, points generally referred to as centroids will be determined. To distinguish between them, names in accordance with those used in QGIS will be quoted in italics.

For the analyses, two objects were chosen. Croatia is a country made up of the continental part and islands. This area was used to present the location of centroids for areas consisting of a few parts. Data were taken from the portal Natural Earth (2009), and calculations were made in Mercator projection.

The second test area was the area of Poland within the borders including administrative units and territorial waters (datasets: A00_Granice_panstwa, A01_Granice_wojewodztw, A02_granice_powiatow i A03_granice_gmin). Data were obtained from the National Register of Boundaries (in Polish: Państwowy Rejestr Granic, PRG) with QGIS plugin (*Pobieracz danych GUGiK*).

Calculations were also made for two US states, i.e., Alabama and Arizona, to compare the results obtained with the developed algorithm with results presented in the article (Rogerson, 2015).

First, calculations were made for Croatia, then for Poland in the administrative borders with division into voivodships, counties and communes and without any division. Then, calculations were made for Poland with territorial waters.

Coordinates of centroids were calculated in QGIS using the methods of determination of the points implemented there. For Croatia, the following methods were used: *centroid*, *mean coordinate(s)*, *point on surface*, *pole of inaccessibility* and *realcentroid* plug-in. For Poland, the same methods were used, as well as the methods of determining the centroid directly from the reference plane developed by the author (*ellipsoidal and spherical centroid*).

In the last method, an area of a quadrilateral on a reference surface was determined bounded by the extreme meridians and parallels of Poland. Then the area was successively divided into smaller and smaller quadrilaterals, and their centres were determined (half of latitude and longitude). For calculations, the quadrilaterals within the borders of Poland were selected. Surface areas of small quadrilaterals were calculated on an ellipsoid and a sphere. Then, the weighted average was calculated with the coordinates of ellipsoidal or spherical centres with weights equal to the surface areas of the quadrilaterals.

The algorithm of the method was developed with the use of the *Model Designer* tool in QGIS. The model is presented in Figure 6.

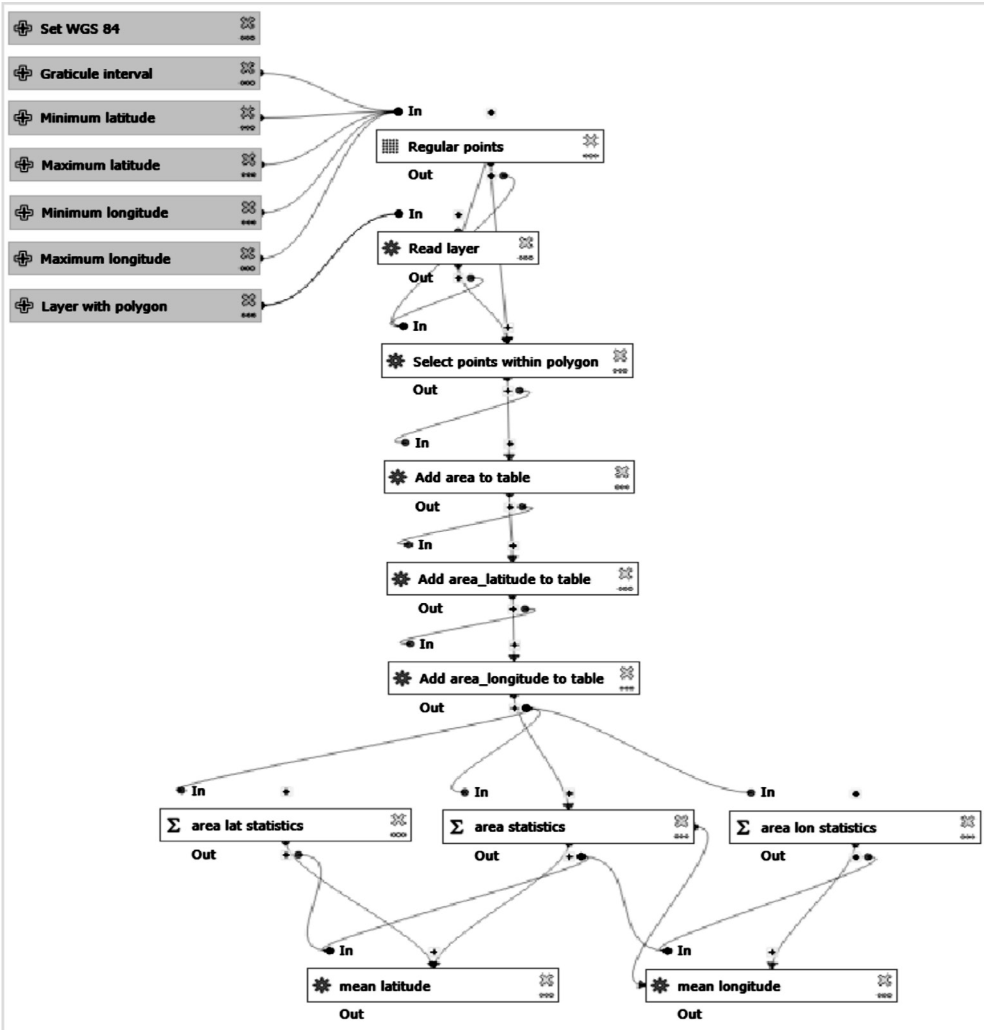

Figure 6. Model of the algorithm for determination

First, one should set the WGS84 system or another ellipsoidal one (WGS84 is the default system). Then we determine the range of the spherical or ellipsoidal quadrilateral in geographical coordinates. The next stage is adding the polygon layer with object borders for which we want to calculate the *ellipsoidal or spherical centroid*. The launched model displays the geographical coordinates of the centroid.

A few QGIS tools are used in the algorithm model to allow to perform subsequent stages of calculations. After entering the data, a set of points is determined, regularly placed inside the quadrilateral, including the given area. The tool *regular points* is used to this end. Figure 7 presents a window of the function *regular points*. To determine the range of data, the following expression was used:

concat(to_string(@minimum_longitude),',',to_string(@maximum_longitude),',',to_string(@minimum_latitude),',',to_string(@maximum_latitude)).

It is also important to define the dependencies between the elements of the model (*dependencies*), in this case this is the dependency on the function *read layer* (Figure 8).

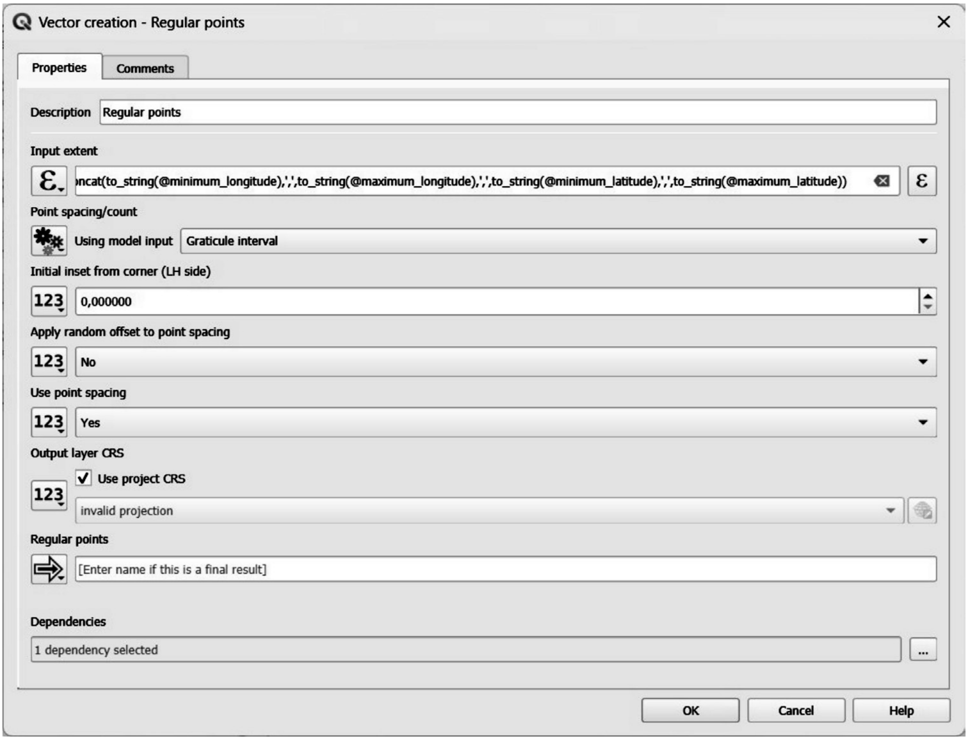

Figure 7. Interface after running the function *regular points*

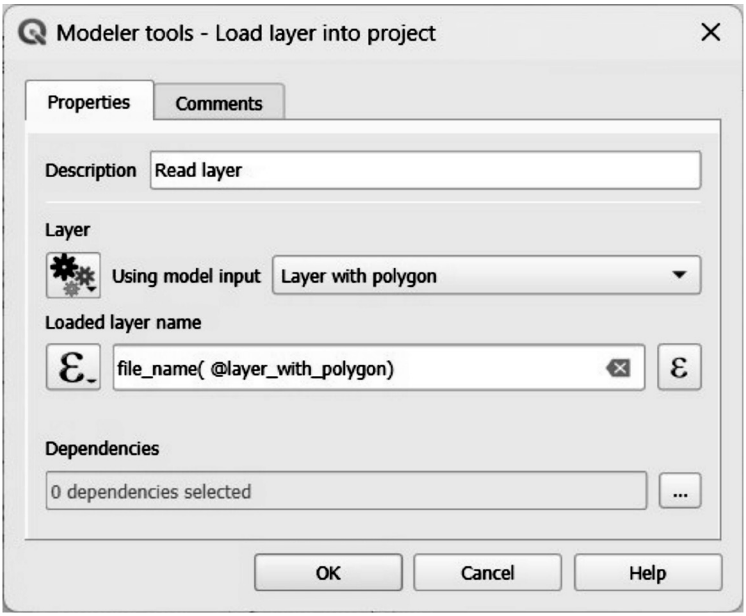

Figure 8. Interface of the function *read layer*

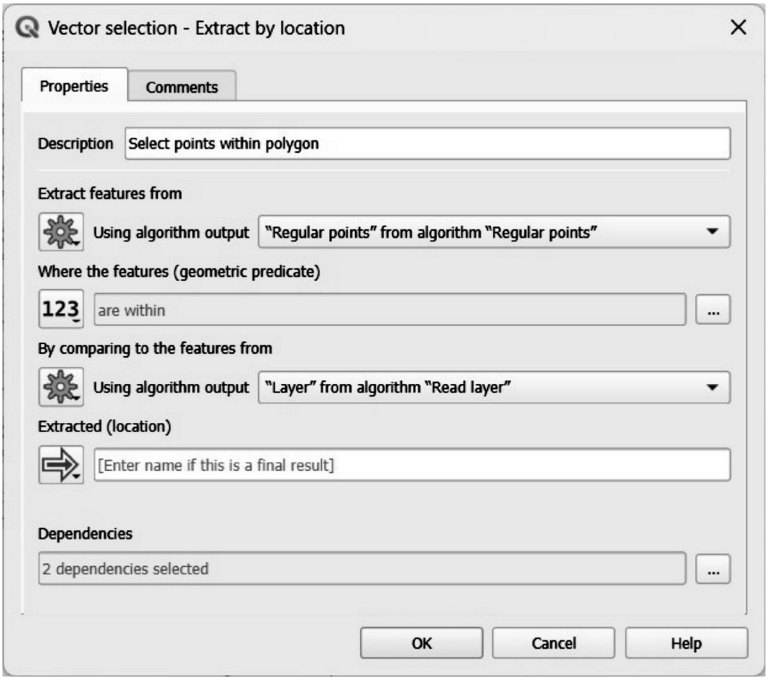

Figure 9. Interface of the function *perform spatial extraction*

It is important to enter the correct expression:

file_name(@layer_with_polygon).

Then points are selected inside the polygon with the function *perform spatial extraction* (Figure 9).

After that columns are added with calculated surface areas and products of surface area and the latitude and longitude. The *surface area calculator* is used to calculate surface areas and products of the surface areas and latitudes, and longitudes. An example expression calculating the product of the surface area on a sphere (one may disregard the square of the radius of the sphere as it is reduced) and the latitude has the following form (for an ellipsoid, it is much more complex):

*$y*pi()/180*(sin($y*pi()/180 + @graticule_interval * pi() /360)-sin($y*pi()/180 - pi()* @graticule_interval /360))* @graticule_interval * pi()/180.*

Surface area statistics are calculated (Figure 11); from them values of attributes are selected and then average coordinate values are calculated.

Finally the mean latitude and longitude are calculated on the basis of sums (Figure 13), for example:

*@area_lat_statistics_SUM *180/ (@area_statistics_SUM *pi()).*

After launching the model, a window pops up, like in Figure 13.

After running the required calculations, the results were compared against each other. The locations of centroids for Poland were also analysed in various projections and for various reference planes.

Calculations for Poland were made in three planar Cartesian coordinate systems valid in Poland: PL-1992, PL-LAEA and PL-LCC. The basic feature of these systems that allowed their use in the calculations is the continuity of projection in the whole area of Poland. PL-1992 is a system based on the Gauss-Krüger projection of ellipsoid GRS80. It is a conformal transverse cylindrical projection with a central meridian at the longitude of $\lambda_0 = 19°$. The scale on the axis meridian is $m_0 = 0.9993$. This system is mainly used to develop a database of

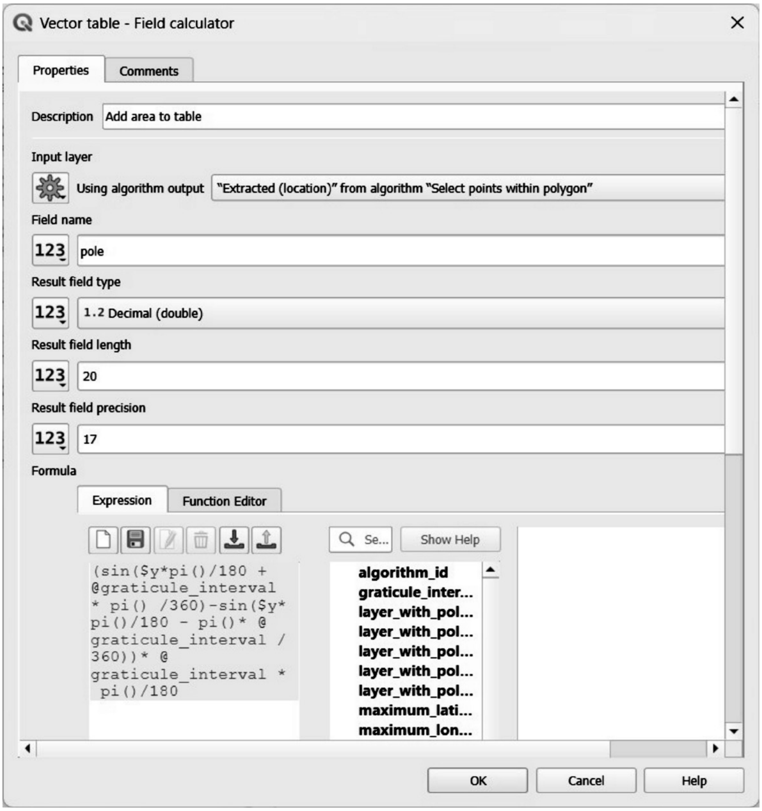

Figure 10. Interface of the function *surface area calculator*

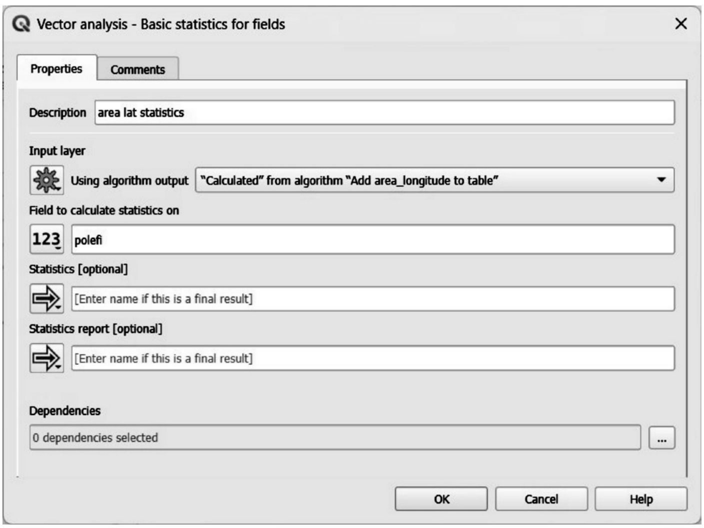

Figure 11. Interface of the function *basic surface area statistics*

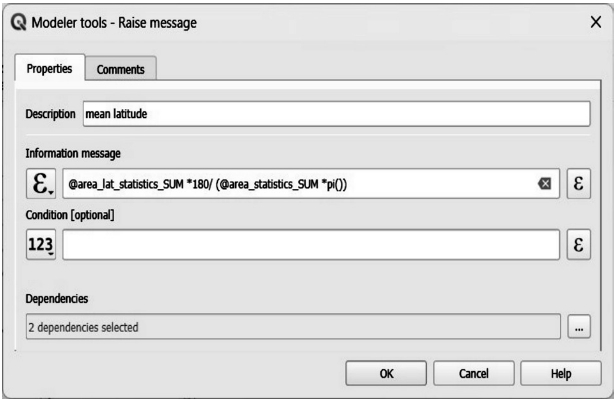

Figure 12. Interface of the designer's tool *Raise message*

Figure 13. Interface that pops up after launching the model

topographical objects BDOT10k. The EPSG number for the system is 2180. PL-LAEA is a system based on the Lambert oblique azimuthal equal-area projection. The main projection point has the following coordinates: φ_0 = 52° and λ_0 = 10°. This system is used for spatial analyses and reporting at the general European level (EPSG code 3035). PL-LCC is based on Lambert conformal conic projection. It has two standard parallels of the following latitudes:

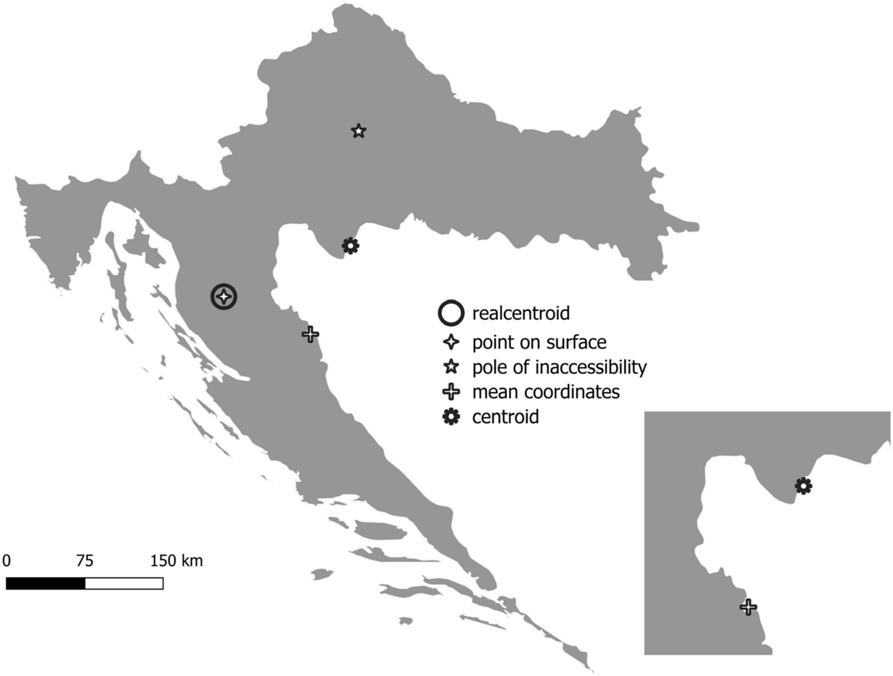

Figure 14. Location of centroids in Croatia

φ_1 = 35° and φ_2 = 65°. It mainly uses general and topographic maps (EPSG code 3034) (Rozporządzenie, 2012).

These systems have various features and distortions but their important common property is that in each of the systems, Poland is within one projection zone. Other systems valid in Poland, i.e., PL-UTM and PL-2000, include a division into meridian zones within Poland, so they are not well-suited for our calculations. The calculations were also made on reference ellipsoids: GRS80 and Krasowski, and on a sphere of an average radius R = 6371 km. The GRS80 ellipsoid is part of the currently binding geodetic reference system PL-ETRF2000. In terms of the use for calculations, important are the main parameters of the ellipsoid GRS80, so the length of the equator semi-axis a = 6 378 137 m and the inverse flattening 1/f = 298.2572221. The Krasowski ellipsoid was part of the Pułkowo ’42 system valid in the USSR and dependent countries, including Poland. The parameters of this ellipsoid are as follows: a = 6378 245 m, 1/f = 298.3. In calculations for the sphere, the mean radius was R = 6371 000 m. Transformations between coordinate systems were performed “on the fly” based on algorithms implemented in QGIS.

5. Results and their analysis

First of all, the location of centroids for Croatia was determined. The following were determined: *centroid*, *mean coordinate(s), pole of inaccessibility, point on surface* and centroid calculated with the plug-in *realcentroid.* As presented in Figure 14 the *centroid* and the *mean coordinate(s)* are outside of the borders of Croatia, the *pole of inaccessibility* is in the norther part and *point on surface* and the centroid calculated with the plug-in *realcentroid* are in the same place in the central part. Figure 14 on the right includes an enlarged fragment with the centroids the location of which against the borders of Croatia is not clearly visible in the figure on the left.

Similar calculations were made for Poland. The same algorithms were used as for Croatia.

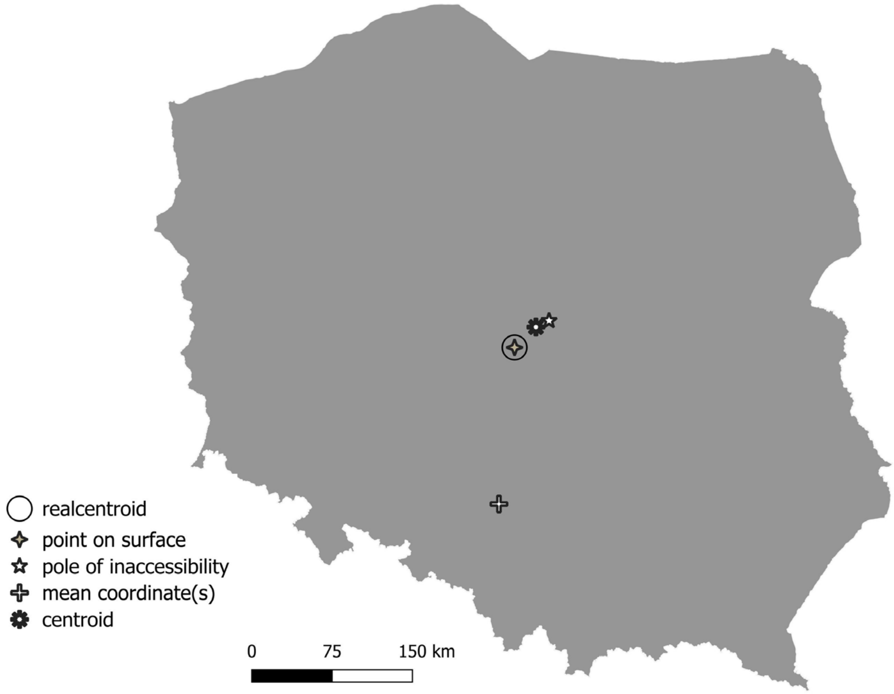

Figure 15. Location of centroids in Poland in administrative borders

The results are presented in Figure 15. Two methods yield the same results – *realcentroid* and *point on surface,* which may suggest that the same algorithms were used. In the further part, the plug-in *realcentroid* will not be used. The location of the *mean coordinate(s)* varies significantly from the other points. This function was thus tested applying a different approach. *Mean coordinate(s)* for Poland were calculated but adding the borders of administrative units: voivodships, counties and then communes. As a result, the *mean coordinate(s)* moved closer to the *centroid,* as illustrated in Figure 16. The function of determining the *mean coordinate(s)* also works in point layers. Thus, subsequent calculations were made with this function. *Centroids* were determined for administrative units, surface areas were also calculated for them. Then the *mean coordinate(s)* was also determined for this set of points, with weights calculated for the surface area. Figure 17 presents the location of these points in relation to the *centroid* and previously calculated *mean coordinate(s)*. It can be seen that the new *mean coordinate(s)* (for voivodships 5, counties 6 and communes 7) are very close to the *centroid* (Figure 17).

We then analysed the impact of the selection of projection on the location of *centroids*. Apart from PL-1992, two other coordinate systems were also selected, i.e., PL-LAEA and PL-LCC. Calculations were made for Poland without a division into administrative units. Since there were slight differences, the results in the form of geographical coordinates are presented in Table 1. The coordinates differ up to several

Table 1. Geographical coordinates of centroids determined in three planar Cartesian coordinate systems

Coordinate system	Latitude	Longitude
PL-1992	52°7’26.0”	19°25’17.4”
PL-LCC	52°7’ 30.8”	19°25’14.0”
PL-LAEA	52°7’25.4”	19°25’05.5”

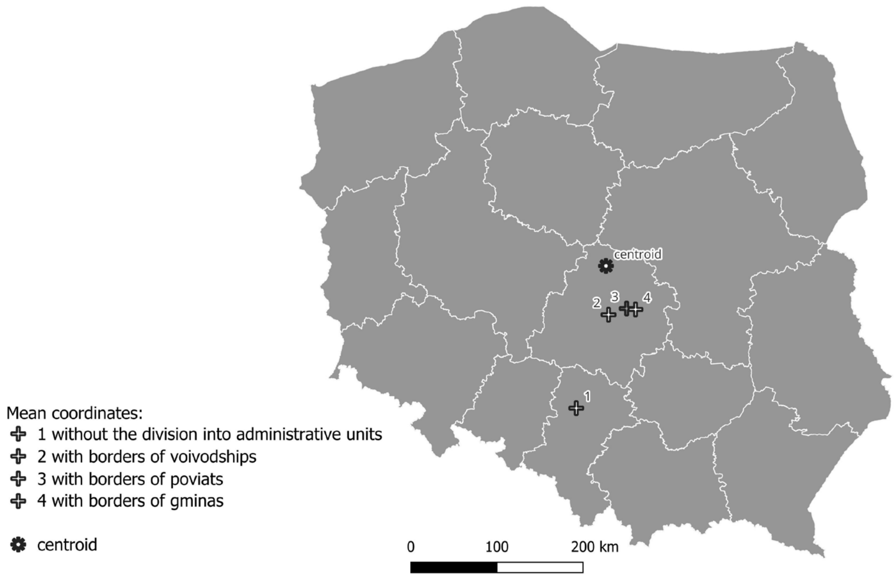

Figure 16. Location of the *mean coordinate(s)* of Poland determined on the basis of polygons: without the division into administrative units (1) and as a result of subsequent addition of borders of voivodships (2), counties (3) and communes (4)

seconds, which means that physically the differences are no greater than a few hundred metres.

Subsequent calculations were made directly on the reference ellipsoid according to the author's own algorithm, presented in Figure 6. First, a set of meridian-parallel quadrilateral centres wase generated, every one degree, then every one minute and finally every 15 seconds. The surface areas of these quadrilaterals on an sphere were calculated, followed by calculations of the mean values of geographical coordinates of the *spherical centroid* for Poland. The obtained results are presented in Table 2.

We can easily observe that the smaller the differences of the size of the quadrilaterals, the smaller the differences between the coordinates.

Calculations were also made assuming as the reference surface the GRS80 and Krasowski ellipsoid. The results of calculations for quadrilateral of 15" sides are presented in Table 3.

Only small differences between the results were obtained. Slightly different parameters of the ellipsoid basically have no impact on the results. Slightly bigger differences occur when we compare the results for the sphere and the ellipsoid.

Similar calculations were made for the area of Poland, including the territorial waters. Such borders are published by the Main Office of

Table 2. Geographical coordinates of spherical centroids for a set of points regularly located within the area of Poland

Quadrilateral side	**Latitude**	**Longitude**
1°	52°11'06.3"	19°21'54.6"
1'	52°5'47.9"	19°24'21.3"
15"	52°5'48.9"	19°24'21.2"

Table 3. Coordinates of ellipsoidal and spherical centroids calculated for various reference surfaces for 15" quadrilaterals

Reference surface	**Latitude**	**Longitude**
GRS80 ellipsoid	52 °5'49.6"	19°24'20.6"
Krasowski ellipsoid	52°5'49.6"	19°24'20.6"
Sphere	52°5'48.9"	19°24'21.2"

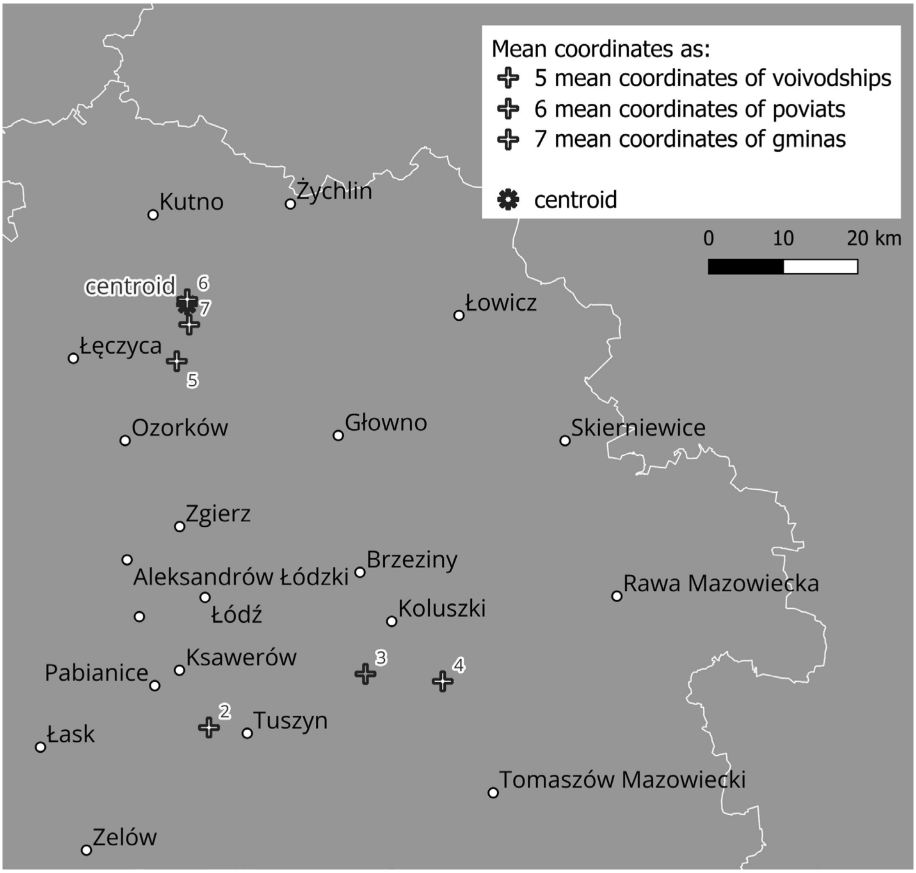

Figure 17. Location of the *mean coordinate(s)* of Poland determined on the basis of the *mean coordinate(s)* of administrative units as a result of subsequent addition of the borders of voivodships (5), counties (6) and communes (7)

Geodesy and Cartography (in Polish, Główny Urząd Geodezji i Kartografii, GUGiK) in the PRG database. Calculations were made on the GRS80 ellipsoid (for 15” quadrilaterals) and on a plane in PL-1992 (EPSG 2180). The differences are quite significant, of a few minutes. Also, if we compare the results from Table 3 and 4 we see differences of a few minutes. This is, obviously, the result of pushing the border north since territorial waters are included in the calculations.

6. Comparison of results

The coordinates of the centre of Poland calculated with the algorithm proposed in the article can be compared with coordinates calculated by Meus (2019) with the methods developed by Różański (2018). Meus obtained the following coordinates of the centre of Poland, taking into account territorial waters:

Table 4. Coordinates of the ellipsoidal centroid for GRS80 ellipsoid for 15” quadrilaterals and the centroid of Poland in PL-1992, taking into account territorial waters

Ellipsoid/coordinate system	Latitude	Longitude
GRS80	52°9’45.8”	19°20’14.9”
PL-1992	52°7’26.0”	19°25’17.4”

Figure 18. Polygons reptesenting the area of two US states: Arizona (on the left) and Alabama (on the right, USGS, n.d. a)

φ = 52°11'27.95" = 52.191097222°, λ = 19°21'19.46" = 19.355405556°. This article presents the following results φ = 52°9'45.8" = 52.162722222°, λ = 19°20'14.9" = 19.337472222°. The respective differences are thus Δφ = 0.028375° and Δλ = 0.017933°.

Results obtained with the developed algorithm were also compared with the results of calculations published in the article by Rogerson (2015). Rogerson (2015) presented results of calculations of geographical centres of the US according to his methods briefly described in the third section (Methods of centroid determination) of this paper. Figure 18 presents the borders of the states. Data were obtained from the portal (USGS, n.d. a). For Arizona and Alabama, Rogerson obtained the following centres of geographical centres: for Alabama φ = 32.7784°, λ = -86.8290° and for Arizona φ = 34.2740°, λ = -111.6582°. The centroid coordinates calculated with the algorithm described in this paper are as follows: for Alabama φ = 32.7396°, λ = -86.8455° and for Arizona φ = 34.2649°, λ = -111.6629°. The differences are: for Alabama - Δφ = 0.0388°, Δλ = 0.0165° and for Arizona - Δφ = 0.0091°, Δ λ = 0.0047°.

USGS provides an approximate location of geographical centres but as a description, e.g., for Alabama 12 miles south west of Clanton in Chilton county and for Arizona 55 miles east-south-east from Prescott in Yavapai county (USGS, n.d. b).

7. Summary and conclusions

The article presented results of calculations of centroids for the area of Poland with application of different methods. It analysed the impact of various reference surfaces (GRS80 and Krasowski ellipsoids and a sphere) and cartographic projections (coordinate systems PL-1992, PL-LAEA, PL-LCC) on the location of centroids. Application of such cartographic projections which do not have large distortions leads to slight differences in results. The use of a sphere or an ellipsoid does not change the location of centroids much either. However, the method of centroids calculation is of great importance. The choice of the method depends on what we want to use the centroids for. Significant is also the correct determination of the borders of the area. Large changes in the borders cause large changes in the location of centroids.

The article proposed a method of calculation of centroids on a sphere and ellipsoid. The method consists of determining smaller and smaller meridian-parallel quadrilaterals inside the area where the centroid is determined. Theoretically, decreasing the quadrilaterals should yield smaller and smaller changes of location of centroids and so we can observe this analysing the obtained results. In this method, the weighted average was determined using the weights of surface areas of smaller and smaller quadrilaterals. With small spherical quadrilaterals we can use a simpler method of weights determination, i.e., calculate the cosine of the latitude of their centres since the elementary surface area on a sphere is described with the formula $dP = R^2cos(\varphi)d\varphi d\lambda$. This was checked by calculations. In this method some quadrilaterals may be cut through by the border of the area. Then one can additionally calculate the surface area of the part within the area. This is, however, difficult to do on an ellipsoid since it requires the use of complicated algorithms of calculating surface areas of ellipsoidal polygons. Disregarding these quadrilaterals is a big simplification but it was assumed that with smaller and smaller quadrilaterals the resulting error will be smaller and the differences between the obtained results will be smaller. A large simplification in the developed algorithms is also determination of the centres of quadrilaterals as the average of the coordinates. The length of the side in the north is shorter than the length in the south. However, also in this case decreasing in subsequent iterations of the size of quadrilaterals will minimize the problem, e.g. for 15” quadrilaterals the differences between these lengths are of a centimetre. Another simplification is the use of WGS84 ellipsoidal coordinates when making calculations on a sphere. This means that the calculations reflect only the impact of differences between the radii of curvatures of the surfaces on the calculated coordinates of centroids. A similar assumption was made for the GRS80 ellipsoid.

The article compared calculation results of centroids for Poland and two US states: Arizona and Alabama. The differences in the obtained coordinates are ca. 0.005 – 0.04°. These are not large differences. USGS provides the location of a geometric centre in a very approximate way by giving an approximate direction and distance from a chosen town. It also gives a justification for these approximated locations. “Because there is no generally accepted definition of a geographic center and no completely satisfactory method of determining it, there may be as many geographic centers of a State or county as there are definitions of the term.” (USGS, n.d. b).

In the case of Poland, the town of Piątek is the traditionally accepted centre. In 2018, a geodetic undertaking ”Honorowy Południk Krakowski” led by the surveyor Mariusz Meus determined that the centre of Poland is located in Nowa Wieś near Kutno. There, they erected a memorial stand physically marking this place (52°11’27.95”, 19°21’19.46”, Meus, 2019). To determine this place, modern calculation and measurement methods were used and territorial waters were taken into account. This caused that the point is a bit farther from Piątek.

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