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TEMPORAL HAWKES PROCESS FOR MODELING SHORT-TERM CLUSTERING OF CRIME EVENTS IN OSTRAVA

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Abstract:
This paper applies a temporal Hawkes process to the analysis of short-term clustering in crime event data. The study uses publicly available records from the Crime Map of the Police of the Czech Republic and focuses on events of general crime in the Ostrava region during the period 2016-2025. A homogeneous Poisson process is used as a baseline model, while the Hawkes process with an exponential kernel is used to capture possible self-exciting behavior in the event sequence. The parameters are estimated by maximum likelihood and the models are compared using log-likelihood, AIC, BIC and residual diagnostics based on the time-rescaling principle. The results show that the temporal Hawkes model provides a substantially better description of the data than the homogeneous Poisson model. The estimated branching ratio is approximately 0.143, which indicates the presence of a measurable but not dominant self-exciting component. The estimated excitation half-life is about 1.11 hours, suggesting that the detected temporal dependence is mainly short-term. The results demonstrate that even a simple one-dimensional Hawkes model can provide an interpretable description of temporal clustering in crime data. At the same time, the analysis highlights the limitations of a stationary temporal model and motivates future extensions including non-stationary background intensity and spatial information.

Key words: crime data; Hawkes process; point process; self-excitation; temporal clustering

INTRODUCTION

Crime is an important social and security-related phenomenon, and its analysis is increasingly based on spatially and temporally referenced data. Such data make it possible to study not only the overall frequency of events, but also their clustering in time and space. Short-term concentrations of events are particularly important, as they may indicate that the occurrence of one event is followed by an increased risk of further events in the near future.

A simple reference model for random events in time is the homogeneous Poisson process. This model assumes a constant intensity and independent events. In the context of crime data, it provides a useful baseline model, but its main limitation is that it cannot capture dependence on the history of the process. If events occur in short temporal clusters, the Poisson model describes this structure only as random variability around the mean rate.

The Hawkes process provides a more suitable framework for modeling temporal clustering. Its basic idea is that each observed event temporarily increases the conditional intensity of the process. The model therefore distinguishes between the baseline background intensity and a short-term self-exciting component. As a result, it provides not only greater flexibility, but also interpretable parameters, in particular the branching ratio and the half-life of the excitation effect.

The aim of this paper is to assess whether a simple temporal Hawkes model provides a better description of crime events than a homogeneous Poisson process. The analysis is based on publicly available data from the Crime Map of the Police of the Czech Republic. It focuses on a selected category of general crime in the Ostrava region during the period 2016-2025. Spatial information is used only to define the analyzed region, while the model itself is formulated as a one-dimensional temporal point process.
The contribution of the paper is a quantitative comparison of the Poisson and Hawkes models using maximum likelihood estimation, information criteria and residual diagnostics. The results show whether a measurable short-term self-exciting component is present in the analyzed data and what limitations remain when a stationary temporal model is used.

LITERATURE REVIEW

Crime analysis has long been concerned with the question of whether events are distributed uniformly in space and time, or whether they form stable or short-term concentrations. In practical applications, hotspot maps, kernel density estimation and other spatial clustering methods are often used to identify areas with increased crime activity [1, 2]. These approaches are intuitive and easy to interpret, but they are mainly descriptive. They show where events are concentrated, but do not by themselves explain whether the observed clustering is related to short-term dependence between individual incidents.
The importance of spatial concentration is also emphasized in the criminology of place. Empirical studies show that a substantial proportion of crime is concentrated in a relatively small number of locations, which motivates place-based prevention and hotspot policing strategies [3, 4]. These studies support the practical relevance of spatial crime analysis, but they also indicate that spatial concentration alone is not sufficient to describe the temporal dynamics of crime events.
In addition to spatial concentration, the temporal structure of crime is also important. Crime events may exhibit regular daily, weekly or seasonal patterns, as well as short-term episodes of increased activity. In criminological literature, this type of behavior is often associated with repeat victimization and near-repeat crime, where the occurrence of one event may increase the risk of further events in nearby time and space [5]. Such behavior cannot be adequately represented by a model that assumes complete independence between events.
The near-repeat mechanism has motivated prospective crime mapping, where recent events are used as indicators of increased future risk [6, 7]. Related predictive policing approaches use historical crime data to support operational decision-making and resource allocation [8]. However, these approaches differ in the degree to which they explicitly model the stochastic dependence between individual events. In theoretical criminology, such dependence is also connected with opportunity structures, routine activities and crime pattern theory [9, 10].
Point processes provide a natural mathematical framework for modeling events in time. The basic model is the homogeneous Poisson process, which assumes constant intensity and independent events [11]. In crime data analysis, it is therefore useful as a simple reference model. However, its limitation is that it does not include any mechanism by which a past event could influence the probability of future events.
This limitation is addressed by Hawkes processes, originally introduced as self-exciting point processes [12, 13]. Their basic idea is that each realized event temporarily increases the conditional intensity of the process. The model therefore separates the background component, describing the baseline occurrence of events, from the excitation component, describing the effect of the process history. A review of Hawkes process applications in different fields is provided by Reinhart [14].
In crime modeling, a key contribution is the work of Mohler and co-authors, who applied self-exciting point processes to crime data and showed their connection with the near-repeat mechanism [15]. The advantage of the Hawkes framework is that it provides not only intensity prediction, but also an interpretable decomposition of the process into baseline and history-dependent components.
Mathematical models of crime have also been developed outside the Hawkes framework. For example, reaction-diffusion and agent-based formulations have been used to describe the emergence, persistence

and displacement of crime hotspots [16, 17]. These models provide useful insight into spatial concentration and repeat victimization, while the Hawkes framework used in this paper focuses directly on the conditional intensity of event occurrence in continuous time.

The parameters of Hawkes processes are commonly estimated by maximum likelihood [18, 19]. This approach allows direct comparison of models defined on the same observation window, such as the homogeneous Poisson process and the temporal Hawkes process. Model assessment is also commonly based on information criteria such as AIC and BIC, which account for both goodness of fit and the number of estimated parameters [20, 21].

From a broader methodological perspective, temporal point processes are classical tools for the statistical analysis of event series [22]. Recent overview literature discusses Hawkes processes as a flexible class of self-exciting point process models with applications across several domains [23, 24].

Spatio-temporal Hawkes processes also exist, modeling not only the event time but also its spatial location [14]. Such models are natural for crime data, but they require a more precise specification of the spatial domain, the spatial triggering kernel and often also a non-stationary background intensity. This paper therefore focuses on a simpler temporal model. The aim is not to develop a complete crime prediction system, but to assess whether temporal self-excitation alone improves the description of the data compared with the homogeneous Poisson process.

METHODOLOGY OF RESEARCH

Data sample and preprocessing

The input data were obtained from the publicly available Crime Map of the Police of the Czech Republic [25]. The data contain registered crime events, including their time, spatial coordinates and event category. This study focuses on category 63, corresponding to general crime, in the Ostrava region during the period from 1 January 2016 to 31 December 2025.

Since the aim of the paper is to assess the temporal structure of the event sequence, spatial information was used only for data filtering. Only records located inside a rectangular bounding box covering the Ostrava region were included in the analysis. After spatial and temporal filtering, the final data set contained $N = 8332$ events. The observation window covered $T = 3653$ days. Event times were converted into a one-dimensional ordered time sequence $0 < t_1 < \ldots < t_N < T$, expressed in days from the beginning of the observation period. If several events had identical timestamps, a very small deterministic jitter was applied for numerical purposes to remove tied event times.

Models

Two temporal point process models were compared. The first one was the homogeneous Poisson process, used as a reference model. It represents the simplest continuous-time model for random event occurrence. In this model, the number of events observed in any time interval depends only on the length of the interval, and event counts in disjoint intervals are independent. For an interval of length Δt, the number of events is Poisson distributed with mean $\mu\Delta t$, where μ is the constant event intensity:

$$P(N(t+\Delta t) - N(t) = k) = \frac{(\mu\Delta t)^k}{k!}\exp(-\mu\Delta t). \quad (1)$$

In the present application, this means that the homogeneous Poisson model assumes a constant average rate of crime events over the whole observation period and no memory of previous events. It therefore cannot distinguish between isolated events and short sequences of events occurring close together in time. For this reason, it is useful mainly as a baseline model: if the Hawkes process provides a substantially better fit, this indicates that the temporal structure of the data is not adequately explained by a constant-rate independent-event model. The only parameter of the Poisson model is the average daily intensity, estimated as $\hat{\mu}_P = N/T$.

The second model was a temporal Hawkes process with an exponential triggering kernel. Its conditional intensity was defined as:

$$\lambda(t \mid \mathcal{H}_t) = \mu + \sum_{t_i<t} \alpha \exp[-\beta(t - t_i)], \tag{2}$$

where:
$\mathcal{H}_t$ denotes the history of the process before time t.
The parameter μ represents the background intensity, while α and β determine the magnitude and decay rate of the self-exciting effect. Each observed event therefore temporarily increases the intensity of future events. The ratio α/β was used as the branching ratio, and the excitation half-life was computed from the estimated parameter β. To ensure a stable regime, the constraint $\alpha/\beta \leq 0.8$ was imposed.

Parameter estimation
The parameters of the Hawkes model were estimated by maximum likelihood. For a point process observed on the interval $[0,T]$, the log-likelihood is given by:

$$\ell(\theta) = \sum_{i=1}^{N} \log \lambda(t_i \mid \mathcal{H}_{t_i}) - \int_0^T \lambda(u \mid \mathcal{H}_u) du. \tag{3}$$

The first term evaluates the model intensity at the observed event times, while the second term penalizes the total intensity over the whole observation window. The negative log-likelihood was minimized in MATLAB using fmincon. To reduce dependence on the initial parameter values, several initial settings were tested, and the solution with the highest log-likelihood was retained. The exponential kernel was evaluated recursively, which speeds up the computation compared with direct summation over all previous events.

Model comparison and diagnostics
The homogeneous Poisson model and the temporal Hawkes model were compared using the log-likelihood, AIC and BIC. These criteria allow direct comparison of models fitted on the same observation window, while AIC and BIC also penalize a higher number of estimated parameters.
As an additional diagnostic, the time-rescaling principle was used. If the point process model is correctly specified, the transformed inter-event intervals should approximately follow the uniform distribution on [0,1]. Therefore, the empirical distribution function of the transformed residuals was compared with the ideal diagonal line. This diagnostic was used to assess whether the fitted Hawkes model captures the main temporal structure of the observed event sequence.

RESULTS OF RESEARCH

The basic overview of the analyzed data is shown in Figure 1 on the left.

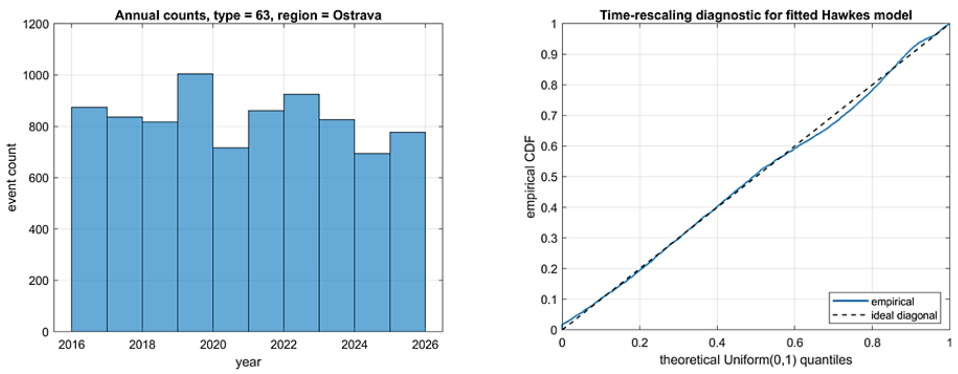

Fig. 1 Annual event counts for the period 2016-2025 and the time-rescaling diagnostic of the fitted Hawkes model. In the diagnostic plot, a well-fitting model should be close to the dashed diagonal line
All text: *annual event counts vary across 2016-2025; the Hawkes time-rescaling curve closely follows the ideal diagonal.*

The annual event counts indicate that the data set covers the whole period 2016-2025 and that the event frequency varies between individual years. This is important for interpreting the results,

because the fitted Hawkes model uses a constant background intensity and does not explicitly capture long-term trends.
The comparison of observed and model-expected daily counts is shown in Figure 2.

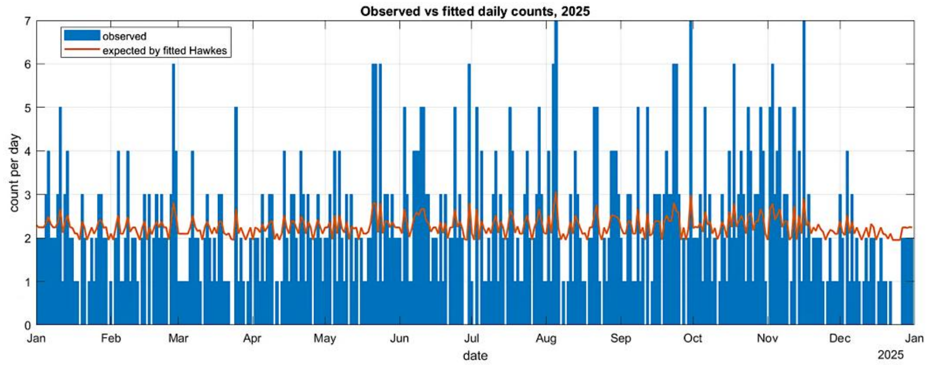

Fig. 2 Observed and model-expected daily event counts in 2025. Bars indicate the observed daily counts, while the curve represents the expected values according to the Hawkes model estimated on the full period 2016-2025
All text: *observed daily crime counts in 2025 fluctuate around the fitted Hawkes expected count, mostly near two events per day.*

The model was estimated on the full observation period, but only the year 2025 is displayed for readability. The bars show the observed daily event counts, while the curve represents the expected number of events according to the fitted Hawkes model. The curve remains mostly within a narrow band around the long-term daily frequency, which is consistent with the fact that the estimated self-exciting component is present but not dominant.
The estimated parameters of the Hawkes model are reported in Table 1.

Table 1
Estimated parameters of the temporal Hawkes model

Parameter	Estimate	Interpretation
$\mu.$	1.9544	background intensity [day^{-1}]
$\alpha.$	2.1477	excitation amplitude
$\beta.$	15.0027	decay rate [day^{-1}]
$\alpha/\beta.$	0.1432	branching ratio
$t_{1/2}.$	1.1088	excitation half-life [hour]

The background intensity is lower than the intensity of the homogeneous Poisson model, because part of the total event frequency is explained by the short-term self-exciting component in the Hawkes model. The branching ratio 0.1432 means that approximately 14.3% of the expected intensity can be attributed to dependence on previous events. The excitation half-life of 1.1088 hours indicates that this effect is mainly short-term.
The comparison of the homogeneous Poisson model and the temporal Hawkes model is summarized in Table 2.

Table 2
Comparison of the fitted Poisson and Hawkes models

Model	Log-likelihood	AIC	BIC
Poisson	-1461.81	2925.62	2932.65
Hawkes	-1184.66	2375.33	2396.41

The Hawkes model achieved a higher log-likelihood and, at the same time, lower AIC and BIC values. This shows that the improvement in fit is not only a consequence of the larger number of parameters, but that the Hawkes model better captures the temporal structure of the analyzed events.
The residual diagnostic is shown in Figure 1 on the right. The empirical curve of the transformed inter-event intervals is close to the ideal diagonal, which supports the conclusion that the model captures a substantial part of the temporal dependence in the data. The remaining deviations can be mainly associated with the simplified model specification, especially the constant background intensity and the absence of a spatial component.
Overall, the results show that the temporal Hawkes model provides a more suitable description of the data than the homogeneous Poisson process. The main benefit of the model is its ability to separate the baseline event intensity from the short-term self-exciting component.

DISCUSSION

The results confirm that the temporal structure of the analyzed events is not well described by a model with constant intensity and independent events. The Hawkes model achieves better log-likelihood, AIC and BIC values, and at the same time provides an interpretable decomposition of the intensity into a background and a self-exciting component. From this perspective, the improvement is not only a formal improvement in fit, but also reflects a model that is more consistent with the assumption of short-term clustering in crime events.
The estimated self-exciting component is measurable, but not dominant. The branching ratio of approximately 0.143 indicates that most of the intensity is still associated with the background component. At the same time, the excitation half-life of approximately 1.11 hours suggests that the detected dependence is mainly short-term. The result should therefore not be interpreted as evidence of long chains of dependent events, but rather as a statistical indication of short episodes of increased activity.
This distinction is important from the point of view of interpretation. A high branching ratio would indicate that a large part of the process is generated by previous events, while the value obtained here suggests a more moderate form of temporal dependence. The model therefore does not describe crime occurrence as a purely self-propagating process. Instead, it indicates that the dominant part of the event intensity is associated with the background level of activity, while a smaller but measurable part is related to short-term temporal clustering. This is a useful result, because it provides a quantitative measure of the relative importance of the two components.
It should be emphasized that the model does not prove causality between individual events. The Hawkes process only describes that, after an event occurs, the conditional intensity of further events is temporarily increased in the data. Such dependence may be related to actual links between incidents, but it may also reflect unobserved factors, such as time of day, type of location, reporting practice or other circumstances that are not explicitly included in this simple model.
For this reason, the fitted model should not be interpreted as a direct operational prediction tool. Its role in this paper is primarily descriptive and diagnostic. It allows one to test whether the independent-event assumption is sufficient and to quantify the extent of short-term temporal dependence. In practical crime analysis, such a model can serve as a preliminary statistical layer before more detailed spatial, contextual or operational information is included.
The main limitation is the constant background intensity. The annual event counts in Figure 1 show that the frequency of events changes over time, while the fitted Hawkes model does not model this longer-term variability separately. Part of the deviations in the residual diagnostic may therefore be caused by this simplified stationary formulation. A natural extension would be to introduce a time-varying background component, for example using daily, weekly or annual periodic effects.
This limitation is particularly relevant for crime data, because the observed intensity may be affected by calendar effects, changes in reporting behavior, changes in police activity, social events or long-term

changes in the urban environment. If such effects are not modeled explicitly, part of the slow temporal variation may be absorbed either into the constant background level or into the excitation component. A non-stationary version of the model could therefore provide a cleaner separation between systematic temporal variation and genuine short-term self-excitation.

Another limitation is that spatial location was used only to select events in the Ostrava region. The model therefore cannot distinguish whether dependent events are also spatially concentrated, or whether the clustering is only temporal. For crime data, a natural next step is therefore a spatio-temporal Hawkes model, which would allow temporal and spatial dependence between events to be described jointly. In this sense, the temporal model used here represents a simple reference step for further analysis.

The use of a purely temporal model is nevertheless useful as a first step. It avoids additional assumptions about the shape of the spatial triggering kernel, boundary effects and the definition of the spatial observation window. This makes the model easier to estimate and interpret. Once the temporal component has been assessed, the same framework can be extended to include spatial distance, local background intensity and possibly different excitation mechanisms for different crime categories.

CONCLUSIONS

This paper applied a temporal Hawkes process to the analysis of short-term temporal clustering of crime events in the Ostrava region during the period 2016-2025. The model was compared with a homogeneous Poisson process, which served as a simple memoryless reference model with independent events.

The results showed that the Hawkes model provides a more suitable description of the data than the homogeneous Poisson model. The improvement was confirmed by a higher log-likelihood and lower AIC and BIC values. The estimated branching ratio of approximately 0.143 indicates the presence of a measurable but not dominant self-exciting component. The excitation half-life of approximately 1.11 hours further suggests that the detected dependence is mainly short-term.

The main contribution of the paper is to show that even a simple one-dimensional Hawkes model can provide an interpretable description of the temporal structure of crime data. The model separates the baseline background intensity from the short-term excitation component and provides parameters with direct interpretative value.

However, the model used in this paper is intentionally limited in scope. It does not include a time-varying background intensity or the spatial location of events in the model formulation itself. Future work should therefore focus on a non-stationary background component, regular temporal patterns and an extension to a spatio-temporal Hawkes model.

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