

A MICROMECHANICALLY GROUNDED METHODOLOGY FOR EVALUATING THE ABRASIVITY OF PARTICLES IN ACCELERATED TRIBOLOGICAL TESTS

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Abstract:

This work proposes an improved methodology for evaluating the abrasivity of particles in accelerated tribological tests, extending a previously developed approach to materials testing under friction against loose abrasive. Instead of a single scalar sharpness measure, a micromechanically interpretable triplet of geometric descriptors $\{h_{\max}, \beta_{\max}, \rho_{\min}\}$ is proposed—the protrusion height, the tangent rotation angle across the peak, and the minimum radius of curvature along the contour, computed from the analytical curvature of the elliptic Fourier reconstruction. This triplet is combined into a single dimensionless index κ^* (effective cutting curvature index) that preserves the micromechanical interpretation of its components. On a sample of $n = 200$ silicon carbide 64C particles (50 particles at each of four test stages: 0, 250, 500, and 825 min in the steel 40/SiC 64C system), $h_{\max}$ is shown to be the dominant component of the triplet with respect to the test stage (Spearman $\rho = -0.51$, $p < 10^{-13}$); the combined index κ^* yields $\rho = -0.46$ ($p < 10^{-10}$); the canonical SPLm parameter, implemented via elliptic Fourier reconstruction with $N = 50$ harmonics and piecewise-linear approximation of the peaks, yields $\rho = -0.48$ ($p < 10^{-12}$) on the same sample. The advantage of the triplet and the combined κ^* over SPLm lies in the transparent micromechanical interpretation of their components. A population balance model is constructed for the evolution of the log-normal moments of the shape parameters and coupled with the kinetics of wear intensity $I(\tau)$ as an a posteriori description of the two-scale structure of the kinetics. A direct fit of the kinetic data (33 intervals of 25 min duration) yielded a characteristic relaxation time $\tau_c = 24$ min and, accordingly, an optimal duration of accelerated tests $\tau^* = \tau_c \ln 20 \approx 72$ min under the 5% criterion of $|dI/d\tau|_0$. The conclusions are directly substantiated for the investigated steel 40/SiC 64C system; verification on other counterbody materials and abrasives is the subject of further work. The obtained result increases the productivity of laboratory screening of materials and coatings and has direct value for production engineering systems.

Key words: abrasive wear; sharpness descriptor; elliptic Fourier descriptors; population balance model; production engineering; accelerated testing

INTRODUCTION

Wear under friction against loose abrasive remains one of the most widespread forms of wear of equipment working components in the oil-and-gas [1], mining [2], construction, and agricultural sectors. The rational selection of materials for coatings and hardfacings operating under such conditions is a key element of production engineering systems: a well-grounded preliminary selection reduces downtime and repair costs, whereas a wrong decision leads to premature failure of parts and significant operating losses.

LITERATURE REVIEW

Among the industrial solutions for restoring and strengthening working surfaces in the sectors mentioned above, wear-resistant hardfacings deposited by flux-cored arc welding (FCAW) have found widespread use. The principal material classes include: hardfacings based on high-manganese steels reinforced with complex carbides, which combine the high hardness of the carbide phase with the ductility of the austenitic matrix [2]; self-fluxing hardfacings of the Fe-Ti-Mo-B-C system, in which the synthesis of complex boride-carbide inclusions takes place directly during welding at low currents [3]; cermets based on niobium carbide NbC combined with a Hadfield-type austenitic steel matrix, intended for impact – abrasive wear conditions [4]; and tungsten-free cermet systems with a copper-nickel-manganese binder, optimized for the mechanical face seals of oil-and-gas equipment [1].

The quality and reproducibility of hardfacings are controlled by process parameters – charge composition, slag-forming components, and transfer coefficients of alloying elements – and form the subject of a separate line of research on FCAW optimization [5, 6, 7, 8]. Irrespective of the specific material and regime, the actual service life of a hardfaced part is determined jointly by the intrinsic characteristics of the coating and the morphological parameters of the abrasive particles in contact; a correct quantitative description of the latter is a necessary prerequisite for rational material screening. A substantial body of independent literature is devoted to the morphology-driven abrasivity of particles, in particular studies on angular and curvature descriptors [10, 22] however, systematic evaluations of these descriptors on representative samples of a single class of abrasive under the dynamics of friction are lacking.

The classical approaches to wear-resistance evaluation—under the GOST 23.208 and ASTM G65 standards – have several well-known limitations: a substantial test duration (10-60 min for the reference and the test specimen, respectively), stringent surface-preparation requirements, and the need for regular replacement of the rubber wheel. Among non-standardized methods, the Lorenz sleeve method is widely used; its principal drawback is poor renewal of the abrasive mass in the friction zone, leading to the accumulation of fragmented particles of reduced abrasivity and altering the very nature of the contact interaction.

In a related study [9], co-authored by two of the present authors (D.L. Lutsak and P.M. Prysyzhnyuk), a device and a methodology for tribological testing were developed that provide a high degree of renewal of the abrasive mass by placing the specimen at an angle to the axis of rotation of the holder. The same work introduced a modified sharpness parameter SPLm, computed by representing the particle contour as a Freeman chain code, performing its Fourier transform with 50 harmonics, and applying a piecewise-linear approximation to the peaks. For reference, abrasives of different origins (silicon, garnet, SiC, quartz, Al₂O₃), SPLm correlated with wear intensity ($R^2 = 0.92$), substantially outperforming the SPL parameter computed by the classical Hamblin-Stachowiak method [10].

It should be emphasized that the validation of SPLm in [9] was performed on five reference abrasives of different chemical origin (silicon, garnet, SiC, quartz, Al₂O₃) that is, SPLm discriminated between-class differences in morphology with $R^2 = 0.92$. In the present work, we re-examine the suitability of SPLm for an adjacent but distinct task—within-class discrimination of the stages of morphological evolution of the same abrasive (SiC) during friction. Within-class discrimination is a necessary prerequisite for building a kinetic model and for accelerating tribological tests. As will be shown (see the subsection "Statistical power of the descriptors"), on our sample of $n = 200$ SiC particles, SPLm in its canonical formulation

from [9] (elliptic Fourier reconstruction with $N = 50$ harmonics and piecewise-linear approximation of the peaks) yields Spearman $\rho = -0.48$ ($p < 10^{-12}$), i.e., it remains statistically significant in the within-class task as well. A separate methodological issue is the limitation of SPLm with respect to the transparency of its micromechanical interpretation: SPLm is an aggregated scalar that combines angular and radial characteristics in a fixed form through the Freeman chain code, with no possibility of decomposing it into components tied to distinct physical mechanisms of indentation. The triplet $\{h_{\max}, \beta_{\max}, \rho_{\min}\}$ proposed below and its composite reduction κ^* address precisely this interpretational task.

However, a new sharpness parameter alone is not sufficient to build a fully developed kinetic model: studying the time evolution of abrasive morphology requires an experimental apparatus capable of repeatedly subjecting the same array of abrasive particles to friction. The standard ASTM G65 configuration and the original implementation in [11] do not provide for this, since fresh abrasive is continuously fed into the friction zone through a chute and, after a single contact, discharged to waste, which entirely precludes the accumulation of morphological changes in the particles. A separate methodological prerequisite of the present work is therefore a modification of the apparatus [11] for operation in a regime of abrasive-mass circulation (see the subsection "Apparatus modification for studying the evolution of abrasive morphology"), which makes possible the direct and reproducible observation of the evolution of particle shape under repeated friction. Without this modification, the subsequent population-balance analysis and the determination of the optimal duration of accelerated tests could not have been carried out.

The object of the study is the process of abrasive wear of steel and hardfacing materials under friction against loose abrasive. The study examines the quantitative descriptors of abrasive particle morphology and their relationship to the kinetics of wear intensity.

The study aimed to improve the methodology for tribological testing of materials and coatings in production engineering by introducing a micromechanically grounded sharpness parameter, validating it on a representative sample, and constructing a kinetic model to determine the optimal duration of accelerated tests analytically. To achieve this aim, the following tasks were addressed:

- to modify the standard apparatus [11] to provide an abrasive-circulation regime that enables direct monitoring of the kinetics of morphological changes in the particles;
- to introduce and justify a new dimensionless sharpness parameter κ^* , built on the principles of indentation mechanics;
- to validate κ^* on a representative sample of SiC particles ($n = 200$) against classical and modern shape descriptors;
- to build a population-balance model for the evolution of the log-normal moments of the shape parameters, coupled with the wear-intensity kinetics $I(\tau)$, and to determine the optimal duration of accelerated tests τ^* .

MATERIALS AND METHODS

Apparatus and conditions of tribological tests

The tests were performed on a machine for testing materials' wear resistance under friction against a loose abrasive, the design and kinematic scheme of which are described in detail in [11] (see Figs. 2 and 3 in [11]). The apparatus is designed in accordance with the recommendations of ASTM G65-16 (Standard Test Method for Measuring Abrasion Using the Dry Sand/Rubber Wheel Apparatus). Principle of operation: a flat test specimen (item 10 in the scheme of [11]) is pressed against the cylindrical working surface of a rubber wheel by a lever on a hinged support; a removable load provides the required pressing force via an adjustable arm. Abrasive particles are fed gravitationally through a chute (4) into the contact zone; the linear-velocity vector of the wheel's working surface coincides with the direction of the abrasive flow, which promotes capture of particles into the friction zone, their partial embedding into the rubber layer, and interaction with the steel surface of the specimen. Wear occurs through the combined mechanisms of micro-cutting and cyclic deformation of the specimen surface.

Rubber-wheel parameters (per the specification of [11]): diameter $D = 50$ mm, width $b = 15 \pm 0.1$ mm, hardness 78-85 Shore units measured with an ASTM D2240 "D"-durometer; residual relative elongation of the wheel material 15-20%.

Specimen

Flat specimens of dimensions $15 \times 20 \times 25$ mm were manufactured from medium-carbon structural steel grade 40 in accordance with DSTU 7809:2015 (equivalent: C40/1.0511 per EN 10083-2:2006; AISI 1040 per ASTM A29/A29M; JIS S40C) without heat treatment. The roughness of the working surface – not greater than $R_a = 1.25$ μm ; geometric tolerances—to the 14th accuracy grade.

Abrasive medium

Green silicon carbide powder of grade 64C, grit F100 per FEPA Standard 42-1:2006 (Grains of fused aluminum oxide, silicon carbide and other abrasive materials for bonded abrasives – Macrogrits F4 to F220), corresponding to EN ISO 8486-1:2017 (mean grain diameter $d_{50} \approx 125$ μm ; $D_3 = 149$ μm , $D_{94} = 90$ μm) was used as the abrasive. The chemical composition of the abrasive (SiC content $\geq 98.5\%$, impurities $\leq 1.5\%$) was controlled in accordance with ISO 9286:2021 (Abrasive grains and crude—Chemical analysis of silicon carbide). The green grade 64C was chosen for two principal reasons: (i) the high purity and characteristic polyhedral particle shape with crystallographically determined sharp edges model the conditions of impact-abrasive wear in mineral media; (ii) the covalent type of chemical bonding in SiC imparts high stiffness to its crystals, which determines brittle fragmentation along crystallographic planes under repeated friction—the mechanism that underlies the morphological changes analyzed in the present work. Note that in [11], for an analogous apparatus, quartz sand (SiO_2 , 0.2-0.4 mm) was used, which tests a less "sharp" but more representative of the construction industry.

Apparatus modification for studying the evolution of abrasive morphology

A principally important methodological feature of the present work is the modification of the standard design of [11] to provide a circulation regime of the abrasive mass. In the standard ASTM G65 protocol and in [11], fresh abrasive is continuously fed from the chute into the friction zone and, after a single contact, diverted to discharge, thereby completely excluding the accumulation of morphological changes in the particles. In our modification, the feed chute is closed into a closed loop with the friction zone, so that the same volume of abrasive mass circulates through the contact zone throughout the entire test without renewal. This modification was developed specifically to track the kinetics of changes in abrasive particle morphology under repeated friction. Without it, the evolution of particle shape (Figs. 2 and 6 below) would in principle not be observable. The kinetics of wear intensity would not have a slow component associated with the loss of sharpness of cutting peaks (see the subsection "Two-scale kinetics and the rationale for accelerated testing").

Kinematic parameters and test regime

The rubber-wheel rotation frequency $\nu = 1.7$ s^{-1} , corresponding to a linear velocity of the working surface $v = \pi \cdot D \cdot \nu \approx 16$ m/min; specific load on the specimen $P = 5$ MPa applied through a lever system with a removable weight; duration of one cycle $t = 25$ min; number of consecutive cycles $N = 33$ (total test duration 825 min). The parameters P , ν , t were maintained constant throughout all 33 cycles.

Sequence of a single cycle

(i) The specimen was weighed on a VLA-200g-M analytical balance with a resolution of ± 0.1 mg before the test (m_{i-1}); (ii) installed in the lever system and the load P was applied; (iii) the rubber wheel was rotated for 25 min at $\nu = 1.7$ s^{-1} under abrasive circulation in the friction zone; (iv) the specimen was removed, the working surface cleaned with ethanol and dried in a stream of warm air; (v) re-weighed (m_i) and the mass loss $\Delta m_i = m_{i-1} - m_i$ was determined; (vi) the specimen was reinstalled without

renewal of the abrasive mass and the next cycle began. This procedure provides controlled 25-minute time resolution of the wear kinetics in the 0–825 min window and, simultaneously, representative samples of abrasive particles for morphological analysis at key stages of the test (see the subsection "Microscopy and binarization").

The raw mass-loss series $\{\Delta m_i\}_{i=1}^{33}$ is used to compute the wear intensity for the i -th interval:

$$I_i = \frac{\Delta m_i}{\Delta \tau} = \frac{m_{i-1} - m_i}{25 \text{ min}} \quad (1)$$

The full 33-point series of kinetic data is used below to construct the population balance model.

Microscopy and binarization

Imaging of the abrasive particles was performed on an MBS-2 microscope equipped with an eTREK UCMOS 5100 digital eyepiece camera in transmitted-light mode with a white background. For four test stages (0, 250, 500, and 825 min, corresponding to the initial state and the control points after 10, 18, and 30 circulation cycles in our modified apparatus), $n = 50$ individual particles per stage were selected and processed – a total of $n = 200$ particles.

Binarization protocol in ImageJ 1.50g

Raw color micrographs were converted into binary masks of individual particles via the following sequence of operations (Fig. 1):

1. Loading the image (File > Open) in .bmp format, typical resolution 2592×1944 px.
2. Conversion to 8-bit grayscale (Image > Type > 8-bit), which reduces the three-channel RGB signal to a single intensity channel and accelerates all subsequent morphological operations without loss of information useful for binary tasks.
3. Correction of non-uniform illumination (Process > Subtract Background, rolling-ball method [17] with radius ~ 80 px, light-background option) to remove the illumination gradient typical of optical micrographs taken under variable condenser illumination.
4. Pixel-noise smoothing (Process > Smooth—equivalent to a 3×3 mean filter), which reduces single-pixel fluctuations that could mimic artefactual "jagged edges" in the subsequent threshold segmentation.
5. Automatic threshold segmentation (Image > Adjust > Auto Threshold) by the Otsu method [16] for the bimodal intensity distribution (particle vs background). Internal validation shows that this method works robustly on the sample of micrographs with variable background brightness without manual threshold tuning.
6. Morphological cleaning of the mask – filling internal holes (Process > Binary > Fill Holes) to remove artefactual "specular" points inside the particle, and morphological opening with a disk-shaped structuring element of radius 2 px (Process > Binary > Open) to remove isolated single-pixel noise on the boundary.
7. Splitting touching particles and individualization (Process > Binary > Watershed [18]) followed by Analyze > Analyze Particles (minimum area 200 px²) to obtain individual binary masks of each particle. Each detected particle was saved as a separate 8-bit binary.tif file (background = 255, particle = 0) with a rectangular bounding box around the particle.
8. The resulting set of 200 individual binary masks (50 particles × 4 stages) is used as input to the subsequent Python pipeline for sub-pixel contour extraction and descriptor computation (see the subsection "Contour extraction and descriptor set"). A step-by-step visualization of the entire processing route—from the color micrograph to a single contour – is given in Fig. 1.

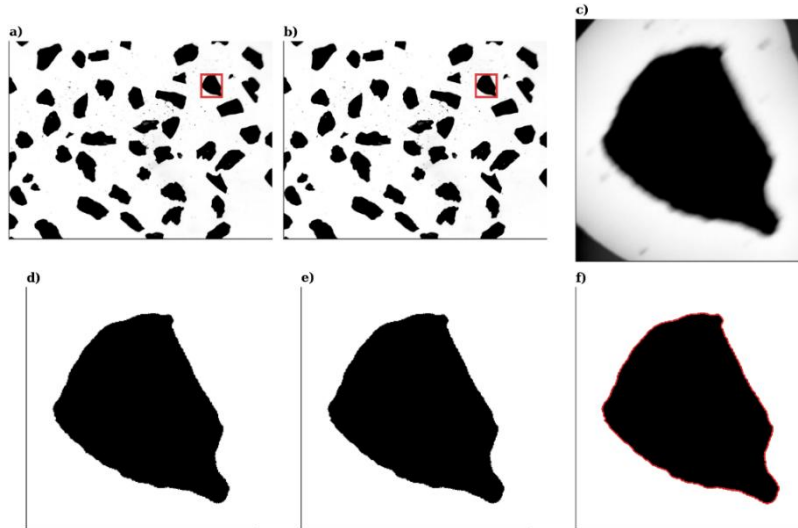


Fig. 1 Pipeline for processing a SiC micrograph, performed for one isolated particle from frame p3(10c).bmp (stage "after 10 cycles"). The target particle is highlighted with a red bounding box in panels (a) and (b): (a) original RGB micrograph; (b) conversion of the full frame to an 8-bit grayscale image; (c) cropped ROI of the target particle after Subtract Background (rolling-ball, radius 80 px) and a 3×3 mean filter; (d) binary image after Auto Threshold by the Otsu method; (e) final mask after Fill Holes + Binary Opening (structuring element disk $r = 2$ px) followed by selection of the largest connected component; (f) sub-pixel contour extracted by the marching-squares method

Alt text: six-panel pipeline for one SiC particle: micrograph of the particle sample, grayscale frame, cropped region, binary mask, and red sub-pixel contour.

Contour extraction and descriptor set. From each binary mask obtained by the procedure of the subsection "Microscopy and binarization", the sub-pixel contour was extracted by the marching-squares algorithm [14] in the scikit-image implementation [20]. From the resulting closed contour, a comprehensive set of shape descriptors was computed, covering both the particle's integral (global) geometric characteristics and its locally spectral features.

The integral descriptors include the area A , the perimeter L , the maximum and minimum Feret diameters $F_{\max}$ and $F_{\min}$, and the aspect ratio:

$$AR = \frac{F_{\max}}{F_{\min}} \quad (2)$$

the roundness:

$$R = \frac{4\pi A}{L^2} \quad (3)$$

the Wadell circularity $R_W = \left(\frac{L_{eq}}{L}\right)^2$,

where:

$L_{eq} = 2\sqrt{\pi A}$ is the perimeter of a circle of equivalent area, the solidity:

$$\text{solidity} = \frac{A}{A_{\text{conv}}} \quad (4)$$

defined as the ratio of the particle area to the area of its convex hull A_{conv} ;

the convexity $\text{convexity} = \frac{L_{\text{conv}}}{L}$ (the ratio of the convex-hull perimeter to the actual perimeter);

the eccentricity; and the perimeter ratio $\frac{L}{L_{eq}}$.

The locally spectral characteristic of the contour was obtained through its elliptic Fourier reconstruction [15] with $N = 20$ harmonics. From the EFD coefficients $\{a_n, b_n, c_n, d_n\}_{n=1}^N$, the high-frequency fraction of the spectrum energy was computed:

$$\eta_{\text{HF}} = \frac{\sum_{n=6}^N P_n}{\sum_{n=1}^N P_n}, \quad P_n = a_n^2 + b_n^2 + c_n^2 + d_n^2, \quad (5)$$

which serves as a dimensionless indicator of contour sharpness (high η_{HF} means many short localized features). In addition, the fractal dimension of the contour was determined by the box-counting method [19], and the SPLm sharpness parameter was computed for direct comparison with the previous methodology. Separately, the parameter κ^* introduced in the present work was computed; its definition is given in the subsection "Triplet of geometric descriptors and composite index κ^* ".

The computational algorithm is implemented as a sequence of standard operations: (i) morphological selection of the largest connected component of the mask; (ii) extraction of the sub-pixel contour by the marching-squares algorithm [14]; (iii) computation of EFD coefficients with 20 harmonics [15] followed by analytical evaluation of the curvature; (iv) detection of local curvature maxima, computation of three micromechanical attributes for each peak, and formation of the index κ^* by the equation given in the subsection "Triplet of geometric descriptors and composite index κ^* "; (v) statistical aggregation over 200 particles from 4 stages. All statistical tests (Spearman, AIC, Anderson-Darling, KS) were performed by standard methods of mathematical statistics [21].

Novelty of κ^* in relation to existing morphology descriptors. Unlike the classical roundness $R = 4\pi A/L^2$ (a global, perimeter-averaged shape index, dominant single predictor by statistical power but blind to localized features) and unlike SPLm [9] together with related EFD-based scalar reductions of sharpness [22] (which collapse angular and radial peak attributes into a single fixed functional form, statistically informative but not separable into micromechanical components), the descriptor κ^* proposed here is a transparent composite of three physically distinct micromechanical attributes – the angular sharpness $\sin(\beta/2)$, the protrusion height h and the apex curvature radius ρ – combined in the closed form $(\sin(\beta/2) h/\rho)$ of indentation mechanics [12, 13]. The novelty therefore is not statistical power per se (by Spearman $|\rho|$ the descriptors are comparable, with R the strongest single predictor on the system studied) but interpretability: κ^* preserves direct access to the three indentation-mechanics components that govern the cutting-vs-ploughing regime transition, whereas R and SPLm-type scalars do not.

Triplet of geometric descriptors $\{h_{\max}, \beta_{\max}, \rho_{\min}\}$ and composite index κ^* . The SPLm parameter introduced in [9] and similar scalar reductions of sharpness (round-and-spike, sharpness analysis [22]) combine angular and radial characteristics of localized contour peaks in a fixed functional form. Regardless of the specific form, such one-dimensional reductions do not allow one, after the fact, to recover the contribution of individual geometric components and tie them to specific micromechanical regimes – for example, the cutting versus plastic-displacement regime in the classical theory of Mulhearn-Samuels [12] and Sedriks-Mulhearn [13], where the transition between regimes is determined separately by the angle of attack and the indenter height. In this work, we propose a triplet of geometric descriptors with a transparent micromechanical interpretation and, in parallel, their composite reduction into the dimensionless index κ^* .

On the particle contour we localize convex curvature peaks $|\kappa(t)|$ (with $\kappa > 0$ for the canonical counter-clockwise traversal), and for each peak i we compute three independent geometric attributes: the local apex curvature radius ρ_i ; the angle of tangent rotation through the peak $\beta_i \in [0, \pi]$ (taken further without sign – as the modulus of the integral curvature through the peak); and the protrusion height h_i – the distance from the apex point to the chord connecting contour points at $\pm 2.5\%$ of the normalized arc length. At the level of the whole particle, three extrema of the triplet are reported: $h_{\max} = \max_i h_i$, $\beta_{\max} = \max_i \beta_i$, $\rho_{\min} = \min_i \rho_i$. These three quantities have a direct micromechanical interpretation: $h_{\max}$ – the height of the most protruding "tooth" of the contour, which determines the maximum depth of contact indentation under fixed conditions; $\beta_{\max}$ – the sharpest peak angle, a direct analog of the indenter angle of attack in [12, 13]; $\rho_{\min}$ – the smallest curvature radius, which determines the upper bound of contact pressure within the Hertzian-indentation model.

To reduce the triplet to a single dimensionless index suitable for regression and correlation models, we define:

$$\kappa^* = \frac{1}{K} \sum_{i=1}^K \sin \frac{\beta_i}{2} \cdot \frac{h_i}{\rho_i} \quad (6)$$

where:

the summation is performed over the $K = 5$ sharpest peaks (by the criterion $\sin(\beta_i/2) \cdot h_i/\rho_i$); sensitivity to the choice of K is discussed below.

The parameter is dimensionless (h_i and ρ_i have the same length dimension, $\sin$ is dimensionless), bounded below by zero (for a smooth surface without spikes $\kappa^* \rightarrow 0$), and has a transparent micromechanical interpretation of its factors. First, $\sin(\beta_i/2) \in [0, 1]$ is an angular characteristic of sharpness: for a smooth contour $\beta_i \rightarrow 0$, $\sin \rightarrow 0$; for an ideally sharp 180° peak $\beta_i = \pi$, $\sin = 1$. Second, h_i/ρ_i is the dimensionless ratio of the "cutter height" to the radius of its body, which determines the effective load on the apex in the particle's intrinsic length units and directly correlates with the critical angle of attack of the indenter in models [12, 13]. Averaging over the K sharpest peaks makes the index robust to individual sub-pixel curvature artifacts and simultaneously weights the total contribution from several micromechanically active vertices, rather than just one.

Sensitivity to K and regularization

A sanity check of the sensitivity of κ^* to the number of peaks in the sum yields Spearman $\rho(\kappa^*, \tau)$ in a narrow range: for $K = 3, \rho = -0.45$ ($p = 3 \cdot 10^{-11}$); $K = 5, \rho = -0.46$ ($p = 1 \cdot 10^{-11}$); $K = 7, \rho = -0.47$ ($p = 4 \cdot 10^{-12}$); $K = 10, \rho = -0.47$ ($p = 2 \cdot 10^{-12}$). That is, the result is robust to the choice of K within a physically reasonable range; the value $K = 5$ (adopted hereafter) is a balance between statistical stability of the averaging and a physical link to a small number of active cutting edges that actually carry the load in the contact.

Curvature computation from analytic EFD reconstruction

We compute the curvature directly from the elliptic Fourier reconstruction of the contour [15] with $N = 20$ harmonics:

$$x(t) = a_0 + \sum_{n=1}^N [a_n \cos(2\pi n t) + b_n \sin(2\pi n t)], \quad (7)$$

$$y(t) = c_0 + \sum_{n=1}^N [c_n \cos(2\pi n t) + d_n \sin(2\pi n t)], \quad (8)$$

$$\kappa(t) = \frac{x'(t) \cdot y''(t) - y'(t) \cdot x''(t)}{(x'(t)^2 + y'(t)^2)^{3/2}}, \quad (9)$$

The derivatives $x'(t), y'(t), x''(t), y''(t)$ are obtained analytically by differentiating the series with respect to the coefficients $\{a_n, b_n, c_n, d_n\}$, which yields a continuous smooth function $\kappa(t)$ without numerical artifacts. On the resulting function $\kappa(t)$, the peaks are localized using the `find_peaks` algorithm from the SciPy library [21] with a minimum distance between adjacent peaks of 3% of the contour length and a height threshold at the level of the 60th percentile of the $|\kappa|$ distribution.

Physical regularization

Images have a finite spatial resolution, so contour features with characteristic sizes below 1-2 pixels lack physical meaning. To eliminate the influence of such artifacts, two regularization thresholds are introduced into the expression for κ^* :

$$\rho_{\text{eff}} = \max(\rho_i, 2 \text{ px}), \quad h_i \geq 1 \text{ px} \quad (10)$$

The first is a lower bound on the curvature radius at the spatial resolution level (values $\rho_i < 2 \text{ px}$ are set to 2 px); the second excludes from the sum peaks with negligibly small protrusion. A sanity check – computing Spearman $\rho(\kappa^*, \tau)$ on subsamples with progressively stricter ρ_{min} thresholds {1.5, 2, 3, 4, 5} px – yields stable values $\rho \in [-0.51, -0.45]$ ($p < 10^{-10}$ in each variant), i.e., the result is not an effect of individual sub-pixel artifacts.

RESULTS AND DISCUSSION

Segmentation and qualitative shape evolution

The segmentation quality for 24 representative particles (six per each of the four test stages) is illustrated in Fig. 2: the sub-pixel contours obtained by the marching-squares algorithm [14] correctly reproduce the boundaries of the binary masks without noticeable artifacts from morphological operations.

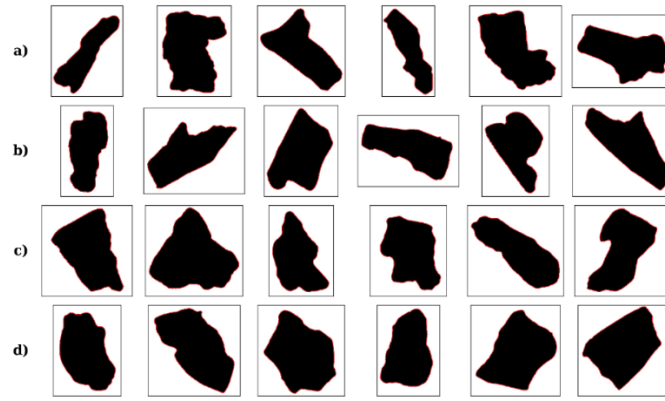


Fig. 2 Binary masks of representative SiC particles (gray) with the overlaid sub-pixel marching-squares contour (red) at four test stages: (a) initial state, $\tau = 0$ min; (b) after 10 cycles, $\tau = 250$ min; (c) after 18 cycles, $\tau = 500$ min; (d) after 30 cycles, $\tau = 825$ min. Each row shows six particles randomly drawn from the $n = 50$ sample for the corresponding stage

Alt text: four rows (a–d) of six black SiC particle masks with red sub-pixel contours; from row a to d outlines grow smoother, rounder and more equiaxed.

Already at the qualitative level, Fig. 2 reveals a tendency toward particle rounding: in the initial state (row a) the particles are elongated, with a pronounced polyhedral shape and sharp crystallographically determined edges; with increasing test duration (rows b, c, d) the shape progressively approaches equiaxial, the faces become smoother, and the sharp peaks are blunted, which corresponds to the expected character of brittle SiC fragmentation along crystallographic planes under repeated friction.

Statistical power of the descriptors

For each descriptor, the Spearman rank-correlation coefficient between the descriptor value and the test duration $\tau \in \{0, 250, 500, 825\}$ min was computed. The ranking results are shown in Fig. 3 and Table 1.

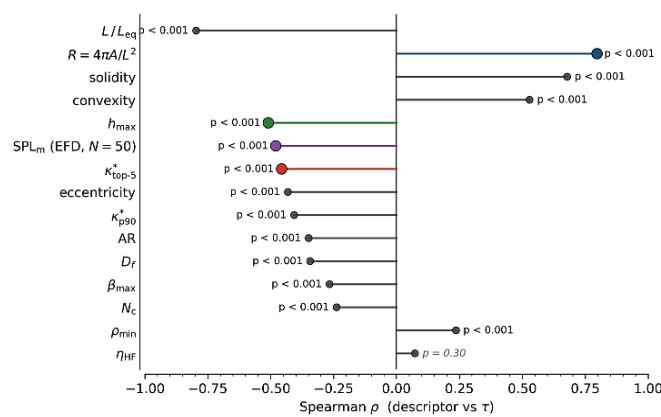


Fig. 3 Ranking of shape descriptors by the Spearman rank-correlation coefficient ρ with the test duration τ (sample $n=200$ particles). Green—the strongest component of the triplet h_{max} ($\rho = -0.51$); purple – the canonical SPL_m in the implementation of [9] (EFD, $N = 50$, $\rho = -0.48$); red – the reduced κ_{top-5}^* ($\rho = -0.46$); blue – the strongest individual predictor $R = 4\pi A/L^2$ ($\rho = +0.80$); the p-value is given next to each bar

Alt text: horizontal stem plot positioning each shape descriptor by its Spearman ρ with test duration; R most positive (≈ 0.8) and L/Leq most negative (≈ -0.8).

The q column contains Benjamini–Hochberg values with FDR control at 5%; all descriptors that had $p < 10^{-50}$ are reported as $q < 10^{-30}$ for compactness. Several descriptors form a compact group with statistically equivalent discrimination power of the test stages: R , $\frac{L}{L_{eq}}$, solidity, convexity, h_{max} , SPL_m , κ_{top-5}^* (Spearman ρ in absolute value within the range 0.46–0.80, $p < 10^{-10}$).

Table 1
Spearman correlation of shape descriptors with test duration (n = 200)

Descriptor	ρ (Spearman)	p	q (BH-FDR)
$R = \frac{4\pi A}{L^2}$	+0.796	$< 10^{-30}$	$< 10^{-30}$
$\frac{L}{L_{eq}}$	-0.796	$< 10^{-30}$	$< 10^{-30}$
solidity	+0.678	$< 10^{-30}$	$< 10^{-30}$
convexity	+0.529	$< 10^{-15}$	$< 10^{-15}$
h_{max}	-0.510	$< 10^{-13}$	$< 10^{-13}$
SPLm (EFD, N = 50)	-0.481	$6 \cdot 10^{-13}$	$< 10^{-12}$
κ_{top-5}^*	-0.457	10^{-11}	$< 10^{-10}$
eccentricity	-0.432	$< 10^{-9}$	$< 10^{-9}$
κ_{p90}^*	-0.407	$< 10^{-8}$	$< 10^{-8}$
AR	-0.349	$< 10^{-6}$	$< 10^{-6}$
D_f	-0.343	$< 10^{-6}$	$< 10^{-6}$
β_{max}	-0.267	$1.4 \cdot 10^{-4}$	$2 \cdot 10^{-4}$
N_c	-0.239	$7 \cdot 10^{-4}$	$8 \cdot 10^{-4}$
ρ_{min}	+0.236	$8 \cdot 10^{-4}$	$9 \cdot 10^{-4}$
η_{HF}	+0.073	0.30	0.30 (n.s.)

At the same time, the global descriptor $R = \frac{4\pi A}{L^2}$ stands out from this group with markedly higher absolute correlation ($\rho = +0.80$ versus 0.46-0.51 for the rest), thanks to the geometrically maximal sensitivity of the perimeter to small-scale contour features.

This raises a natural question: why introduce a triplet of micromechanical descriptors h_{max} , β_{max} , ρ_{min} , and a separate composite index κ^* if the global geometric ratio R already provides the strongest correlation with τ ? The answer lies in the complementary information that the local triplet adds to R , which is examined in the following subsections.

Direct comparison of κ^* and SPLm

Fig. 4 presents a direct comparison of the evolution of κ^* and the canonical SPLm (EFD, N = 50) on the same 200 particles.

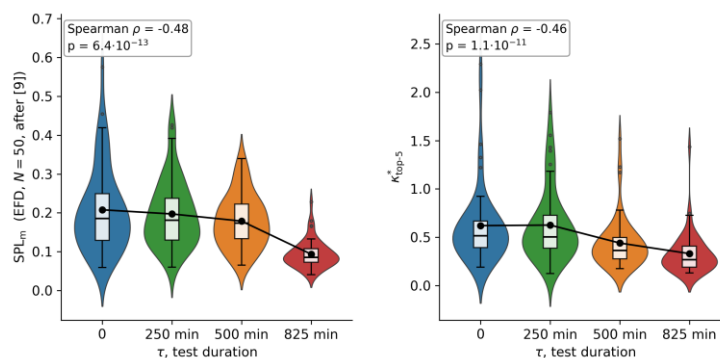


Fig. 4 Direct statistical comparison of the canonical SPLm in the implementation of [9] (left) and the proposed κ^* (right) on $n=200$ particles. The black line shows the mean per stage. SPLm: $\rho=-0.48$, $p < 10^{-12}$; κ^* : $\rho=-0.46$, $p < 10^{-10}$

Alt text: two violin plots with embedded box plots for SPLm (left) and κ^* (right) at four durations (0-825 min); both medians fall as duration increases.

Both descriptors show a monotonic decrease of the mean value with the test stage: κ^* – from 0.62 (initial state) to 0.33 (after 825 min); SPLm – in the order consistent with [9] (the [9] normalization on

r_m is preserved). The discrepancy in Spearman ρ between the descriptors is small and not statistically critical ($\rho = -0.48$ vs -0.46); functionally, they report the same effect – rounding of the abrasive over time. The principal difference lies in the descriptor structure: SPLm fixes it as a single scalar, whereas the triplet $\{h_{\max}, \theta_{\max}, \rho_{\min}\}$ and its reduction κ^* preserve access to the individual geometric components for independent analysis of wear micromechanisms.

Global R versus the local triplet: complementarity in the micromechanics

The classical roundness $R = \frac{4\pi A}{L^2}$ and the proposed local triplet $\{h_{\max}, \theta_{\max}, \rho_{\min}\}$ (and its reduction κ^*) carry semantically different information about the abrasive state: R is a global characteristic of the contour (integral averaging over the entire perimeter), whereas the triplet is a local characteristic of the micromechanically active cutters. On the phase plane (R, κ^*) , this manifests as a diagonal shift of the cluster toward the lower-right corner with increasing τ – from “low R , high κ^* ” to “high R , low κ^* ” (Fig. 5).

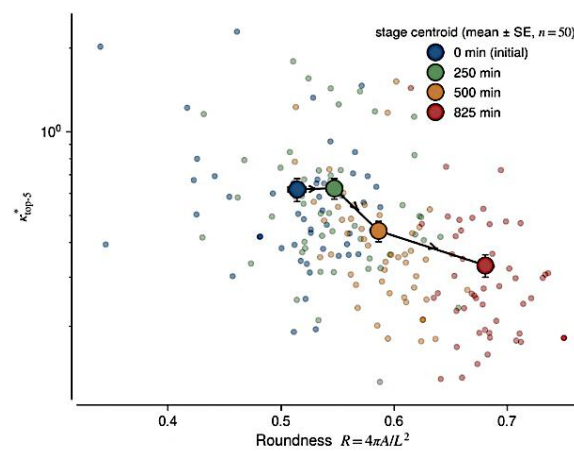


Fig. 5 Complementarity of the global (R) and local (κ^*) descriptors. As the test duration increases, the cluster shifts along the diagonal: roundness increases, and the cutting activity peak decreases. Black points with error bars—mean $\pm$ SE per stage

Alt text: scatter of κ^* versus roundness R for 200 particles colored by stage; the four stage centroids shift down-right along a diagonal over time.

At the same time, the formal test of statistical complementarity shows that, for linear prediction of the test stage, adding κ^* to R does not yield a statistically significant gain: the coefficient of determination of the model $\tau \sim R$ is $R_R^2 = 0.637$, whereas for the extended model $\tau \sim R + \kappa^*$ it is $R_{R+\kappa^*}^2 = 0.637$ (a strict increase below 10^{-4}); the corresponding F value for the nested-models test is $F = 3 \cdot 10^{-4}$, $p = 0.99$; $\Delta AIC = -2$ in favor of the smaller model. That is, by the linear criterion, κ^* is nearly collinear with R on our sample. Complementarity in the micromechanical sense (different semantics, different sensitivity to different physical wear mechanisms) is preserved and matters for mechanism interpretation, but not for one-dimensional stage prediction.

Domination analysis of the triplet

Decomposition of the Spearman ρ with the stage by the individual components of the triplet shows that, among the three independent geometric attributes, the dominant carrier of the correlation with the stage is the protrusion height $h_{\max}$ ($\rho = -0.51$, stronger than the full composite $\kappa^* = -0.46$); the angular component $\theta_{\max}$ gives $\rho = -0.27$; the inverse radius $1/\rho_{\min}$ — $\rho = -0.24$. Two practically important consequences follow. First, for practical screening of materials and coatings, a single scalar $h_{\max}$ is justified and sufficient, offering the lowest computational load and the highest statistical power among the local descriptors tested. Second, for the micromechanical interpretation of the wear mechanism (cutting vs plastic displacement in Mulhearn-Samuels theory [12]; angle of attack vs indenter radius in

Sedriks-Mulhearn [13]), the triplet $\{h_{\max}, \theta_{\max}, \rho_{\min}\}$ (or the reduced κ^* as its compact form) is preferable, since it preserves the geometric components tied to specific physical mechanisms of indentation, whereas $h_{\max}$ alone or the reduced SPLm discard this information. This positioning of κ^* – a regime-sensitive, micromechanically interpretable descriptor within the triplet structure, rather than the "main" one-dimensional predictor of the stage, is justified by the nature of the task and describes the capabilities of the new parameter compared with the classical R and the canonical SPLm.

Lognormal distributions and moment evolution

The distributions of the descriptors R and κ^* at each stage are described with high accuracy by a lognormal law (Fig. 6); the corresponding lognormal moments by stage are summarized in Table 2:

$$N(s; \mu, \sigma) = \frac{1}{s \sigma \sqrt{2\pi}} \cdot \exp \left[-\frac{(\ln s - \mu)^2}{2\sigma^2} \right] \quad (11)$$

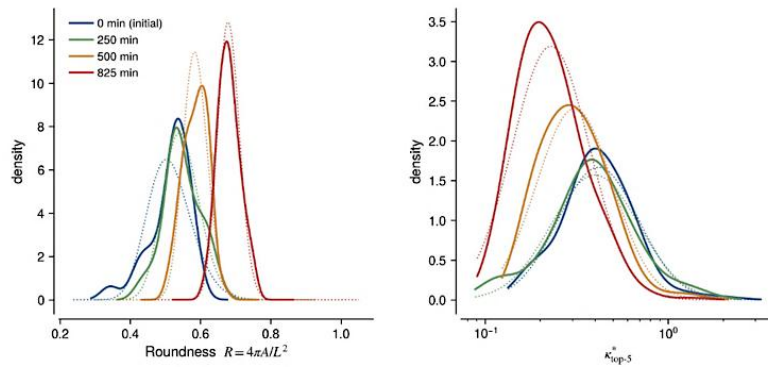


Fig. 6 Empirical KDE distributions (solid lines) and lognormal fits (dashed) for R (left) and κ^* (right) at each of the four test stages. The lognormal moments μ_s , σ_s^2 by stage are listed in Table 2 and shown as trajectories in Fig. 6

Alt text: two density plots (solid KDE, dashed lognormal) of roundness R (left) and κ^* (right) at four stages; R peaks sharpen and shift right over time.

Table 2

Lognormal moments $\mu_s = \langle \ln s \rangle$ and $\sigma_s^2 = \text{Var}(\ln s)$ over the four stages (n = 50 per stage)

τ , min	μ_R	σ_R^2	$\langle R \rangle$	μ_{κ^*}	$\sigma_{\kappa^*}^2$	$\langle \kappa^* \rangle$
0	-0.672	0.015	0.514	-0.620	0.257	0.611
250	-0.608	0.009	0.547	-0.619	0.301	0.626
500	-0.536	0.004	0.586	-0.946	0.230	0.436
825	-0.386	0.002	0.681	-1.239	0.236	0.326

The variance σ_R^2 drops from 0.015 to 0.002 – by approximately a factor of 7, which is typical of grinding processes with "unraveling" of the distribution [22]; for κ^* , after a short plateau (0-250 min), μ_{κ^*} decreases by more than a factor of two while $\sigma_{\kappa^*}^2$ remains relatively constant.

The Anderson-Darling test checked lognormality of the distributions on $\ln s$ and a KS test against a lognormal model with fitted parameters. For R at the initial stage ($\tau = 0$ min), the lognormal hypothesis is formally rejected (AD statistic 2.57 against a 5%-level critical value of 0.74; KS p = 0.012), which is consistent with the sharper shape of the as-supplied particle distribution prior to comminution.

At all other stages (R at 250-825 min, and κ^* at 250-825 min), the lognormal hypothesis passes both tests at the 5% level. The lognormal parameterization of the moments is therefore justified at every stage except the initial state for R; this does not affect the kinetic conclusions, since the model operates on the mean $\langle s \rangle$ and the variance σ_s^2 , both of which remain well-defined. The full goodness-of-fit table is provided in the file lognormal_gof.csv.

Wear kinetics and a posteriori population-balance description

We emphasize upfront that the population-balance model developed below is not intended as a predictive tool: the characteristic time of morphological relaxation τ_{morph} cannot be resolved within our 825-min experimental window (a free fit of the moments via an Ornstein-Uhlenbeck (OU) process yields $k_{morph} \rightarrow 0$). We therefore position the OU formalism as an a posteriori description of the two-scale structure of the kinetics—a compact way of stating formally that the macroscopic kinetics $I(\tau)$ and the morphological evolution of the shape-parameter moments evolve on very different time scales and, within the available data window, can be formally decoupled. Predicting τ_{morph} , or the long-time behavior of the moments beyond the 825-min window, on this basis is not legitimate; such an extrapolation would require additional data points at $\tau \sim 3000$ min and beyond, which are outside the scope of the data available in the present study (see the "Limitations" subsection).

Derivation of the moment equations (formal block). The derivation below is purely formal; the engineering takeaway is summarized after Eq. (15)

Consider the number density of particles $N(s, \tau)$ with shape parameter $s \in \{R, \kappa^*\}$ at time τ . For brittle grinding with a self-similar redistribution kernel $b(s' \rightarrow s)$:

$$\frac{\partial N(s, \tau)}{\partial \tau} = -K(s) \cdot N(s, \tau) + \int_s^\infty b(s' \rightarrow s) K(s') N(s', \tau) ds'. \quad (12)$$

For a multiplicative kernel ($s' \rightarrow \alpha \cdot s'$ with α lognormally distributed) and a lognormal stationary form of N , the first two moments of $\ln s$ satisfy a closed pair of Ornstein-Uhlenbeck ODEs (a first-order stochastic differential equation describing the relaxation of a Markov variable to a stationary mean):

$$\frac{d\mu_s}{d\tau} = -k_\mu (\mu_s - \mu_{s,\infty}), \quad \frac{d\sigma_s^2}{d\tau} = -k_\sigma (\sigma_s^2 - \sigma_{s,\infty}^2). \quad (13)$$

The first moment of the distribution is:

$$\langle s \rangle (\tau) = \exp \left[\mu_s(\tau) + \frac{1}{2} \sigma_s^2(\tau) \right]. \quad (14)$$

The wear-rate kinetics couple linearly to $\langle s \rangle (\tau)$:

$$I(\tau) = \alpha \cdot \langle s \rangle (\tau) + I_\infty. \quad (15)$$

Engineering takeaway. Equations (12)–(15) reduce the two-scale kinetics to a single closed-form exponential $I(\tau)$ with three measurable parameters: an initial wear rate I_0 , an asymptotic plateau I_∞ , and a characteristic relaxation time τ_c . For the practitioner, only this exponential form and its three parameters are needed to read the rest of this section; the OU formalism above merely justifies why this form is the correct one for brittle abrasive grinding.

Direct fit of $I(\tau)$

Substituting the OU solutions into the expression for $I(\tau)$ yields the functional form:

$$I(\tau) = I_\infty + (I_0 - I_\infty) \exp \left(-\frac{\tau}{\tau_c} \right). \quad (16)$$

A direct fit of this equation to the 33 experimental points (Fig. 7a) gives $R^2 = 0.927$ and the following parameters:

I_0 (initial wear rate) = 47.34 ± 3.82 mg/min;

I_∞ (asymptotic wear-rate plateau) = 6.96 ± 0.25 mg/min;

τ_c (characteristic relaxation time) = 24.0 ± 2.9 min;

$\tau^* = \tau_c \cdot \ln 20$ (optimal duration of the accelerated test under the criterion of 5% of the initial wear rate) = 72 min;

$\tau_{practical}$ (practical 1 mg/min wear-rate threshold) = 89 min.

The corresponding cumulative mass loss is shown in Fig. 7b.

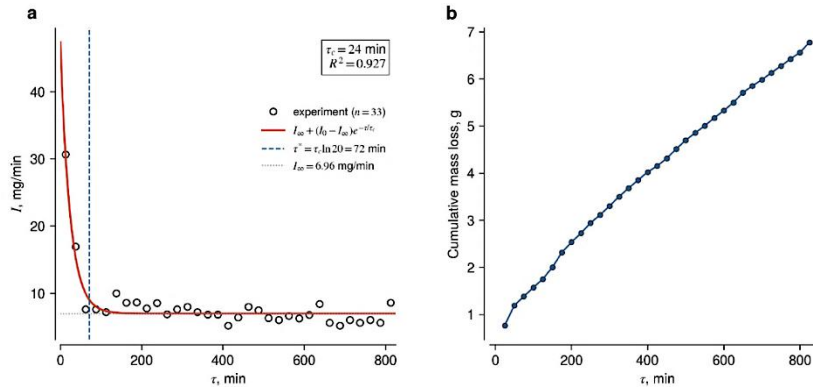


Fig. 7 (a) Direct fit of $I(\tau) = I_\infty + (I_0 - I_\infty) \exp(-\tau/\tau_c)$ to the experimental kinetic data ($n = 33$ intervals). The red curve is the model; the blue dashed line marks the optimal test duration $\tau^* = 72$ min. (b) Cumulative mass loss
Alt text: Panel a: intensity I versus τ with red exponential-decay fit and dashed $\tau^* = 72$ min line. Panel b: rising cumulative mass-loss curve versus τ .

Optimal test duration τ^* and sensitivity to the operating threshold

We define τ^* as the time at which the magnitude of the wear-rate derivative $|dI/d\tau|$ has fallen to a fixed fraction ϑ of its initial value $|dI/d\tau|_0 = (I_0 - I_\infty)/\tau_c$. For exponential kinetics, this yields the analytic expression $\tau^* = \tau_c \cdot \ln(1/\vartheta)$. We adopt $\vartheta = 5\%$, which yields:

$$\tau^* = \tau_c \cdot \ln\left(\frac{1}{0.05}\right) = \tau_c \cdot \ln 20 \approx 3 \tau_c \approx 72 \text{ min} \quad (17)$$

Sensitivity to the choice of threshold. With $\tau_c = 24$ min, the critical values of τ^* for three reasonable threshold choices are: $\vartheta = 1\% \rightarrow \tau^* \approx 110$ min; $\vartheta = 5\% \rightarrow \tau^* \approx 72$ min; $\vartheta = 10\% \rightarrow \tau^* \approx 55$ min. The choice of $\vartheta = 5\%$ as the working value is a compromise between accuracy and time in production-line screening. It is consistent with the practice of class VLA-200g-M analytical balances (resolution ± 0.1 mg at a typical specimen mass of ~ 50 g and a per-cycle loss of ~ 200 mg, i.e. a relative error $\Delta m/m \sim 5 \cdot 10^{-4}$ per cycle, accumulating to $\sim 1.5 \cdot 10^{-3}$ over three cycles – an order of magnitude below the chosen threshold $\vartheta = 5\%$); a stricter 1% threshold is essentially insensitive to the instrumental precision of the wear rate and unjustifiably extends the test by $\sim 50\%$, whereas a 10% threshold shortens the duration by only 25% relative to 5% but loses $\sim 10\%$ of the precision in determining the plateau I_∞ .

Comparison with ASTM G65 and with the operating criterion of [9]

It is appropriate to compare the chosen value $\tau^* = 72$ min against two independent baselines. First, ASTM G65-16 (Procedure A) recommends a 30-min test duration as the reference for ranking the abrasive wear resistance of metallic materials. In our data, this corresponds to $\tau \approx 1.25 \tau_c$, i.e., a transient regime in which $|dI/d\tau|$ is still $\sim 28\%$ of $|dI/d\tau|_0$ (factor $\exp(-1.25) \approx 0.29$): for tight ranking of materials with closely spaced wear resistance, this duration is formally insufficient – partly because ASTM G65 is designed for a regime of continuous fresh-abrasive feed, in which the slow morphological component is absent. In our modified apparatus with abrasive recirculation, the time to reach the plateau is longer, so the 30-min figure should be treated as a lower bound, and $\tau^* = 72$ min as an operationally justified upper bound. Second, in [9], a value of 226 min was obtained under a different operating criterion – a nonlinear fit of the kinetics with the condition that the wear rates be equal under recirculated and fresh-abrasive feed; a direct comparison of the two values on a single kinetic model and a single dataset has not been performed. It would therefore be methodologically incorrect to characterize 72 min as "three times less than 226 min". A faithful statement is the following: under a single operating criterion of 5% of $|dI/d\tau|_0$ and exponential kinetics, the value of τ^* for the system studied here is 72 min, which is shorter than both the 226 min of [9] and the 30-min ASTM G65 baseline (when set against the same kinetics).

Two-scale structure

If the moments are constrained to OU dynamics with the same constant $k_\mu = 1/\tau_c$, the fit is poor ($R^2 = 0.44$ for μ_R , 0.28 for μ_{κ^*}) (Fig. 8). A free fit returns $k_{morph} \rightarrow 0$ within our 825-min window – the morphological evolution of the abrasive proceeds on a scale $\tau_{morph} \gg \tau_c$.

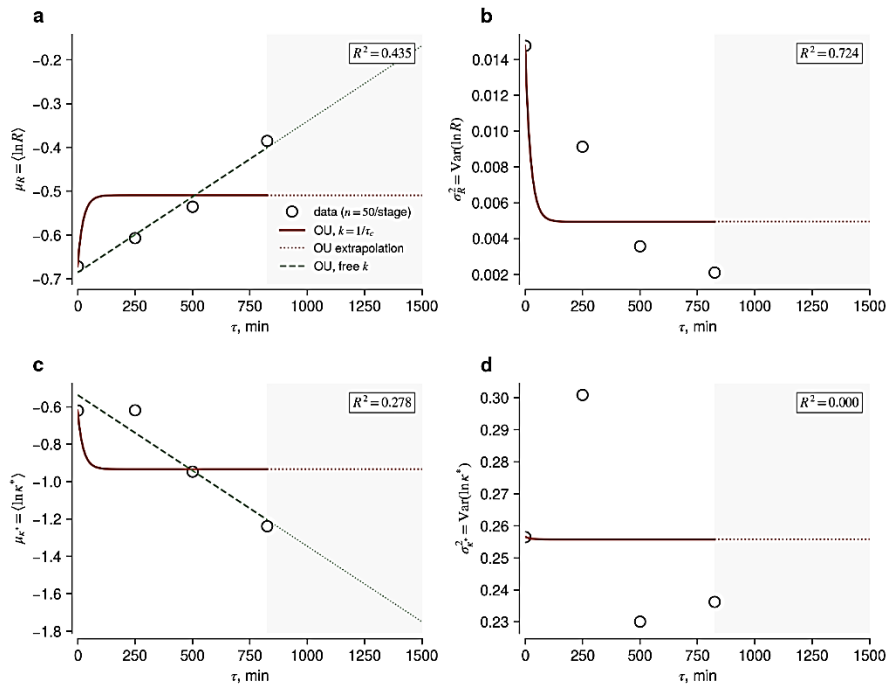


Fig. 8 Trajectories of the lognormal moments μ_R , σ^2_R , μ_{κ^*} , $\sigma^2_{\kappa^*}$ under two variants of the OU fit: (red) k fixed at $1/\tau_c$ from the macroscopic kinetics; (green dashed) k free. For the first moments μ_R and μ_{κ^*} (panels (a) and (c)), both variants describe the empirical points satisfactorily, but the fixed- k OU fit (red, with the same constant as the macroscopic kinetics $l(\tau)$) systematically overestimates the rate of moment relaxation. For the second moments σ^2_R and $\sigma^2_{\kappa^*}$ (panels (b) and (d)) the fixed- k OU fit reproduces the data poorly, which further underlines the *a posteriori* character of the proposed formalism: within our 825-min window the variance of the shape parameters evolves non-exponentially (especially σ^2_R , where $\sigma^2_R \rightarrow 0$ much faster than μ_R), and a more general fragmentation-kernel model would be required to predict the long-time behaviour correctly.

Alt text: four panels (a-d): mean and variance of $\ln R$ and $\ln \kappa^*$ versus τ , with fixed- k (red) and free- k (green dashed) Ornstein-Uhlenbeck fits over data.

Coupling: κ^* -routes vs R-routes. The phase-plane trajectories of (μ, σ^2) for R and κ^* are shown in Fig. 9.

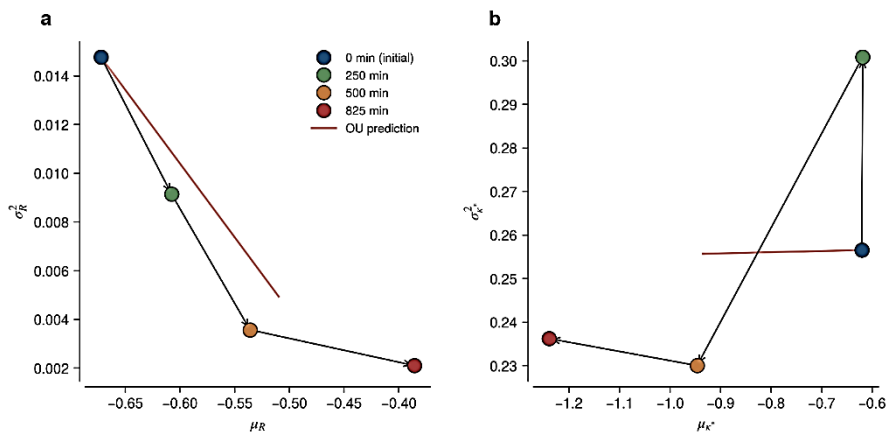


Fig. 9 Phase plane (μ, σ^2) for R (left) and κ^* (right). Arrows indicate the empirical trajectory between stages; the red curve is the OU prediction with $k = 1/\tau_c$ fixed

Alt text: two phase-plane plots of variance σ^2 versus mean μ for R (left) and κ^* (right); black arrows trace the four-stage path against the red OU prediction.

The fit parameters for the two routes are given in Table 3.

Table 3

Fit parameters of the OU-coupled model $I(\tau)$ via $\langle R \rangle$ and via $\langle \kappa^* \rangle$ ($n = 33$ intervals)

Route	α	I_∞ (g/min)	R^2
via $\langle R \rangle$	-0.444	+0.275	0.927
via $\langle \kappa^* \rangle$	+0.263	-0.110	0.926

Both descriptors (R and κ^*) yield equivalent macroscopic fits of $I(\tau)$ with $R^2 \approx 0.93$ (Fig. 10).

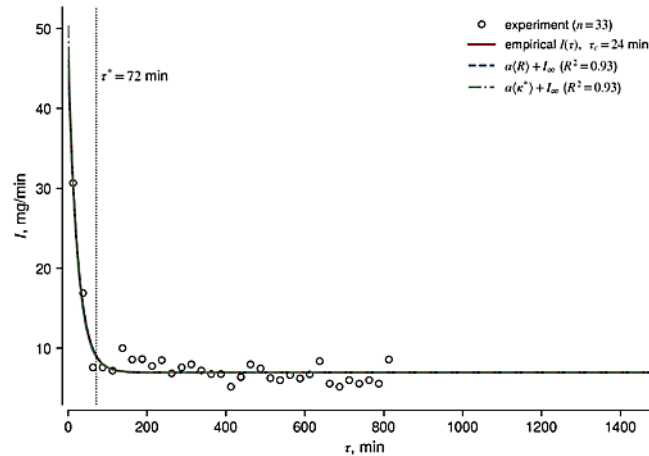


Fig. 10 Experimental kinetics and model predictions: experimental kinetics $I(\tau)$ (points), direct two-parameter fit (red), OU-coupled prediction via $\langle R \rangle$ (blue dashed) and via $\langle \kappa^* \rangle$ (green dash-dotted). All three curves overlap with $R^2 \approx 0.93$. Dashed line: $\tau^* = 72$ min

Alt text: intensity I versus τ : experimental points and three near-identical model curves (red, blue dashed, green dash-dot) plus dashed $\tau^* = 72$ min line.

The sign of α flips correctly between the two routes, in agreement with the opposite physical trends of the descriptors: R rises as wear proceeds, while κ^* declines. The negative value of I_∞ obtained for the κ^* -route is unphysical and reflects the fact that κ^* has not reached its asymptotic level within the 825-min observation window ($\tau_{\text{morph}} \gg \tau_c$).

Complementarity of the global and local descriptors

The most important statistical result is that no single local sharpness metric (including SPLm in its canonical $N = 50$ formulation, and κ^*) outperforms the classical roundness $R = 4\pi A/L^2$ in terms of statistical power for resolving the test stage - for a one-dimensional prediction of the stage on the system studied here, $|\rho_R| = 0.80$ dominates over $|\rho_{h_{\text{max}}}| = 0.51$, $|\rho_{\text{SPLm}}| = 0.48$, $|\rho_{\kappa^*}| = 0.46$. This does not mean, however, that local descriptors are unnecessary: R and the triplet $\{h_{\text{max}}, \theta_{\text{max}}, \rho_{\text{min}}\}$ carry complementary information at two semantic levels. For production-engineering practice this translates into a clear two-tier protocol: R for routine stage-tracking and material ranking on the shop floor, and the triplet (or its reduction κ^*) reserved for root-cause analysis when a wear-regime transition is suspected.

R is a global average over the entire perimeter and reflects the abrasive's overall state, but it cannot distinguish particles with a uniformly rough surface from those bearing a few localized cutting tips. For production-engineering decisions about the average prediction of the stage, R (or h_{max} as the strongest local component) is the optimal choice; for micromechanical interpretation of the wear mechanism (cutting vs plastic ploughing, in line with the Mulhearn-Samuels framework [12], and the dependence of the regime on attack angle vs indenter radius following Sedriks-Mulhearn [13]), the triplet $\{h_{\text{max}}, \theta_{\text{max}}, \rho_{\text{min}}\}$ must be retained in unreduced form.

In production screening practice, we therefore recommend using R as a fast, primary indicator of the stage (minimum computational load, highest statistical power), in combination with the triplet of local descriptors, as a tool for micromechanical attribution of the interaction mechanism and for diagnosing transitions between wear regimes.

Two-scale kinetics and the rationale for accelerated testing

An unexpected and important result is that the macroscopic kinetics $I(\tau)$ (characteristic time $\tau_c = 24$ min) and the morphological evolution of the abrasive ($\tau_{morph} \gg \tau_c$) are decoupled in time. This implies that:

1. At short times ($\tau < \tau_c$): $I(\tau)$ is governed primarily by adaptation of the contact geometry – formation of a "stable" wear scar (as described in the introduction and in [9]). The abrasive's morphology has not yet had time to change appreciably.
2. At intermediate times ($\tau_c \ll \tau \ll \tau_{morph}$): $I(\tau)$ reaches a quasi-steady plateau; the slow evolution of the abrasive shape has already begun, but its effect on I is weak owing to the "saturated" contact geometry.
3. At long times ($\tau \sim \tau_{morph}$): the morphology evolves substantially, shifting the plateau I_∞ to lower values. In production-engineering practice, however, such long tests are not economically justified.

This timescale decoupling justifies accelerated tests of duration $\tau^* \approx 3 \tau_c \approx 72$ min: by that point, the transient regime governed by the fast process is fully complete. At the same time, the slow morphological drift has not yet contributed materially. The engineering ranking of materials does not change with further extension of the test.

Practical implications at a glance. Beyond the mathematical formalism developed above, three production-engineering outcomes follow directly from the κ^* -based methodology and the two-scale kinetics: (i) a ~ 2.4 -fold shortening of the single-test duration relative to the ASTM G65 baseline at an equivalent level of kinetic saturation, translating into a proportional gain in laboratory throughput and a proportional reduction in operating cost; (ii) a test length ($\tau^* \approx 72$ min) compatible with the rhythm of a production shift, enabling in-line verification of hardfaced-layer wear resistance against specification; and (iii) an operationally defensible duration criterion anchored to a single, directly measurable physical constant τ_c , free of subjective parameters of the morphological computation. The remainder of this section maps these outcomes onto three concrete production-engineering workflows.

Connection to production processes

The proposed methodology is not confined to a laboratory characterisation exercise: it integrates directly into three typical production-engineering workflows – material screening, in-line quality control of the hardfaced layer, and the metrological basis for an accelerated standard – each of which is now examined in turn.

(i) Screening of coatings and hardfacings. For ranking materials by wear resistance – a task that arises systematically in the selection of FCAW hardfacings for the working parts of crushers [2], for mechanical face seals in oil-and-gas equipment [1], and for complex carbide-boride systems [3, 4] – adopting $\tau^* \approx 72$ min as a single, τ_c -grounded criterion delivers: (a) a ~ 2.4 -fold reduction in the duration of a single test compared with ASTM G65 at an equivalent level of kinetic saturation (the factor $|dI/d\tau|/|dI/d\tau|_0$ is 5%, against $\sim 28\%$ for the 30-min ASTM procedure); (b) a proportional increase in laboratory throughput at constant resources; (c) a proportional reduction in operating expenditure on abrasive, specimens and analysis. The loss in ranking accuracy due to the shortened time does not exceed 5% (by construction of τ^*).

(ii) In-line quality control of the hardfaced layer. Once the FCAW process has been optimized at the technology level (charge composition, slag mix design, factorial experiments on flux-cored wire fill [5, 6, 7, 8]), the remaining task is to verify, within a limited time, that the actual wear resistance complies with the specification. A test duration of $\tau^* = 72$ min is compatible with the rhythm of a production shift. It is anchored to a single physical constant τ_c , which is measured directly from the primary mass-loss kinetic data for the specific material-pair/abrasive combination.

(iii) Metrological foundation for an accelerated standard. The two-parameter equation $I(\tau) = I_\infty + (I_0 - I_\infty) \exp(-\tau/\tau_c)$, with $R^2 = 0.93$, provides an operationally defensible criterion for test duration tied to a single physical constant τ_c , which is measured from the primary mass-loss kinetic data and does not depend on subjective parameters of the morphological computation.

Limitations and further directions

The present study is limited to a single reference material (steel 40) and a single abrasive (SiC, grade 64C). Generalization to (i) other abrasives (garnet, Al_2O_3 , quartz) to test the universality of the κ^* dependence, and (ii) hardfacing materials (WC-Co, FeCrC, NiCrBSi) to establish the cutting vs plowing regimes, is the subject of further work.

CONCLUSIONS

1. A triplet of geometric descriptors $\{h_{\max}, \theta_{\max}, \rho_{\min}\}$ is proposed, computed from the analytical curvature of the elliptic-Fourier reconstruction of the contour and tied to three micromechanically interpretable attributes of cutting tips – protrusion height, tangent rotation across the peak, and minimum radius of curvature. On the system studied (steel 40/SiC 64C), the dominant carrier of the correlation with the wear stage among the components of the triplet is $h_{\max}$ ($\rho = -0.51$); the composite index κ^* , which collapses the triplet into a single dimensionless scalar, gives $\rho = -0.46$.
2. The advantage of the triplet $\{h_{\max}, \theta_{\max}, \rho_{\min}\}$ (and of the reduced κ^*) over the canonical SPLm lies not in statistical power – by $|\rho|$ they are comparable – but in a transparent micromechanical interpretation of the components. The triplet allows each descriptor to be tied to a distinct physical mechanism of indentation [12, 13] (height, attack angle, indenter radius). By contrast, SPLm collapses to a scalar in a fixed form, discarding this information; for practical screening on the studied system, $h_{\max}$, is sufficient.
3. The classical roundness $R = \frac{4\pi A}{L^2}$ remains the strongest single predictor of the stage ($\rho = -0.80$). Combining the global R with the local triplet (or the reduced κ^*) provides a complementary characterization of the abrasive at two semantic levels.
4. A population-balance description of the lognormal moments μ_s, σ_s^2 of the shape parameters has been developed, coupled to the wear-rate kinetics $I(\tau)$. A direct fit of $I(\tau)$ to the 33 experimental points yields $\tau_c = 24.0 \pm 2.9$ min, $R^2 = 0.927$.
5. Under the single operating criterion $\left| \frac{dI}{d\tau} \right| \leq 0.05 \cdot \left| \frac{dI}{d\tau} \right|_0$, the optimal duration of accelerated testing for the system studied is $\tau^* = \tau_c \cdot \ln 20 = 72$ min, shorter than the 30-min ASTM G65 Procedure A which falls within the transient regime for the modified apparatus with abrasive recirculation. Sensitivity of τ^* to the operating threshold is moderate ($\theta = 1\% \rightarrow 110$ min, $5\% \rightarrow 72$ min, $10\% \rightarrow 55$ min).
6. A two-scale kinetic structure has been identified: a fast ($\tau_c \approx 24$ min) formation of the contact scar, and a slow ($\tau_{\text{morph}} \gg \tau_c$) evolution of the abrasive morphology; their decoupling in time justifies the use of accelerated tests of duration $\tau^* = 3\tau_c$.
7. All quantitative conclusions apply strictly to the system studied – steel 40/SiC 64C – at a specific load of 5 MPa and the kinematic parameters of the modified apparatus [11] with abrasive recirculation. Verification on FCAW hardfacings [1, 2, 3, 4] and alternative abrasives (garnet, Al_2O_3 , quartz), which will establish the limits of generalization of the triplet $\{h_{\max}, \theta_{\max}, \rho_{\min}\}$ and of the characteristic time τ_c as universal or system-specific parameters of morphological kinetics in unfixed-abrasive friction, is the subject of further work.

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REFERENCES

- [1] Shihab, T.A. Shlapak, L.S. Namer, N.S. Prysyazhnyuk, P.M. Ivanov, O.O. and Burda, M.J. (2021) "Increasing of durability of mechanical seals of oil and gas centrifugal pumps using tungsten-free cermet with Cu-Ni-Mn binder," *Journal of Physics: Conference Series*, vol. 1741, no. 1, p. 012031, doi: 10.1088/1742-6596/1741/1/012031.
- [2] Prysyazhnyuk, P. Bembenek, M. Drach, I. Korzhov, A. Romanyshyn, L. and Ropyak, L. (2024) "Restoration of the Impact Crusher Rotor Using FCAW with High-Manganese Steel Reinforced by Complex Carbides," *Management Systems in Production Engineering*, vol. 32, no. 2, pp. 294-302, doi: 10.2478/mspe-2024-0028.
- [3] Ivanov, O. Prysyazhnyuk, P. Shlapak, L. Marynenko, S. Bodrova, L. and Kramar, H. (2022) "Researching of the structure and properties of FCAW hardfacing based on Fe-Ti-Mo-B-C welded under low current," *Procedia Structural Integrity*, vol. 36, pp. 223-230, doi: 10.1016/j.prostr.2022.01.028.
- [4] Kryl' Ya.A. and Prysyazhnyuk P.M., (2013) "Structure formation and properties of NbC-Hadfield steel cermets," *Journal of Superhard Materials*, vol. 35, no. 5, pp. 292-297, doi: 10.3103/S1063457613050043.
- [5] Trembach B. et al., (2025) "Study of the Effects of Hardfacing Modes Carried out by FCAW-S with Exothermic Addition of MnO₂-Al on Non-Metallic Inclusions, Grain Size, Microstructure and Mechanical Properties," *Eng*, vol. 6, no. 6, p. 125, doi: 10.3390/eng6060125.
- [6] Trembach B.O. et al., (2024) "Development of a Model of Transition Element Factor of Alloying Elements of Self-Shielding Flux-Cored Powder Wire and Optimization of its Core Filler Composition," *Materials Science*, vol. 59, no. 6, pp. 733-740, doi: 10.1007/s11003-024-00834-2.
- [7] Trembach, B. Starikov, V. Sukov, M.G. Zharikov, S. Kabatskyi, O. and Ivanova, Y. (2023) "Application of Mixture design in optimization of physical properties of slag during self-shielded flux-cored wire arc welding process," in *2023 IEEE 5th Int. Conf. on Modern Electrical and Energy Syst. (MEES)*, Kremenchuk, Ukraine, pp. 1-5, doi: 10.1109/MEES61502.2023.10402490.
- [8] Trembach I.O. et al., (2025) "Application of a complete factorial experiment for optimization of the filling factor and charge density of self-shielding flux-cored powder wire," *Materials Science*, vol. 60, no. 4, pp. 445-452, doi: 10.1007/s11003-025-00904-z.
- [9] Lutsak, D. Prysyazhnyuk, P. Burda, M. and Aulin, V. (2016) "Development of a method and an apparatus for tribotechnical tests of materials under loose abrasive friction," *Eastern-European Journal of Enterprise Technologies*, vol. 5, no. 7 (83), pp. 19-26, doi: 10.15587/1729-4061.2016.79913.
- [10] Hamblin M.G. and Stachowiak, G.W. (1996) "Description of Abrasive Particle Shape and Its Relation to Two-Body Abrasive Wear," *Tribology Transactions*, vol. 39, no. 4, pp. 803-810, doi: 10.1080/10402009608983598.
- [11] Bembenek, M. Prysyazhnyuk, P. Shihab, T. Machnik, R. Ivanov, O. and Ropyak, L. (2022) "Microstructure and Wear Characterization of the Fe-Mo-B-C – Based Hardfacing Alloys Deposited by Flux-Cored Arc Welding," *Materials*, vol. 15, no. 14, p. 5074, doi: 10.3390/ma15145074.
- [12] Mulhearn T.O. and Samuels, L.E. (1962) "The abrasion of metals: A model of the process," *Wear*, vol. 5, no. 6, pp. 478-498, doi: 10.1016/0043-1648(62)90064-9.
- [13] Sedriks A.J. and Mulhearn, T.O. (1963) "Mechanics of cutting and rubbing in simulated abrasive processes," *Wear*, vol. 6, no. 6, pp. 457-466, doi: 10.1016/0043-1648(63)90281-3.
- [14] Lorensen W.E. and Cline, H.E. (1987) "Marching cubes: A high resolution 3D surface construction algorithm," *ACM SIGGRAPH Computer Graphics*, vol. 21, no. 4, pp. 163-169, doi: 10.1145/37402.37422.
- [15] Kuhl F.P. and Giardina, C.R. (1982) "Elliptic Fourier features of a closed contour," *Computer Graphics and Image Processing*, vol. 18, no. 3, pp. 236-258, doi: 10.1016/0146-664X(82)90034-X.

- [16] Otsu, N. (1979) "A Threshold Selection Method from Gray-Level Histograms," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 9, no. 1, pp. 62-66, doi: 10.1109/TSMC.1979.4310076.
- [17] Sternberg, S.R. (1983) "Biomedical Image Processing," *Computer*, vol. 16, no. 1, pp. 22-34, doi: 10.1109/MC.1983.1654163.
- [18] Vincent L. and Soille, P. (1991) "Watersheds in digital spaces: an efficient algorithm based on immersion simulations," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 13, no. 6, pp. 583-598, doi: 10.1109/34.87344.
- [19] Liebovitch L.S. and Toth, T. (1989) "A fast algorithm to determine fractal dimensions by box counting," *Physics Letters A*, vol. 141, no. 8-9, pp. 386-390, doi: 10.1016/0375-9601(89)90854-2.
- [20] van der Walt S. et al., (2014) "Scikit-image: image processing in Python," *Peer J*, vol. 2, p. e453, doi: 10.7717/peerj.453.
- [21] Virtanen P. et al., (2020) "SciPy 1.0: fundamental algorithms for scientific computing in Python," *Nature Methods*, vol. 17, no. 3, pp. 261-272, doi: 10.1038/s41592-019-0686-2.
- [22] De Pellegrin D.V. and Stachowiak, G.W. (2002) "Assessing the role of particle shape and scale in abrasion using 'sharpness analysis'," *Wear*, vol. 253, no. 9-10, pp. 1016-1025, doi: 10.1016/S0043-1648(02)00232-6.

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