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EXPERIMENTAL ANALYSIS OF THERMO-MECHANICAL EFFECTS ON GEOTHERMAL PILES

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Abstract: The growing global population necessitates sustainable energy solutions to meet escalating energy demands. Geothermal energy, abundant and continuous, ranks as a leading alternative after solar energy. Integrating geothermal energy into building systems requires specialized structures. For shallow geothermal applications, thermo-active piles offer a cost-effective and efficient solution regarding both investment and operational expenses. Distinct from traditional piles, these energy piles enable thermal exchange between the pile and surrounding soil, supporting building heating, cooling, and hot water needs. Their design mandates a coupled thermo-hydro-mechanical analysis. This study presents a numerical model of a field-scale energy pile experiment, validated through stress and temperature data collected during the experiment..
Keywords: Energy Pile , 3D model, Mathematical Models **,** Information System

Case study on the numerical modelling of energy piles

Energy piles serve a dual purpose by functioning both as structural foundation elements and as components of geothermal heating and cooling systems, harnessing the earth's renewable thermal energy. This integrated approach has become increasingly prominent in the design of sustainable buildings due to its advantages in both environmental impact and long-term cost efficiency. Numerical simulations play a key role in evaluating the thermal behavior and mechanical stability of these systems under varying operational and climatic scenarios. Meanwhile, the global population is growing at a rapid pace, currently approaching 8 billion, with projections suggesting an increase to around 9.8 billion by the year 2050 [1]. This population growth is directly contributing to a surge in worldwide energy consumption. Today, nearly 60% of global energy needs are still being met through the use of fossil fuels [2**]**
The global dependence on fossil fuels poses two major challenges: the acceleration of environmental degradation through greenhouse gas emissions, and the finite nature of these energy sources. To address these concerns, geothermal energy has emerged as a promising and sustainable alternative. Geothermal energy—defined as the heat stored within the Earth's crust—is considered the second most plentiful renewable energy source worldwide, surpassed only by solar energy.
A major advantage of geothermal energy lies in its continuous availability, which is unaffected by weather conditions, making it a stable and reliable option for energy generation [3]. Because of its low environmental footprint and long-term sustainability, geothermal energy is increasingly being viewed as a key player in the transition to clean energy systems.
Among the various methods used to exploit shallow geothermal energy, thermo-active piles—commonly known as energy piles—have become particularly noteworthy [4]. These piles differ from traditional foundation elements by incorporating embedded pipe networks that facilitate heat exchange between the underground environment and the building structure. A heat pump regulates the temperature of the circulating fluid, allowing the system to provide heating, cooling, and domestic hot water based on seasonal demands.
A considerable body of research has been conducted to examine how energy piles respond under thermal and mechanical loading. Their thermal behavior has been explored through field-based experiments [5], [6], [7], and the findings have been further validated through advanced numerical modeling techniques [8], [9].

In this paper, a detailed numerical simulation was carried out to replicate the behavior of an energy pile tested in the field in Houston, Texas. The model was calibrated and validated using experimental data on temperature changes and mechanical stresses, establishing a robust platform for future analysis of energy pile performance under real-world operating conditions

Field experiment Energy and Pile Characteristics

The pile, constructed using the Augered Cast-In Place (ACIP) method, has a diameter of 45.7 cm and a total length of 15.24 m. To monitor the pile's mechanical behavior, vibrating wire strain gauges were installed at five different depths along the reinforcement cage (Figure 1).
The pile incorporates high-density polyethylene (HDPE) pipes to facilitate heat exchange between the ground and the structure. These pipes have a diameter of 25.4 mm and a wall thickness of 3.2 mm. A single U-shaped pipe configuration is positioned centrally within the pile. The heat transfer fluid circulating within the pipe is composed of 80% water and 20% antifreeze. During testing, the fluid circulated at a constant flow rate of 0.88 m/s. The thermal conductivity of the pipe material is 0.42 W/m/°C.
The energy pile has a modulus of elasticity of 36 GPa and a linear thermal expansion coefficient of 1.29×10^{-5} με/°C. The instrumentation was placed at depths of -2.7 m, -5.7 m, -8.7 m, -11.6 m, and -14.5 m, respectively.

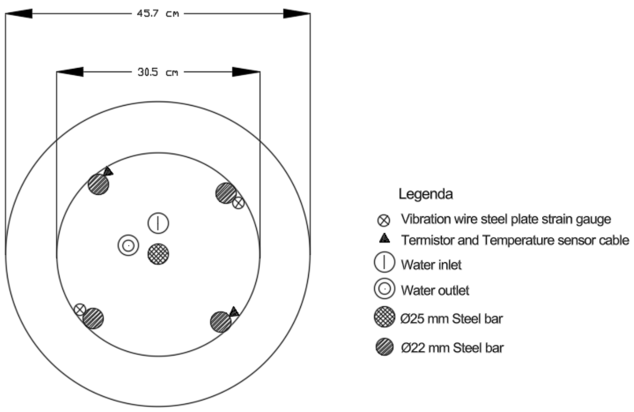

Fig. 1 - Energy Pile instrumentation

Soil properties

Based on the findings from the geotechnical investigations conducted at the site, the subsurface profile consists of a clay layer extending to a depth of 9.8 meters, beneath which a sand stratum is located. The groundwater table was identified at a depth of 3.7 meters below the surface. The geotechnical parameters utilized in the numerical analysis are presented in Table 1.

Table - 1

Soil parameter

Ground layer	Terzaghi(Mpa)	Poisson	Kohezyon (kPa)	Internal friction angle (˚)
Clay	45	0.495	114	0
Sand	75	0.3	5	43

Purpose of the Experiment

As part of this study, the energy pile was subjected to a static load test in the field, and its vertical load capacity was determined to be 2558 kN. After the static load was removed, cyclic thermal loading was applied. This thermal loading consisted of five thermal cycles and lasted for a total of six weeks. The temperature of the fluid circulated through the pipes inside the pile is shown in Figure 2.

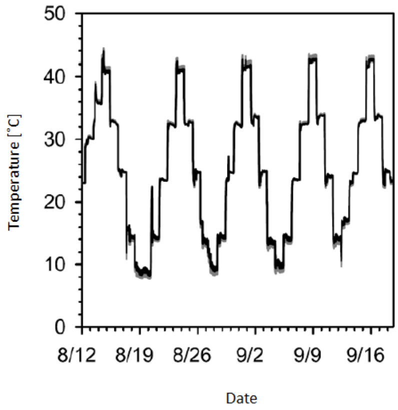

Fig. 2 - Thermal loading cycles

Physico-Mathematical Models for the Simulation of Transfer Processes in Geothermal Piles

Analytical methods are commonly employed in the study of geothermal heat pump systems because they offer a balance of simplicity and computational speed. These models typically assume that heat transfer occurs primarily through conduction, often disregarding the effects of natural groundwater movement. Most analytical frameworks are grounded in either the infinite line source (ILS) model or the cylindrical heat source theory, both of which provide reliable solutions under conduction-dominated conditions.

$$\frac{T_r - T_e}{T_i - T_e} = \sum_{n=1}^{\infty} \frac{2J_1(\varepsilon_n)}{\varepsilon_n[J_0^2(\varepsilon_n) + J_1^2(\varepsilon_n)]} J_0\left(\frac{r}{R}\varepsilon_n\right) \exp(-\varepsilon_n^2 Fo) \tag{1}$$

These analytical models are particularly useful for on-site evaluation of geothermal systems through Thermal Response Tests (TRT), which typically span 12 to 60 hours. However, their applicability is limited when it comes to long-term performance analysis, as significant thermal effects tend to emerge only after around 1.6 years of system operation, depending on the hydrogeological and operational characteristics of the site.
In the infinite line source (ILS) model, which assumes no groundwater movement, the system cannot reach a thermal steady state—the temperature anomaly continues to grow indefinitely over time. To overcome these limitations, numerical tools like COMSOL can be used to simulate long-term system behavior. These simulations account for axial heat transfer and the impact of groundwater flow on the temperature distribution along the surfaces of geothermal foundation elements, offering a more accurate representation of real-world conditions.

Model “Infinite Line Source” (ILS) *Reg* [1].

The Infinite Line Source (ILS) model stems from Lord Kelvin’s theoretical formulation of a heat source [11] and is frequently employed in the analysis of ground heat exchanger systems. It treats a linear heat source as a sequence of infinitesimally spaced point sources. The approach starts by determining the temperature response at a point within an infinite, uniform medium using the classic point source solution. The cumulative thermal influence is then obtained by integrating the effects of all individual point sources along the line, leading to an accurate temperature estimation at the desired location (Figure 3-1).
The following general assumptions were made for the derivation of this model:

1. The soil is considered a homogeneous medium, and its thermophysical properties are assumed to remain constant with temperature

2. The medium has a uniform initial temperature
3. The heating rate per unit length is constant from the start until the end of the thermal loading

The infinite line heat source model is the analytical solution of the conductive heat transfer equation in a homogeneous and isotropic medium, expressed in radial coordinates according to [2] [3]

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t} \tag{1}$$

where :

- $T = T(r, t)$: temperature of the medium, a function of radial distance r and time t
- **r**: radial distance from the heat source (m)
- **ρ**: density of the medium (kg/m^3)
- **c_p**: specific heat capacity at constant pressure (J/kg·K)
- **k**: thermal conductivity of the medium (W/m·K)
- **t**: time (s)

To solve the equation, it is assumed that the system is initially at a constant temperature (T(r, t=0) = T_0), and that the boundaries located at an infinite distance from the heat source (r=∞) always remain at a constant temperature (T(r=∞, t) = T_0). The surrounding medium is considered homogeneous and isotropic

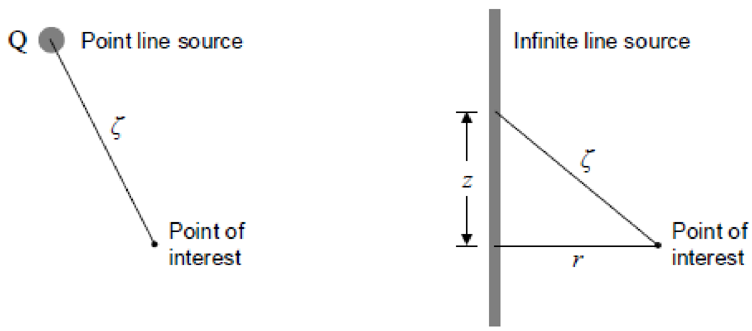

Fig. 3 - Point Line Source" and "Infinite Line Source" models according to [1]

In their 1948 study, Ingersoll and Plass [14] showed that the temperature variation at a given time t and at a radial distance ζ from a point heat source emitting a constant heat flux q can be determined using the following expression:

$$\Delta T(t) = \left(\frac{q\alpha}{k}\right)\left(\frac{1}{2\sqrt{\pi\alpha t}}\right)^3 exp\left(-\frac{\zeta^2}{4\alpha t}\right) \tag{3}$$

To estimate the temperature change induced by an infinite linear heat source, the following integral expression is employed. As before, the line source is modeled as a continuous sequence of adjacent point sources, with each point contributing to the thermal field at the observation location. The distance from each point source to the point of interest is given by 2ζ=r+z, as illustrated in Fig. 3.

$$\Delta T = \int_{-\infty}^{+\infty}\left(\frac{q\alpha}{k}\right)\left(\frac{1}{2\sqrt{\alpha\pi t}}\right)^3 e^{-(r^2+z^2)/4\alpha r}dz = \frac{q}{4\pi k t}e^{\frac{-r^2}{4\alpha t}} \tag{4}$$

Since the heat source continuously emits from time 0 to t, the temperature change at the point of interest can be estimated as:

$$\Delta T = \frac{q}{4\pi k}\int_0^t \frac{e^{\frac{-r^2}{4\alpha t}}}{t}dt \tag{5}$$

Assuming an infinite linear heat source located at r=0r = 0r=0 and represented by a constant heat flux per unit length, and given the prescribed initial and boundary conditions, the solution to equation (3.5) can be expressed as the following temperature increment ΔT(r,t)=T(r,t)−T0, which depends on the distance r from the source and time t

$$\Delta t(r,t) = -\frac{q}{4\pi k}\mathrm{Ei}\left(-\frac{r^2}{4\alpha t}\right) \tag{6}$$

where,

- Ei(u): exponential integral
- q: heat flux injected per unit length of the borehole

The exponential integral in equation (3.6) can be derived as the following Taylor series expansion [4].

$$E_i(u) = \gamma + \ln(u) + \sum_{i=1}^{\infty}\frac{u^t}{i \cdot i!} \tag{7}$$

where γ is the Euler–Mascheroni constant (γ = 0.57721…).
For large values of t/r^2, Ei(u) can be approximated as:

$$\Delta T(r_b,t) = \frac{q}{4\pi k}\left[\ln\left(\frac{4\alpha t}{r_b^2}\right) - \gamma\right] \tag{8}$$

The relationship between the measured average fluid temperature (T_f) during a thermal response test and the temperature at the borehole wall is:

$$T_f(t) = T_0 + qR_b + \frac{q}{4\pi k}\left[\ln\left(\frac{4\alpha t}{r_b^2}\right) - \gamma\right] \tag{9}$$

where,

- T_0: initial (undisturbed) ground temperature,
- R_b: borehole thermal resistance.

Equation (9) can be expressed in the form $T_f(t) = m \ln t + n$. Determining the ground's thermal conductivity can be achieved by analyzing the slope of the average fluid temperature curve plotted against the logarithm of time. This slope serves as the key parameter for estimating the thermal conductivity value:

$$m = \frac{q}{4\pi k} \tag{10}$$

The thermal conductivity of the ground can be evaluated using this slope, according to the following relation:

$$k = \frac{q}{4\pi m} \tag{11}$$

The ILS model does not account for the finite length of typical heat exchangers. The finite length causes an edge effect, which results in a reduction of the temperature change at the ends of the

line. This axial effect becomes significant at approximately t≈H^2/180αt for a single line source, where H is the length of the heat exchanger. This limiting time is even shorter if there is interaction between multiple line sources [5].
The Infinite Line Source (ILS) model does not consider the finite length of real heat exchangers. Because of this finite length, an edge effect occurs, leading to a decrease in temperature variation near the ends of the heat exchanger. This axial effect becomes significant at around $t \approx \frac{H^2}{180\alpha t}$ for a single line source, where H represents the length of the heat exchanger. When multiple line sources interact, this characteristic time becomes even shorter [16].
Representing the heat exchanger as a linear heat source does not provide accurate results for short time periods and in the case of vertical heat exchangers with large diameters, such as thermal piles (which also function as heat exchangers).
The Infinite Line Source (ILS) model does not account for the finite length of actual heat exchangers. Due to this finite length, an edge effect arises, causing a reduction in temperature variation near the ends of the exchanger. This axial effect becomes significant at approximately $t \approx \frac{H^2}{180\alpha t}$ for a single line source, where H is the length of the heat exchanger. When multiple line sources interact, this characteristic time is even shorter [16].
Modeling the heat exchanger as an infinite linear heat source is therefore inaccurate for short time periods and for vertical heat exchangers with large diameters, such as thermal piles, which also serve as heat exchangers.

Standard Finite Line Source Model (FLS) [6]

Traditionally, heat transfer in the porous soil medium without groundwater flow is described by the heat conduction equation as follows [6] :

$$\rho c \frac{\partial T}{\partial t} - \nabla(\lambda\, \nabla T) = \dot{q}_V \tag{12}$$

where:

- T is the temperature [°C or K],
- t is time [s],
- k is the thermal conductivity of the soil [W/m·K],
- $\dot{q}_V$ is the volumetric internal heat source term [W/m^3].
- **λ** – Thermal conductivity of the porous medium [W/m·K]
- **ρc** – Volumetric heat capacity of the porous medium [J/m^3·K], (where ρ is the density and c is the specific heat capacity of the bulk medium)

$$\rho c = n\rho_w c_w + (1-n)\rho_s c_s \tag{13}$$

- $\boldsymbol{\rho_s c_s}$ – Volumetric heat capacity of the solid phase in the aquifer (weighted by its proportion) [J/m^3·K] (ρ_s = density of the solid, c_s = specific heat capacity of the solid)
- $\rho_w c_w$ – Volumetric heat capacity of the water phase in the aquifer (weighted) [J/m^3·K] (ρ_w - density of water, c_w - specific heat capacity of water)

The solution of the partial differential equation for heat transfer from a source in a porous medium with an initial uniform temperature T_0 is given by the relation:

$$\Delta T(x,y,z,t) = \frac{Q}{4\pi\lambda r} erfc\left[\frac{r}{\sqrt{4at}}\right] \tag{14}$$

The temperature difference in the borehole, ΔT=T0−T, where T0 is the initial uniform temperature and T is the local temperature, is related to the heat Q extracted or injected from the borehole. The thermal diffusivity is defined as (a=$\frac{\lambda}{\rho c}$), where λ is the thermal conductivity and ρc is the volumetric heat capacity of the porous medium. The distance $r = \sqrt{x^2 + y^2 + (z - z')^2}$ to the heat source, located along the z-axis at coordinates (0,0,z′).
The Finite Line Source (FLS) model can be expressed as follows (H.Y. Zeng, N.R. Diao, Z.H. Fang, 2002): [7], [8].

$$\Delta T_{FLS}(x, y, z, t) = \frac{q_L}{4\pi\lambda}\left[\int_0^H \frac{1}{r} erfc \frac{1}{\sqrt{4at}} dz' - \int_{-H}^0 \frac{1}{r} erfc \frac{r}{\sqrt{4at}} dz'\right] \tag{15}$$

Numerical Model

Standard Finite element numerical model

A **3D** numerical simulation of the energy pile was carried out using COMSOL Multiphysics (refer to Figure 3). The computational domain spans 50 meters in width and length, and 40 meters in depth. The lateral boundaries were modeled with vertical sliding supports to permit horizontal movement, while the **bottom boundary** was **fully restrained** to simulate anchorage. All boundaries were treated as thermally insulated, except for the top surface, which was exposed to ambient conditions for thermal exchange.
An initial uniform temperature of 21.6°C was assigned to both the pile and the surrounding soil, based on in-situ temperature readings. The Mohr-Coulomb model was applied to simulate the mechanical behavior of the clay and sand layers, whereas the energy pile was represented as a linear elastic body. Thermal loading was introduced using the first of five measured thermal cycles (Fig. 4). The model began with an initial geostatic stress distribution to reflect realistic subsurface pressure conditions.

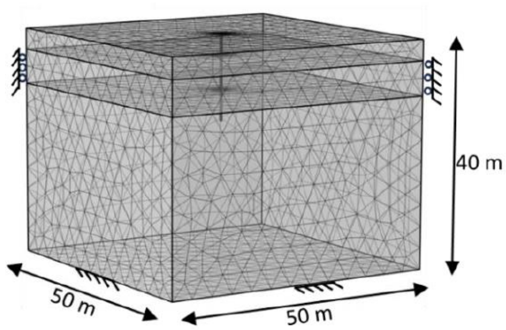

Fig. 4 - Numerical model and boundary conditions

Comparison of the field experiment and numerical model results

In the field experiment, temperature measurements along the energy pile were compared with numerical simulation results for the initial three days, as illustrated in Figure 4. Temperature and strain data were collected using vibrating wire strain gauge sensors, positioned as shown in Figure 1. The values presented in the graphs represent the averages from two sensors placed symmetrically within the pile.
Due to manufacturing and installation tolerances, slight positional differences occurred in the placement of pipes and sensors. Specifically, the radial distance of the sensors from the pile's center ranged between 11.7 cm and 12.9 cm. Consequently, the numerical model was calibrated to reflect this range, with results labeled as D1 and D2 corresponding to 11.7 cm and 12.9 cm,

respectively. As shown in Fig. 4, the numerical temperature profiles closely match the experimental data, demonstrating good agreement.
Thermal-induced stresses calculated by the finite element model are presented in Figure 5. With the exception of one sensor reading at a depth of approximately 8.7 meters, which recorded a slightly higher stress than predicted, the overall measured stress values align well with the model outcomes, further validating the model's reliability.

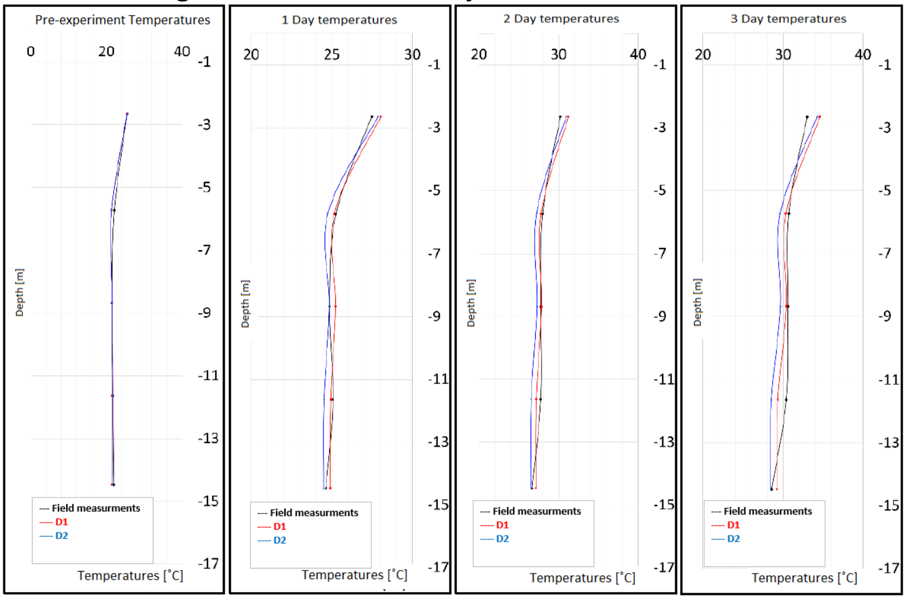

Fig. 5- Measured and Calculated Temperatures Along the Pile Before and During Thermal Loading

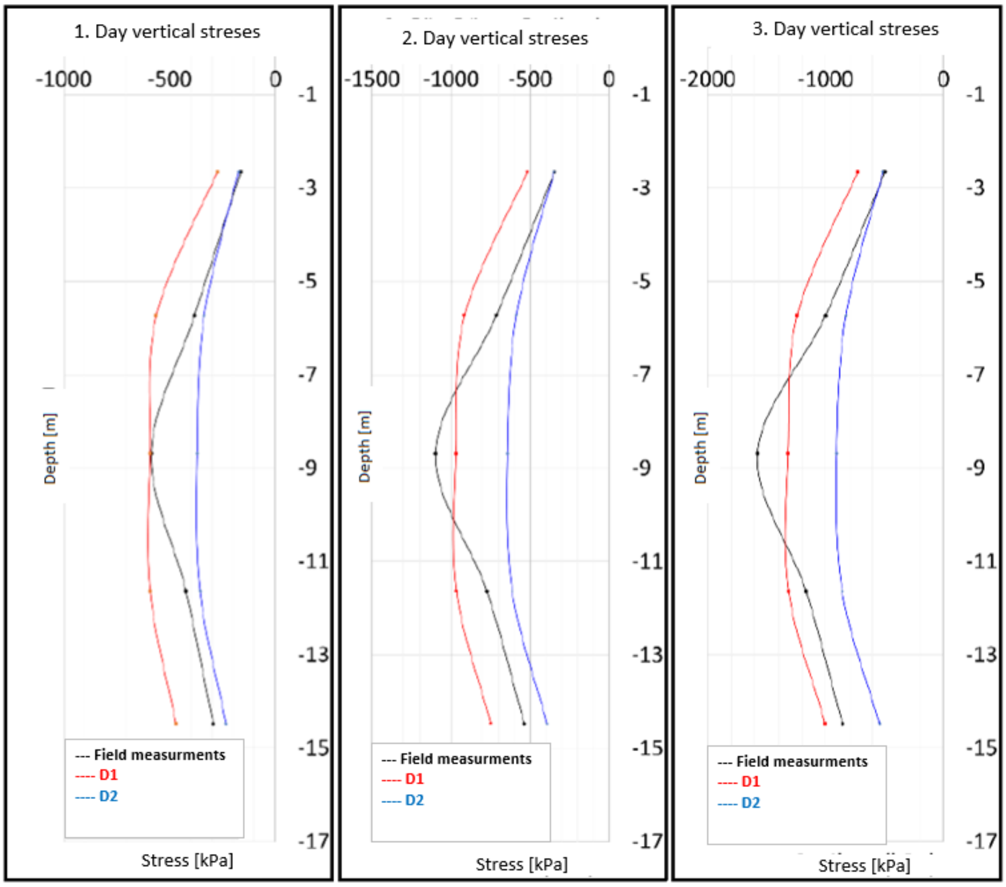

Fig. 6 - Measured and calculated temperatures along the pile during thermal loading

Conclusions

As a result of the numerical analyses conducted, the thermomechanical behavior of the pile under thermal effects was investigated. The temperatures and stresses measured along the pile in the field were compared with the results of the numerical model. The thermal fluid circulated through the pipes within the pile led to an increase in the pile's temperature. The temperature increases measured in the field were consistent with those calculated by the finite element numerical model. The temperature rise within the pile caused thermal expansion, which, depending on the boundary conditions of the pile, induced additional axial stresses along its length. Except for the sensor located at a depth of 8.7 m, the stress increases measured in the field due to thermal expansion were in agreement with those obtained from the numerical model. The discrepancy at this depth can be attributed to a change in sensor positioning during pile installation in the field

Important Considerations in the Numerical Modeling of Energy Piles: There are several important factors that must be taken into account when numerically modeling energy piles. The first is the definition of thermal boundary conditions, which must ensure that the surface temperatures of both the pile and the surrounding soil are consistent with those obtained from field experiments. Secondly, the positioning and installation of sensors in the field-tested pile is critical. Sensors should be placed on the reinforcement in such a way that their positions are not affected by external influences or the construction process. A third important consideration, particularly in water-saturated soils, is the need to account for the thermal properties of water, as it significantly influences heat transfer behavior.

As a result of the numerical analyses, the thermo-mechanical behavior of the energy pile subjected to thermal loading was thoroughly investigated. The temperature and stress distributions along the pile obtained from field measurements were compared against the predictions of the finite element model. The circulation of a thermal fluid through embedded pipes resulted in a temperature increase within the pile. These temperature rises, as recorded on-site, showed strong consistency with the outputs of the numerical simulations.

The increase in temperature led to thermal expansion of the pile material, which, depending on the imposed boundary conditions, induced additional axial stresses along the pile's length. With the exception of the sensor located at a depth of 8.7 meters, the thermally induced stress increases measured in the field correlated well with those predicted by the numerical model. The deviation observed at this depth is likely attributable to a displacement or misalignment of the sensor during field installation. Important Considerations in the Numerical Modeling of Energy Piles: There are several important factors that must be taken into account when numerically modeling energy piles.

1. **Thermal Boundary Conditions**: Accurate modeling demands thermal boundary conditions that reflect the surface temperatures of both the pile and surrounding soil, in alignment with experimental data.
2. **Sensor Placement and Installation**: The accuracy of model validation heavily depends on the correct positioning of sensors. Sensors should be affixed to the reinforcement in such a manner that minimizes the risk of misplacement due to construction activities or external factors.
3. **Soil Saturation and Water Properties**: In water-saturated soils, it is essential to incorporate the thermal properties of water, which can significantly affect heat transfer mechanisms within the system.

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