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MODELLING OF STRESS INTO HUMIDIFIED POROUS LAYER UNDER INFLUENCE OF MICROWAVE IRRADIATION

PART II THE GENERAL MODEL EQUATIONS

Volodymyr YUZEVYCH - Leading researcher, Professor, D.Sc., Department of Theoretical Principles of Mechanical Desrtructions: Karpenko Phisico-Mechanical Institute of NAS Ukraine,
e-mail: volodymyr.yuzevych@lnu.edu.ua

Taras HOLUBETS – Math of Irst category, M.Sc., Department of Physico-Mechanical Fields:Pidstryhach Institute of Applied Problems of Mechanics and Mathematics of NAS Ukraine, email:taras_holubets@yahoo.com

Abstract: In this part of paper are examined interrelated processes of heat and mass transfer, which take into account the nature of interphase interaction in a moistened porous media. A closed system of balance diffusion equations for the liquid phase and components of the gas mixture of water vapour and dry air is reviewed. The energy or heat balance equation, which contains the intensity of sources of evaporation or condensation during the phase transformation of liquid into water vapour, is analyzed. The phenomena of dielectric relaxation into a heterogeneous porous layer under influence of symmetric electromagnetic field of microwave frequency is modelled by an isotropic (non-magnetic) dielectric with weak or insignificant conductivity and strongly depends from pore saturation by liquid .With the help of the spatial averaging method, as it was described in the previous part of paper, taking into account the interphase interaction the equations for mechanical balance of are proposed, in which the exchange integrals of the moment of impulse and mechanical forces are applied. The macroscopic equations of mechanical balance into the solid and liquid phases are proposed, from where the analytical expressions for an effective, thermal and moisture stresses in the porous wetted layer are obtained using the accepted approximations and taking into account the known relations for stresses in the solid and liquid phases.

Keywords: microwave irradiation, porous media, heat and mass transfer, mechanical stress

1. Introduction

The mathematical modeling of coupled diffusional, electromagnetic and mechanical phenomena [1] into ingomogeneos wetted porous media is quite a complicated and troublesome process. First of all we have been oriented on the well know mathematical model [2,3] for describing of heat and mass transfer phenomena in muliphase porous media under influence of microwave irradiation. Second, for correct interpretation the physical phenomena of microwave heating has been defined the effective dielectric properties [4] of wetted porous material. Third, for the proper calculation of mechanical stress and strain into porous body under conditions of internal loading have been concentrated on the receiving of analytical expressions [5,6] for describing the mechanical state of porous body. In summary to this part of paper we have been formulated the conditions of compatibility for proposed systems of interconnected equations.

2. The object of studies

Let's define the object of studies. This is the one dimensional porous wetted plate with length of L (Fig.1) which is under influence of external symmetrical microwave irradiation of fixed frequency and constant power. The an ideal conditions of convective drying are realized through the constant velocity flow of hot air at the external surfaces of the plate during all time of microwave treatment.

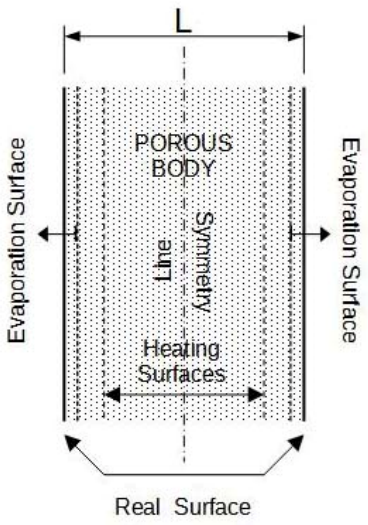


Fig.1 - Depiction of porous plate sample with boundary surfaces

We are modeling such one dimensional drying system mathematically by the introduction of dynamical boundary conditions. There is confirmed by the other authors [7-9] that conditionally it is possible to highlight the two plate areas during of microwave treatment of porous plate sample: the wetted and heated zones. The term of zone is meaning as the one dimensional length from the internal surface to the real fixed physical boundaries of porous body in the direction of external environment. The moving that zones is interconnected due to dependence of diffusion coefficients in the internal volume of sample from humidity as well as the time dependence of surface heat and mass transfer coefficients from the time of drying on the real physical boundaries of mentioned porous sample.

To describe the diffusive processes in a porous plate, we have applied the well known [2,3] averaged locally (R.E.V. - Representative Averaged Volume [1,2]) heat and mass transport equations written in the general form as

$$(\rho C_p)_{\text{eff}}\partial_t T + \left[\mathrm{C}_p^{v}\langle J_v^x\rangle^G + \mathrm{C}_p^{a}\langle J_a^x\rangle^G + \mathrm{C}_p^{L}\langle J_L^x\rangle^L\right]\cdot\partial_x T \quad = \partial_x[\lambda_{\text{eff}}\partial_x T] - \langle r\rangle\dot{I}$$
$$\partial_t\langle c_L\rangle^G + \partial_x\langle\vec{J}_L\rangle = \dot{I} \quad \partial_t\langle c_v\rangle^G + \partial_x\langle\vec{J}_v\rangle = -\dot{I} \qquad (1)$$
$$\partial_t\langle c_a\rangle^G + \partial_x\langle J_a\rangle = 0$$

where $(\rho C_p)_{\text{eff}} = \sum_\sigma \langle c_\sigma\rangle^\sigma C_p^\sigma$ and $\lambda_{\text{eff}} = \sum_\sigma \theta_\sigma\langle\lambda_\sigma\rangle^\sigma$, where $\sigma = \{S, L, G\}$ is the index of phase, are the effective thermal characteristics of porous material, $\dot{q}$ and $\dot{I}$ is the heat and mass sources intensity correspondingly, C_p^σ and $\langle\lambda_\sigma\rangle^\sigma$ are the specific capacity and thermo-conduction of σ-phase accordingly and $\langle r\rangle$ is the latent heat of vaporization.

Expressions for concentrations

$$\langle \mathrm{c}_L\rangle^L = \langle\varphi\rangle\eta_L\langle\rho_L\rangle^L\ \langle \mathrm{c}_v\rangle^G = \langle\varphi\rangle(1-\eta_L)\langle\rho_v\rangle^G, \langle \mathrm{c}_a\rangle^G = \langle\varphi\rangle(1-\eta_L)\langle\rho_a\rangle^G$$

and flows of phases (components) into the porous plate are the following

$$\langle J_L^x\rangle = -\langle\rho_L\rangle^L\frac{k_{\text{in}}k_{\text{rL}}}{\mu_L}\partial_x P_L \quad \langle J_\alpha^x\rangle = -\langle\rho_\alpha\rangle^G\frac{k_{\text{in}}k_{\text{rG}}}{\mu_G}\partial_x P_G - \langle\rho_G\rangle^G D_{\text{eff}}\partial[\langle\rho_v\rangle^G/\rho_G],$$

here $\psi_L = [k_{\text{in}}k_{\text{rL}}]/\mu_L$ and $\psi_G = [k_{\text{in}}k_{\text{rG}}]/\mu_G$, where k_{in} and $k_{r\alpha}$, $\alpha = \{L, G\}$ are the intrinsic permeability of skeleton and liquid phase correspondingly, $\langle\rho_L\rangle^L = \rho_L$ and $\langle\rho_\alpha\rangle^G = p_\alpha M_\alpha/\mathrm{RT}$, where $\alpha = \{v, a\}$ are the corresponding densities of liquid phase and components of gas mixture, μ_L and μ_G is dynamical viscosity, $D_{\text{eff}} = [(M_v M_a)/M_G^2]D_G^{\text{eff}}$ is the model diffusion coefficient, P_L and P_G are the values of pressures into liquid and gas phases, P_{C} is a capillary pressure. Here M_v and M_a are the molar mass of water vapor and dry air respectively.

3. The constitutive or material equations

We assume that**:** 1. Pressure balance in liquid and gas phase is $P_L = P_G - P_C$, where $P_C(\eta_L) = \sigma\sqrt{k_{\text{in}}/\langle\varphi\rangle}J(\eta_L)$ is the capillary pressure and $J(\eta_L)$ is the dimensionless characteristic of porous material named as J-Laverett [10] function; 2. The Daltone's law $p_v + p_a = P_G$, where p_β is the

partial pressures of gas mixture components ($P_G = \rho_G \mathrm{RT}/M_G$, where $M_G = 1/([\langle\rho_v\rangle^G/\langle\rho_G\rangle^G]/M_v + [\langle\rho_a\rangle^G/\langle\rho_G\rangle^G]/M_a)$ is the molar mass of mixture) is taking place; 3. The equation of sorption equilibrium $P_c = -\rho_L \mathrm{RT} Ln(\varphi)$, here $\varphi = p_v/p_{vs}$ is the relative humidity and p_{vs} is the partial pressure of water vapor into the gas phase is satisfied.

Modification of heat and mass equations consist of: 1. A transition from the set of variables $\{\eta_L, T, P_G\}$ to $\{\eta_L, T, x_v\}$ according to Rault's law [2,3] ($x_\alpha = p_\alpha/P_G$,($x_v + x_a = 1$), where x_α is molar fraction of components; 2. The intensity of evaporation sources in the internal linear volume and at the surfaces of the porous plate is determined in the different ways

$$\dot{I}^{(\mathrm{int})} = -\frac{1}{2}(\partial_t[\langle c_L\rangle^L - \partial_t\langle c_v\rangle^G] + \partial_x[\langle J_L^x\rangle - \langle J_v^x\rangle]),$$
$$\dot{I}^{(\mathrm{ext})} = -\frac{(\rho C_p)_{\mathrm{eff}}}{\langle r\rangle}\partial_t T + \frac{1}{\langle r\rangle}\partial_x[\langle\lambda_{\mathrm{eff}}\rangle\partial_x T] + \frac{\dot{q}}{\langle r\rangle};$$

3. An expression for the effective Burger [11] diffusion coefficient of the gas mixture into the pores of porous sample is modified by authors including of molar fraction x_v of water vapor into porous media and proposed into form

$$D_G^{\mathrm{eff}} = \frac{1}{\tau}\frac{\langle\varphi\rangle(1-\eta_L)}{x_v - \langle\varphi\rangle(1-\eta_L)(x_v-1)}D_v^a, \quad (2)$$

where D_v^a is the diffusion coefficient of water vapor into dry air, τ is the tortuosity factor.

For the internal volume

$$(\rho C_p)_{\mathrm{eff}} - \frac{1}{2}\langle r\rangle\partial_t[\langle c_L\rangle^L - \langle c_v\rangle^G] = \partial_x\left[\lambda_{eff}\partial_x T\right] + \frac{1}{2}\partial_x[\langle J_L^x\rangle + \langle J_v^x\rangle],$$
$$\partial_t[\langle c_L\rangle^L + \langle c_v\rangle^G] = -\partial_x[\langle J_L^x\rangle + \langle J_v^x\rangle], \quad (3)$$
$$\partial_t\langle c_a\rangle^G = -\partial_x\langle J_a^x\rangle$$

and surface of plate

$$\partial_t\langle c_L\rangle^L + \frac{(\rho C_p)_{\mathrm{eff}}}{\langle r\rangle}\partial_t T = -\partial_x\langle J_L^x\rangle + \frac{1}{\langle r\rangle}\partial_x[\lambda_{\mathrm{eff}}\partial_x T] + \frac{\dot{q}}{\langle r\rangle}, \quad (4)$$
$$\partial_t\langle c_v\rangle^G - \frac{(\rho C_p)_{\mathrm{eff}}}{\langle r\rangle}\partial_t T = -\partial_x\langle J_v^x\rangle - \frac{1}{\langle r\rangle}\partial_x[\lambda_{\mathrm{eff}}\partial_x T] - \frac{\dot{q}}{\langle r\rangle},$$
$$\partial_t\langle c_a\rangle^G = -\partial_x\langle J_a^x\rangle$$

the main or original system of heat and mass transport equations (1) are modified through the linear combination of existing equations.

4. Modeling of heat and mass transfer under microwave treatment

Let's denote the real not moving hard body surfaces of porous plate by the symbol S (Fig.1). Then dynamically changed heating and evaporation surfaces we mark as S_1 and S_2 respectively at the initial and the current moments of time during microwave drying. Into the near-surface zones of evaporation Δh_{S_1} and heating Δh_{S_2}, which are delimited by the differences of the length between external body surfaces S and surfaces of evaporation S_1 and heating S_2 respectively, the solutions of the system of heat and mass equations and quantity of microwave heating sources are assumed to be constant with coordinate changes. It is because along the defined length of near-surface zones of plate the quantities of physical variables into system of equations (3) and (4) takes from the set of constant boundaries values of physical variables for heat and mass transfer equations at the surfaces S_1 and S_2 from the side of an internal surface S_0 of symmetry. So, the physical variables into evaporation and heating near-surface zones may only change dynamically with time during of drying period.

The internal and external diffusion processes can be described by the modified system of diffusion equations (3) and (4) with taking into account the defined below extended effective diffusion coefficient (2) of the gas mixture into pores of modeled one-dimensional porous

sample according to the corresponding expressions for the liquid and gas mixture components flows

$$\langle J_L^x\rangle = \psi_L\rho_L\left(\frac{\partial p_c}{\partial \eta_L} - \frac{1}{x_v}\frac{\partial p_v}{\partial \eta_L}\right)\partial_x\eta_L - \psi_L\rho_L\left(\frac{\partial p_c}{\partial T} - \frac{1}{x_v}\frac{\partial p_v}{\partial T}\right)\partial_x T + \psi_L\rho_L\frac{P_G}{x_v}\partial_x x_v,$$
$$\langle J_v^x\rangle = -\psi_G\frac{1}{x_v}\langle\rho_v\rangle^G\frac{\partial p_v}{\partial \eta_L}\partial_x\eta_L - \psi_G\frac{1}{x_v}\langle\rho_v\rangle^G\frac{\partial p_v}{\partial T}\partial_x T + \left(\psi_G\frac{P_G}{x_v}\langle\rho_v\rangle^G - D_{\text{eff}}\langle\rho_G\rangle^G\right)\partial_x x_v, \quad (5)$$
$$\langle J_a^x\rangle = -\psi_G\frac{1}{x_v}\langle\rho_a\rangle^G\frac{\partial p_v}{\partial \eta_L}\partial_x\eta_L - \psi_G\frac{1}{x_v}\langle\rho_a\rangle^G\frac{\partial p_v}{\partial T}\partial_x T + + \left(\psi_G\frac{P_G}{x_v}\langle\rho_a\rangle^G + D_{\text{eff}}\langle\rho_G\rangle^G\right)\partial_x x_v.$$

The thicknesses of the near-surface zones evaporation an heating relatively to surface of symmetry S_0 (Fig.1) are described by the corresponding lengths

$$\Delta h_1 = \frac{1}{h_m}\psi_G\frac{p_v}{x_v}|_{S_0} \text{ and } \Delta h_2 = \frac{1}{h_T}\lambda_{\text{eff}}|_{S_0},$$

where h_m and h_T are the constant surface mass and heat exchange coefficients under convective heating conditions and S_0, as it was mentioned below, is the surface of symmetry of the porous plate.

4.1 Boundary conditions

For the liquid phase we takes $\langle J_L^x\rangle|_{S_1^-} = J_L^*|_{S_1^+}$. The flows of components of gas mixture at boundary surfaces $\langle J_v^x\rangle|_{S_1^-} = J_v^*|_{S_1^+}$ and $\langle J_a^x\rangle|_{S_1^-} = J_a^*|_{S_1^+}$, where signs {+} or {-} means as the right or left limit to the surface accordingly. This boundary conditions are defined relatively to the heat are mass diffusion on the surface S_0 of symmetry under additional conditions: 1. There is no liquid flow ($P_L|_{S_1} = 0$) and $P_C|_{S_1} = P_G|_{S_1}$ on the surface S_1 of evaporation; 2. The phase transformation of liquid density $\langle\rho_L\rangle^L|_{S_1^+} = \langle\rho_v\rangle^G|_{S_1^+}$ into water vapor on the boundary surfaces S_1 with corresponding near-surface length $\Delta h_{S_1} = \{1/2\,|\text{-L}+\Delta h_1|, 1/2\,|\text{L-}\Delta h_1|\}$ at constant wetting occurs instantly; 2. Heating sources along the near-surface S_2 layer with the length of $\Delta h_{S_2} = \{1/2\,|\text{-L}+\Delta h_2|, 1/2\,|\text{L-}\Delta h_2|\}$ are uniform.

Then

$$J_L^* = -\frac{h_T^*}{\langle r\rangle}\left(T_{|S_1} - T_{|\text{amb}}\right),$$
$$J_v^* = -\langle\varphi\rangle(1-\eta_L)\frac{\langle\rho_a\rangle^G}{\langle\rho_v\rangle^G}h_m^*\left(\langle\rho_v\rangle^G{}_{|S_1} - \langle\rho_v\rangle^G{}_{|\text{amb}}\right) - \frac{1}{T}\langle\rho_a\rangle^G\frac{D_v^a}{\lambda_G}\frac{h_T^*}{\langle r\rangle}\left(T_{S_1} - T_{|\text{amb}}\right) + + \langle\varphi\rangle(1-\eta_L)\frac{1}{x_v}\langle\rho_a\rangle^G\left(h_m^* + \frac{x_v}{1-x_v}\frac{M_v}{M_g}D_S^a\right)\left(x_{v|S_1} - x_{v|\text{amb}}\right), \quad (6)$$
$$J_a^* = -\langle\varphi\rangle(1-\eta_{sL})h_m^*\left(\langle\rho_v\rangle^G{}_{|S_1} - \langle\rho_v\rangle^G{}_{|\text{amb}}\right) - \left(\langle\rho_v\rangle^G\frac{D_v^a}{\lambda_G}\frac{1}{T} + 1\right)\frac{h_T^*}{\langle r\rangle}\left(T_{|S_1} - T_{|\text{amb}}\right) + \langle\varphi\rangle(1-\eta_L)\frac{1}{x_v}\langle\rho_v\rangle^G\left(h_m^* - \frac{M_a}{M_g}D_S^a\right)\left(x_{v_{S_1}} - x_{v|\text{amb}}\right),$$

here $h_m^* = \left[\psi_G\frac{p_v}{x_{v|S_1}}/\psi_G\frac{p_v}{x_{v|S_0}}\right]h_m$ and $h_T^* = \left[\lambda_{\text{eff}|S_1}/\lambda_{\text{eff}|S_0}\right]h_T$ are the variable with time heat and mass transfer coefficients, $\text{D}_\text{S}^\text{a} = \langle\varphi\rangle^{1/3}(1-\eta_\text{L})^{7/3}\text{D}_\text{v}^\text{a}$ is the well known Millington-Quirk [12] coefficient of diffusion for dry air as component of gas mixture into the pores of material at surfaces S_2 of evaporation and $\text{D}_\text{v}^\text{a}$ is the diffusion water vapor into dry air coefficient.

4.2 Modeling of microwave heating

From the point of view of the cluster approach, the polarization operator of the porous (composite) medium was defined [13] previous in the terms of the local magnitudes of the electromagnetic field (E.M.F), according to the properties of which an expression for the generalized complex effective dielectric constant of the heterogeneous material is obtained [14] in the harmonic approximation. The properties of such dielectric constant were studied depending on the choice of the local geometry of the porous material, the phase and volume content of the constituents or components [4]. Thus, in the coordinate space of variables, the averaged equations of the electromagnetic field with effective dielectric and low conductive properties, which are characterized by the frequency of microwave irradiation and the volumetric content of phase components, are obtained.

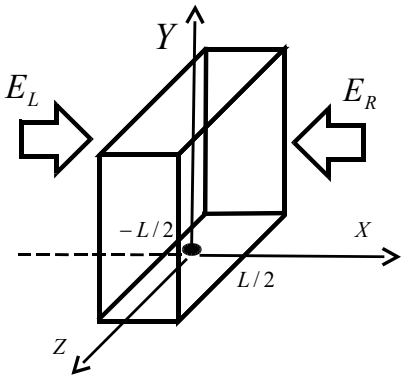


Fig.2 - Schematic representation of microwave dried porous plate

According to the local spatial method the averaged Maxwell [13] equations of E.M.F. have been obtained in the harmonic approximation. In this paper we examine the problem of electromagnetic wave propagation into inhomogeneous media with weakly variable wave properties [15] in the coordinate and time spaces of variables. The solution of the wave equation in the term of electric and magnetic field is found by approximate methods under the conditions of weak variability of the volume and frequency characteristics of the porous (multiphase) wetted medium [14] and can be applied into the microwave range of frequencies. The case of the one-dimensional symmetric microwave irradiation of an unlimited flat plate of finite thickness is described.

According to the boundary conditions for the components and derivatives of the field, the transmission and reflection coefficients of the transverse plane electromagnetic (T.E.M.)[14] wave was obtained [16]. The general solution in electric and magnetic field strengths is considered. In this way, the power of the energy flow in a non-homogeneous porous wetted plate was calculated and, as a consequence of the law of conservation of energy, an expression for the intensity of internal heat sources was received.

As it was described in the paper [16] for a flat plate in terms of electric field strengths, the wave equation will have the form

$$\partial_x^2\langle\vec{E}_*^t\rangle(\vec{x}) + k_0^2\left[\bar{n}_\omega^{\text{eff}}(\vec{x},t)\right]^2\langle\vec{E}_*^t\rangle(\vec{x}) = 0$$

here $\bar{n}_\omega^{\text{eff}}(\vec{x},\omega) = \bar{k}_\omega^{\text{eff}}(\vec{x},t)/k_0 = \sqrt{\bar{\varepsilon}_\omega^{\text{eff}}(\vec{x},t)}$ is the complex refractive index, $\bar{k}_\omega'(\vec{x},t)$ is the wave vector into the porous (inhomogeneous) linear media, $k_0 = \omega\sqrt{\mu_0\varepsilon_0} = \omega/c_0$ (where $c_0 = 1/\sqrt{\mu_0\varepsilon_0}$ is the light velocity in vacuum) is the wave vector of electromagnetic irradiation in the vacuum, $\omega = 2\pi f$ is the corner frequency of electromagnetic field (here f is the lineal frequency), μ_0 and ε_0 are the magnetic and electric constants in the vacuum correspondingly.

We have been founded the solution of the wave equation, as it is specified in the work [16] , according to the method of W.K.B. (Wentzel-Kramers-Brillouin) in the form

$$\langle E_{*y}^t\rangle(x) = \frac{1}{\sqrt{\bar{n}_\omega^{\text{eff}}(x,t)}}\begin{pmatrix} A(t)\sqrt{\bar{n}_\omega^L(t)}\text{Exp}\left[-\text{ik}_0\int_{-L/2}^{x}\bar{n}_\omega^{\text{eff}}(x,t)\,\text{dx}\right] + \\ +B(t)\sqrt{\bar{n}_\omega^R(t)}\text{Exp}\left[\text{ik}_0\int_{L/2}^{x}\bar{n}_\omega^{\text{eff}}(x,t)\,\text{dx}\right]\end{pmatrix},$$

where $\bar{n_\omega^\alpha}(t) = \bar{n_\omega}(\pm L/2\,,t), \alpha = \{L,R\}$ are the constant values of refractive index on the edges of porous plate, $A(t)$ and $B(t)$ are the unknown functions of time.

Under applying of boundary conditions [16] to the components and derivatives of E.M.F. at the edges of the porous plate we get the time-varying coefficients

$$A(t) = E_L \frac{T_L^t - (E_R/E_L)\delta(t) T_R^t R_L^t \gamma_{\mathrm{RL}}(t)}{1 - [\delta(t)]^2 R_L^t R_R^t},$$
$$B(t) = E_R \frac{T_R^t - (E_L/E_R)\delta(t) T_L^t R_R^t \gamma_{\mathrm{LR}}(t)}{1 - [\delta(t)]^2 R_L^t R_R^t},$$

where $T_\alpha^t = 2/[1 + \bar{n}_\omega^\alpha(t)]$ and $R_\alpha^t = [1 - \bar{n}_\omega^\alpha(t)]/[1 + \bar{n}_\omega^\alpha(t)]$ (here $\alpha = \{L, R\}$ is the index, which corresponds to the L-left and R-right edges of the plate respectively), for transmission and reflection of a plane wave at the boundaries of the plate in the accepted notations. Here E_L and E_R are amplitudes of electric field at the surfaces and $\delta(t)$ is the small dimensionless parameter. In the approximation of the large plate L thicknesses according to [14] for the power of microwave irradiation in the porous plate we have

$$\langle P_x^t \rangle(x) = P_{\mathrm{in}} |\bar{n}_\omega(t)| |T(t)|^2 \mathrm{Re}\left[\sqrt{\frac{\bar{n}_\omega^{\mathrm{eff}}(x,t)}{\bar{n}_\omega^{\mathrm{eff}}(x,t)}}\right] \times \left(\mathrm{Exp}\left[-2\int_{-L/2}^{x} \alpha(x,t)\,\mathrm{dx}\right] + \right.$$
$$\left. + \mathrm{Exp}\left[-2\int_{x}^{L/2} \alpha(x,t)\,\mathrm{dx}\right]\right)$$

where $P_{\mathrm{in}} = E_{0^2}/2\,\eta_0$ is the power of external microwave irradiation, $\bar{n}_\omega(t) = \bar{n}_\omega^{\mathrm{eff}}(\pm L/2\,, t)$ and $T(t) = 2/[1 + \bar{n}_\omega(t)]$ are the boundary values of refractive index and transmission coefficient for an electromagnetic wave.

The intensity of heat sources of electromagnetic (microwave) heating is obtained by the default relation

$$\dot{q}^t(x) = -\partial \langle P_x^t \rangle$$

and all this gets the following view

$$\dot{q}^t(x) = 2P_{\mathrm{in}} |\bar{n}_\omega(t)| |T(t)|^2 \alpha(x,t) \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{1 + \mathrm{tg}^2 \delta_\omega(x,t)}}\right)} \times$$
$$\times \left(\mathrm{Exp}\left[-2\int_{-L/2}^{x} \alpha(x,t)\,\mathrm{dx}\right] + \mathrm{Exp}\left[-2\int_{x}^{L/2} \alpha(x,t)\,\mathrm{dx}\right]\right), \qquad (7)$$

where $\mathrm{Re}\left[\sqrt{\frac{\bar{n}_\omega^{\mathrm{eff}}(x,t)}{\bar{n}_\omega^{\mathrm{eff}}(x,t)}}\right] = \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{1 + \mathrm{tg}^2 \delta_\omega(x,t)}}\right)}$ is the actual definition.

Given the ratio $k_0 \bar{n}_\omega^{\mathrm{eff}}(x,t) = \beta(x,t) - i\alpha(x,t)$, where $\alpha(x,t) = -k_0 \mathrm{Im}\left[\sqrt{\bar{n}_\omega^{\mathrm{eff}}(x,t)}\right]$ and $\beta(x,t) = k_0 \mathrm{Re}\left[\sqrt{\bar{n}_\omega^{\mathrm{eff}}(x,t)}\right]$ are the coefficients of the absorption and transmission of a plane electromagnetic wave, we obtain the effective quantities of wave length $\lambda_\omega^{\mathrm{eff}}(x,t)$ and penetration depth $D_p^{\mathrm{eff}}(x,t)$ for such wave process

$$\lambda_\omega^{\mathrm{eff}}(x,t) = \lambda_0 \frac{\sqrt{2}}{\sqrt{\bar{\varepsilon}_\omega^{\mathrm{eff}(1)}}} \frac{1}{\sqrt{\sqrt{1 + \mathrm{tg}^2 \delta_\omega(x,t)} + 1}},\ D_p^{\mathrm{eff}}(x,t) = \frac{\mathrm{c}_0}{\omega} \frac{\sqrt{2}}{\sqrt{\bar{\varepsilon}_\omega^{\mathrm{eff}(2)}}} \frac{1}{\sqrt{\sqrt{1 + \mathrm{tg}^2 \delta_\omega(x,t)} - 1}},$$

under conditions of executing of the $2\alpha\beta = k_0^2 \sqrt[3]{\mathrm{tg}^2 \delta_\omega}\,\mathrm{Re}\left[\bar{\varepsilon}_\omega^{\mathrm{eff}}\right]$ rationed relation, where $tg_\omega^{\mathrm{eff}}(x,t) = \bar{\varepsilon}_\omega^{\mathrm{eff}(2)} / \bar{\varepsilon}_\omega^{\mathrm{eff}(1)}$ is effective tangent of dielectric losses.

4.3 Mechanical stress under moisture and thermal loading

For an approximate description of the stress-strain state of the wetted porous plate the continuum approach is used. In this case, the heterogeneous porous medium is modelled by the set of three coexisting and interacting continuums of material points: skeleton (solid phase), water (liquid phase) and two-component gas mixture of water vapor and dry air (gas phase). Microscopic balance equations of momentum or stresses are considered [5], for which the following approximations are adopted in the theoretical model of the stress-strain state: 1. The porous skeleton is isotropic and the fluid phase is homogeneous; 2. Deformations of the porous skeleton

are small and the work of deformation forces against the external load can be neglected; 3. It is also ignored by the phase transformation of the skeleton and the transfer of impulse into the solid phase due to the phase transformation.

For a spatially isotropic homogeneous solid phase (framework or skeleton) in a porous medium under the condition of the absence of external force loading, the quasi-static problem of linear thermoelasticity is formulated in the form of relations

$$\hat{T}_{\alpha\beta,\beta} + \hat{I}F_\alpha = 0, \;\; F_\alpha = -\langle\varphi\rangle\left[\frac{\partial P_G}{\partial x_\alpha} - \eta_L\frac{\partial P_C}{\partial x_\alpha}\right] + \langle\varphi\rangle P_C\frac{\partial \eta_L}{\partial x_\alpha},$$

where $\hat{T}^* = (1-\langle\varphi\rangle)\left(\hat{t}_S' - \langle K\rangle\langle\alpha_S^T\rangle(T-T_0)\hat{I}\right)$ is the leading stress tensor, F_α is the components of internal (pondermotore) forces due to gas pressure P_G, capillary pressure P_C and liquid pore saturation η_L under the averaged constant porosity of body $\langle\varphi\rangle$, T_0 is the initial equilibrium temperature and $\hat{I}$ is the diagonal tensor.

In the case of a one-dimensional porous plate (Fig. 2), which is under the influence of internal microwave heating according to conditions of compatibility of Saint-Venant deformations [5], the system of equations for the stress component of the tensor takes the form

$$\frac{\partial^2 T_{\rm xx}^*}{\partial x^2} + \frac{\partial F}{\partial x} = 0,$$

$$\frac{\partial^2 T_{\rm yy}^*}{\partial x^2} + \frac{1}{1-\langle\nu\rangle}\left(\langle E\rangle\frac{\partial^2 L}{\partial x^2} + \langle\nu\rangle\frac{\partial F}{\partial x}\right) = 0, \frac{\partial^2 T_{\rm zz}^*}{\partial x^2} + \frac{1}{1-\langle\nu\rangle}\left(\langle E\rangle\frac{\partial^2 L}{\partial x^2} + \langle\nu\rangle\frac{\partial F}{\partial x}\right) = 0.$$

Force loads into the internal area of the plate are also determined

$$\Phi^*(x) = -\int_0^x F(x)\mathrm{dx} = -[\Phi(x) - \Phi(0)]\,, L^*(x) = L(x) - L(0) \qquad (8)$$

where

$$\Phi(x) = -\int F(x)\mathrm{dx} = \langle\varphi\rangle[P_G - \eta_L \mathrm{P}_C]\,, \;\; L(x) \equiv -(1-\langle\varphi\rangle)\langle K\rangle\langle\alpha_S^T\rangle(T-T_0)$$

are the quantity of humidity and thermal stress and $\Phi(0)$ and $L(0)$ are corresponding constant values on the internal surface of symmetry (see Fig.2 is the surface S_0 at $x = 0$) for the linear porous plate .

According to the symmetry conditions, the boundary conditions for a plate of a finite L thickness are applied: 1. There are no internal forces $\Phi^*(0) = L^*(0) = 0$ along the plane of symmetry of the plate; 2. The boundary surfaces of the plate are free from external loads $\int_{-L/2}^{L/2} T_{\rm xx}^*\mathrm{dx} = \int_{-L/2}^{L/2} T_{\rm yy}^*\mathrm{dx} = \int_{-L/2}^{L/2} T_{\rm zz}^*\mathrm{dx} = 0$; 3. The derivative of stresses along the plane of symmetry $\frac{\partial T_{\rm xx}^*(0)}{\partial x} = \frac{\partial T_{\rm yy}^*(0)}{\partial x} = \frac{\partial T_{\rm zz}^*(0)}{\partial x} = 0$ is equal to zero number.

Then the solution of the equations of the quasi-linear thermoelastic problem under the action of internal (moisture and thermal) loads has the form of stresses

$$T_{\rm xx}^* = \Phi^*(x) - \left\langle\Phi^*(x)\right\rangle \;\; T_{\rm yy}^*(x) = T_{\rm zz}^*(x) = \frac{\langle E\rangle}{1-\langle\nu\rangle}\left[M^*(x) - \left\langle M^*(x)\right\rangle\right]$$

and deformation

$$E_{\rm yy}^*(x) = E_{\rm zz}^*(x) = \frac{1}{\langle E\rangle}\left[(1-\langle\nu\rangle)T_{\rm yy}^*(x) - \langle\nu\rangle T_{\rm xx}^*(x)\right] + L(x) - L(0),$$

$$E_{\rm xx}^* = \frac{1}{\langle E\rangle}\left[T_{\rm zz}^* - 2\langle\nu\rangle T_{\rm yy}^*\right] + L(x) - L(0),$$

correspondingly, where $M^*(x) = L^*(x) - \frac{\langle v\rangle}{\langle E\rangle}\Phi^*(x)$ is an expression for the total internal forces, and $\langle M^*\rangle = \frac{1}{L}\int_{-L/2}^{L/2} M^*(x)\mathrm{dx}$ or $\langle \Phi^*\rangle = \frac{1}{L}\int_{-L/2}^{L/2} \Phi^*(x)\mathrm{dx}$ refers to the averaged effort values.

5. The main mechanical thermo-diffusion equations

In such way, we obtain a system of closed equations of heat and moisture diffusion

$$(\rho C_p)_{\text{eff}}\partial_t T(x,t) - \frac{1}{2}\langle r\rangle\partial_t[\langle c_L\rangle^L - \partial_t\langle c_v\rangle^G] = \partial_x[\lambda_{\text{eff}}\partial_x T(x,t)] + \\ + \frac{1}{2}\langle r\rangle\vec{\nabla}\cdot\ [\langle J_L^x\rangle - \langle J_v^x\rangle] + \dot{q}^t(x),$$

$$\partial_t\langle c_L\rangle^G + \partial_x\langle \vec{J}_L\rangle = \dot{I}\ \ \partial_t\langle c_v\rangle^G + \partial_x\langle \vec{J}_v\rangle = -\dot{I},$$

$$\partial_t\langle c_a\rangle^G = -\partial_x\langle J_a^x\rangle$$

into the described above modification (6) and (7) for the parts of porous plat with the corresponding boundary conditions (6), the solutions of which can be found according to the obtained analytical expression (7) for heat sources

$$\dot{q}^t(x) = 2P_{\text{in}}|\bar{n}_\omega(t)||T(t)|^2\alpha(x,t)\sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{1+\text{tg}^2\delta_\omega(x,t)}}\right)}\times$$

$$\times\left(\text{Exp}\left[-2\int_{-L/2}^{x}\alpha(x,t)\,\mathrm{dx}\right] + \text{Exp}\left[-2\int_{x}^{L/2}\alpha(x,t)\,\mathrm{dx}\right]\right).$$

Then we can determine the stress-strain state of the body with the values calculated relatively to the plane of symmetry

$$\Phi(x) = -\int F(x)\mathrm{dx} = \langle\varphi\rangle\left[P_G(x,t) - \eta_L \mathrm{P}_C(x,t)\right],$$

$$L(x) = -(1-\langle\varphi\rangle)\langle K\rangle\langle\alpha_S^T\rangle[T(x,t) - T_0]$$

according to defined above humidity and thermal internal forces loadings.

The closed system of equations is obtained by fulfilling the conditions of compatibility which takes into account the weak variability of volume (phase), dielectric (wave) properties of the three-phase porous humidified media

$$\frac{1}{\theta_\sigma(x,t)}\frac{\partial\theta_\sigma(x,t)}{\partial t} << \omega_0 \text{ and } \frac{1}{\bar{n}_\omega^{\text{eff}}(x,t)}\frac{\partial\bar{n}_\omega^{\text{eff}}(x,t)}{\partial x} << k_\omega^{\text{eff}}(x,t)$$

as well as conditions of applying the approximation of the effective macroscopic field

$$\lambda_\omega^{\text{eff}}(x,t) = \frac{2\pi v_\omega^{\text{eff}}(x,t)}{\omega_0} >> l$$

in the expressions for determining of the effective electrophysical properties of the porous material [4] according to the method of local spatial averaging.

Here $k_\omega^{\text{eff}}(x,t) = 2\pi/\lambda_\omega^{\text{eff}}(x,t)$ and $v_\omega^{\text{eff}}(x,t) = c_0/\bar{n}_\omega^{\text{eff}}(x,t)$ wave vector and phase velocity of electromagnetic (T.E.M. – Transverse Electro Magnetic [14]) wave propagation in the simulated environment, $\bar{n}_\omega^{\text{eff}}(x,t)$ is the effective value of refractive index, $\theta_\sigma(x,t)$ is the volume fraction of σ- phase, ω_0 is the angle frequency, l is the characteristic (R.E.V. – Represenatative Elementere Volume [1]) length of averaged volume.

6. Conclusions

The main goal of this part of paper lies to mathematical description of mechanical thermodiffusion process by the closed system of differential equations. The new in this work is that the proposed boundary conditions for describing of the heat and mass transport processes are

written into dynamically form and takes into account the time moving of near-surface zones of evaporation and heating during microwave drying period. The given form of heat and mass transfer equations can be applied to these boundary conditions through diffusion properties of the two component gas mixture (water vapor and dry air) and strongly depends on the rate of evaporation water at the porous plate surface due to influence microwave irradiation. Also, we are received an analytical expression for calculating of dielectric losses for microwave power into porous humidified plate which have not been used previous and applies these the first time for calculation of heat sources during microwave treatment. The relations for modeling of mechanical stress and strain were obtained according to defined humidity and thermal internal loading. In such way the interconnected diffusion, electromagnetic and mechanical phenomena are divided and represented into separated mathematical forms. Let's describe the results of numerical simulation of mechanical thermodiffusion phenomena by the demonstrated equation in the next part of the paper.

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