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UTILIZING THE STRUT-AND-TIE MODEL FOR THE DESIGN AND ANALYSIS OF DEEP BEAMS

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Abstract: Reinforced concrete deep beams, distinguished by a low shear span-to-depth ratio, exhibit complex, nonlinear strain distributions that render conventional beam theory inadequate. The Strut-and-Tie Model (STM) offers a sophisticated yet practical framework for analyzing and designing such structures by conceptualizing internal stresses through truss like systems of compressive struts and tensile ties. This paper presents a dual-method approach - a detailed manual calculation and an advanced case study using CAST software providing a step-by-step tutorial that bridges theoretical principles with practical application. The study meticulously explores critical phases, including force path determination, nodal classification, and reinforcement optimization, adhering to Eurocode 2 standards. Results demonstrate that STM achieves high accuracy in predicting both local and global stress responses, while ensuring structural safety and efficiency. The strong correlation between manual and computational outcomes underscores the method's reliability and adaptability for complex structural challenges, reinforcing its value in modern engineering design.

Keywords: Deep beam, shear strength, strut-and-tie model, CAST.

1. Introduction

Reinforced concrete deep beams are structural components frequently used in buildings, bridges, and other critical infrastructures. Their low shear span-to-depth ratio causes a highly nonlinear strain distribution, making traditional sectional design approaches inadequate. These beams are particularly vulnerable to shear-dominated failures, requiring robust design methodologies to ensure both safety and reliability.

Research has introduced various methods to improve shear strength prediction for deep beams. One prominent approach is the Strut-and-Tie Model (STM), which conceptualizes the internal stress flow as a truss system. Diagonal struts represent compressive stresses in concrete, while ties—which may include both longitudinal and web reinforcements—resist tensile forces. Nodal zones, where struts and ties converge, play a critical role in transferring forces and maintaining equilibrium. This visualization aids engineers in analyzing complex stress paths, particularly in discontinuity regions (D-regions), where standard beam theory does not apply.

Khaled Megahed's study emphasizes machine learning (ML) as a promising tool, demonstrating that models like CatBoost surpass conventional code-based models (ACI318, EC2) in accuracy and reliability. Megahed's findings also highlight the potential of probabilistic optimization, such as Bayesian parameter estimation, to reduce variability and improve predictions [1].

Similarly, Panatchai Chetchotisak and colleagues propose refined strut-and-tie models (STM) tailored for deep beams, incorporating empirical data and probabilistic methods to enhance prediction accuracy. Their research shows that integrating multiple load-bearing mechanisms—diagonal struts, horizontal ties, and vertical reinforcements—provides a comprehensive understanding of stress transfer in these structures [2].

Additionally, Wael Kassem's work on corbels underscores the need for compatibility-based STM approaches to capture complex failure mechanisms such as nodal crushing and diagonal splitting. This model offers improved agreement with experimental results compared to traditional shear-friction approaches [3].

For dapped-end beams, Bashar S. Mohammed et al. address the limitations of PCI code provisions in accurately predicting failure loads. Their study demonstrates that varying reinforcement schemes, including diagonal reinforcements, can significantly improve structural performance [4].

Building upon these foundations, this paper presents a dual-method approach to deep beam design and analysis, combining both manual calculations and computational simulations using CAST software. The manual approach provides clarity on the theoretical framework of the Strut-and-Tie Model, while the software-based analysis enhances efficiency and allows for more complex scenarios to be addressed. Together, these methods form a comprehensive workflow for structural engineers, enabling precise modeling of stress flow, effective reinforcement placement, and robust nodal zone verification.

The study adopts key findings from previous research to refine and optimize the STM application process. The inclusion of empirical validation through software-generated results strengthens the model's credibility for real-world applications. Specific attention is given to force equilibrium, stress distribution across varying thicknesses, and reinforcement anchorage, all critical aspects for ensuring structural safety. Additionally, this paper functions as a detailed tutorial, guiding engineers through each stage of the STM process—from defining struts, ties, and nodal zones to optimize design parameters in compliance with Eurocode 2 [5] standards. By integrating both theoretical and practical perspectives, the study aims to bridge the gap between academic research and field implementation, contributing to the advancement of reliable and efficient deep beam design strategies.

Building upon these foundations, this paper presents a dual-method approach that integrates both manual calculations and computational simulations using CAST software. Manual calculations offer detailed insights into STM theory, including the definition of struts, ties, and nodal zones, as well as equilibrium verification. Meanwhile, software-based simulations streamline the analysis of more complex scenarios and provide empirical validation of the model's performance.

A central contribution of this research is the structured tutorial provided for implementing the STM in practice. Key stages of the design process—such as optimizing reinforcement placement, ensuring stress equilibrium across beam thickness, and verifying compliance with Eurocode 2 standards—are comprehensively addressed. This integration of theoretical and empirical approaches enhances the credibility and applicability of the STM.

Despite its inherent assumptions, such as the uniaxial force behavior in struts and ties and the neglect of concrete's tensile strength, the STM remains a reliable tool for the design of D-regions. As these regions often experience significant shear strains and variations in normal stress, the model's capacity to balance simplicity with predictive precision is invaluable. Advances in probabilistic modeling and experimental research continue to refine the STM, ensuring that it remains a cornerstone of efficient and safe deep beam design.

2. The Strut-and-Tie Approach

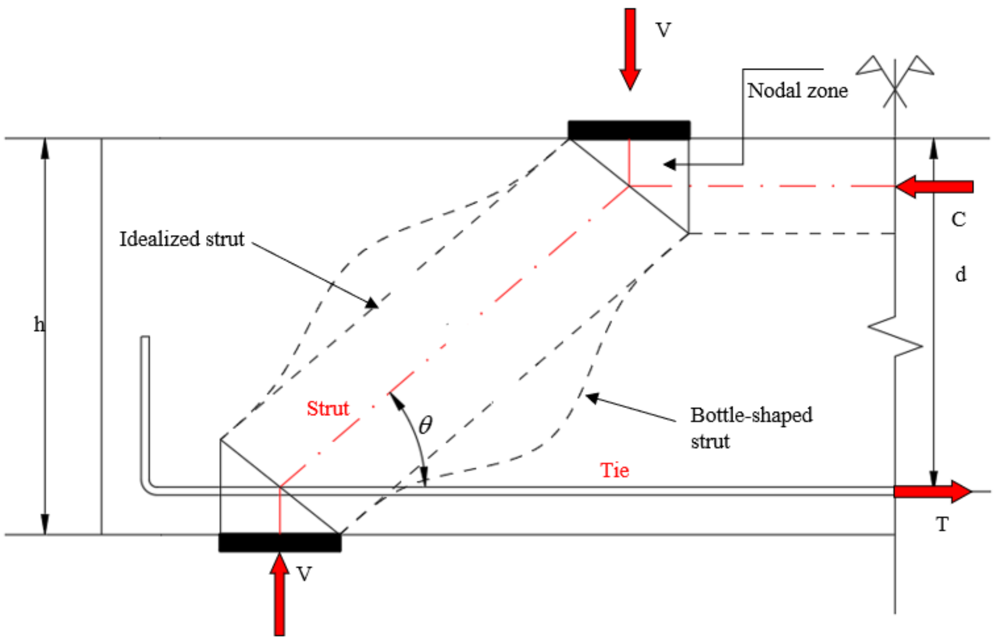


Fig. 1 - Elements of a Strut-and-Tie Model for a Deep Beam

The Fig. 1 illustrates the primary components of the Strut-and-Tie Model (STM) for a deep beam. It consists of the following key elements:

1. Strut: Represented as an idealized and bottle-shaped diagonal compression element that transfers compressive forces within the concrete. Struts carry compressive stress along their length and form a vital part of the load transfer mechanism.
2. Tie: A tension element located at the bottom of the beam, typically reinforced with steel, which resists tensile forces generated by the load.
3. Nodal Zone: The area where struts and ties intersect, playing a crucial role in transferring and balancing internal forces. Proper detailing and stress distribution in this zone are essential to maintain structural integrity.
4. Load Application (V): The vertical load is applied at the top of the beam, initiating the internal force flow.
5. Support Reactions (T and C): At the beam's ends, tension and compression reactions are depicted. The left side shows compression (C), while the right side indicates tensile reaction (T).
6. Angle (θ): The angle between the strut and the horizontal axis, indicating the strut's orientation in the beam's stress distribution.

This diagram demonstrates how STM conceptualizes force paths through the beam, aiding in the design and analysis of deep beams by providing a clear visualization of internal stress flow.

Creating and Refining a Strut-and-Tie Model (STM)

The first step in developing a Strut-and-Tie Model (STM) is to outline the paths where stresses flow through the structure. These paths represent how forces move within the system, ensuring they don’t cross or interfere with each other. Once these paths are mapped, they are depicted as force representations in the STM framework. Additional struts and ties may be added to maintain force equilibrium as required by the structure. Struts should align with the principal directions of compressive forces, while ties and reinforcement are arranged either parallel or perpendicular to the structural boundaries. Proper placement of tie centerlines should account for adequate concrete cover and allow for multiple layers of reinforcement when needed.

After constructing the basic model, the next step involves calculating the forces within the struts and ties. For straightforward designs, this can often be done with basic trigonometry. Initial dimensions for struts and ties are estimated through preliminary analysis based on minimum allowable stress criteria. To ensure greater accuracy, the model may require iterative refinements as additional structural details and load conditions are considered. These iterations help to enhance the model’s precision and overall reliability.

Challenges and Optimizations in STM Design

Selecting the optimal STM configuration for a specific loading scenario can be challenging, as there are often several valid approaches to model the structure. Elastic finite element analysis often serves as an effective starting point, providing insights into the preliminary STM layout by predicting stress flow patterns. However, this method doesn’t fully account for changes that occur when the concrete cracks and the ties begin to carry tensile forces. Once the structure stabilizes, the STM orientation typically remains steady until the reinforcement starts to yield. At this point, further adjustments may be needed as the load approaches its maximum.

STM configurations derived from elastic stress fields are often suitable but not always fully effective, as they may overlook the stress redistribution that occurs upon cracking. The most accurate model is the one requiring minimal strain energy, which is achieved by reducing strain energy as much as possible.[6]

$$\text{Strain energy} = \sum F_i l_i \varepsilon_{mi} \tag{1}$$

where

F_i is the force in the i^{th} strut or tie,

l_i is the length of i^{th} member,

ε_{mi} is the mean strain in the i^{th} member.

In simpler terms, the most efficient model typically features the shortest length of ties that remain unyielded.

The angle between struts and ties should be sufficiently large to prevent strain incompatibility, ensuring that ties stretching and struts compressing do not occur in nearly the same direction. This angle should not be less than 35°.[6]

The Strut-and-Tie Method relies on the lower-bound theorem of plasticity and is only valid if the structure possesses sufficient ductility to allow the truss mechanism to fully develop. According to Eurocode 2 [5], this ductility requirement can typically be satisfied by using Class B or Class C reinforcement

2.1. Strength of struts

According to Eurocode 2 [5], the design shear strength of a strut without longitudinal reinforcement, $V_{Rd,c}$ should be determined based on the effective compressive strength of the concrete [5]. This calculation accounts for factors such as the characteristic compressive strength of the concrete, the geometry and orientation of the strut, and any applicable reduction factors to account for imperfections or non-uniform stress distributions.

$$V_{Rdc} = f^{s}_{cd,eff} A_{strut} \tag{2}$$

A_{strut} : Effective cross-sectional area of the strut.

a) Where strength for a concrete strut in a region with transverse compressive stress or no transverse stress (Fig. 2)may be calculated as:

$$f^{s}_{cd,eff} = f_{cd} \tag{3}$$

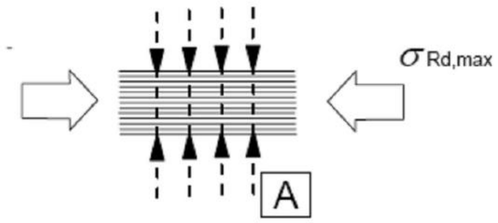


Fig. 2 - Design strength of concrete struts without transverse tension [5]

b) The design strength for concrete struts should be reduced in cracked compression zones (Fig. 3) and, unless a more rigorous approach is used, may be calculated as:

$$f^{s}_{cd,eff} = 0.6\nu f_{cd} \tag{4}$$

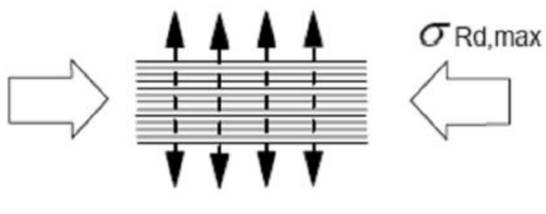


Fig. 3 - Design strength of concrete struts with transverse tension [5]

Where

ν: Reduction factor accounting for the strut's geometry, confinement, and imperfections, and it can be calculated as:

$$\nu = \left[1 - \frac{f_{ck}}{250}\right] \quad (\ f_{ck}\ \text{ in } MPa\ ) \tag{5}$$

f_{ck}: Characteristic compressive strength of the concrete.

γ_c : Partial safety factor for concrete (usually 1.5 unless otherwise specified).

The value of ν depends on the conditions of the strut:

- For a well-confined prismatic strut (uniform compression):
  - $\nu = 1.00$ (no reduction).
- For a bottle-shaped strut (stress dispersion with transverse tensile forces):
  - $\nu \approx 0.6 \div 0.8$ depending on the confinement provided by transverse reinforcement.
- For an unconfined or poorly confined strut:
  - $\nu \leq 0.6$ to account for significant imperfections or splitting stresses.

2.2. Nodal zones

Smeared nodes do not require verification if the reinforcement is adequately anchored to extend across the entire stress field. However, it is essential to check the concrete stresses in singular nodes due to the high stress concentrations present in these regions. Singular nodes are categorized into four main types based on the stress distribution: C-C-C, C-C-T, C-T-T, and T-T-T nodes (Fig. 4), where "C" represents compressive forces and "T" represents tensile forces.

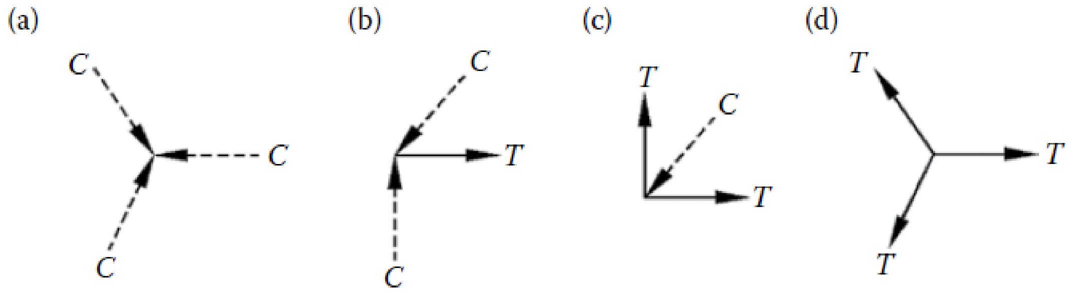


Fig. 4 - Nodes classification: (a) C − C − C node, (b) C − C − T node, (c) C − T − T node, and (d) T − T − T node.

The effective design compressive strength of the concrete in a nodal zone $f_{cd,eff}$, is determined by applying reduction factors to account for stress concentration, non-uniform stress distribution, and the geometry of the nodal zone. The effective compressive strength [5] is given as:

$$f_{cd,eff} = k_n \nu \frac{f_{ck}}{\gamma_c} \tag{6}$$

where k_n is reduction factor depending on the type of nodal zone and the forces acting on it. Eurocode 2 [5] defines nodal zones based on the combination of compressive and tensile forces at the node:

- **C-C-C Node**: A node where all forces are compressive.
  - $k_1 = 1.00$ (no reduction needed, as stress is uniformly compressive).

- **C-C-T Node**: A node with two compressive forces and one tensile force.
  - $k_2 = 0.85$ (reduction for moderate stress concentration).
- **C-T-T Node**: A node with one compressive force and two tensile forces.
  - $k_3 = 0.75$ (greater reduction due to higher stress concentration and risk of cracking).
- **T-T-T Node**: A node where all forces are tensile.
  - Tensile stresses are carried entirely by reinforcement, as concrete tensile strength is not considered.

2.3. Reinforced ties

The nominal capacity of a tie, $F_{tie,d}$, should be considered as $F_{tie,d} = A_s \cdot f_{yd}$; where A_s is the area of steel and f_{yd} denotes the design yield strength of the reinforcing steel and is determined as f_{yk} / γ_s (f_{yk}: Characteristic yield strength of the reinforcement (e.g., 500 MPa for standard steel, γ_s: Partial safety factor for reinforcement, typically 1.15 unless otherwise specified).

The width of a tie (b_{tie}) must be determined to ensure the safety of the connected node, while adhering to a defined upper limit ($b_{tie} \leq b_{node}$, where b_{tie} is the effective width of the tie and b_{node} is width of the nodal zone).

To ensure node safety, the effective width of the tie must satisfy:

$$\sigma_c \leq f_{cd,eff} \tag{7}$$

where σ_c represents compressive stress transferred to the node by tie.

The tie width influences σ_c by determining the contact area between the tie and the nodal zone. A wider tie reduces stress concentration, ensuring that σ_c remains within permissible limits.

2.3.1. Practical determination of tie width

The tie with is determined as the smaller of:

- The actual physical width required for reinforcement placement, including minimum concrete cover and bar spacing:

  $$b_{tie} = 2 \cdot c + (n-1) \cdot s + n \cdot d_b \tag{8}$$

  Where *c* is concrete cover, *n* is the number of reinforcement bars, *s* is the clear bar spacing and d_b is the bar diameter.
- The width required to satisfy nodal stress limits:

  $$b_{tie} = \frac{F_{tie,d}}{f_{cd,eff} \cdot h_{tie}} \tag{9}$$

  Where h_{tie} is the height of the tie.
- The width of the nodal zone (b_{node}): The tie width cannot exceed the physical boundaries of the node.

3. Design example of deep beam

The beam, measuring 6000 mm x 1600 mm x 500 mm as shown in Fig. 5, is supported by 700 mm x 700 mm columns spaced at 5300 mm centers. It carries a 500 mm x 500 mm bearing plate with applied actions $G_k = 1500kN$ and $Q_k = 500kN$, positioned 850 mm from one of the supports. Determine the required reinforcement, assuming C35/45 concrete and $f_{yk} = 500MPa$. $c_{nom} = 25mm$.

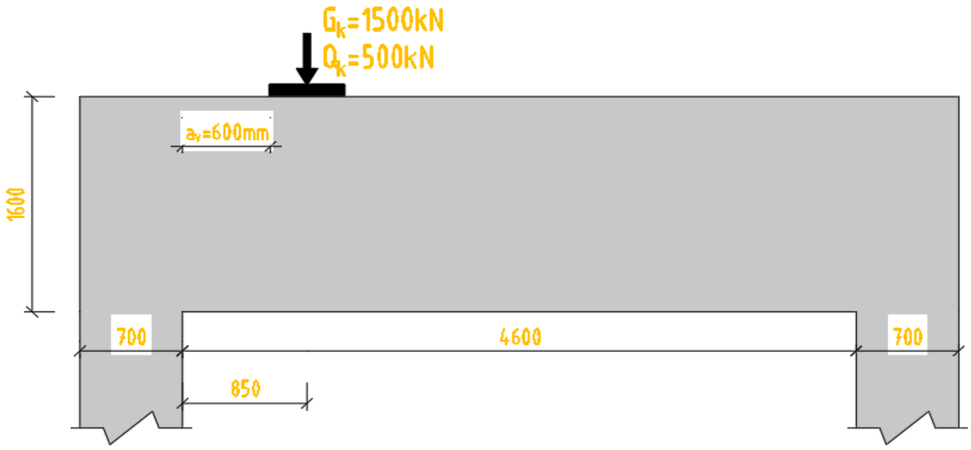


Fig. 5 - Deep beam

a) Define D-regions

Upon inspection, the entire deep beam is identified as consisting mostly of D-regions.

b) Proposed STM

ULS load, $F = 1500 \cdot 1.35 + 500 \cdot 1.5 + 6.0 \cdot 1.6 \cdot 0.45 \cdot 25 \cdot 1.35 = 2921kN$ (self-weight assumed to act at node 2)

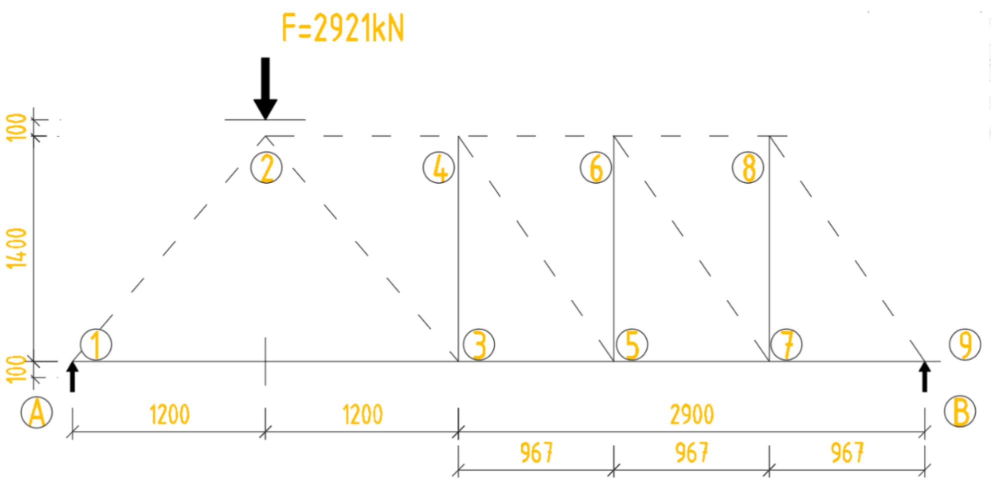


Fig. 6 - Proposed STM

Fig. 6 presents a potential Strut-and-Tie Model (STM) for the deep beam, along with its dimensions. The design provides a 100 mm clearance at both the top and bottom to the centerlines of compression strut C24 and tie T13. The depth of tie T13 is calculated as approximately 0.12h [6], which equals 192 mm, and is rounded up to 200 mm for simplicity.

Forces:

Assess the moment with reference to point B.

$$V_A = 2921 \cdot 4.1 / 5.3 = 2259.64kN$$

$$V_B = 2921 - 2259.64 = 661.36kN$$

$$N_{12} = -2976.12kN \;\; \textbf{strut}$$

$$N_{13} = 1936.84kN \;\; \textbf{tie}$$

$$N_{34} = N_{56} = N_{78} = 661.36kN \;\; \textbf{ties}$$

c) Verify the bearing stresses

At node 2, under load F

$$\sigma_{Ed} = \frac{2921 \cdot 10^3}{500 \cdot 500} = 11.68MPa$$

$$\text{CCC node: } \sigma_{Rd} = 1 \cdot \left(1 - \frac{35}{250}\right) \cdot 0.85 \cdot \frac{35}{1.5} = 17.06MPa \quad (10)$$

$$\sigma_{Ed} < \sigma_{Rd}$$

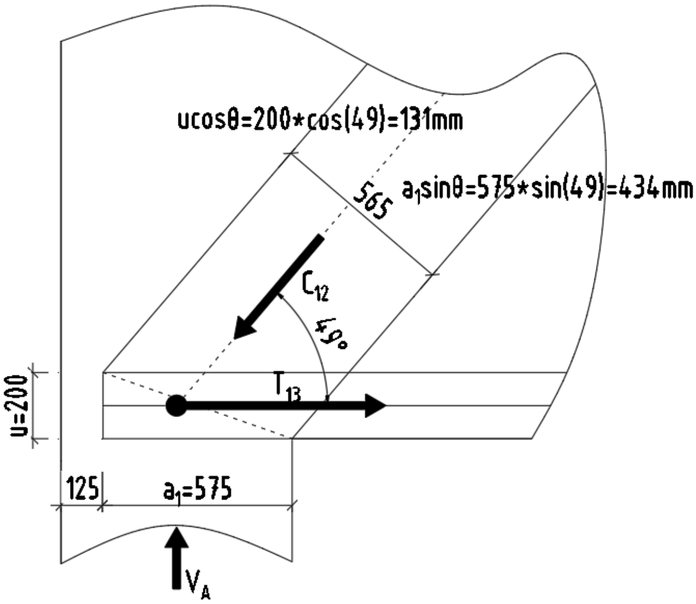
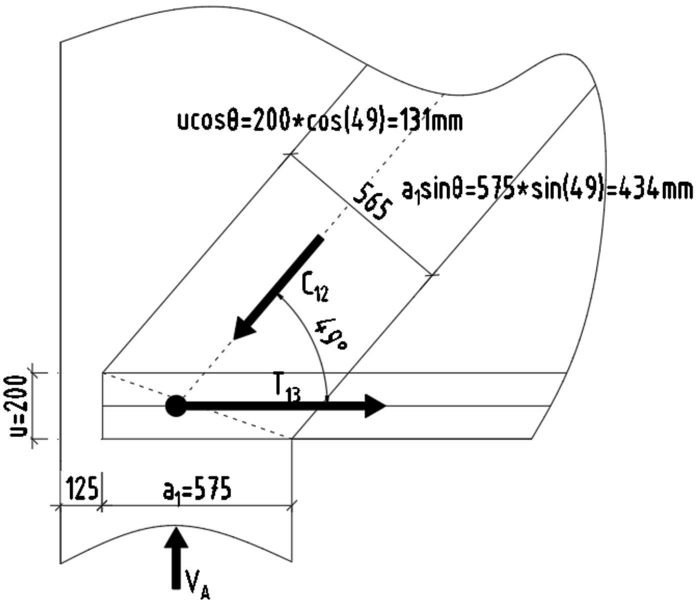


Fig. 7 - Geometry at support A

At node 1 located at support A (see Fig. 7)



$$a_1 = 700 - c_{nom} - 2s_0$$
$$s_0 = 50mm$$
$$a_1 = 700 - 25 - 2 \cdot 50 = 575mm$$
$$\sigma_{Ed} = \frac{2259.64 \cdot 10^3}{575 \cdot 450} = 8.73MPa$$

CCT node: $\sigma_{Rd} = 0.85 \cdot \left(1 - \frac{35}{250}\right) \cdot 0.85 \cdot \frac{35}{1.5} = 14.5MPa$ (11)

$$\sigma_{Ed} < \sigma_{Rd}$$

At node 9 located at support B

$$a_9 = 575mm$$
$$\sigma_{Ed} = \frac{661.36 \cdot 10^3}{575 \cdot 450} = 2.56MPa$$

CCT node: $\sigma_{Rd} = 0.85 \cdot \left(1 - \frac{35}{250}\right) \cdot 0.85 \cdot \frac{35}{1.5} = 14.5MPa$

$$\sigma_{Ed} < \sigma_{Rd}$$

d) Ties

$$T_{13} \rightarrow N_{13} = 1936.84kN$$
$$A_{s,req'd} = \frac{N_{13}}{f_{yd}} = \frac{1936.84 \cdot 10^3}{\frac{500}{1.15}} = 4459mm^2$$

Try $8Ø28$ ($A_{s,eff} = 4926mm^2$) in two layers i.e. $2 \times 4Ø28$ with $50mm$ concrete cover.

Check anchorage

For $Ø28$, the required anchorage is calculated assuming a straight bar in "good" conditions within C35/45 concrete. $l_{bd} = 920mm$.

Average anchorage length available beyond the face of the compression strut

= bearing + extended node - cover - u-bar diameter

$= 700 + 200/2 - 25 - 16 = 759mm$ =>not good

$$l_{bd} = 920 \cdot \frac{4455}{4926} = 823mm$$

$l_{bd} > 759mm$ =>not good

Upon inspection, ensure bobs are provided at the ends of the reinforcement bars.

$$T_{34} \rightarrow N_{34} = 661.36kN$$

$A_{s,req'd} = \frac{N_{34}}{f_{yd}} = \frac{661.36 \cdot 10^3}{\frac{500}{1.15}} = 1521mm^2$ per tie

Try two-legged stirrup $\varnothing 16/250mm$ with $A_s = 1609mm^2$.

e) Struts

Verify the strength of the direct strut in the left-hand shear span.

Check the stress in strut 1-2 at the bottom node. Transverse reinforcement is required if the design stress in the inclined strut at the bottom node exceeds its design strength under transverse tension, i.e., if $\sigma_{Ed} \geq \sigma_{Rd}$

The strut's maximum width is determined by:

$$a_{12} = a_1 \sin(\theta) + u\cos(\theta) \quad (12)$$

$$a_{12} = 575\sin(49) + 200\cos(49) = 565mm$$

$$\sigma_{Ed} = \frac{2976.12 \cdot 10^3}{565 \cdot 500} = 10.53MPa$$

$$\sigma_{Rd} = 0.6 \cdot \left(1 - \frac{35}{250}\right) \cdot 0.85 \cdot \frac{35}{1.5} = 10.23MPa \quad (13)$$

$$\sigma_{Ed} > \sigma_{Rd}$$

Therefore, the calculated shear or transverse anti-bursting reinforcement is required.

Bursting forces (bottle-shaped ties)

In this scenario, the design strength of the strut at the bottom node can be increased to match the design strength of a CCT node ($\sigma_{Rd,bot} = 0.85 \cdot (1 - f_{ck}/250) f_{cd}$) by incorporating transverse reinforcement, as outlined in expression (14), where applicable.

Check strut 1-2

$$N_{12} = -2976.12kN$$

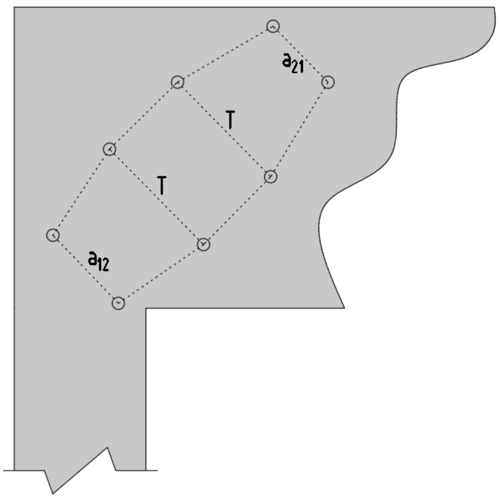


Fig. 8 - Bursting forces, T, in strut 1-2

$$T = 1/4 \cdot (1 - 0.7 \cdot a / H) \cdot F \quad (14)$$

where

T = bursting force in the strut.

a = width of strut at end (a_{21} or a_{12}).

F =Compression (strut) force. The main axial force carried by the strut between nodes in the Strut-and-Tie Model.

H = strut length.

To maximize T (by minimizing a/H), consider the minimum value of a, specifically a_{21} at node 2, which is smaller than a_{12} at node 1. This is because k_1 for the CCC node at node 2 is significantly greater than k_1 for the CCT node at node 1.

$$a_{21} = \frac{F}{t \cdot \sigma_{Rd,\max}}$$

where

$$F = N_{12} = 2976.12kN$$

$$t = 500mm$$

$$\sigma_{Rd,\max} = k_1 \cdot \nu \cdot f_{cd} \tag{15}$$

$$\sigma_{Rd,\max} = 1 \cdot \left(1 - \frac{35}{250}\right) \cdot 0.85 \cdot \frac{35}{1.5} = 17.1MPa \tag{16}$$

$$a_{21} = \frac{2976.12 \cdot 10^3}{500 \cdot 17.1} = 348.1mm$$

$$H = 1844mm$$

$$a/H = 348.1/1844 = 0.19$$

$$T = 1/4 \cdot (1 - 0.7 \cdot 0.19) \cdot 2976.12 = 645.07kN$$

$$A_{s,reqd} = \frac{T}{f_{yd}} = \frac{645.07 \cdot 10^3}{\frac{500}{1.15}} = 1484mm^2$$

Positioned between 0.2H and 0.5H from the loaded surface.

i.e. $1484mm^2$ to be placed over $0.3 \cdot 1846 = 554mm \equiv 2679mm^2/m$ horizontally and vertically.

Try $\O 16/150mm$ with $A_s = 1340.5mm^2$ both ways both sides. ($2681mm^2/m$ both ways)

Bursting or "transverse" reinforcement is typically provided in at least two orthogonal directions around the strut region. Even though the primary tension may appear in one main orientation, local stress redistributions near the node can introduce multi-directional tensile forces. Placing reinforcement both horizontally and vertically (over the specified depth or "smeared" region) is therefore a common, conservative way to ensure that all the transverse (bursting) forces are properly resisted.

Struts in the right-hand shear span

Check the stress in strut 8-9 at the bottom node.

The strut's maximum width is determined by:

$$N_{89} = -803.7kN$$

$$a_{89} = a_9 \sin(\theta) + u \cos(\theta)$$

$$a_{89} = 575 \sin(55) + 200 \cos(55) = 586mm \tag{17}$$

$$\sigma_{Ed} = \frac{803.7 \cdot 10^3}{586 \cdot 500} = 2.74MPa$$

$$\sigma_{Rd} = 0.6 \cdot \left(1 - \frac{35}{250}\right) \cdot 0.85 \cdot \frac{35}{1.5} = 10.23MPa \quad (18)$$

$$\sigma_{Ed} < \sigma_{Rd}$$

In this analysis, the stress in strut 8-9 at the bottom node has been evaluated. The strut's maximum width is determined using geometry and force equilibrium. Based on the calculations, the design stress in the strut is significantly lower than the allowable design strength of the strut. Due to the relatively lower load demand in this span, no additional verification or transverse anti-bursting reinforcement is required. This confirms that the struts in the right-hand span experience significantly reduced loading compared to those on the left-hand span.

f) Check shear

As per PD 6687 [7], shear should be verified when $a_v > 1.5d$ (a_v the distance between the load and the support).

For LHS (left hand section) $a_v = 850 - 500/2 = 600mm$ (see Fig.5)

For RHS (right hand section) $a_v = 4600 - 850 - 500/2 = 3500mm$

$d =$ effective depth $= 1500mm$

For LHS, $a_v < 1.5d$, so no shear design is required.

For RHS, $a_v > 1.5d$, so shear design is required:

Shear design for RHS

$$\beta = \frac{a_v}{2d} = \frac{3500}{2 \cdot 1500} = 1.17 \quad (19)$$

$$\beta V_{Ed} = 1.17 \cdot 661.36 = 773kN$$

$$A_{sw} \geq V_{Ed} / f_{ywd} \cdot \sin(\alpha) \quad (20)$$

$A_{sw} = \frac{773 \cdot 10^3}{(500/1.15)} \cdot \sin(55°) = 1456.4mm^2$ to be provided in the middle $0.75a_v$

$$A_{sw} = 1456.4/(0.75 \cdot 3.50) = 555mm^2/m$$

Try two-legged stirrup $Ø12/250mm$ with $A_s = 905mm^2/m$.

g) Minimum Reinforcement

In deep beams, the minimum area of horizontal and vertical reinforcement that needs to be provided in each face is $0.002A_c mm^2/m$ which equals $1600mm^2/m$ in each face [5].

Provide min $Ø16/125mm$ each face with $A_s = 1609mm^2$.

h) Summary of reinforcement requirements

Tie: $8Ø28$.

Horizontal reinforcement: $Ø16/125mm$ (minimum).

Vertical reinforcement: $Ø16/125mm$ (minimum).

Blue is for rebar in Fig. 9.

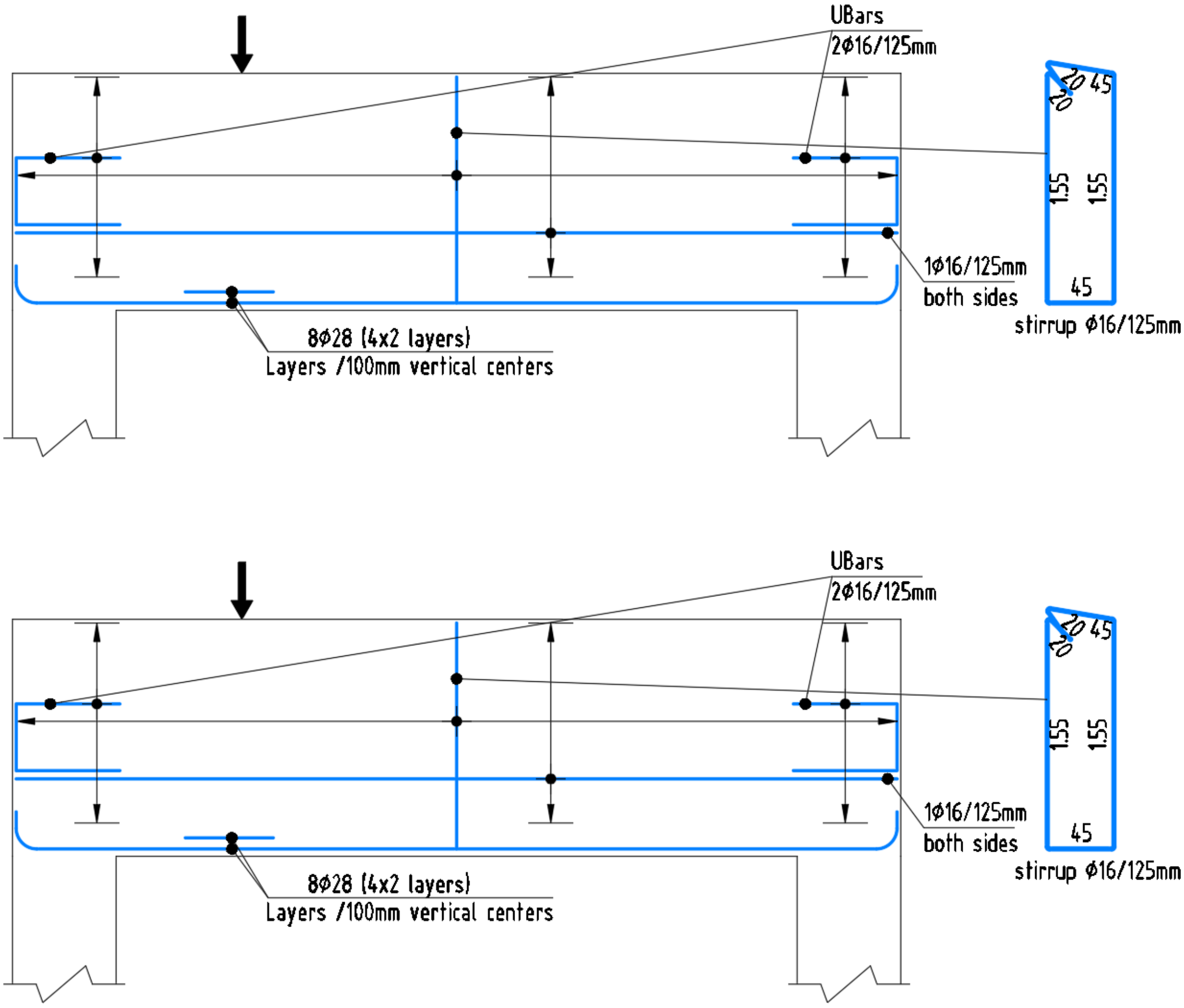


Fig. 9 - Summary of reinforcement for deep beam

This deep beam is reinforced with eight Ø28 mm main (longitudinal) bars arranged in 2 layers of 4 bars each, providing the primary tension/compression capacity along the beam’s span. Around these main bars, Ø16 mm stirrups at 125 mm centers and Ø16 mm U-bars at 125 mm centers are placed to provide shear resistance and concrete confinement.

The Ø16 mm reinforcement meets or exceeds the standard code requirements for minimum shear and bursting reinforcement, ensuring reliable crack control and overall structural integrity in the deep beam.

4. Example of Strut-and-Tie Modeling Using Specialized Software

This chapter illustrates the application of the strut-and-tie method to solve a deep beam problem. While manual calculations can be used (see in chapter 3), the example employs CAST [8] to demonstrate the capabilities of computer-aided design tools for strut-and-tie modeling.

In this example, CAST [8] is utilized to construct the truss model, analyze it, and determine the stresses in struts, ties, and nodal zones. The program requires the user to input key parameters, including material properties, dimensions of struts and ties, the number of rebars in ties, and node classifications [9]. It then automatically calculates nodal zone dimensions based on the hydrostatic concept or predefined nodal zone specifications[10].

To model the problem effectively, the following steps have been taken:

Step 1: Define the outer boundary.

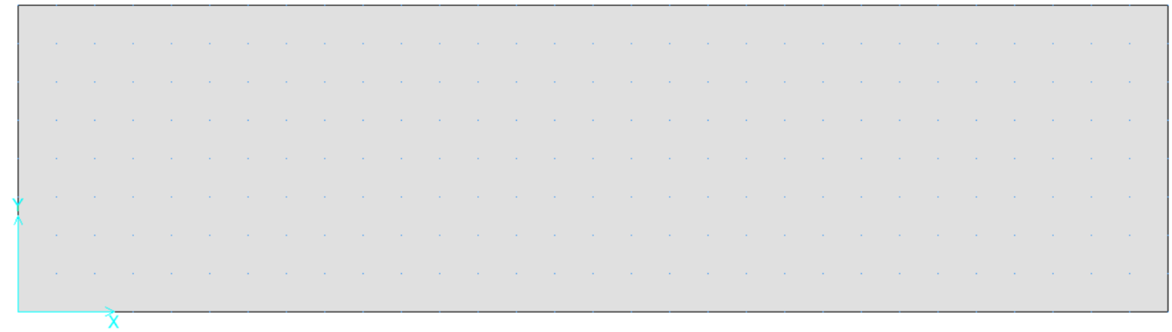


Fig. 10 - Outer boundary

Step 2: Construct strut-and-tie model

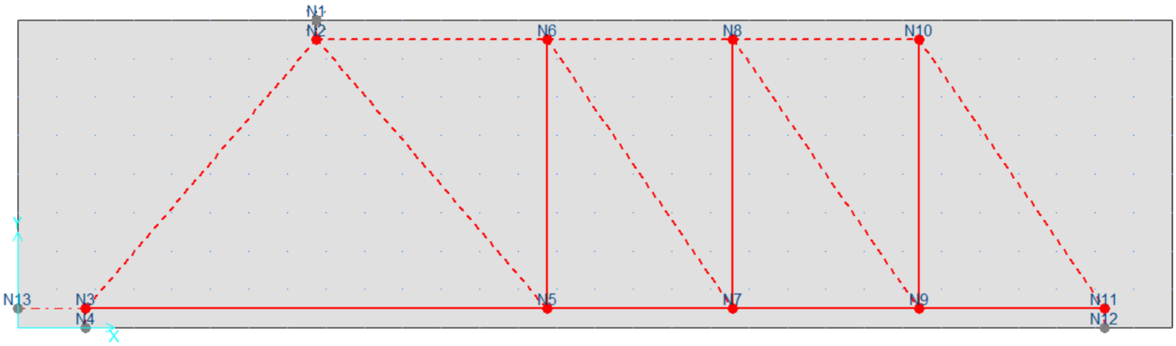


Fig. 11 - Node and elements

In this strut-and-tie (STM) model, the deep beam is subdivided into nodes (N1, N2, N3, etc.) and elements (E12, E13, E14, etc.) that represent struts or ties. The red lines in the figure show the assumed load paths: diagonal elements for inclined struts/ties and vertical or horizontal ones for reinforcement or boundary conditions.

Step 3: Apply loads and boundary supports

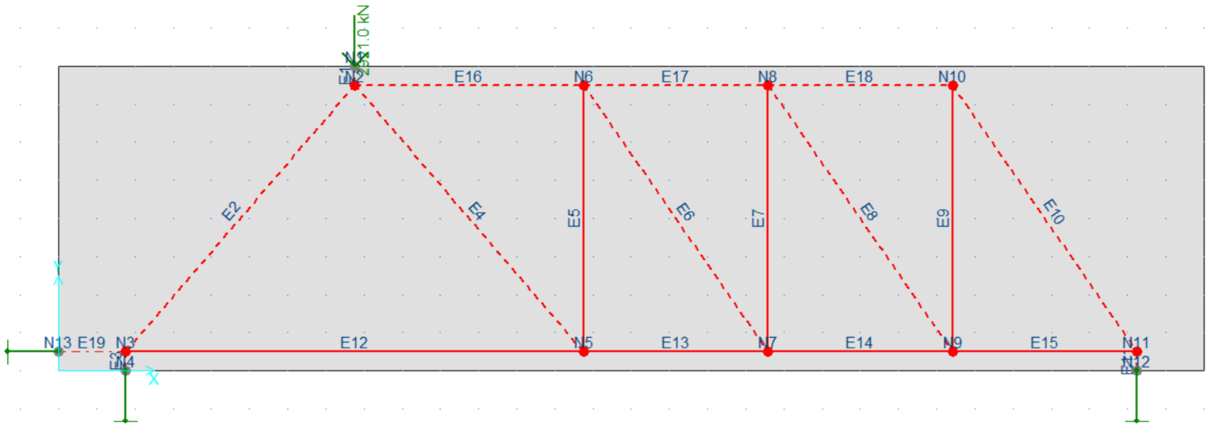


Fig. 12 - Model with loads and supports

To ensure the model stability, it is necessary to introduce supports at node N13, N4 and N12.

The vertical load is applied on the top side N1.

Because the beam is modeled in CAST, the structure can become kinematically unstable if there is no restraint against horizontal movements (i.e., it can "slide" left or right as a rigid body). Therefore, a horizontal restraint is introduced at node N13 to prevent this free lateral motion, ensuring that the software can find a unique, stable equilibrium solution for the internal forces in the struts and ties.

Step 4: As illustrated in Fig. 13 Fig. 13 - Defining prismatic concrete struts, designate all struts as prismatic struts, except for strut E2, which should be classified as a bottle-shaped strut. Prismatic struts and bottle-shaped struts are defined using their respective properties, such as concrete compressive strength, efficiency factors, and strength reduction factors. For strut E2, specific parameters for bottle-shaped struts are applied, as detailed in the previous chapter (equation (13)) and shown in the configuration settings in the figure.

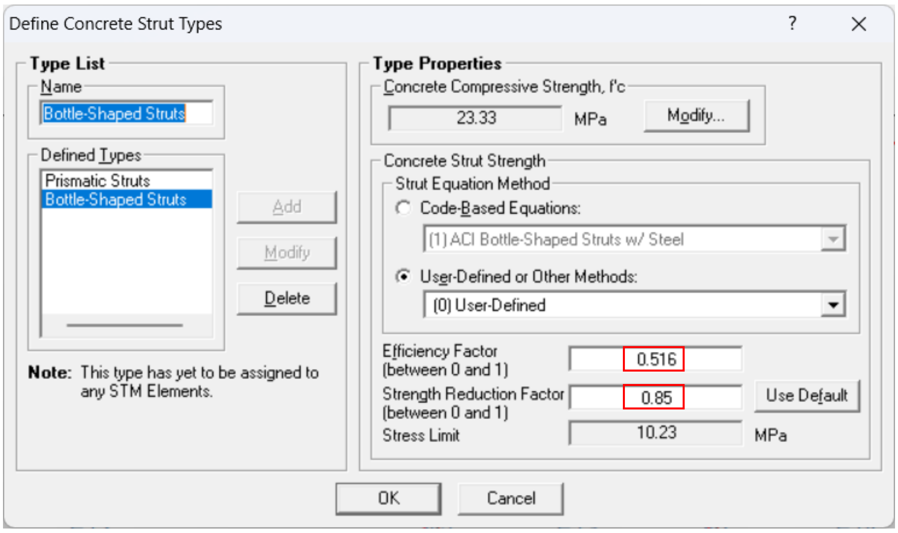


Fig. 13 - Defining bottle-shaped concrete struts

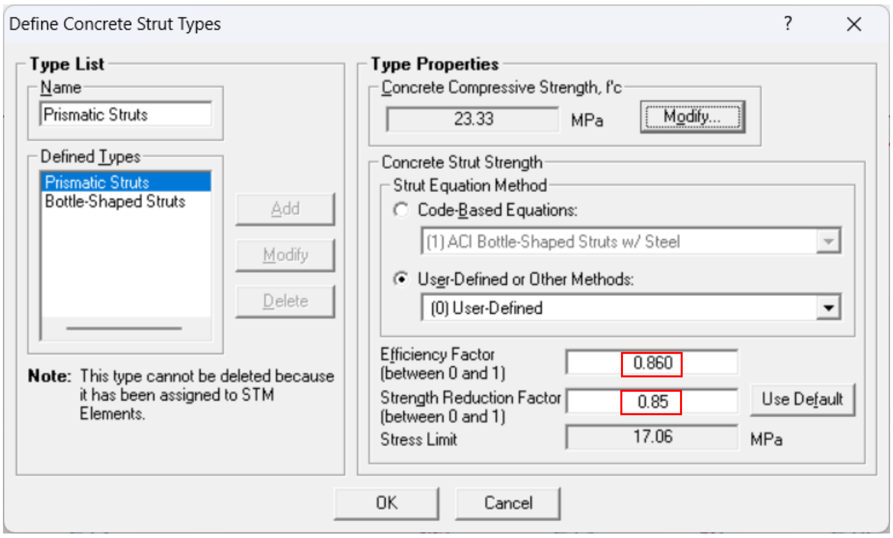


Fig. 13 - Defining prismatic concrete struts

Tie E12, E13, E14 and E15 are defined as non-prestressed rebar (8Ø28) as shown in Fig.14.

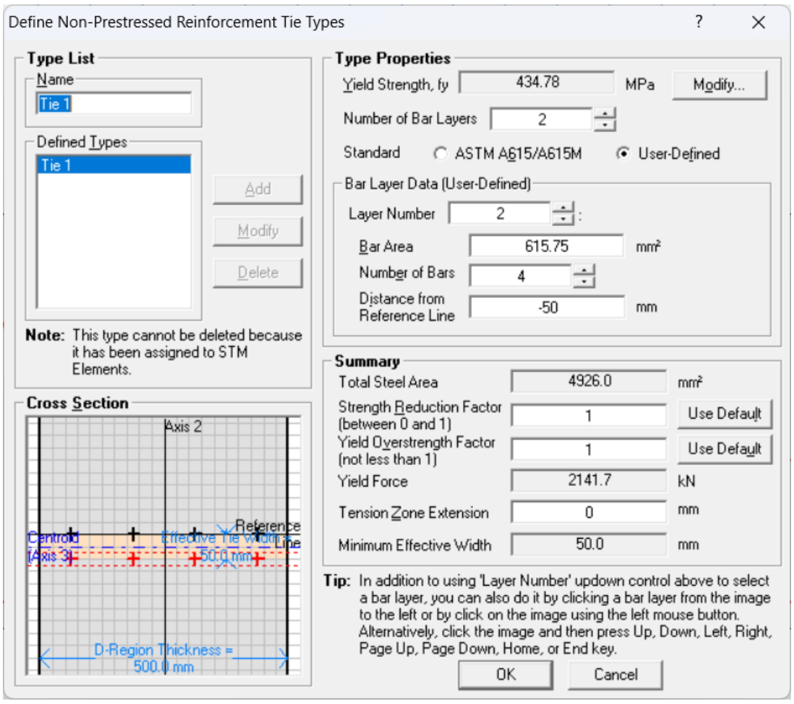


Fig. 14 - Properties of ties

Step 5: Assign width of struts and ties

In this example, width of ties E2, E4 and E10 are 565 mm, and the width of the remaining struts is 300 mm.

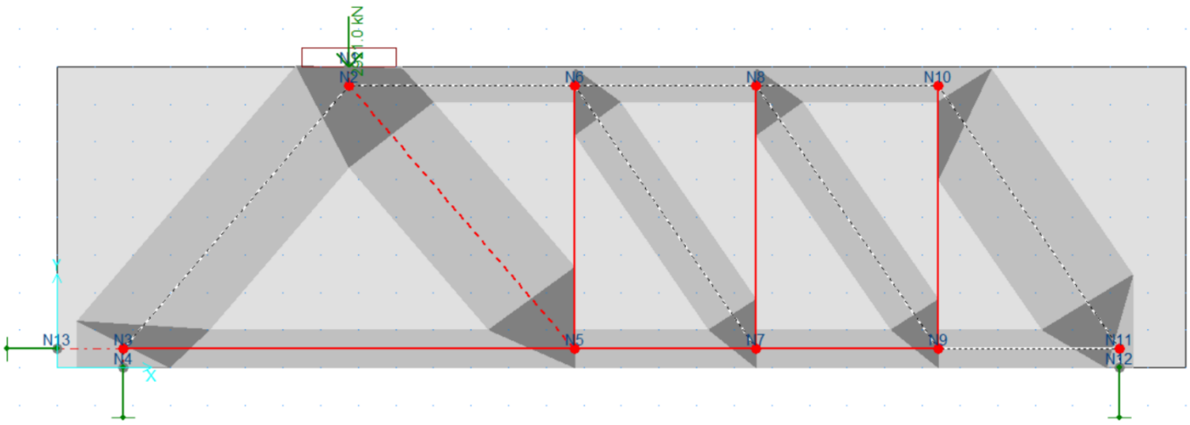


Fig. 15 - Strut-and-tie dimension

Step 6: Assign node types

The user is required to manually assign a node type to each node in the model. In this example, assign a CCC node to N2 and CCT nodes to N3, N5, N7, N9, and N11. It is important to note that only the nodes intended for verification by the program need to be specified. The program will calculate the maximum stress based on the assigned node types.

Step 7: Run analysis and check results

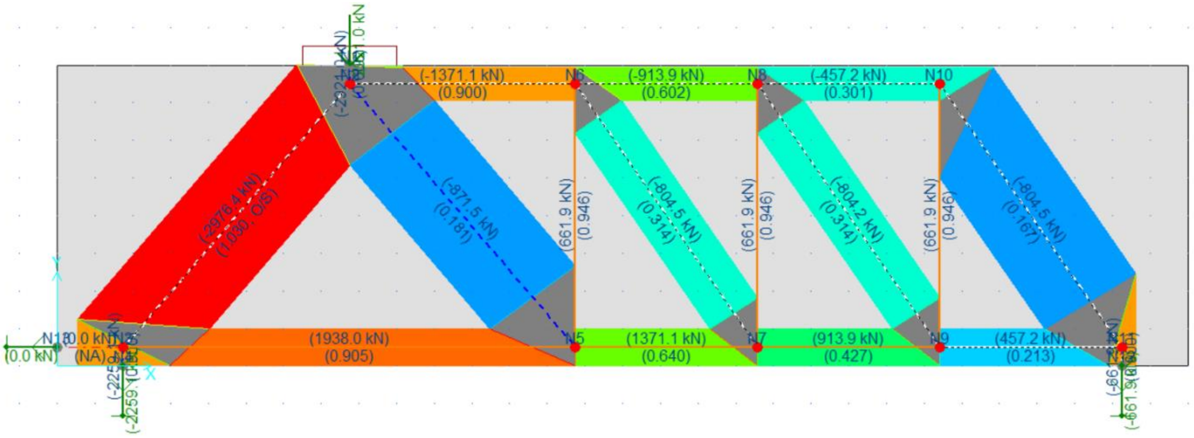


Fig. 16 - Analysis results for forces in truss model and utilized stress ratio

In this analysis output from the strut-and-tie (STM) or truss model, each colored element represents a strut or tie, with numbers indicating the internal force (kN) and a dimensionless "utilized stress ratio." The red diagonal on the left, for example, is a heavily stressed strut transferring large compressive loads.

One of the elements, E19, appears here as a "stabilizing" or "dummy" truss member and shows no internal forces. In CAST software, these stabilizers are introduced to prevent kinematic instability, as explained in step 3. If E19 is removed, the software detects an unstable mechanism and issues an error, preventing the analysis from running. Thus, even though E19 carries no load, its presence is structurally necessary within the model to achieve a stable solution.

Following the analysis, the computed results closely aligned with the manual calculations, confirming both the precision of the computational approach and the model's suitability for representing structural behavior. The similarity in outcomes underscores the method's efficiency and accuracy in replicating traditional manual analysis. Additionally, this agreement bolsters confidence in the chosen input parameters and modeling assumptions, affirming that both methods are reliable for practical design applications.

Step 8: Check nodal zone stress

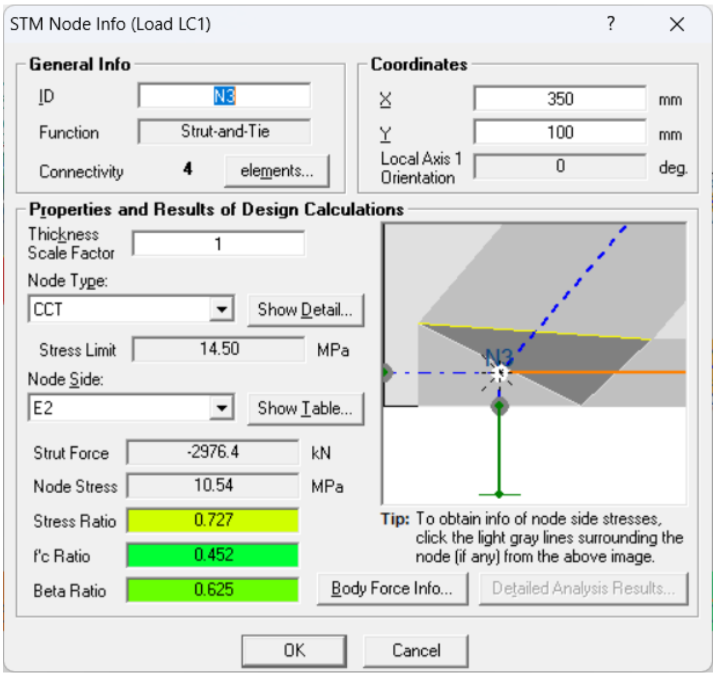


Fig. 17 - Nodal zone stress check

At this stage, only nodes with a pre-defined node type will display the results of the nodal zone stress check. By right-clicking on a node, a window like Fig. 17 will appear. It is important to note that the program determines the nodal zone dimensions based on the specified strut and tie dimensions. The nodal stress is then calculated by dividing the computed forces in the corresponding struts and ties intersecting at the node by their cross-sectional area [11].

Table 1

Bearing stress comparison between manual calculations and CAST results

Node	Type	N [kN]	σ_{Ed} Manual [MPa]	σ_{Ed} CAST [MPa]	σ_{Rd} [MPa]	Stress Ratio	Error
2	CCC	-2921	11.68	11.68	17.06	68%	0%
1	CCT	2259	8.73	8.73	14.5	60%	0%
9	CCT	661.36	2.56	2.56	14.5	18%	0%

The manual and CAST calculations agree perfectly (0% error) across all three nodes, indicating the models used for bearing stress are consistent. In each case, the calculated stress remains below the design capacity (i.e., the Stress Ratio is under 70%), suggesting an acceptable safety margin for the bearing stress at all nodes. Node 2 (CCC) exhibits the highest stress ratio at 68%, followed by Node 1 (CCT) at 60%, and Node 9 (CCT) at 18%.

Table 2

Ties stress comparison between manual calculations and CAST results

Ties	N [kN]	σ_{Ed} Manual [MPa]	σ_{Ed} CAST [MPa]	σ_{Rd} [MPa]	Stress Ratio	Error
T_{13}	1936	393.02	393.42	435	90%	0.10%
T_{35}	1369.7	278.06	278.34	435	64%	0.10%
T_{57}	913.3	185.40	185.53	435	43%	0.07%
T_{34}	661.36	411.04	411.51	435	95%	0.11%

All ties remain within their design capacity ($\sigma_{Rd} = 435MPa$), though T_{34} and T_{13} are each utilized at a relatively high level (95% and 90% respectively). T_{35} sits at 64% utilization, while T_{57} is the lowest at 43%. The extremely small percentage errors (roughly 0.1% or less) confirm that the manual and CAST results match closely, indicating consistent modeling and no major discrepancies in the calculations.

Table 3

Ties stress comparison between manual calculations and CAST results

Struts	N [kN]	σ_{Ed} Manual [MPa]	σ_{Ed} CAST [MPa]	σ_{Rd} [MPa]	Stress Ratio	Error
S_{12}	2976.12	10.53	10.54	10.23	103%	0.05%
S_{89}	803.7	2.74	2.85	10.23	28%	3.75%

Strut S_{12} slightly exceeds its design capacity at 103%, indicating an overstress. Because S_{12} is subject to bursting forces (often referred to as bottle-shaped ties), it should be recalculated using the appropriate detailing and reinforcement provisions. By contrast, Strut S_{89} remains well within capacity (28%). Although the manual and CAST calculations largely agree, the key takeaway is that S_{12} strut is overstress and its bottle-shaped tie behavior require special design considerations (detailed in chapter 3).

5. Conclusions

This study confirms the Strut-and-Tie Model (STM) as a reliable and effective approach for the design and analysis of reinforced concrete deep beams and other discontinuity regions. By transforming complex stress distributions into simplified truss-like systems of struts, ties, and nodal zones, STM ensures precise force equilibrium and optimal distribution. The method's ability to model and analyze D-regions, where traditional beam theory no longer applies, demonstrates its adaptability and relevance for modern structural challenges.

This research has demonstrated the efficacy and reliability of the Strut-and-Tie Model (STM) in the design and structural analysis of reinforced concrete deep beams, particularly within discontinuity regions (D-regions) where conventional beam theory proves insufficient. The study employed a dual-method approach, consisting of both manual calculations and computational analysis using CAST software, to ensure a rigorous and comprehensive evaluation of the model's performance. The key stages of the investigation can be summarized as follows:

1. Methodological Framework: The Strut-and-Tie Model was systematically applied, conceptualizing the internal force distribution through an idealized truss network of compressive struts, tensile ties, and nodal zones. This method facilitated the accurate visualization and assessment of stress flow, thereby enhancing the analytical capacity for complex structural regions.
2. Manual Calculation and Code Application: A detailed tutorial was developed using a design example using manual calculation, adhering to Eurocode 2 standards. The analysis included the determination of forces within struts and ties, equilibrium verification, and nodal stress assessment. Special attention was given to reinforcement configuration and anchorage to ensure structural integrity and compliance with design requirements.
3. Computational Validation: The manual example was further modeled using CAST software to illustrate how computer-aided tools can streamline complex analyses. The tutorial demonstrated the software's capabilities in automating stress verification, node classification, and reinforcement optimization, thereby enhancing both precision and efficiency in real-world scenarios.
4. Comparative Analysis: A systematic comparison between the manual and software-generated results revealed a remarkable consistency, with negligible error margins across all stress components. Noteworthy findings included an overstress condition in a bottle-shaped strut, highlighting the necessity of tailored design adjustments for critical elements.

Through its dual focus on theoretical principles and practical implementation, this study functions not only as a demonstration of STM's reliability but also as a comprehensive guide for engineers seeking to apply the method in practice. The integration of computational tools enhances the learning experience by providing hands-on insights into iterative model refinement, stress analysis, and compliance verification.

The findings emphasize the importance of detailed modeling, accurate input parameters, and rigorous nodal checks to mitigate stress concentrations and optimize reinforcement strategies. This tutorial-based approach equips practitioners with the skills to confidently apply STM to a wide range of structural challenges. Looking ahead, further advancements in STM can be achieved through probabilistic modeling and expanded experimental data, thus increasing the method's adaptability and predictive precision. By bridging theory, computation, and practice, STM remains a cornerstone of modern structural engineering design.

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