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More On Weak Soft Axioms

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Abstract. In this paper, we continue studying the properties of weak soft axioms discussed and studied in [8]. We initiate and explore soft semi-R_0 spaces at soft point in terms of soft semi-open sets and study its characterizations and properties. It is interesting to mention that this soft contains the soft semi-closure of each of its soft point singletons. We also define soft semi-R_1 spaces at soft point and discuss some of its characterizations.

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1. Introduction

To solve complicated problems having uncertainties and incomplete information, a new mathematical tool named as soft set theory was initiated by Molodstov [19]. A lot of researches have been done to find out the different structures of soft set theory and its applications, especially to decision making problems, measurement criteria for sound quality and establishment of remarkable method for the classification of natural textures. For details, one is refer to [14, 15, 16, 17, 18, 19, 20, 21]. Shabir and Naz [22] initiated and discussed soft topological spaces. Different algebraic structures, axioms and properties of soft topology and soft topological spaces are studied and discussed in [1, 4, 6, 10, 11, 12, 13].
B. Chen [2, 3] defined and discussed the basic properties of soft semi-open and soft semi-closed sets in soft topological spaces. Later, S. Hussain [5] discussed the different characterizations and properties using soft semi-open(closed) sets. In [7], S. Hussain defined and discussed

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new form of continuity called soft pu-semi-continuity via soft semi-open set in soft topological spaces. Moreover the concepts of soft-pu-semi-open and soft pu-semi-closed functions are introduced and discussed many of their characterizations and properties. It is interesting to mention that the soft functions defined and discussed in [7] are the generalization of soft functions explored in [4]. S. Hussain [9] defined and explored the characterizations and properties of soft regular open (closed) as well as soft semi-regular sets in soft topological spaces. In [8], S. Hussain introduced and explored the properties and characterizations of soft semi-T_i and soft semi-D_i(for $i = 0, 1, 2$) spaces at soft point by analyzing the relationship among them.
The main aim of this paper is to initiate and explore soft-semi-R_0 spaces at soft point in terms of soft-semi-open sets. It is interesting to mention that this soft contains the soft-semi-closure of each of its singletons. We study and discuss the properties of soft-semi-R_0 spaces at soft point. We also define soft-semi-R_1 spaces at soft point and discuss the relation between two newly defined spaces. We also discuss some of its characterizations.

2. Preliminaries

To make our paper self contained, first we recall some definitions from [1, 2, 3, 5, 6, 7, 8, 9, 17, 18, 19, 20, 22], .

Definition 2.1. *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).*

Definition 2.2. *For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A)\tilde{\subseteq}(G, B)$. In this case (G, B) is said to be a soft super set of (F, A).*

Definition 2.3. *Two soft sets (F, A) and (G, B) over a common universe X are said to be soft equal, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).*

Definition 2.4. *The union of two soft sets of (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}.$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.5. *The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted $(F, A)\tilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e)\tilde{\cap}G(e)$, for all $e \in C$.*

Definition 2.6. *The relative complement of a soft set (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$ where $F' : A \to P(U)$ is a mapping given by $F'(\alpha) = U\backslash F(\alpha)$, for all $\alpha \in A$.*

Definition 2.7. *Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if*
(1) Φ , $\tilde{X}$ belong to τ.
(2) the union of any number of soft sets in τ belongs to τ.
(3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.8. *Let (X, τ, E) be a soft topological space over X. Then*
(1) the soft interior of soft set (F, E) over X is denoted by $(F, E)^{\circ}$ and is defined as the union of all soft open sets contained in (F, E). Thus $(F, E)^{\circ}$ is the largest soft open set contained in (F, E).
(2) the soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of (F, E). Clearly $\overline{(F, E)}$ is the smallest soft closed set over X which contains (F, E).

Definition 2.9. *Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. The soft set (F, E) is called a soft point in $\tilde{X}$, denoted by e_F, if for the element $e \in E$, $F(e) \neq \phi$ and $F(e^{'}) = \phi$, for all $e^{'} \in E - \{e\}$.*

Definition 2.10. *The soft point e_F is said to be in the soft set (G, E), denoted by $e_F \tilde{\in} (G, E)$, if for the element $e \in E$, $F(e) \subseteq G(e)$. Note that if $e_F \tilde{\in} (G, E)$, then $e_F \tilde{\notin} (G, E)^c$.*

Definition 2.11. *Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then (F, E) is called soft semi-open set if and only if there exists a soft open set (G, E) such that $(G, E) \tilde{\subseteq} (F, E) \tilde{\subseteq} \overline{(G, E)}$. The set of all soft semi-open sets is denoted by $SSO(X)$. Note that every soft open set is soft semi-open set.*
A soft set (F, E) is said to be soft semi-closed if its relative complement is soft semi-open . Equivalently there exists a soft closed set (G, E) such that $(G, E)^{\circ} \tilde{\subseteq} (F, E) \tilde{\subseteq} (G, E)$. Note that every soft closed set is soft semi-closed set.

Definition 2.12. *Let (X, τ, E) be a soft topological space over X.*
(i) soft semi-interior of soft set (F, E) over X is denoted by $int^s(F, E)$ and is defined as the union of all soft semi-open sets contained in (F, E).
(ii) soft closure of (F, E) over X is denoted by $cl^s(F, E)$ and is the intersection of all soft semi-closed super sets of (F, E).

Definition 2.13. *Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X and soft point $e_F \tilde{\in} \tilde{X}_E$. Then (G, E) is said to be a soft semi-neighborhood of e_F, if there exists a soft semi-open set (F, E) such that $e_F \tilde{\in} (F, E) \tilde{\subseteq} (G, E)$.*

3. Properties of Weak Soft Axioms

Now we define:

Definition 3.1. *Let (X, τ, E) be a soft topological space over X, $A \subseteq X$ and (F, A) be a soft set over X. Then soft semi-kernal of (F, A) denoted by $Sker^s(F, A)$ and is defined as the intersection of all soft semi-open soft supersets of (F, A).*
That is, $Sker^s(F, A) \tilde{=} \tilde{\bigcap} \{(L, A) \tilde{\in} SSO(X) : (F, A) \tilde{\subseteq} (L, A)\}$.

Example 1. Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\tau = \{\tilde{\Phi}, \tilde{X}, (F, A)\}$, where $(F, A)\tilde{=}\{\{h_1\}, \{h_2\}\}$ is a soft sets over X with soft points: $e_1(F) = \{h_1\}$, $e_2(F) = \{h_2\}$.
Then τ defines a soft topology on X and hence (X, τ, A) is a soft topological space over X. Soft closed sets are $\tilde{\Phi}$, $\tilde{X}$, $\{\{h_2, h_3\}, \{h_1, h_3\}\}$. Calculations show that soft semi-open sets are $SSO(X)\tilde{=}\{\{\{h_1\}, \{h_1, h_2\}\}, \{\{h_1\}, \{h_2, h_3\}\}, \{\{h_1, h_3\}, \{h_2\}\}, \{\{h_1, h_2\}, \{h_2\}\}, \{\{h_1, h_3\}, \{h_1, h_2\}\}, \{\{h_1, h_3\}, \{h_2, h_3\}\}, \{\{h_1, h_2\}, \{h_1, h_2\}\}, \{\{h_1, h_2\}, \{h_2, h_3\}\}, \{\{h_1\}, \{h_2\}\}, \tilde{X}\}$.
Let us take $(G, A)\tilde{=}\{\{h_1\}, \{h_3\}\}$. Clearly, $Sker^s(G, A)\tilde{=}\{\{h_1\}, \{h_2, h_3\}\}$.

Proposition 1. *Let (X, τ, E) be a soft topological space over X, (F, A) be a soft set over X and e_G be a soft point in $\tilde{X}_E$. Then $Sker^s(F, A)\tilde{=}\{e_G\tilde{\in}\tilde{X}_E : cl^s(\{e_G\})\tilde{\cap}(F, A)\tilde{\neq}\tilde{\Phi}\}$.*

Proof. Let $e_G\tilde{\in}Sker^s(F, A)$ and suppose on the contrary that $cl^s(\{e_G\})\tilde{\cap}(F, A)\tilde{=}\tilde{\Phi}$. Thus $e_G\tilde{\notin}(cl^s(\{e_G\}))^c$, where $(cl^s(\{e_G\}))^c$ is a soft semi-open set such that $(F, A)\tilde{\subseteq}(cl^s(\{e_G\})^c$. This in not possible, since $e_G\tilde{\in}Sker^s(F, A)$. Hence $cl^s(\{e_G\})\tilde{\cap}(F, A)\tilde{\neq}\tilde{\Phi}$.
Suppose $e_G\tilde{\in}X$ with $cl^s(\{e_G\})\tilde{\cap}(F, A)\tilde{\neq}\tilde{\Phi}$ and assume contrarily that $e_G\tilde{\notin}Sker^s(F, A)$. Then by definition, there exists a soft semi-open set (U, A) such that $(F, A)\tilde{\subseteq}(U, A)$ and $e_G\tilde{\notin}(U, A)$. Let $e_H\tilde{\in}cl^s(\{e_G\})\tilde{\cap}(F, A)$. Thus, (U, A) is a soft semi nbd of e_H such that $e_G\tilde{\notin}(U, A)$. This contradiction proves as requires. Hence the proof.

Definition 3.2. *A soft topological space (X, τ, E) over X is said to be a soft semi-R_0 space, if (U, A) contains the soft semi closure of each of its soft point singletons, for every soft semi-open set (U, A).*

Example 2. Let us consider the soft topological spaces (X, τ, A) as in Example 1. Calculations show that the space (X, τ, A) is not soft-semi-R_0.

Proposition 2. *Let (X, τ, E) be a soft topological space over X and soft points $e_G, e_K\tilde{\in}\tilde{X}_E$. Then $e_K\tilde{\in}Sker^s(\{e_G\}) \Leftrightarrow e_G\tilde{\in}cl^s(\{e_K\})$.*

Proof. Let $e_G\tilde{\in}cl^s(\{e_K\})$ and contrarily suppose that $e_K\tilde{\notin}Sker^s(\{e_G\})$. Then there exists a soft semi-open set (U, A) such that $e_G\tilde{\in}(U, A)$ and $e_K\tilde{\notin}(U, A)$. Thus $e_G\tilde{\notin}cl^s(\{e_K\})$. A contradiction. The converse follows similarly. This completes the proof.

Proposition 3. *Let (X, τ, E) be a soft topological space over X and e_G, e_H are any two soft points soft contains in $\tilde{X}_E$. Then the following statements are equivalent:*
(1) $Sker^s(\{e_G\})\tilde{\neq}Sker^s(\{e_H\})$.
(2) $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$.

Proof. (2) $\Rightarrow$ (1). Let $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$. Then there exists $e_L\tilde{\in}\tilde{X}_E$ such that $e_L\tilde{\in}cl^s(\{e_G\})$ and $e_L\tilde{\notin}cl^s(\{e_H\})$. This implies that there exists a soft semi-open set (U, A) such that $e_L\tilde{\in}(U, A)$ and thus $e_G\tilde{\in}(U, A)$, $e_H\tilde{\notin}(U, A)$. Therefore $e_H\tilde{\notin}Sker^s(\{e_G\})$ and hence $Sker^s(\{e_G\})\tilde{\neq}Sker^s(\{e_H\})$.
(1) $\Rightarrow$ (2). Let $Sker^s(\{e_G\})\tilde{\neq}Sker^s(\{e_H\})$. Then there exists a soft point $e_L\tilde{\in}\tilde{X}_E$ such that $e_L\tilde{\in}Sker^s(\{e_G\})$ and $e_L\tilde{\notin}Sker^s(\{e_H\})$. $e_L\tilde{\in}Sker^s(\{e_G\})$ implies that $\{e_G\}\tilde{\cap}cl^s(\{e_L\})\tilde{\neq}\tilde{\Phi}$ and hence $e_G\tilde{\in}cl^s(\{e_L\})$. Also $e_L\tilde{\notin}Sker^s(\{e_H\})$ implies that $\{e_H\}\tilde{\cap}cl^s(\{e_L\})\tilde{=}\tilde{\Phi}$. $e_G\tilde{\in}cl^s(\{e_L\})$ follows that $cl^s(\{e_G\})\tilde{\subseteq}cl^s(\{e_L\})$ and $\{e_H\}\tilde{\cap}cl^s(\{e_L\})\tilde{=}\tilde{\Phi}$. Thus $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$. The proof is completed.

Theorem 3.1. *Let (X, τ, E) be a soft topological space over X and e_G, e_H are soft points soft contains in $\tilde{X}_E$. Then the following statements are equivalent:*
(1) (X, τ, E) is soft semi-R_0 space.
(2) $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\}) \Rightarrow cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}$.

Proof. (1) $\Rightarrow$ (2). Let e_G, e_H are soft points soft contains in $\tilde{X}_E$ such that $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$. Therefore, there exist a soft point $e_L\tilde{\in}cl^s(\{e_G\})$ such that $e_L\tilde{\notin}cl^s(\{e_H\})$ or there exist $e_L\tilde{\in}cl^s(\{e_H\})$ such that $e_L\tilde{\notin}cl^s(\{e_G\})$. Since $e_L\tilde{\in}cl^s(\{e_G\})$, then there exists a soft semi-open set (U, A) such that $e_L\tilde{\in}(U, A)$ and $e_H\tilde{\notin}(U, A)$ implies that $e_G\tilde{\in}(U, A)$. Thus $e_G\tilde{\notin}cl^s(\{e_H\})$. Hence $e_G\tilde{\in}(cl^s(e_H))^c\tilde{\in}SSO(X)$ follows that $cl^s(\{e_G\})\tilde{\subseteq}(cl^s(\{e_H\}))^c$ and $cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}$. Similarly we can prove that $cl^s(\{e_H\})\tilde{\subseteq}(cl^s(\{e_G\}))^c$ and $cl^s(\{e_G\})\tilde{\cap}cl^s(\{y\})\tilde{=}\tilde{\Phi}$.
(2) $\Rightarrow$ (1). Suppose (U, A) is a soft semi-open set in (X, τ, E) and a soft point $e_G\tilde{\in}(U, A)$. To show our required result, we prove that $cl^s(\{e_G\})\tilde{\subseteq}(U, A)$. Suppose $e_H\tilde{\notin}(U, A)$ implies $e_H\tilde{\in}(U, A)^c$. Thus $e_G\tilde{\neq}e_H$ and hence $e_G\tilde{\notin}cl^s(\{e_H\})$, which follows that $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$. Since $cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}$, then $e_H\tilde{\notin}cl^s(\{e_G\})$. Thus $cl^s(\{e_G\})\tilde{\subseteq}(U, A)$. The proof is completed.

Theorem 3.2. *Let (X, τ, E) be a soft topological space over X and e_G, e_H are soft points soft contains in $\tilde{X}_E$. Then the following statements are equivalent:*
(1) (X, τ, E) is soft semi-R_0 space.
(2) $Ker_\gamma(\{e_G\})\tilde{\neq}Sker^s(\{e_H\}) \Rightarrow Sker^s(\{e_G\})\tilde{\cap}Sker^s(\{e_H\})\tilde{=}\tilde{\Phi}$.

Proof. (1) $\Rightarrow$ (2). Suppose that (X, τ, E) is soft semi-R_0 space. Using the result of Proposition 3, it is enough to show that $Sker^s(\{e_G\})\tilde{\cap}Sker^s(\{e_H\})\tilde{=}\tilde{\Phi}$. Contrarily suppose that a soft point $e_L\tilde{\in}Sker^s(\{e_G\})\tilde{\cap}Sker^s(\{e_H\})$, then by Proposition 2, $e_G\tilde{\in}cl^s(\{e_L\})$. Since (X, τ, E) is soft semi-R_0 space, by Theorem 3.1, $e_G\tilde{\in}cl^s(\{e_G\})$ implies that $cl^s(\{e_G\})\tilde{=}cl^s(\{e_L\})$. In a similar fashion, we get $cl^s(\{e_G\})\tilde{=}cl^s(\{e_L\})\tilde{=}cl^s(\{e_H\})$. This contradiction gives as desired.
(2) $\Rightarrow$ (1). We use Theorem 3.1 to prove (1). By Proposition 3, $Sker^s(\{e_G\})\tilde{\neq}Sker^s(\{e_H\})$ implies $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$. Hence $Sker^s(\{e_G\})\tilde{\cap}Sker^s(\{e_H\})\tilde{=}\tilde{\Phi}$. It follows that

$$cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}.$$

Since $e_L\tilde{\in}cl^s(\{e_G\})$, then $e_G\tilde{\in}Sker^s(\{e_L\})$ and hence $Sker^s(\{e_G\})\tilde{\cap}Sker^s(\{e_L\})\tilde{\neq}\tilde{\Phi}$. By (2), we have $Ker_\gamma(\{e_G\})\tilde{=}Sker^s(\{e_L\})$. Therefore $e_L\tilde{\in}cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})$ follows

$$Sker^s(\{e_G\})\tilde{=}Sker^s(\{e_L\})\tilde{=}Sker^s(\{e_H\}).$$

Then, we obtain a contradiction. Hence

$$cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}.$$

By Proposition 3, (X, τ, E) is soft semi-R_0 space. This completes the proof.

Theorem 3.3. *Let (X, τ, E) be a soft topological space over X. Then the following statements are equivalent:*
(1) (X, τ, E) is soft semi-R_0 space.
(2) For any soft-semi-closed set (U, A) and $e_G\tilde{\notin}(U, A)$, there exists soft semi-open set (V, A) such that $(U, A)\tilde{\subseteq}(V, A)$ and $e_G\tilde{\notin}(V, A)$.

(3) $(U,A)\tilde{\cap}cl^s(\{e_G\})\tilde{=}\tilde{\Phi}$, *for any soft semi-closed set* (U,A) *and* $e_G\tilde{\notin}(U,A)$.
(4) For any two soft points e_G, e_H *soft contains in* $\tilde{X}_E$ *such that* $e_G\tilde{\neq}e_H$, *one of* $cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}$ *and* $cl^s(\{e_G\})\tilde{=}cl^s(\{e_H\})$ *holds.*

Proof. (1) $\Rightarrow$ (2). Let (U,A) be any soft semi-closed set such that $e_G\tilde{\notin}(U,A)$. By (1), $cl^s(\{e_G\})\tilde{\subseteq}(U,A)^c$. Let us take $(V,A)\tilde{=}(cl^s(\{e_G\}))^c$, then $(V,A)\tilde{\in}SSO(X)$. Therefore

$$(U,A)\tilde{\subseteq}(V,A) \quad \text{and} \quad e_G\tilde{\notin}(U,A).$$

(2) $\Rightarrow$ (3). Let (U,A) be a soft semi-closed set and $e_G\tilde{\notin}(U,A)$. By assumption, there exists soft semi-open set (V,A) such that $(U,A)\tilde{\subseteq}$ and $e_G\tilde{\notin}(V,A)$. $(V,A)\tilde{\in}SSO(X)$ implies that $(V,A)\tilde{\cap}cl^s(\{e_G\})\tilde{=}\tilde{\Phi}$ and $(U,A)\tilde{\cap}cl^s(\{e_G\})\tilde{=}\tilde{\Phi}$.
(3) $\Rightarrow$ (4). For any two soft points e_G, e_H soft contains in $\tilde{X}_E$ such that $e_G\tilde{\neq}e_H$, consider $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$. Then there exists a soft point $e_M\tilde{\in}cl^s(\{e_G\})$ such that $e_M\tilde{\notin}cl^s(\{e_H\})$(or $e_M\tilde{\in}cl^s(\{e_H\})$ such that $e_M\tilde{\notin}cl^s(\{e_G\})$). Therefore there exists soft semi-open set (U,A) such that $e_H\tilde{\notin}(U,A)$ and $e_M\tilde{\in}(U,A)$. Thus $e_G\tilde{\in}(U,A)$. Hence $e_G\tilde{\notin}cl^s(\{e_H\})$. By assumption, we have $cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}$. Similarly, we can prove the otherwise.
(4) $\Rightarrow$ (1). Suppose that (U,A) be a soft semi-open set and $e_G\tilde{\in}(U,A)$. For each $e_H\tilde{\notin}(V,A)$ with $e_G\tilde{\neq}e_H$ and $e_G\tilde{\notin}cl^s(\{e_H\})$ implies that $cl^s(\{e_G\})\tilde{\neq}cl^s(\{e_H\})$. By assumption, for each $e_H\tilde{\in}(U,A)^c$ $cl^s(\{e_G\})\tilde{\cap}cl^s(\{e_H\})\tilde{=}\tilde{\Phi}$ implies $cl^s(\{e_G\})\tilde{\cap}(\cup_{e_H}\tilde{}\,\tilde{(}U,A)\in cl^s(\{e_H\}))\tilde{=}\tilde{\Phi}$. Also for $e_H\tilde{\in}(U,A)^c$, implies $cl^s(\{e_H\})\tilde{\subseteq}(U,A)^c$ and thus $(U,A)^c\tilde{=}(\tilde{\cup}_{e_H}\tilde{\in}(U,A)^c)cl^s(\{e_H\})$. Thus, we have
$(U,A)^c\tilde{\cap}cl^s(\{e_G\})\tilde{=}\tilde{\Phi}$ and $cl^s(\{e_G\})\tilde{\subseteq}(U,A)$. This implies that (X,τ,E) is soft semi-R_0 space. Hence the proof.

Theorem 3.4. *Let* (X,τ,E) *be a soft topological space over X. Then the following are equivalent:*
(1) (X,τ,E) *is a soft semi-*R_0 *space.*
(2) For any nonempty soft open set (F,A) *over X and soft semi-open set* (U,A) *such that* $(F,A)\tilde{\cap}(U,A)\tilde{\neq}\tilde{\Phi}$, *there exists soft semi-closed set* (M,A) *such that* $(F,A)\tilde{\cap}(M,A)\tilde{\neq}\tilde{\Phi}$ *and* $(M,A)\tilde{\subseteq}(U,A)$.
(3) $(U,A)\tilde{=}\tilde{\bigcup}\{(M,A):(M,A)^c\tilde{\in}SSO(X)$ *and* $(M,A)\tilde{\subseteq}(U,A)\}$, *for soft semi-open sets* (U,A).
(4) $(M,A)\tilde{=}\tilde{\bigcap}\{(U,A):(U,A)\tilde{\in}SSO(X)$ *and* $(M,A)\tilde{\subseteq}(U,A)\}$, *for soft semi-closed sets* (M,A).
(5) $cl^s(\{e_H\})\tilde{\subseteq}Sker^s(\{e_H\})$, *for any* $e_H\tilde{\in}\tilde{X}_E$.

Proof. (1) $\Rightarrow$ (2). Suppose (F,A) be a nonempty soft open set over X and soft semi-open set (U,A) such that $(F,A)\tilde{\cap}(U,A)\tilde{\neq}\tilde{\Phi}$. Then $e_G\tilde{\in}(F,A)\tilde{\cap}(U,A)$ implies e_G soft belongs to a soft-open set (U,A). Therefore $cl^s(\{e_G\})\tilde{\subseteq}(U,A)$. Take $(M,A)\tilde{=}cl^s(\{e_G\})$. Then (M,A) is soft semi-closed, $(M,A)\tilde{\subseteq}(U,A)$ and $(F,A)\tilde{\cap}(M,A)\tilde{\neq}\tilde{\Phi}$.
(2) $\Rightarrow$ (3). Suppose that (U,A) be a soft semi-open set. Then $\tilde{\bigcup}\{(M,A):(M,A)^c\tilde{\in}SSO(X)$ and $(M,A)\tilde{\subseteq}(U,A)\}\tilde{\subseteq}(U,A)$. Suppose $e_G\tilde{\in}(U,A)$. Then there exists soft semi-closed set (M,A) such that $e_G\tilde{\in}(M,A)$ and $(M,A)\tilde{\subseteq}(U,A)$ implies

$$e_G\tilde{\in}(M,A)\tilde{\subseteq}\tilde{\bigcup}\{(M,A):(M,A)^c\tilde{\in}SSO(X) \quad \text{and} \quad (M,A)\tilde{\subseteq}(U,A)\}.$$

Hence $(U,A)\tilde{=}\tilde{\bigcup}\{(M,A):(M,A)^c\tilde{\in}SSO(X)$ and $(M,A)\tilde{\subseteq}(U,A)\}$.
(3) $\Rightarrow$ (4). The proof directly follows.

(4) $\Rightarrow$ (5). Suppose that $e_H\tilde{\in}\tilde{X}_E$ and $e_L\tilde{\notin}Sker^s(\{e_H\})$. Then there exists soft semi-open set (W,A) such that $e_H\tilde{\in}(W,A)$ and $e_L\tilde{\notin}(W,A)$. This implies that $cl^s(\{e_L\})\tilde{\cap}(W,A)\tilde{=}\tilde{\Phi}$. By supposition $(\bigcap\{(U,A) : (U,A)\tilde{\in}SSO(X)$ and $cl^s(\{e_L\})\tilde{\subseteq}(U,A)\})\tilde{\cap}(W,A)\tilde{=}\tilde{\Phi}$. Therefore, there exists soft semi-open set (U,A) such that $e_H\tilde{\notin}(U,A)$ and $cl^s(\{e_L\})\tilde{\subseteq}(U,A)$. Hence

$$cl^s(\{e_H\})\tilde{\cap}(M,A)\tilde{=}\tilde{\Phi} \quad \text{and} \quad e_L\tilde{\notin}cl^s(\{e_H\}).$$

Thus we have $cl^s(\{e_H\})\tilde{\subseteq}Sker^s(\{e_H\})$.
(5) $\Rightarrow$ (1). Suppose that (U,A) be a soft semi-open set and $e_H\tilde{\in}(U,A)$. Suppose $e_L\tilde{\in}Sker^s(\{e_H\})$, then $e_H\tilde{\in}cl^s(\{e_L\})$ and $e_L\tilde{\in}(U,A)$ implies that $Sker^s(\{e_H\})\tilde{\subseteq}(U,A)$. Hence we have $e_H\tilde{\in}cl^s(\{e_H\})\tilde{\subseteq}Sker^s(\{e_H\})\tilde{\subseteq}(U,A)$. Which implies (1). This completes the proof.

Theorem 3.5. *Let (X,τ,E) be a soft topological space over X. Then the following are equivalent:*
(1) (X,τ,E) is a soft semi-R_0 space.
(2) For any soft point $e_H\tilde{\in}\tilde{X}_E$, $cl^s(\{e_H\})\tilde{=}Sker^s(\{e_H\})$.

Proof. (2) $\Rightarrow$ (1). This follows form Theorem 3.4.
(1) $\Rightarrow$ (2). Suppose that (1) holds and by Theorem 3.4, $cl^s(\{e_H\})\tilde{=}Sker^s(\{e_H\})$. Suppose $e_L\tilde{\in}Sker^s(\{e_H\})$ then $e_H\tilde{\in}cl^s(\{e_L\})$ and Proposition 3 implies that $cl^s(\{e_H\})\tilde{=}cl^s(\{e_L\})$. Hence $e_L\tilde{\in}cl^s(\{e_H\})$ and therefore $Sker^s(\{e_H\})\tilde{\subseteq}cl^s(\{e_H\})$. The other implication follows form Theorem 3.4-(5). Therefore $cl^s(\{e_H\})\tilde{=}Sker^s(\{e_H\})$. Hence the proof.

Theorem 3.6. *Let (X,τ,E) be a soft topological space over X. Then the following are equivalent:*
(1) (X,τ,E) is a soft semi-R_0 space.
(2) For any two soft points $e_H,e_L\tilde{\in}\tilde{X}_E$, $e_H\tilde{\in}cl^s(\{e_L\}) \Leftrightarrow e_L\tilde{\in}cl^s(\{e_H\})$.

Proof. (2) $\Rightarrow$ (1). Let (M,A) be a soft semi soft semi-open set and $e_H\tilde{\in}(M,A)$. For if $e_L\tilde{\notin}(M,A)$, then $e_H\tilde{\notin}cl^s(\{e_L\})$ and hence $e_L\tilde{\notin}cl^s(\{e_H\})$. This shows that $cl^s(\{e_H\})\tilde{\subseteq}(M,A)$. Therefore (X,τ,A) is a soft semi-R_0 space.
(1) $\Rightarrow$ (2). Suppose that (1) holds. Let $e_H\tilde{\in}cl^s(\{e_L\})$ and (U,A) be a soft semi-open set such that $e_L\tilde{\in}(U,A)$. By (1), $e_H\tilde{\in}(U,A)$. Thus every soft semi-open set (M,A) such that $e_L\tilde{\in}(M,A)$ implies $e_H\tilde{\in}(M,A)$. Therefore $e_L\tilde{\in}cl^s(\{e_H\})$. This completes the proof.

Theorem 3.7. *Let (X,τ,E) be a soft topological space over X. Then the following are equivalent:*
(1) (X,τ,E) is a soft semi-R_0 space.
(2) $Sker^s((M,A))\tilde{=}(M,A)$, for soft semi-closed set (M,A).
(3) $Sker^s(\{e_H\})\tilde{\subseteq}(M,A)$, for soft semi-closed set (M,A) and $e_H\tilde{\in}(M,A)$.
(4) $Sker^s(\{e_H\})\tilde{\subseteq}cl^s(\{e_H\})$, for any soft point $e_H\tilde{\in}\tilde{X}_E$.

Proof. (1) $\Rightarrow$ (2). This follows directly from Theorem 3.4.
(2) $\Rightarrow$ (3). Clearly $Sker^s(F,A)\tilde{\subseteq}Sker^s(G,A)$, for $(F,A)\tilde{\subseteq}(G,A)$. Thus by assumption, we have $Sker^s(\{e_H\})\tilde{\subseteq}Sker^s(M,A)\tilde{=}(M,A)$.
(3) $\Rightarrow$ (4). As $Sker^s(\{e_H\})$ is soft semi-closed and soft contains e_G. Thus by assumption, $Sker^s(\{e_H\})\tilde{\subseteq}cl^s(\{e_H\})$.
(4) $\Rightarrow$ (1). We prove this by using Theorem 3.6. By Proposition 2, $e_H\tilde{\in}cl^s(\{e_L\})$ implies $e_L\tilde{\in}Sker^s(\{e_H\})$. Since $e_H\tilde{\in}cl^s(\{e_L\})$ and $cl^s(\{e_L\})$ is soft semi-closed, our assumption follows that

$e_L \tilde{\in} Sker^s(\{e_H\}) \tilde{\subseteq} cl^s(\{e_H\})$. Hence $e_H \tilde{\in} cl^s(\{e_L\})$ follows that $e_L \tilde{\in} cl^s(\{e_H\})$. The converse follows directly. Hence (X,τ,E) is soft semi-R_0 space. Hence the proof.

Definition 3.3. *Let (X,τ,E) be a soft topological space over X. Then (X,τ,E) is said to be a soft semi-R_1 space, if for any two soft points e_F and e_G in $\tilde{X}_E$, there exists soft semi-open sets (L,A) and (M,A) such that $cl^s(e_F) \tilde{\subseteq} (L,A)$, $cl^s(e_G) \tilde{\subseteq} (M,A)$ and $(L,A) \tilde{\cap} (M,A) \tilde{=} \tilde{\Phi}$.*

Example 3. Let us consider the soft topological spaces (X,τ,A) as in Example 1. Calculations show that the space (X,τ,A) is not soft-semi-R_1.

Theorem 3.8. *Let (X,τ,E) be a soft topological space over X. Then (X,τ,E) is soft semi-R_1 space implies (X,τ,E) is soft semi-R_0 space.*

Proof. Let (L,A) be soft semi-open and e_F be soft point in (L,A). Since $e_F \tilde{\notin} cl^s(e_G)$ and if $e_G \tilde{\notin} (L,A)$, then $cl^s(e_F) \tilde{\neq} cl^s(e_G)$. Therefore, there exists a soft semi-open (V,A) such that $cl^s(e_G) \tilde{\subseteq} (V,A)$ and $e_F \tilde{\notin} (V,A)$. This follows that $e_G \tilde{\notin} cl^s(e_F)$. Thus $cl^s(e_F) \tilde{\subseteq} (L,A)$. Hence (X,τ,E) is soft semi-R_0 space. This completes the proof.

The Proof of the following Theorem directly follows for Proposition 3 and is therefore omitted.

Theorem 3.9. *Let (X,τ,E) be a soft topological space over X. Then the following statements are equivalent.*
(1) (X,τ,E) is soft semi-R_1 space.
(2) For any two soft points e_G, e_H soft contains in $\tilde{X}_E$ with $Sker^s(\{e_G\}) \tilde{\neq} Sker^s(\{e_H\})$, there exist soft semi-open sets (L,A) and (M,A) such that $cl^s(\{e_G\}) \tilde{\subseteq} (L,A)$ and $cl^s(\{e_H\}) \tilde{\subseteq} (M,A)$ with $(L,A) \tilde{\cap} (M,A) \tilde{=} \tilde{\Phi}$.

Theorem 3.10. *Let (X,τ,E) be a soft topological space over X and e_F, e_H are soft points in $\tilde{X}_E$. Then the following statements are equivalent.*
(1) (X,τ,E) is soft semi-R_1 space.
(2) $e_F \tilde{\in} cl^s(\{e_H\})$ follows that e_F and e_H have soft disjoint soft semi-open neighborhoods.

Proof. $(\Rightarrow)$ Suppose that (X,τ,E) is soft semi-R_1 space and soft point $e_F \tilde{\in} (cl^s(\{e_H\}))^c$. This implies that $cl^s(\{e_F\}) \tilde{\neq} cl^s(\{e_H\})$. Therefore, e_F and e_H have soft disjoint soft semi-open neighborhoods.
$(\Leftarrow)$ First, we prove that (X,τ,E) is soft semi-R_0 space. Suppose (U,A) be a soft semi-open set and $e_F \tilde{\in} (U,A)$. Let $e_H \tilde{\notin} (U,A)$. Then $cl^s(\{e_H\}) \tilde{\cap} (U,A) \tilde{=} \tilde{\Phi}$ and $e_F \tilde{\notin} cl^s(\{e_H\})$. There exists soft semi-open sets $(U,A)_{e_F}$, $(V,A)_{e_H}$ such that $e_F \tilde{\in} (U,A)_{e_F}$, $e_H \tilde{\in} (V,A)_{e_H}$ and $(U,A)_{e_F} \tilde{\cap} (V,A)_{e_H} \tilde{=} \tilde{\Phi}$. Therefore, $cl^s(\{e_F\}) \tilde{\subseteq} cl^s((U,A)_{e_F})$ and $cl^s(\{e_F\}) \tilde{\cap} (V,A)_{e_H} \tilde{\subseteq} cl^s((U,A)_{e_F}) \tilde{\cap} (V,A)_{e_H} \tilde{=} \tilde{\Phi}$. Hence $e_H \tilde{\notin} cl^s(\{e_F\})$. This follows that $cl^s(\{e_F\}) \tilde{\subseteq} (U,A)$. Consequently, (X,τ,E) is soft semi-R_0 space. Now we prove that (X,τ,E) is soft semi-R_1 space. Suppose that $cl^s(\{e_F\}) \tilde{\neq} cl^s(\{e_H\})$. Then there exists $e_K \tilde{\in} cl^s(\{e_F\})$ such that $e_K \tilde{\notin} cl^s(\{e_H\})$. Therefore, there exist soft semi-open sets $(M,A)_{e_K}$ and $(N,A)_{e_H}$ such that $e_K \tilde{\in} (M,A)_{e_K}$ and $e_H \tilde{\in} (N,A)_{e_H}$ with $(M,A)_{e_K} \tilde{\cap} (N,A)_{e_H} \tilde{=} \tilde{\Phi}$. Since $e_K \tilde{\in} cl^s(\{e_F\})$, $e_F \tilde{\in} (M,A)_{e_K}$ and (X,τ,E) is soft semi-R_0 space, then $cl^s(\{e_F\}) \tilde{\subseteq} (M,A)_{e_K}$, $cl^s(\{e_H\}) \tilde{\subseteq} (N,A)_{e_H}$ with
$(M,A)_{e_K} \tilde{\cap} (N,A)_{e_H} \tilde{=} \tilde{\Phi}$. This implies that (X,τ,E) is soft semi-R_1 space. Hence the proof.

Theorem 3.11. *Let (X, τ, E) be a soft topological space over X and e_F, e_H are soft points in $\tilde{X}_E$. Then (X, τ, E) is soft semi-R_1 space if and only if $cl^s(\{e_F\}) \tilde{\neq} cl^s(\{e_H\})$ implies that e_F and e_H have soft disjoint soft semi-open neighborhoods.*

Proof. Necessity: This is straightforward.
Sufficiency: Let $e_F \tilde{\in} (cl^s(\{e_H\}))^c$. Then $cl^s(\{e_F\}) \tilde{\neq} cl^s(\{e_H\})$ and our hypothesis follows that e_F and e_H have soft disjoint soft semi-open neighborhoods. Therefore, Theorem 3.10 follows that (X, τ, E) is soft semi-R_1 space. This completes as required.

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References

[1] B. Ahmad, S. Hussain, *On some structures of soft topology, Mathematical Sciences, **6(64)** (2012), 7 pages.*

[2] B. Chen, *Soft local properties of soft semi-open sets, Discrete Dynamics in Nature and Society, Vol. 2013, Article ID 298032, 6 pages.*

[3] B. Chen, *Soft semi-open sets and related properties in soft topological spaces, Applied Mathematics and Information Sciences, **7(1)**(2013), 287-294.*

[4] S. Hussain, *On some soft functions, Mathematical Science Letters, **4(1)**(2015), 55-61.*

[5] S. Hussain, *Properties of soft semi-open and soft semi-closed sets, Pensee Journal, **76(2)**(2014), 133-143.*

[6] S. Hussain, *A note on soft connectedness, Journal of Egyption Mathematical Society, **23**(2015), 6-11.* http://dx.doi.org/10.1016/j.joems.2014.02.003.

[7] S. Hussain,*On some generalized soft mappings, Hacettepe Journal of Mathematics and Statistics, **45(3)** (2016), 743-754.*

[8] S. Hussain, *On some properties of weak soft axioms, European Journal of Pure and Applied Mathematics, **10(2)** (2017) 199-210.*

[9] S. Hussain, *On soft Regular-open(closed) sets in soft topological spaces, Journal of Applied Mathematics and Informatics, **36(1-2)**(2018) 59-68.*

[10] S. Hussain, B. Ahmad, *Soft separation axioms in soft topological spaces, Hacettepe Journal of Mathematics and Statistics, **44(3)** (2015), 559-568.*

[11] S. Hussain, B. Ahmad, *Some properties of soft topological spaces, Computers and Mathematics with Applications, **62** (2011) 4058-4067.*

[12] S. Hussain, H. F. Akiz and A. I. Alajlan, *On soft real point matrices and their operations*, Moroccan J. of Pure and Appl. Anal. (MJPAA), 4(1) (2018) 9-16.

[13] S. Hussain, H. F. Akiz and A. I. Alajlan, *On algebraic properties of soft real points*, Fixed Point Theory and Applications, Article number: 9(2019), 1-20

[14] Y. B. Jun, C. H. Park, *Applications of soft sets in ideal theory of BCK/BCI-algebras, Information Science, **178** (2008) 2466-2475.*

[15] Y. B. Jun, C. H. Park, *Applications of soft sets in Hilbert spaces, Iranian Journal of Fuzzy Systems, **6(2)** (2009) 55-86.*

[16] B. Kostek, *Soft set approach to subjective assesment of sound quality, IEEE Conference, **I**,669-676(1998).*

[17] P . K. Maji, R. Biswas, R. Roy, *An application of soft sets in a decision making problem, Computers and Mathematics with Applications, **44** (2002), 1077-1083.*

[18] P. K. Maji, R. Biswas, R. Roy, *Soft set theory, Computers and Mathematics with Applications, **45** (2003) 555-562.*

[19] D. Molodtsov, *Soft set theory first results, Computers and Mathematics with Applications, **37** (1999), 19-31.*

[20] D. Molodtsov, V.Y. Leonov, D.V. Kovkov, *Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya, **9(1)** (2006), 8-39.*

[21] D. Pie, D. Miao, *From soft sets to information systems, Granular computing, 2005 IEEE Inter. Conf., **2** , 617-621.*

[22] M. Shabir, M. Naz, *On soft topological spaces, Computers and Mathematics with Applications, **61** (2011) 1786-1799.*