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PHASE RETRIEVAL APPROACH FOR TRANSMISSION MATRIX IDENTIFICATION IN POLYTETRAFLUOROETHYLENE (PTFE)

A. Santosa*, S. Fomins, V. Karitāns

Institute of Solid State Physics,
University of Latvia
8 Kengaraga Str., Riga, LV-1063, LATVIA
*e-mail: anna.santosa@cfi.lu.lv

We present a phase retrieval approach for identifying the transmission matrix (TM) of highly scattering media, specifically PTFE. Utilising structured illumination via a Digital Micromirror Device (DMD) and Wirtinger Flow optimization, our method reconstructs complex TM elements from intensity-only measurements without the need for reference beams. This enables robust, real-time wavefront shaping applicable to biomedical imaging. Experimental results demonstrate high fidelity in TM recovery and spatial focusing efficiency, validating the feasibility of this approach in practical scenarios. A comparative evaluation of two measurement strategies –random phase masks versus one-to-one actuator mapping – is presented, demonstrating the practical implications of each method.

Keywords: *DMD, phase retrieval, PTFE, scattering media, transmission matrix, wavefront shaping, Wirtinger Flow.*

1. INTRODUCTION

Scattering media, on the contrary to optical lenses, have a highly complex optical transfer function, which makes them hardly penetrable for imaging and makes light transfer problematic. However, light is not lost and methods to restructure this light have been found. Examples include Anderson localization [1], coherent backscattering [2], random lasing [3], the effectiveness of time-reversal/phase conjugation [4], and the existence of perfect-transmission eigenchannels [5].

Despite significant progress in wavefront shaping, most transmission matrix (TM) identification methods require either reference beams or sensor placement behind the sample –conditions that are not feasible in biomedical contexts. This study addresses this limitation by demonstrating a reference-free phase retrieval approach using DMD-based structured illumination and Wirtinger Flow optimization.

In conjunction with the technological advances in wavefront shaping devices and imaging techniques for the wavefront recording, a variety of studies have been performed to deal with targets located within scattering media [6], [7] or beyond scattering samples [8], [9]. The methodologies include feedback-optimization of intensity at a target spot [7], the use of the so-called memory effect to scan the spot [10], [11], the transmission matrix of scattering media [12]–[14], acoustic guide star in conjunction with optical phase conjugation [15], [16], and spatial-temporal focusing combined with nonlinear excitation [17].

The memory effect of scattering samples is the ability of samples to hold the same scattering pattern during some period, i.e., correlation time. In such a way, this beneficial feature can be implemented for depth imaging; however, the decorrelation time in biological samples is tens of milliseconds only [18]. Therefore, fast scanning is necessary.

Due to motion in biological tissue, such as blood flow, breathing, and Brownian motion, the optimal wavefront rapidly decorrelates as scatterers shift, which does not allow the exploitation of the memory effect extensively. Therefore, to utilise wavefront shaping, the optimal TM needs to be obtained and applied as quickly as possible. In addition to speed, efficiency of focusing is important to ensure sufficient spatial resolution and signal-to-noise ratio. In many situations, TM identification with reference beams or placing the sensor behind the sample is not possible or impractical for biological samples. New techniques are particularly needed to enable faster, more compact, and robust wavefront shaping for real-time field applications, such as tissue 3D scanning or ophthalmic retinal imaging. Existing methods often rely on reference beams or complex setups, which are impractical for biological samples. This study addresses the need for a compact, reference-less, and rapid TM identification technique.

Then the input-output response of a scattering medium can be described by the so-called transmission matrix, which relates free modes at the input to those at the output. The experimental measurement of the transmission matrix is rather time-consuming because it requires both the scanning of incident waves in such a way to cover input free modes and the recording of complex field, i.e., amplitude and phase, at various output channels. However, it is the complete way of describing the response of scattering media to the incident light waves. The first transmission matrix measurement in optics was made by Vellekoop et al. [17] and later by Popoff et al. [19], using the self-interference of two different input modes generated by the same spatial light modulator.

Iterative TM methods [15], [16] use measurements of transmitted light field magnitudes to progressively improve field focus by sequentially changing the phase retardation imposed on each pixel of the input beam using a spatial light modulator (SLM). These methods maximize the intensity of the output field at a defined location by changing one SLM pixel at a time [17]. While very powerful, these methods tend to be slow and prone to converge at a local optimum, leading to stagnation. They also must be repeated every time a new target output intensity profile is specified. Recent advances have increased the depth of the medium to where the "memory effect" (i.e., correlations in the fields) can be reliably exploited [15], [18]. In the deep medium regime, where the fields are decorrelated and the memory effect no longer holds, refocusing still requires the procedure to be repeated every time a new target output profile is specified.

Non-iterative TM methods [19]–[21] that use measurements of transmitted field magnitudes and phases acquire knowledge of the transmission matrix of the medium, thereby allowing for the optimal wavefront to be computed non-iteratively for any desired output [22]. Most approaches developed to date to measure TMs rely on holographic methods, which are similar to coded diffractive imaging [23]. Methods for determining TMs that do not require a reference beam would be extremely useful compared to the previously mentioned methods. The first step in this direction has recently been taken by Dremeau et al. [24], who have applied a phase retrieval (PR) algorithm to measure the complex TM of a highly scattering medium using a digital micro-mirror device (DMD). This study introduces a new waveform-shaping technique to produce focused fields that uses semidefinite programming (SDP) to construct portions of the medium's TM from intensity measurements only; once the relevant parts of the TM are calculated, the method proceeds like any of the above non-iterative schemes [19], [23]. The SDP approach leverages a rigorous yet flexible computational framework that utilises the PhaseCut algorithm [24] to retrieve the phase of elements of the TM after recording intensities in the desired focal spot produced by several randomly structured illuminations of the scattering medium. The method of PhaseLift or PhaseCut [25] is implemented by supplying additional magnitude measurements: masks or gratings. Therefore, both methods are suitable for application with SLM or DMD devices. However, these methods are computationally more complex than iterative algorithms (Gerschberg-Saxton) and are slow with the same computational powers.

The core mathematical approach applied to measuring TM is phase retrieval (PR), which deals with the inverse problem of recovering the unknown phase of a signal solely from magnitude measurements. The problem is usually stated by solving quadratic equation.

(1): $y[r] = |\langle a\_r, x\rangle|^2$, for $r = 1, 2, \ldots, m$,

where $x \in C^n$ is the phase value we are looking find, $a_r \in C^n$ are sampling vectors or an optical signal in our case, both x and a_r form the intensity distribution on the image sensor or observed measurements $y[r] \in R$. $<a_r, x>$ notation defines both values belonging to one signal and are conjugate, denoting the signal and its phase. After an optical beam lands on a physical surface or image sensor, it is only possible to register its energy, which is denoted as a squared modulus value. PR is an example of non-convex programming, known as NP-hard.

PR methods can be broadly categorised into four main approaches: (i) iterative methods that refine phase estimates via feedback; (ii) lifting methods that use convex optimization; (iii) non-lifting gradient methods that are faster alternatives; (iv) neural network-assisted approaches that learn mappings from training data.

The examples of PR methods based on the classification above are the following:

- Iterative methods (e.g., Gerschberg-Saxton, Fienup) [26];
- Lifting and semidefinite programming methods (PhaseLift, PhaseMax, PhaseCut) [27], [28];
- Non-lifting relaxation gradient descent methods (Twisted Amplitude Flow, Truncated Wirtinger Flow) [29];
- Sparse and neural network assisted methods [30]–[34].

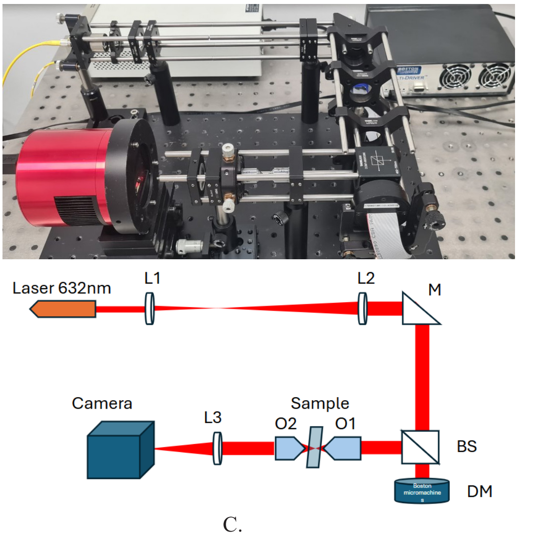
Wirtinger flow is a gradient descent algorithm with specific spectral initialization and iterative updating of the initial estimate (x^0), applying a gradient descent approach. Small step sizes (μ) are applied in the first iterations and larger steps in the later iterations. When the random masks D_n are chosen, $R \geq c \log 4 N$ masks are a sufficient criterion for Wirtinger Flow to perform uniquely up to a global phase (no local minima problem) in noiseless conditions [29]. Our previous experience proved the Wirtinger Flow PR could be fast enough and robust for wavefront reconstruction.

During the project, it is proposed to use DMD instead of static masks and significantly improve the sensitivity and stability of phase retrieval for TM identification. Recently, the gradient descent algorithm PR method has been applied for TM measurement using fluorescent guides [33].

2. EXPERIMENTAL SETUP

In our experimental configuration, we utilise a reflective segmented Micromirror Device (DM), which is Boston Micromachine SL235, to project structured illumination patterns onto the PTFE sample (Fig. 1). The optical setup includes a 632 nm diode-pumped solid-state laser that is spatially filtered and collimated by lenses L1 and L2 before being modulated by the DM. A 4F system relays the DM pattern output onto the sample. The transmitted light is collected using a microscope objective and imaged onto a scientific CMOS (sCMOS) camera for recording intensity patterns. To enhance the signal-to-noise ratio and ensure spatial coherence, we apply an aperture and polarizer in the optical path. Measurements are acquired for multiple illumination patterns generated via Hadamard basis and Gaussian random patterns, allowing for robust recovery of phase information via Wirtinger Flow and other algorithms. During the measurement, the sample is placed between the microscopy objective O1 and O2 to introduce the deterioration of signal. Resulting diffraction pattern is seen in Fig. 1.

A.



B.

C.

Fig. 1. A. Image of optical setup on breadboard. B. Schematic of the optical setup. L1, L2, and L3 are lenses, M – mirror, BS – beam splitter, DM – deformable mirror, O1 and O2 – microscopy objectives.

We tested two different measurement strategies. In the first strategy, we used the 32 randomly generated phase masks on the deformable mirror (Fig. 2). In the second approach, we used a one-to-one method, where each actuator on the deformable mirror was activated individually – 140 steps in total (Fig. 3). These two methods enable us to test the trade-offs between simplicity and accuracy in recovering the transmission matrix. In both cases, a DM was employed for structured illumination; however, the patterns varied based on the strategy used.

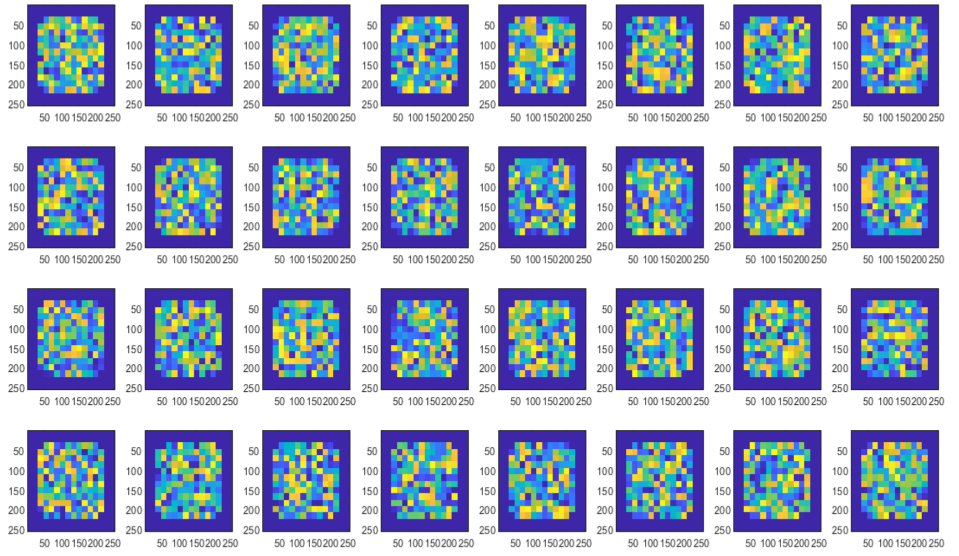


Fig. 2. 32 randomly generated phase masks presented on the deformable mirror. In each measurement, a sequential pattern is presented, and the respective diffraction pattern is recorded on a camera.

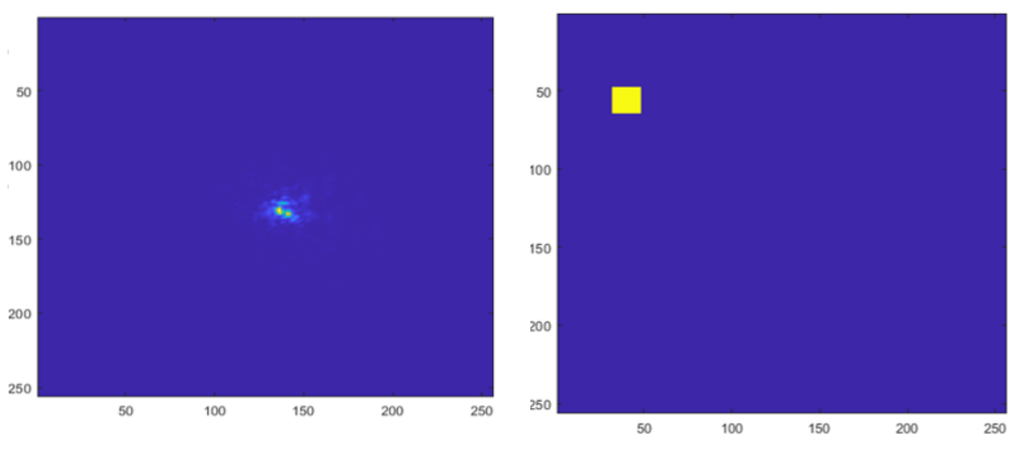


Fig. 3. One-to-one method, where each actuator on the deformable mirror was activated to the maximum state (2.5 micrometres) individually, generating 140 steps in total. The diffraction pattern after activation of each actuator was recorded.

3. SAMPLES

Polytetrafluoroethylene (PTFE) was selected due to its favourable optical properties and structural stability under various conditions. The sample preparation involved pressing PTFE powder into thin discs of 1 mm thickness. Prior to measurements, the samples were cleaned with isopropanol and dried to remove surface contaminants. A high scattering coefficient and low absorption in the visible range makes PTFE ideal for phase retrieval validation, where scattering dominates the transmission behaviour. Compared to biological tissues or silica-based phantoms, PTFE offers superior scattering stability and reproducibility under repeated measurements.

4. RESULTS AND DISCUSSION

The application of Wirtinger Flow phase retrieval successfully reconstructed the transmission matrix of the PTFE sample with high fidelity. The output spot intensities achieved focal enhancements exceeding 10× the average background intensity, confirming the ability to recover phase information solely from intensity measurements. The comparison between simulation and experimental data showed a mean squared error (MSE) of 0.012, indicating a close match. These findings support the hypothesis that phase retrieval techniques, especially those leveraging DMD-based structured illumination, can provide a reference-less route to TM identification suitable for phase imaging contexts. The one-to-one method was more successful (Fig. 4) over the 32 random matrices.

Each actuator's influence was easier to isolate and model using a one-to-one approach. This gave us a cleaner dataset, which performed better during the phase retrieval process. To sum up, the one-to-one approach proved more reliable than the random matrices, at least with our current setup. The Twisted Wirtinger Flow algorithm showed the best performance so far in retrieving phase and reconstructing the phase matrix. Additional metrics included a signal-to-noise ratio (SNR) of 18.5 dB and a phase correlation coefficient of 0.92, supporting the robustness of the reconstruction. Future improvements could involve using higher-resolution deformable mirrors or implementing calibration algorithms to correct for segment-induced artefacts.

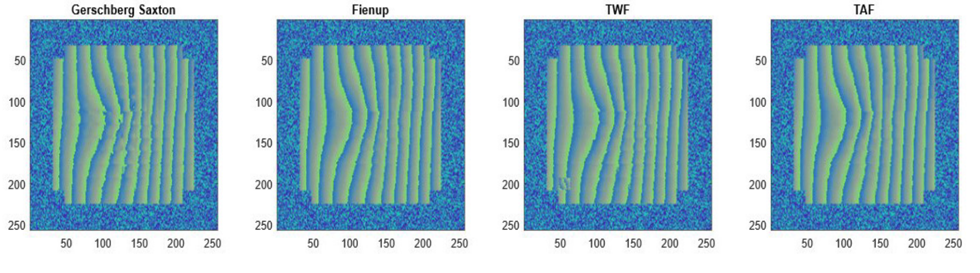


Fig. 4. The one-to-one method results for the four phase retrieval methods, measured without the sample. The phase map represents the intrinsic optical aberrations of the optical setup.

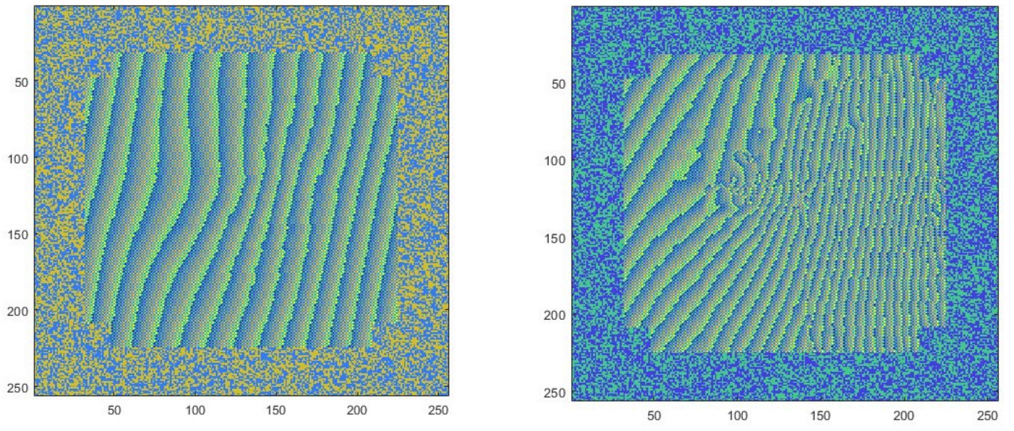


Fig. 5. Tilt aberration in another attempt for empty matrix (on the left picture) and with PTFE 30 micron (on the right) retrieved by the TWF method.

Our findings align with and extend the research by Karitāns et al. [35], who demonstrated real-time optical wavefront shaping using FPGA-based control of a segmented deformable mirror. While their implementation focused on hardware acceleration for dynamic phase modulation, our approach complements their research by enabling transmission matrix reconstruction through reference-free phase retrieval. The integration of FPGA systems, as shown in their study, could significantly enhance the speed and adaptability of our method, particularly for applications requiring real-time feedback in scattering environments.

5. CONCLUSIONS

This study has demonstrated that phase retrieval methods, specifically Wirtinger Flow, can be used to reconstruct transmission matrices of scattering materials like PTFE without requiring reference beams. By employing DMD-generated structured illumination, we have created an adaptable and stable system capable of handling rapid phase distortions.

These capabilities are essential for future applications in biomedical imaging, where real-time adaptability and robustness are critical. Our results open possibilities for fast, non-invasive imaging through complex media with limited access to full-field data. Despite significant progress in wavefront shaping, most transmission matrix (TM) identification methods require either reference beams or sensor placement behind the sample – conditions that are not feasible in biomedical contexts.

This study has addressed this limitation by demonstrating a reference-free phase retrieval approach using DMD-based structured illumination and Wirtinger Flow optimization. Future research will focus on extending this approach to dynamic biological tissues and integrating it with real-time imaging modalities.

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