

**QUANTITATIVE ASSESSMENT MATHEMATICAL MODEL
FOR MILITARY COMBAT ENGAGEMENT
BY USING STOCHASTIC MODELING AND SIMULATION**

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Abstract: *This paper presents an integrated model for analyzing combat engagement between two armor units and predicting battle outcomes through stochastic analysis. The proposed model examines the key elements of both one-on-one and three-on-two combat, incorporating technical and tactical elements such as double armor, superior firing accuracy, and ambush conditions. The conceptual framework is based on classical Markov chains, which serve as mathematical tools for simulating processes that evolve over time under conditions of uncertainty. These models are founded on the principle that a system's future state is determined by its current state, rather than by the sequence of events that produced it. Consequently, the system's transition history is embedded in the current state matrix, which serves as the basis for modelling and simulation subsequent transition processes.*

Keywords: Stochastic analyse, Markov chain, mathematical model, combat engagement

1. Introduction

The aim of the research conducted is to develop a quantitative mathematical model capable of calculating the outcome of an engagement between two units under varying conditions by applying stochastic processes.

Stochastic modeling is used in fields where uncertainty is a defining characteristic of systems or phenomena. In a stochastic process, the evolution of the system is random, depending on probability distributions, which determines the specific hazard of a fight to be incorporated into the mathematical model. Stochastic processes describe systems in which deterministic dynamics are influenced by random variables, resulting in behavior that reflects the interaction between predictable structure

and inherent uncertainty.

In the proposed conceptual model, we start from the use of classical Markov chains as mathematical tools to simulate processes that evolve over time under the influence of uncertainty.

In a Markov chain, the current state of the system is independent of past transient states of the system's evolution. In other words, the system's present state is not determined by its historical trajectory or prior behavior. This characteristic, known as the Markov property, may appear restrictive in that stipulates that the probability of a current event depends solely on the most recent state. However, it effectively allows the entire historical evolution of the system to be encapsulated within its recent memory.

A basic illustration of a Markov process, used in our mathematical model, is the random movement through the nodes of a graph, where each transition represents a move from one node to one of its successor

nodes with a given probability, independently of the path previously followed (figure 1). Understanding this process provides the conceptual foundation of the proposed mathematical model.

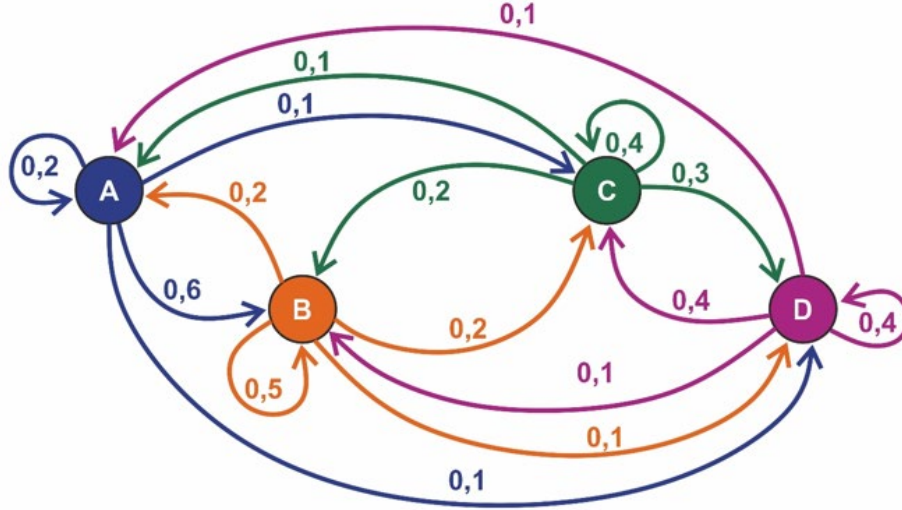


Figure 1: The proposed Markov chain by authors for mathematical model

In a Markov chain or process, at any time, the system can change or remain in its state. Each transition is described by a probability distribution. The sum of the transition probabilities for a state A_i to all possible states is equal to unity.

The transition matrix corresponding to a four-state Markov chain is:

$$(M) = \begin{pmatrix} p_{A \rightarrow A} & p_{A \rightarrow B} & p_{A \rightarrow C} & p_{A \rightarrow D} \\ p_{B \rightarrow A} & p_{B \rightarrow B} & p_{B \rightarrow C} & p_{B \rightarrow D} \\ p_{C \rightarrow A} & p_{C \rightarrow B} & p_{C \rightarrow C} & p_{C \rightarrow D} \\ p_{D \rightarrow A} & p_{D \rightarrow B} & p_{D \rightarrow C} & p_{D \rightarrow D} \end{pmatrix}$$

$$(M) = \begin{pmatrix} 0.2 & 0.6 & 0.1 & 0.1 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.4 & 0.3 \\ 0.1 & 0.1 & 0.4 & 0.4 \end{pmatrix} \quad (1)$$

where:

$p_{A \rightarrow A}$ represent the probability that the system remains in state A ,

$p_{A \rightarrow B}$ represent the probability that the system will transit from state A to state B .

$p_{X \rightarrow Y}$ represent the probability that the system will transit from state X to state Y , where X and Y could be A , B or C .

In the military field, Markov chains are used to analyze the dynamics of armed conflicts,

offering a stochastic alternative to deterministic models, such as Lancaster equations.

Within the proposed mathematical framework, classical Markov chains serve as the primary mathematical tools for modeling and simulating processes whose evolution over time is shaped by uncertainty.

The general goal of using Markov models in the military field is to maximize the probability of mission success by transforming tactical uncertainty into manageable mathematical variables.

2. The Conceptual Model

2.1. Combat engagement

From a NATO doctrinal perspective, combat engagement represents the tactical level interaction in which opposing forces employ combat power to generate the desired effects against an adversary, an assigned objective or target. In AJP-3.9, NATO Allied Joint Doctrine for Joint Targeting [1], combat engagement is defined as an action against an adversary, in accordance with the law of armed conflict.

In order to support the analysis and to predict

the evolution of such interactions, we developed this stochastic model, based on a rigorous mathematical framework.

The model was developed by incorporating the rules of engagement applicable to conventional warfare, as established in NATO procedures.

Combat engagement is closely linked to the decision-making cycle within NATO planning and execution processes. It evolves through successive decision points, shaped by situational awareness, mission objectives, force capabilities, risk, anticipated effects and operational uncertainty. The integration of the three concepts: combat engagement, rules of engagement and decision-making process provide a doctrinal basis for stochastic modelling. By incorporating probabilistic state transitions, doctrinal constraints and decision criteria, the model enables a rigorous analysis of combat dynamics within the NATO operational context.

2.2 The Fundamental Model: Markov Chain with Absorbing States Model

In order to set the right parameters of mission success for the proposed stochastic mathematical framework, we used in our research the NATO dynamic targeting model F2T2EA and the indicators of measurement in correlation with the OODA loop (Observe-Orient-Decide-Act) [1]. Within this context, two primary categories of indicators are used to characterize the state of the system: Measures of Performance, and Measures of

Effectiveness. In addition, the F2T2EA dynamic targeting model was analyzed in correlation with the OODA loop used in the decision-making process, as described in Air and Missile Defense (AMD) kill chain. The six functions described in the dynamic targeting process are:

1. Find (detection);
2. Fix (localization);
3. Track (prioritize);
4. Target (select method);
5. Engage (order engagement);
6. Combat assessment (strike)

In this study, the F2T2EA targeting cycle [1] was employed as a conceptual framework to simulate the battlefield environment, with the phases compressed and parameterized through assigned probability values, enabling the stochastic representation and subsequent quantitative evaluation of combat effects between two opposing armored units.

Modeling F2T2EA as a Markov chain is an effective analytical method for assessing the probability of mission success. Each stage is treated as a state, and the transition from one stage to another depends on transition probability. If any link fails (for example, the target is lost during the *Track* phase), the process returns to the start or is abandoned, making the entire system sensitive to a failure rate at a critical point.

Within the model, we can compress some stages, and we define the following stages and probability of success (table 1).

Table 1 Stages and associated probabilities

Stage	Description	Probability of Success
Fing & Fix	Detection and localisation of the target	0.9
Track & Target	Maintaining track, weapon allocation	0.9
Engage	Order engagement and strike	0.8
Asses	Assessing the target and report results	0.8

By using this compression, we can make an analogy between the four-nodes Markov model shown in *Figure 1* and the combat model of two tanks subunits.

The matrix of probabilities of success is given by the history of previous operations or the experience of the military analyst. If any stage fails, the system can go into a

"Mission Failed" state or return to a previous stage (for example, if contact is lost, it returns to *Find* stage). Simulations of this type help us decide where to invest more: better sensors (increase target detection and location) or more precise ammunition (increase target destruction).

A Markov chain with absorbing states is a system in which, once the process reaches a certain state, it cannot exit it. So, we will understand an absorbing state, the state in which the probability of remaining in itself is equal to one:

$$p_{A_i \rightarrow A_i} = 1 \quad (2)$$

An absorbing chain is defined by the presence of at least one absorbing state reachable from any transient state, together with the condition that the process cannot remain in transient states indefinitely.

Suppose we have a confrontation in which two tanks, **A** and **B**.

The following states are defined:

- **State 0**: Both tanks are operational (battle continues).
- **State 1**: Tank **A** has destroyed tank **B** (**A** wins - absorbing state).
- **State 2**: Tank **B** has destroyed tank **A** (**B** wins - absorbing state).
- **State 3**: Both tanks are destroyed (lethal draw - absorbing state).

3. Integrated Combat Model

3.1. The One-on-One Duel

Let's: p_a – be the probability that tank **A** hits and destroys tank **B** in a single shot, and p_b – be the probability that tank **B** destroys tank **A**.

The transition probabilities from **State 0** to the next shot are:

- $p_{01} = p_a(1 - p_b)$: only **A** hits;
- $p_{02} = p_b(1 - p_a)$: only **B** hits;
- $p_{03} = p_a \cdot p_b$: both hit simultaneously;
- $p_{00} = (1 - p_a)(1 - p_b)$: both miss the shot (the battle continues).

The probability that tank **A** wins is given by the relationship:

$$p(A \text{ win}) = \frac{p_a(1-p_b)}{1-(1-p_a)(1-p_b)} \quad (3)$$

This simple model allows commanders to estimate likely casualties before committing units to a specific sector.

3.2. The Three-on-Two Duel

3.2.1. The Three-on-Two Model with Equal Odds

When we move from a one-on-one duel to a group confrontation (e.g. three tanks against two), the Markov model becomes more complex because the number of possible states increases exponentially. In this scenario, we are no longer just talking about who wins, but about how many units survive from each group.

In this case, the system is described by the pair (n_A, n_B) , where:

- n_A represents the number of units of tank **A**: (3, 2, 1, 0)
- n_B represents the number of units of tank **B**: (2, 1, 0)

We note that for the given example we have 12 possible states.

The fight ends when:

$$n_A = 0 \text{ or } n_B = 0 \quad (4)$$

states that we will call absorbing.

Let us assume that both sides have an equal probability of accuracy:

$$p_a = p_b = 0,3 \quad (5)$$

Key conclusions for the 3 vs 2 model:

1. *Disproportionate numerical advantage*: Although group **A** has only one more tank, its probability of victory is over 80%. This mathematically confirms Lanchester's square law: the firepower of a force increases with the square of the number of units.

2. *Accuracy effect*: As accuracy increases, the chances of a lethal draw (both sides destroy each other in the same round) increase slightly, but group **A** retains its dominance.

3. *Economy of force*: The Markov model shows that it is much cheaper to win a battle through numerical superiority at the beginning than by trying to compensate for a numerical deficit with better technology.

4. *Numerical superiority* provides an asymmetric (not just linear) advantage, with group *A* having a probability of success over 80% even at equal accuracy level.

3.2.2. *The Three-on-Two Model with Different Accuracy Levels*

We ask ourselves what group *B* (the two tanks) should do to balance the odds against group *A* with its three tanks.

To compensate for the numerical inferiority

(two against three), the tanks in group *B* must have significantly higher individual accuracy than those in group *A*.

According to the Markov simulation model based on simultaneous firefights, table 2 shows the thresholds needed to balance the odds at 50%. If the tanks in group *A* have standard accuracy, here is how accurate group *B* must be to not lose the fight:

Table 2 Thresholds for a balanced battle

Group A accuracy	Required accuracy for Group B	Performance multiplier
10% (poor)	$\cong 22\%$	2,2 times
20% (average)	$\cong 46\%$	2,3 times
30% (good)	$\cong 75\%$	2,5 times
40% (very good)	$\cong 95\%$	$\cong$ impossible

Tactical conclusions for the 3 vs 2 model with different accuracies:

- *Technological limit*: If group *A* (the larger force) has an accuracy of over 40%, it is mathematically almost impossible for group *B* to balance the situation with accuracy alone, because even with 100% accuracy, the volume of fire of the three tanks will destroy group *B* before it eliminates all targets;
- *Asymmetry*: To compensate for a numerical deficit of just one unit, group *B* must be more than twice as accurate as the opponent.
- *Square law*: These data confirm that in modern warfare, quantity has its own meaning: a linear increase in the number of units requires an exponential increase in technology to counter it.

3.2.3. *The Three-on-Two Model with Superior Armour*

We can refine the model by answering the question: How would adding a superior armour system (which would make a tank withstand two hits instead of one) change the situation?

Introducing superior armour (the ability to withstand two hits instead of one) radically changes the Markov dynamics, transforming the unit from a binary live-dead state into a three-state system:

Intact → Damaged → Destroyed

We will analyse how armour influences the odds in a battle between two dual-state tanks against three standard tanks, assuming an equal accuracy of 30% for both sides.

Table 3 Win probability with superior armour

Configuration	Win probability	Result
3 vs 2 (no armour)	$\cong 18\%$ for group B	Almost certain defeat
3 vs 2 (with x 2 armour)	$\cong 62\%$ for group B	probably win

In this scenario, group *B* (the two tanks) has a dual survivability system. For a tank in group *B* to be eliminated, it must be hit

twice, while tanks in group *A* are destroyed on the first hit (table 3).

From the given example it is observed that

the benefit of armour is cumulative and is more effective than accuracy:

- *Fire absorption*: In the first rounds, Group **A** fires with all three barrels, but does not eliminate any units from **B** (it only brings them to a state of damage);
- *Firepower maintenance*: Since the tanks of Group **B** survive the first hit, they retain the ability to fire with both units for a longer time;
- *Attrition effect*: Group **A** loses units (and implicitly firepower) as soon as it is hit, while Group **B** loses its firepower only after receiving significant damage.
- *Break-even point*: From a military point of view, this Markov model demonstrates that:
 - *Armor beats accuracy*: It is often cheaper and safer to double the resistance of a unit than to try to double its accuracy (which is limited by physics and sensors);
 - *Shock Force*: A hard-to-destroy unit can destroy numerically superior forces simply by maintaining its presence on the battlefield for several iterations of the firefight;
 - *Conclusion*: If group **B** (two units) has double armour, it becomes the favourite over group **A** (three units), completely reversing the odds from 18% to over 60%. For a tank in group **B** to be eliminated, it must be hit twice, while tanks in group **A** are destroyed on the first hit (table 3).

3.2.4. The Three-on-Two Model with Tactical Surprise (Ambush)

Another working problem concerns how we

want to analyze how tactical surprise influences the balance of forces (the fact that one group fires first), this problem being specific to ambush.

Tactical surprise (ambush) works as a force multiplier in the Markov model, because it allows one side to make the first move without the enemy being able to respond in that iteration. This breaks the symmetry of the battle and can completely nullify the numerical advantage of the opponent.

In a standard Markov model, both sides fire simultaneously. In an ambush, the model includes a “zero iteration”:

Step 0: Group **B** fires on Group **A**.

Probable result: If the accuracy is 30% there is a high chance that at least one tank in Group **A** will be eliminated or damaged before it can fire.

New starting state: The actual fight basically starts as a 2 vs 2 or even 2 vs 1, where group **B** already has the initiative.

If the accuracy is 30% and group **B** fires first, the probability that group **A** will lose a unit immediately is:

$$p_a = 1 - (1 - 0,3)^2 = 0,51 \text{ (51\%)} \quad (6)$$

If group **A** starts the fight with only two units left (a state that occurs in 51% of cases), their chances of victory drop drastically, according to Lancaster's square law.

According to military tactical studies, the first hit is often the decisive factor.

Table 4 Win probability in an ambush

Configuration	Win probability	Result
Direct combat	$\cong 18\%$ for group B	Group A dominates with superior firepower
Ambush (B fires first)	$\cong 55 - 60\%$ for group B	Group B becomes the favourite despite being outnumbered

In table 4 is described how the situation changes for a group **B** of two tanks ambushing a group **A** of three tanks:

Tactical conclusions for the ambush model:

- *Shock state*: In advanced Markov models, surprise not only eliminates units, but also reduces the enemy's accuracy in subsequent rounds;

- *Disorganization*: Over 1-2 iterations, the accuracy of group **A** can drop from 30% to 10% due to confusion and the need to identify the source of fire;

- *Decision paralysis*: The ambushed unit goes into the TARGET SEARCH Markov states instead of ENGAGE, giving group **B** time for a second salvo of fire;

• *Tactical surprise* allows a smaller force to achieve victory by eliminating the enemy's numerical advantage in the first round. In a two-on-three situation, a successful ambush turns group **B** from a sure loser into a favourite, with over a 55% chance of

success.

At this point, we can integrate all the elements presented above: 3 vs 2, armour and ambush) to see what happens with the proposed mathematical model (table 5).

Table 5 The cumulative effect of tactical elements on combat

Tactical element	Markov impact	Effect on combat
Ambush	Elimination on the first move	Group B fires first. With 50% accuracy, group A starts the battle with 1,5 tanks (on average), not 3.
Double armour	State multiplication	The tanks in B withstand 2 hits. Even if A fires, it does not immediately remove units from battle, keeping B's firepower intact
Superior accuracy	Transition	With an accuracy of 60% (compared to enemy's 30%), group B destroys the remaining units in Group A in maximum two rounds

If group B combines all the advantages, the probability of victory goes from a disastrous 18% threshold to a guaranteed success of over 90%.

Conclusions for the integrated simulation model where, in a Markov model in which group **B** consists of two units, has double the armour, double the accuracy and holds the initiative (fires first):

1. Round 0 (Ambush):

Group **B** fires and critically destroys/damages at least 1-2 enemy tanks.

2. Round 1:

Group **A**, now numerically reduced and in shock, retaliates. Their hits hit **B**'s heavy armour, which is not destroyed.

3. Round 2:

Group **B** executes the second salvo, ending the fight.

The final probability of victory for the group is greater than 95%.

4. Conclusions

The mathematical model based on stochastic processes, namely by using Markov

processes with absorption states, presents a tool for calculating the effects of a battle. This model allows the use of different states of the system and the integration of technical and tactical elements, ensuring an adequate understanding of the simulation of a battle between two units.

The study provided for two armor subunits demonstrates the scalability of the proposed mathematical model.

The analysis of the preceding examples using absorbing Markov chains demonstrates that the system inevitably reaches a final (absorbing) state, confirming the certainty of absorption.

The presented mathematical model can be extended for different combat scenarios by using software tools for stochastic analysis and prediction.

The proposed model can be further developed by transitioning from the four-state model to a six-state model corresponding to Drones Combat Engagement and their integration in the Joint Target Cycle. Nevertheless, this requires the

use of specialized software or the writing of a customized program.

Another potential direction for future research involves the integration of artificial

intelligence elements within the Combat Engagement or Joint Targeting Cycle to reduce decision-making time and improve target prioritization and allocation.

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