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ADDING ATMOSPHERIC TURBULENCE TO A MATHEMATICAL MODEL OF AN AIRCRAFT

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Abstract: *Regardless of the design, all types of aircraft are affected by atmospheric turbulence and air currents. Depending on the size, type and altitude of the flight, atmospheric disturbances affect them differently. The same disturbance acting on an unmanned aerial vehicle flying at high altitude, for example, a Predator UAV, and on a quadcopter flying a few meters above the ground, will have in the first case minimal or no impact on the flight parameters, and hence on the correct performance of the task, while in the case of a quadcopter it can even lead to a collision with the ground or a nearby obstacle (building, tree, etc.).*
The article examines the types of translational and turbulent atmospheric disturbances. The reasons for their occurrence are described. The article uses a mathematical model of an aircraft with six degrees of freedom. To create it, Euler's standard differential equations for the change in the amount of momentum and the theorem for the change in angular momentum were used. It is examined in detail how atmospheric disturbances should be added to the differential equations describing the aircraft. It is studied how wind gusts are converted into forces that affect the flight parameters. Depending on the type and size of the aircraft, these forces are of different magnitude and have different effects on the behavior of the aircraft in flight.

Keywords: Dryden model, atmospheric turbulence, nonlinear mode

1. Introduction

For accurate modelling of the motion of an aircraft [3], it is necessary to take into account the changes in the parameters of the surrounding environment when changing the altitude and flight speed, as well as to account for wind gusts and atmospheric turbulence. This requires making an accurate model of the atmosphere, which can be added to the aircraft model.

2. Atmospheric Turbulence Model

In atmospheric modelling, the most widely used model is the International Standard Atmosphere [2]. It assumes a uniform average dependence of the physical parameters of the air environment on height for all latitudes and longitudes of the Earth's geoid and for all seasons. This model is suitable for calculating and comparing the characteristics of different aircraft in the same conditions, but does not allow predicting the probabilistic characteristics of aircraft when used in different geographical regions and different seasons.

The reason for the occurrence of air currents is the uneven distribution of air pressure in the atmosphere. The disproportionate distribution of atmospheric pressure leads to the appearance of air currents. The season, the location above the

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Earth's surface, the relief of the area and the presence of vapor in the atmosphere are the factors that influence the degree to which the air is heated by the sun. In this case, significant temperature differences arise between the different layers in the horizontal and vertical directions, leading to changes in density and pressure. Therefore, the thermal energy transmitted by the sun is transformed into air particles' motion which occurs at different speeds and in various directions. If the particle velocity exceeds a certain limit, the motion becomes turbulent. Such motion is random and is accompanied by significant vortex formation [4].

Air currents can be classified in two categories: vertical and horizontal.

Vertical currents include updrafts and downdrafts, cloud currents, and eddies. The effect of these currents on aircraft flight depends on the magnitude of the vertical component of the air velocity, the spatial region occupied by the current itself, the aircraft's flight speed, etc. Horizontal currents include winds, wind layers, gusts, waves, and eddies that are constant in magnitude and direction.

Updrafts occur when air heated by contact with the earth's surface rises. Such currents are often observed on quiet summer afternoons. The speed of rise of warm air sometimes reaches 10÷12 m/s. When one of the wings of the aircraft is affected, the distribution of the lifting force changes and the aircraft rolls. At the moment of entering and exiting such a flow, the aircraft receives a sudden boost. With an upward flow, the aircraft receives an increase in lifting force, and with a downward flow, the lifting force decreases and a tendency to collapse appears.

Upward and downward flows are also observed when moving in clouds. In this case, within the boundaries of the cloud, the air rises up, and between the clouds it descends down.

Individual layers of air differ in temperature, humidity and density, therefore they begin to move relative to each other, as a result of which wind layers are formed. It is known that waves form at the boundary of two air layers moving relative to each other, which can be dangerous for the aircraft. Falling into such an area, the aircraft begins to shake.

Gusts of wind are also observed with all types of air currents. They are characterized by sharp changes in speed and direction of movement. Gusts vary in size and direction. Moreover, the greater the average speed of the current, the more intense the gust. Depending on the size and direction of the gust, the aircraft begins to experience sharp rises and falls, and also experiences lateral slip.

The speed of vertical wind gusts in the troposphere and low layers of the stratosphere can reach 10÷12 m/s with a turbulent layer strength of up to 1000 m and in the horizontal plane up to 200,000 m. The strongest turbulence is observed at an altitude of 7÷10,000 m.

Horizontal wind gusts, as well as horizontal currents, have significantly higher speeds than vertical ones, and with increasing height their speed increases. The speed of such wind gusts at an altitude of 7÷10,000 m can reach 20 m/s, and at higher altitudes and higher values.

Every air flow is accompanied by vortex formation. Moreover, the vortex's rotation speed is directly proportionate to the flow's speed. Air turbulence, which results from the contact of the air with the Earth's surface, thermal motion, the contact between layers of air moving at different speeds, etc., leads to the appearance of vortices.

Vortices caused by the turbulent motion of the air commonly have a horizontal rotation axis, although their rotation can sometimes occur around a vertical axis.

In modelling atmospheric disturbances, the wind speed can be represented as:

$V_{wind} = V_1 + V_c$, (1)

where: V_1 is the constant component of the wind, V_c is a random variable.

The movement of the atmosphere is a

turbulent process, which means that at any given moment the speed and direction of the wind are random. Turbulence characterizes the variable component of the wind speed. The constant component at low altitudes is approximately described by the power law:

$$\frac{V_1}{V_0} = \left(\frac{H_1}{H_0}\right)^n \quad (2)$$

where: V_1- is the mean value of the wind velocity at a certain altitude H_1;
V_0 is the average value of the constant component of the wind at the reference height H_0;
n is an indicator dependent on weather conditions.
Typical values that can be used are H_0=10 m, at this altitude the speed is V_0=3÷4 m/s, with exponent n=0.15÷0.2.
These values are valid for altitudes of 300÷500 m.
For a standard atmosphere, according to [2], the constant component of the wind speed can be approximated by the following equations:

$$V_1 = V_{9,15}\frac{H^{0,2545} - 0,4097}{1,3470} \qquad (0< \text{ H } < 300 \text{ m});$$

$$V_1 = 2.86586 \ V_{9,15} \ (\text{H} \geq 300 \text{ m}), (3)$$

where $V_{9.15}$ is the wind speed at height 9,15 m: $V_{9.15} = 1(m/s)$.
The impact of the direction of the constant component of the wind on the flight of the aircraft is shown in Fig. 1 and is calculated using the following expressions:

$$V_{xwind} = V_1\cos(\psi_w - \pi)\cos\psi + V_1\sin(\psi_w - \pi)\sin\psi$$
$$V_{cwind} = -V_1\cos(\psi_w - \pi)\sin\psi + V_1\sin(\psi_w - \pi)\cos\psi$$

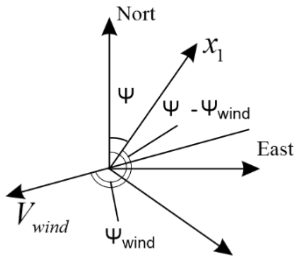

Figure 1: Reading the direction of the constant component of the wind

Statistical methods, which employ the assumptions of stationarity, uniformity and isotropicity, define the wind velocity's variable component. Stationarity implies independence from time. Isotropicity implies independence from direction, which is not true for low-altitude flights over rough terrain.
Uniformity implies independence of between the points and the averaging interval:

$$V_c = V_c(X,t) = V_c(V,t) \quad (4)$$

The spatial distribution of wind is seen as frozen, and considering the time, it is assumed that the aircraft pierces this distribution (Taylor's hypothesis). To estimate isotropic turbulence, two components of the random wind speed are used: the component along the tangent to the trajectory V_t and normal constituent V_n.
The results which are obtained experimentally are subjected to statistical processing in the form of simple approximation using empirical formulas, in which the spectral densities are fractional rational functions of the frequency [1]. In this way, the wind effects are seen as white noise passed through classical structure linear shaping filters. The most common are the empirical formulas for the correlation functions of the longitudinal and transverse components of the turbulent disturbances of the form:

$$R_t = \sigma^2 e^{-\frac{|S|}{L}};$$
$$R_n = \sigma^2 \left(1 - \frac{|S|}{2L} e\right)^{\frac{|S|}{L}}, \qquad (5)$$

in which:
σ - turbulent motion dispersion;
L - turbulence scale;
S - distance between two points in the turbulence field.

The transition from correlation functions to spectral densities as a function of spatial angular frequency is performed by applying the Wiener - Hinchin theorem:

$$S_t(\Omega) = \sigma^2 \frac{L}{\pi} \frac{2}{1+\Omega^2 L^2};$$
$$S_n(\Omega) = \sigma^2 \frac{L}{\pi} \frac{1+3\Omega^2 L^2}{\left(1+\Omega^2 L^2\right)^2}. \qquad (6)$$

It usually lies within the range of 150÷1500 m, and the recommended [1] average values are L=300 m at H≥300 m and L=H at H≤300 m. These values are recommended as a result of measurements of the overloads created during flight in a turbulent atmosphere:

$$L_y = L_z = \frac{H}{\left(0{,}177 + 0{,}000823h\right)^{1{,}2}}. \qquad (7)$$

The distribution of the moment values for the components of turbulent disturbances is described by a normal distribution law. For example, for the component along the X axis the probability density is given by [1]:

$$P(W_x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(W_x - W_{x0})^2}{2\sigma^2}}. \qquad (8)$$

3. Research on the Influence of Atmospheric Disturbances on UAV Movement

To model a turbulent atmosphere, it is necessary to express the spectral densities in real scale, which requires modeling a random stationary process with a normal distribution.

First, it is necessary to use the relationship between the spatial (Ω) and temporal angular velocities (ω), the distance X and the flight speed V [1] (9):

$$\Omega = \frac{\omega t}{x} = \frac{\omega}{V}, \qquad (9)$$
$$S_{t(\omega)} = \sigma^2 \frac{L}{\pi V} \frac{2}{1+\frac{\omega^2 L^2}{V^2}} = \sigma^2 \frac{a}{\pi} \frac{2}{1+\omega^2 a^2},$$
$$S_{n(\omega)} = \sigma^2 \frac{L}{\pi V} \frac{1+3\omega^2 \frac{L^2}{V^2}}{\left(1+\frac{\omega^2 L^2}{V^2}\right)^2} = \sigma^2 \frac{a}{\pi} \frac{1+3\omega^2 a^2}{\left(1+\omega^2 a^2\right)^2},$$

where: $a = \frac{L}{V}$ the time constant of the shaping filter.

Signals with the desired spectral densities are obtained from white noise with spectral density after passing through shaping filters with the following frequency characteristics [1]:

$$K_{t(j\omega)} = \sigma \sqrt{\frac{a}{\pi S_0} \frac{1.41}{1+ja\omega}},$$
$$K_{n(j\omega)} = \sigma \sqrt{\frac{a}{\pi S_0} \frac{1+j1.73a\omega}{\left(1+ja\omega\right)^2}}. \qquad (10)$$

The tangential component is obtained after passing white noise through an aperiodic unit with time constant a. The normal component is obtained after passing the signal through three series-connected filters – two aperiodic ones with the same time constant and a forcing unit with time constant $\sqrt{3}\frac{L}{V}$.

The final version of the atmospheric disturbance modeling algorithm is obtained as a combination of the stochastic and deterministic models.

Taking into account the influence of atmospheric disturbances on the aircraft motion model is reduced to adding the wind speed components to the road speed components in the force and moment expressions [6]. The specific independent arguments are airspeed, glide angle and angle of attack. To take atmospheric disturbances into account, it is necessary to modify these three arguments appropriately. To simulate the influence of atmospheric disturbances on the flight, a nonlinear mathematical model of an aircraft was compiled. To create it, Euler's standard differential equations for the change in the amount of motion $m\frac{d\vec{V}}{dt} = \sum \vec{F}$ were used, where $\frac{d\vec{V}}{dt}$- absolute acceleration in an inertial (stationary) coordinate system, F - resultant of external forces, and the theorem for the change in kinetic momentum $\frac{d\vec{K}}{dt} = \sum \vec{M}$, where K- vector of kinetic momentum and M- resultant vector of external moments. Coupled $Mx_1y_1z_1$ and velocity $Mx_ay_az_a$ coordinate systems were used.

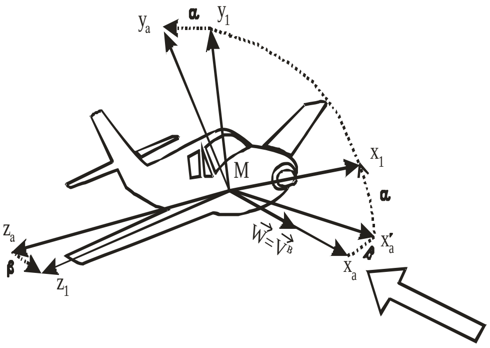

Figure 2: Coordinate systems used

If the velocity vector in the coupled coordinate system(ccs) is represented as the sum of the atmospheric disturbance vector and the velocity vector in the undisturbed atmosphere, we obtain:

$$\begin{aligned} \vec{V}_{ccs} &= \vec{V}_1 + \vec{V}_a \\ V_{xccs} &= V_{x1} + V_{ax} \\ V_{yccs} &= V_{y1} + V_{yx} \\ V_{zccs} &= V_{z1} + V_{ax} \end{aligned} \tag{11}$$

where: $\vec{V}_1$ - velocity vector in an undisturbed atmosphere in the coupled coordinate system;

$\vec{V}_a = \vec{V}_{det.} + \vec{V}_{st.}$ – atmospheric disturbance vector;

$\vec{V}_{det.}$ - deterministic component of atmospheric disturbances;

$\vec{V}_{st.}$ - stochastic component of atmospheric disturbances.

Replacing V_{xccs} , V_{yccs}, V_{zccs} from (11) in the projections of the theorem for changing the amount of motion along the axes the associated coordinate system $Mx_1y_1z_1$ [5] is obtained:

$$\begin{aligned} m\ \ \frac{dV_{xccs}}{dt} + \omega_y V_{zccs} - \omega_z V_{yccs}\Big) &= \sum F_x \\ m\ \ \frac{dV_{yccs}}{dt} + \omega_z V_{xccs} - \omega_x V_{zccs}\Big) &= \sum F_y \\ m\ \ \frac{dV_{zccs}}{dt} + \omega_x V_{yccs} - \omega_y V_{xccs}\Big) &= \sum F_z \end{aligned} \tag{12}$$

Differential equations (12) are transformed into the form:

$$\begin{aligned} \dot{V}_{x1} &= \frac{F_x}{m} - \omega_y V_{zccs} + \omega_z V_{yccs} - \dot{V}_{xa} \\ \dot{V}_{y1} &= \frac{F_y}{m} - \omega_z V_{xccs} + \omega_x V_{zccs} - \dot{V}_{ya} \\ \dot{V}_{z1} &= \frac{F_z}{m} - \omega_x V_{yccs} + \omega_y V_{xccs} - \dot{V}_{za} \end{aligned} \tag{13}$$

From equations (13) get V_{xccs} , V_{yccs}, V_{zccs} with their equal values from (11) and we get:

$$\begin{aligned} \dot{V}_{x1} &= \ \frac{F_x}{m} - \omega_y V_{zccs} + \omega_z V_{yccs}\Big) + \big(\omega_z V_{ya} - \omega_y V_{za} - \dot{V}_{xa} \\ \dot{V}_{y1} &= \ \frac{F_y}{m} - \omega_z V_{xccs} + \omega_x V_{zccs}\Big) + \big(\omega_x V_{za} - \omega_z V_{xa} - \dot{V}_{ya} \\ \dot{V}_{z1} &= \ \frac{F_z}{m} - \omega_x V_{yccs} + \omega_y V_{xccs}\Big) + \big(\omega_y V_{xa} - \omega_x V_{ya} - \dot{V}_{za} \end{aligned} \tag{14}$$

Atmospheric effects can be represented as forces. In equations (14) the accelerations $\dot{V}_{xa}, \dot{V}_{ya}, \dot{V}_{za}$ are taken as forces normalized by the mass of the object. The second summands on the right-hand side of equations (14) can be replaced by:

$$\begin{aligned} F_{xa} &= m\big(\omega_z V_{ya} - \omega_y V_{za} - \dot{V}_{xa}\big) \\ F_{ya} &= m\big(\omega_x V_{za} - \omega_z V_{xa} - \dot{V}_{ya}\big) \\ F_{za} &= m\big(\omega_y V_{xa} - \omega_x V_{ya} - \dot{V}_{za}\big) \end{aligned} \tag{15}$$

Therefore, after putting equations (15) into equations (14) we get:

$$\begin{aligned} \dot{V}_{x1} &= \frac{1}{m}(F_x + F_{xa}) - \omega_y V_{z1} + \omega_z V_{y1} \\ \dot{V}_{y1} &= \frac{1}{m}\big(F_y + F_{ya}\big) - \omega_z V_{x1} + \omega_x V_{z1} \\ \dot{V}_{z1} &= \frac{1}{m}(F_z + F_{za}) - \omega_x V_{y1} + \omega_y V_{x1} \end{aligned} \tag{16}$$

Expressions (16) have the general form. From equations (16) we can obtain differential equations for V, α, β (17):

$$\begin{aligned} \dot{V} &= \dot{V}_{x1}\cos\alpha\cos\beta + \dot{V}_{y1}\sin\alpha\cos\beta + \dot{V}_{z1}\sin\beta; \\ \dot{\alpha} &= \frac{\dot{V}_{x1}\sin\alpha - \dot{V}_{y1}\cos\alpha}{V\cos\beta}; \\ \dot{\beta} &= \frac{\dot{V}_{z1} - \dot{V}\sin\beta}{V\cos\beta}; \end{aligned} \tag{17}$$

To take into account the influence of atmospheric disturbances on the flight parameters, it is sufficient to integrate only the three equations (17). Equations (16) do not need to be integrated, and after calculation their right-hand sides are taken as values for $\dot{V}_{x1}, \dot{V}_{y1}, \dot{V}_{z1}$ в (17).

Figure 2 shows the wind projections along the axes of the coupled coordinate system, obtained from expressions (11). The graphs in it were obtained when modeling a horizontal flight at an altitude of 1000 m and a flight speed of 100 m/s.

4. Conclusion

The angle between the flight control and the wind direction shown in Fig. 1 is $\psi - \psi_{wind} = 30^{\circ}$. The graphs were obtained after integrating equations (17), in which the expressions (11÷16) were substituted.

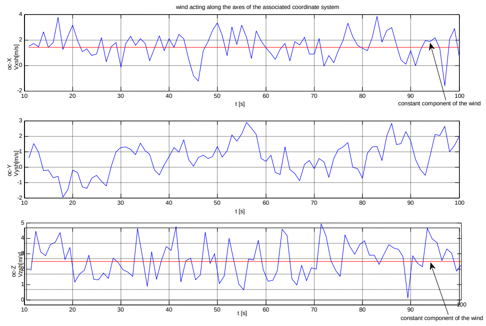

Figure 3: Wind modeling along the axes of the coupled coordinate system

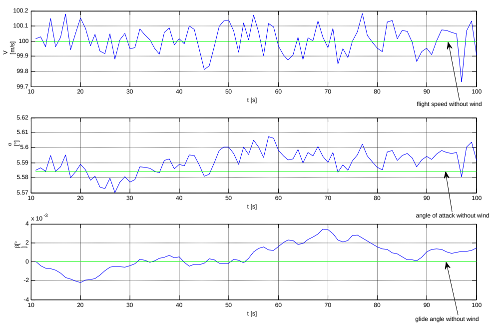

Figure 4: Influence of turbulence on flight Parameters

Figure 3 and figure 4 shows how the wind from Fig. 1 affects the flight parameters.

1. The impact of atmospheric turbulence on flight parameters has been modeled.
2. A model of the forces caused by atmospheric turbulence and acting along the axes of the UAV has been obtained.
3. The resulting model allows for modeling atmospheric disturbances of different strength and direction and for investigating their impact on UAV flight.

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