sciendo International Conference KNOWLEDGE-BASED ORGANIZATION
Vol. XXXI No 3 2025

MATHEMATICAL MODEL OF THE COMBAT USE OF AN AIRCRAFT GUN AGAINST A GROUND TARGET

Milen ATANASOV, Yavor VARBANOV

"Georgi Benkovski" Bulgarian Air Force Academy, Dolna Mitropolia, Bulgaria
matanasov@af-acad.bg, yavorvarbanovaf@gmail.com

Abstract: *The introduction of the latest achievements of science and technology in combat aviation allows for the development and implementation of a general algorithm for the combat use of various groups of weapons, as well as the improvement of avionics crucial for the execution of combat missions. The use of a general algorithm in aviation combat complexes will reduce the duration of the pilot's decision –making process, as well as facilitate their work with the aircraft's weapons control systems. Additionally, this will shorten the time needed to solve combat tasks and determine the areas of initial conditions for the combat use of weapons. For this purpose, a mathematical model of the aiming process for firing with aircraft guns at ground targets using suspended targeting pods has been developed. A general ballistic model, an aircraft model, a targeting system model and an atmosphere model have been created. The areas of possible conditions for the combat use of aircraft guns against ground targets have been determined and an analysis has been carried out.*

Keywords: Mathematical Model, Aircraft Gun, Target

1. Introduction

Maximum effectiveness of aircraft weapons (AW) against air and ground targets is achieved with a precisely defined mutual position of the weapons and the target. During combat use, the actual and required initial conditions for firing the AW are determined, the parameters for controlling the aircraft and the position of the weapon are calculated, and the boundaries of the areas for permitted firing are displayed on the head-up display.

Aiming at air and ground targets is carried out in three stages: bringing the aircraft into the initial point of attack, executing the attack and exiting the attack. At each stage, a specific problem is solved, described by the corresponding mathematical equations. The vector equations for aiming at the stage of attacking the target are determined, assuming that combat manoeuvring is completed [1].

2. Mathematical Model of the Aiming Process for Firing an Aircraft Gun at a Ground Target

To create a mathematical model of the combat use of an aircraft gun against a ground target, the vector-kinematic method is used, determining the position of the aircraft at the point of intersection of the target with the aircraft weapon at the moment of firing [1]:

$$\overline{D}_0 = \overline{D}_c + \bar{r}_c - \overline{L}_0 - \overline{S}_t \qquad (1)$$

Where:

$\overline{L}_0$ is the vector of the base of the weapon;

$\overline{D}_0$ - vector of the distance to target at the moment of firing;

$\overline{D}_c$ - vector of the calculated impact point

DOI: 10.2478/kbo-2025-0073

$\bar{S}_t$ - vector determining the motion of the target;

$\bar{r}_c$ – vector determining the deviation of the projectile of a flexible aircraft gun.

The vector equation (1) is presented in scalar form in the aircraft body-fixed coordinate system $Ox_1y_1z_1$, with point O being the centre of masses of the aircraft.

When firing a flexible aircraft gun (from a ground attack aircraft) **at a mobile ground target**, the vector equation (1) takes the form [1]:

$$\begin{bmatrix} D_{0x1} \\ D_{0y1} \\ D_{0z1} \end{bmatrix} = \left[A_{\eta\zeta\xi}^{(x_1y_1z_1)}\right]^T \begin{bmatrix} \eta \\ \zeta \\ 0 \end{bmatrix} + C_r \begin{bmatrix} \cos\beta_y \sin\alpha \sin\beta' \cos\varepsilon' - \sin\beta_y \sin\varepsilon' \\ \sin\beta_y \cos\beta' \cos\varepsilon' + \cos\beta_y \cos\alpha \sin\beta' \cos\varepsilon' \\ \cos\beta_y \cos\alpha \sin\beta' \sin\varepsilon' + \cos\beta_y \sin\alpha \cos\beta' \cos\varepsilon' \end{bmatrix} - \left[A_{\eta\zeta\xi}^{(x_1y_1z_1)}\right]^T \begin{bmatrix} W_\eta \\ W_\zeta \\ W_\xi \end{bmatrix} T - A_1^{(xDyDzD)} \begin{bmatrix} \dot{D} \\ \omega_{zD} D \\ -\omega_{yD} D \end{bmatrix} T - \begin{bmatrix} L_{0x1} \\ L_{0y1} \\ L_{0z1} \end{bmatrix}. \qquad (2)$$

as the transposed matrix $\left[A_{\eta\zeta\xi}^{(x_1y_1z_1)}\right]^T$ shows the transition between the navigation basis $O\eta\zeta\xi$ and the aircraft body-fixed coordinate system $Ox_1y_1z_1$. T is the time it takes the projectile to reach the target.

When firing a flexible aircraft gun (from a ground attack aircraft) **at a stationary ground target**:

$$\begin{bmatrix} D_{0x1} \\ D_{0y1} \\ D_{0z1} \end{bmatrix} = \left[A_{\eta\zeta\xi}^{(x_1y_1z_1)}\right]^T \begin{bmatrix} \eta \\ \zeta \\ 0 \end{bmatrix} + C_r \begin{bmatrix} \cos\beta_y \sin\alpha \sin\beta' \cos\varepsilon' - \sin\beta_y \sin\varepsilon' \\ \sin\beta_y \cos\beta' \cos\varepsilon' + \cos\beta_y \cos\alpha \sin\beta' \cos\varepsilon' \\ \cos\beta_y \cos\alpha \sin\beta' \sin\varepsilon' + \cos\beta_y \sin\alpha \cos\beta' \cos\varepsilon' \end{bmatrix} - \begin{bmatrix} L_{0x1} \\ L_{0y1} \\ L_{0z1} \end{bmatrix}. \qquad (3)$$

When firing a fixed aircraft gun (from a ground attack aircraft) **at a mobile ground target**:

$$\begin{bmatrix} D_{0x1} \\ D_{0y1} \\ D_{0z1} \end{bmatrix} = \left[A_{\eta\zeta\xi}^{(x_1y_1z_1)}\right]^T \begin{bmatrix} \eta \\ \zeta \\ 0 \end{bmatrix} - \left[\left[A_{\eta\zeta\xi}^{(x_1y_1z_1)}\right]^T \begin{bmatrix} W_\eta \\ W_\zeta \\ W_\xi \end{bmatrix} + A_1^{(xDyDzD)} \begin{bmatrix} \dot{D} \\ \omega_{zD} D \\ -\omega_{yD} D \end{bmatrix} \right] T. \qquad (4)$$

where D is the current distance to the target and W is the ground speed of the aircraft.

When firing a fixed aircraft gun (from a ground attack aircraft) **at a stationary ground target**:

$$\begin{bmatrix} D_{0x1} \\ D_{0y1} \\ D_{0z1} \end{bmatrix} = \left[A_{\eta\zeta\xi}^{(x_1y_1z_1)}\right]^T \begin{bmatrix} \eta \\ \zeta \\ 0 \end{bmatrix}. \qquad (5)$$

In the general case, at the moment of firing, the target sighting angles $\beta_{t1,0}$ and $\varepsilon_{t1,0}$ relative to the body-fixed coordinate system $Ox_1y_1z_1$ are determined by the formulas [1]:

$$\beta_{t1,0} = \operatorname{arctg}\left(\frac{D_{0z1}}{D_{0x1}}\right); \qquad \varepsilon_{t1,0} = \arcsin\left(\frac{D_{0y1}}{D_0}\right), \qquad (6)$$

3. Results of the Conducted Research

3.1. Firing a Flexible Gun at a Ground Target

The study examines the use a flexible aircraft gun for engaging both mobile and stationary ground targets under the following conditions: dive angle $\lambda = -20^0$, aircraft angle of attack $\alpha=1^0$, yaw angle $\beta_y=0$, target speed V_t = 60 km/h, target heading angle $\psi_t=30^0$, target azimuth angle $\beta^{/}=0$, target position angle $\varepsilon^{/}=-15^0$, aircraft speed V from 240 m/s to 300 m/s, flight height H from 400m to 900 m. It is assumed that the firing is carried out from a suspended pod, whose base vector $\bar{L}_0$ has

the values: L_{x1}=0, L_{y1}=1m, L_{z1}=2,5 m.
As a result of the mathematical modelling, the angles $\beta_{t1,0}$ и $\varepsilon_{t1,0}$ are calculated, determining the position of the target at the moment of firing and the firing distance D_c.

3.1.1. Firing a Flexible Aircraft Gun at a Mobile Ground Target

In Table 3 it can be seen that the firing conditions that satisfy the conditions for the permitted distance D_c=800 – 1200 m) are for heights H from 500m to 600m with a dive angle λ= -20^0.
The calculated angles $\beta_{t1,0}$ and $\varepsilon_{t1,0}$ are below the limit values of the sighting head C17 VG.

Table 1: Aiming azimuth angle $\beta_{t1,0}$ when firing a flexible gun at a mobile ground target

λ= -20^0, V_t= 60 km/h, ψ_t=30^0, $\beta^/$=0; $\varepsilon^/$=-15^0						
$B_{t1,0}$, degr.	**V=200, m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	-	-	-	-	-	-
500	-0,98	-0,97	-0,96	-0,95	-0,94	-0,93
600	-0,99	-0,98	-0,97	-0,96	-0,94	-0,93
700	-	-	-	-	-	-
800	-	-	-	-	-	-
900	-	-	-	-	-	-

Table 2: Aiming distance angle $\varepsilon_{t1,0}$ when firing a flexible gun at a mobile ground target

λ= -20^0, V_t= 60 km/h, ψ_t=30^0, $\beta^/$=0; $\varepsilon^/$=-15^0						
$\varepsilon_{t1,0}$, degr.	**V=200, m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	-	-	-	-	-	-
500	-12,81	-12,54	-12,29	-12,03	-11,79	-11,57
600	-12,97	-12,69	-12,42	-12,17	-11,93	-11,71
700	-	-	-	-	-	-
800	-	-	-	-	-	-
900	-	-	-	-	-	-

Table 3: Permitted firing distance with a flexible gun at a mobile ground target

λ= -20^0, V_t= 60 km/h, ψ_t=30^0, $\beta^/$=0; $\varepsilon^/$=-15^0						
D_c, m	**V=200, m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	759,96	765,61	771,37	776,80	781,90	787,18
500	946,60	953,65	960,72	967,82	974,56	980,93
600	1131,22	1140,17	1148,68	1157,15	1165,21	1172,86
700	1314,33	1324,47	1334,83	1344,40	1353,88	1363,30
800	1494,89	1506,87	1518,39	1529,76	1540,70	1551,54
900	1673,46	1686,88	1699,84	1712,63	1725,30	1737,55

3.1.2. Firing a Flexible Gun at a Stationary Ground Target

In this case, the conditions that satisfy the conditions for the permitted distance are also for heights H from 500m to 600m inclusive (Table 6).

The calculated angles $\beta_{t1,0}$ and $\varepsilon_{t1,0}$ when firing a flexible gun at a stationary ground target are smaller than those at a mobile target.

Table 4: Aiming azimuth angle $\beta_{t1,0}$ when firing a flexible gun at a stationary ground target

λ= -20^0, $\beta^/$=0; $\varepsilon^/$=-15^0						
$\beta_{t1,0}$, degr.	**V=200, m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	-	-	-	-	-	-
500	-0,28	-0,29	-0,29	-0,30	-0,30	-0,30
600	-0,26	-0,26	-0,27	-0,27	-0,28	-0,28
700	-	-	-	-	-	-
800	-	-	-	-	-	-
900	-	-	-	-	-	-

Table 5: Aiming distance angle $\varepsilon_{t1,0}$ when firing a flexible gun at a stationary ground target

λ= -20^0, $\beta^/$=0; $\varepsilon^/$=-15^0						
$\varepsilon_{t1,0}$, degr.	**V=200, m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	-	-	-	-	-	-
500	-12,20	-11,96	-11,72	-11,48	-11,25	-11,05
600	-12,33	-12,07	-11,82	-11,58	-11,36	-11,15
700	-	-	-	-	-	-
800	-	-	-	-	-	-
900	-	-	-	-	-	-

Table 6: Permitted firing distance with a flexible gun at a stationary ground target

λ= -20^0, $\beta^/$=0; $\varepsilon^/$=-15^0						
D_c, m	**V=200, m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	772,51	778,00	783,62	788,89	793,83	798,96
500	962,99	969,83	976,70	983,61	990,15	996,32
600	1151,74	1160,44	1168,71	1176,94	1184,75	1192,15
700	1339,33	1349,16	1359,24	1368,50	1377,69	1386,82
800	1524,67	1536,31	1547,49	1558,53	1569,12	1579,63
900	1708,35	1721,39	1733,96	1746,38	1758,67	1770,53

3.2. Firing a Fixed Gun at a Ground Target

The study is carried out for firing a fixed aircraft gun at a mobile and stationary ground target under the same conditions as when firing a flexible gun, as the angles $\beta^/$=0, $\varepsilon^/$=0 and L_{x1}=0, L_{y1}=0, L_{z1}=0.

3.2.1. Firing a Fixed Gun at a Mobile Ground Target

In Tables 7-9 and on Fig. 1 it can be seen that the firing conditions that satisfy the conditions for the permitted distance are for a height H= 400 m with a dive angle λ= -20^0, with the values of the calculated angles $\beta_{t1,0}$ and $\varepsilon_{t1,0}$ being respectively in the ranges from -0,97^0 to -0,62^0 and from -4,48^0 to -2,79^0.

Table 7: Aiming azimuth angle $\beta_{t1,0}$ when firing a fixed gun at a mobile ground target

λ =-20, degr d=23 mm						
$\beta_{t1,0}$, degr	**V=200 m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	-0,70	-0,69	-0,67	-0,65	-0,64	-0,62
500	-	-	-	-	-	-
600	-	-	-	-	-	-
700	-	-	-	-	-	-
800	-	-	-	-	-	-
900	-	-	-	-	-	-

Table 8: Aiming distance angle $\varepsilon_{t1,0}$ when firing a fixed gun at a mobile ground target

λ =-20, degr d=23 mm						
$\varepsilon_{t1,0}$, degr	**V=200 m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	-3,05	-3,00	-2,94	-2,89	-2,84	-2,79
500	-	-	-	-	-	-
600	-	-	-	-	-	-
700	-	-	-	-	-	-
800	-	-	-	-	-	-
900	-	-	-	-	-	-

Table 9: Permitted firing distance with a fixed gun at a mobile ground target

λ =-20, degr d=23 mm						
D_c, m	V=200 m/s	220	240	260	280	300
H=400, m	1065,02	1067,45	1070,11	1072,57	1074,87	1077,04
500	1318,58	1322,48	1326,04	1329,68	1333,04	1336,14
600	1565,68	1570,80	1575,88	1580,57	1584,91	1589,28
700	1804,92	1811,47	1817,95	1824,04	1830,10	1835,48
800	2036,08	2044,23	2052,33	2059,83	2067,31	2074,49
900	2258,70	2268,65	2278,11	2287,58	2296,57	2305,34

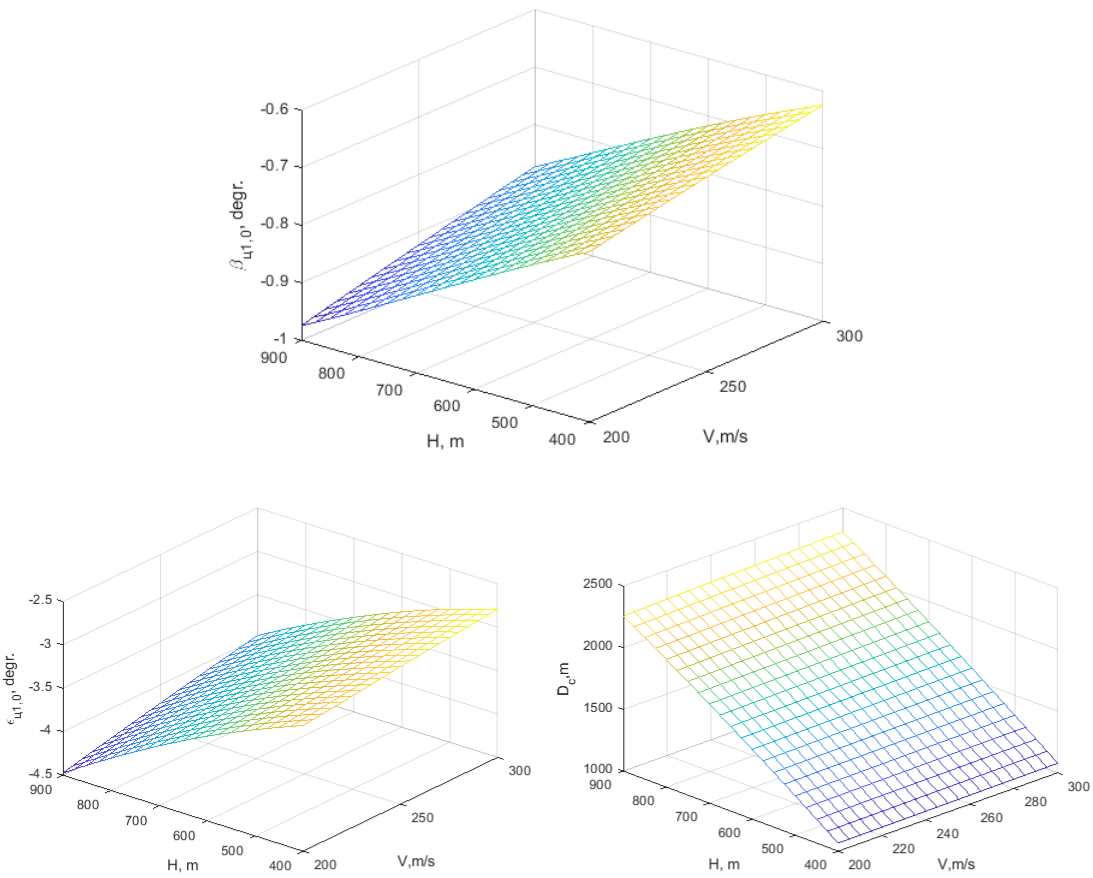

Figure 1: Target sighting angles $\beta_{t1,0}$ и $\varepsilon_{t1,0}$ and distance D_c when firing at a ground target

3.2.2. Firing a Fixed Gun at a Stationary Ground Target

In table 10 and table 11 the angle $\varepsilon_{t1,0}$ and the firing distance D_c are presented.

The conditions that satisfy the requirements for the permitted distance are the same as when firing at a mobile target (H=400 m, Table 9).

In this case, the calculations were performed in calm weather (U=0 km/h) as a result of which the angle $\beta_{t1,0} = 0^0$.

The angles $\varepsilon_{t1,0}$ when firing at a mobile target are greater than those when firing at a stationary target (Table 8, Table 10).

Table 10: Aiming distance angle $\varepsilon_{t1,0}$ when firing a fixed gun at a stationary ground target

λ =-20, degr d=23 mm						
$\varepsilon_{t1,0}$, degr	**V=200 m/s**	**220**	**240**	**260**	**280**	**300**
H=400, m	-2,60	-2,56	-2,52	-2,48	-2,44	-2,40
500	-	-	-	-	-	-
600	-	-	-	-	-	-
700	-	-	-	-	-	-
800	-	-	-	-	-	-
900	-	-	-	-	-	-

Table 11: Permitted firing distance with a fixed gun at a stationary ground target

λ =-20, degr d=23 mm						
D_c, m	V=200 m/s	220	240	260	280	300
H=400, m	1085,94	1087,92	1090,14	1092,18	1094,08	1095,86
500	1346,32	1349,63	1352,62	1355,71	1358,53	1361,11
600	1600,90	1605,30	1609,68	1613,68	1617,34	1621,07
700	1848,22	1853,92	1859,57	1864,85	1870,12	1874,72
800	2087,90	2095,11	2102,30	2108,88	2115,47	2121,76
900	2319,35	2328,31	2336,77	2345,27	2353,29	2361,09

4. Conclusions

1. A general ballistic model for various groups of weapons has been developed.
2. The areas of initial conditions for combat use of weapons against air and ground targets by a multirole aircraft have been determined.
3. The study shows that under the considered conditions when firing a flexible aircraft gun, it is necessary to take into account the deviation of the projectile due to the deviation of the axis of the gun from the axis of the aircraft.

Acknowledgements

The research in this article was carried out in fulfilment of Task 3.1.4. “Research, development and improvement of munitions” from the National Scientific Program “Security and Defence”, adopted by RMS No. 731 of 21.10.2021. and according to Agreement No. D01-74/19.05.2022.

References

[1] Atanasov, M. Matematichesko modelirane na boĭnoto izpolzvane na aviatsionnite sredstva za porazyavane. ВВВУ Георги Бенковски. Pleven. 2022, 169. ISBN 978-954-713-152-1.

[2] Postnikov, A.G., Chuiko, V.S. External ballistics of unguided aircraft missiles and projectiles. M, M, 1985.

[3] Veremeyenko, K. K., Golovinsky, A. N., Insarov, V. V. Control and guidance of unmanned maneuverable aerial vehicles based on modern information technologies. M., Fizmatlit. 2003.