International Conference KNOWLEDGE-BASED ORGANIZATION
Vol. XXVI No 3 2020

SOME CLASSES OF SURFACES GENERATED BY BLENDING INTERPOLATION ON A TRIANGLE WITH ONE CURVED SIDE

Alina BABOȘ

"Nicolae Bălcescu" Land Forces Academy, Sibiu, Romania
alina_babos_24@yahoo.com

Abstract: *The blending interpolation has many practical applications. Remind that blending interpolation is to interpolate a function at an infinite set of points: segments, curves, surfaces, etc. Thus, if one gives the contour of an object by such elements (segments, curves, surfaces) using a blending interpolation, we can generate a surface that contains the given contour. Hence, we can construct a surface (a blending function interpolant) which matches a given function and certain on its derivatives on the boundary of a plane domain (rectangle, triangle, etc. The aim of this paper is to construct some surfaces which satisfy some given condition on the boundary of a domain that can be decomposed in triangles with one curved side. We construct some new surfaces using some Lagrange, Hermite, Birkhoff and Nielson type operators.*

Keywords: Blending interpolation, interpolation operators, surface generation

1. Introduction

In paper [8] the authors consider "a standard triangle $\widetilde{T}_h$, having the vertices $V_1 = (h,0), V_2 = (0,h)$ and $V_3 = (0,0)$, two straight sides Γ_1, Γ_2, along the coordinate axes, and the third side Γ_3 (opposite to the vertex V_3), which is defined by the one-to-one functions *f* and *g*, where *g* is the inverse of the function *f*, i.e., $y = f(x)$ and $x = g(y)$, with $f(0) = g(0) = h$ and F a real-valued function defined on $\widetilde{T}_h$".

In paper [1] we introduce " an Lagrange operator which interpolate the function *F* on cathetus, on the curved side, but also on a interior line of the triangle $\widetilde{T}_h$".

We considered the case when the interior line is a median. Then in [2] we introduce "Hermite and Birkhoff type operators which interpolate a given function and some of its derivatives on median of the same triangle".

The aim of this paper is to construct some "surfaces which satisfy some given condition on the boundary of a domain that can be decomposed in triangles with one curved side" (see [6]). We construct some new surfaces using some Lagrange, Hermite, Birkhoff and Nielson type operators introduced in [1], [2] and by authors in [6], [8].

DOI: 10.2478/kbo-2020-0110
© 2015. This work is licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 3.0 License.

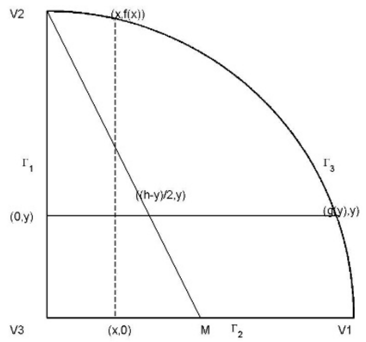

Figure 1:Triangle $\widetilde{T}_h$

2. Surfaces generated by Lagrange, Hermite, Birkhoff and Nielson type operators

In [8] "Supposing that F is a real-valued function defined on $\widetilde{T}_h$,and it has all partial derivatives needed". We consider four types of interpolation operators defined on $\widetilde{T}_h$.

- the Lagrange interpolation operators L_1^y and L_2^x defined by [8] and [1]: "

$$(L_1^y F)(x,y)=\frac{f(x)-y}{f(x)}F(x,0)+\frac{y}{f(x)}F(x,f(x)), \quad (1)$$

$$(L_2^x F)(x,y)=\frac{(2x-h+y)(x-g(y))}{(h-y)g(y)}F(0,y)+\frac{4x(x-g(y))}{(h-y)(h-y-2g(y))}F\left(\frac{h-y}{2},y\right)+\frac{x(2x-h+y)}{g(y)(2g(y)-h+y)}F(g(y),y),$$

- the Hermite interpolation operators H_3^y and H_2^x, given by [8] and [1]:

$$(H_3^y F)(x,y)=\frac{[f(x)-y]^2[f(x)+2y]}{f^3(x)}F(x,0)+\frac{y[f(x)-y]^2}{f^2(x)}F^{(0,1)}(x,0)+\frac{y^3[3f(x)-2y]}{f^3(x)}F(x,f(x))+\frac{y^2[y-f(x)]}{f^2(x)}F^{(0,1)}(x,f(x)),$$

$$(H_2^x F)(x,y)=\frac{(2x-h+y)^2[x-g(y)]^2}{(y-h)^3g^3(y)}\cdot[g(y)(y-h)+2x(y-h-2g(y))]\cdot F(0,y)+\frac{x(2x-h+y)^2[x-g(y)]^2}{(y-h)^2g^2(y)}F^{(1,0)}(0,y)+\frac{16x^2[x-g(y)]^2}{(h-y)^3[y-h-2g(y)]^3}\cdot[(h-y)(y-h-2g(y))-2(2x-h-y)^2(y-h-g(y))]\cdot F\left(\frac{h-y}{2},y\right)+\frac{8x^2[x-g(y)]^2(2x-h+y)}{(h-y)^2[y-h-2g(y)]^2}\cdot F^{(1,0)}\left(\frac{h-y}{2},y\right)+\frac{x(2x-h+y)^2}{g^3(y)[2g(y)-h+y]^3}\cdot[g(y)(10g(y)-3h+3y)+2x(h-y-4g(y))]\cdot F(g(y),y)+\frac{x^2[x-g(y)](2x-h+y)^2}{g^2(y)[2g(y)-h+y]^2}\cdot F^{(1,0)}(g(y),y),$$

"

- the Birkhoff interpolation operators B_3^x and B_3^y, given by [8]:
"

$$(B_3^x F)(x,y) = F(g(y),y) + [x-g(y)]F^{(1,0)}(0,y),$$
$$(B_3^y F)(x,y) = F(x,f(x)) + [y-f(x)]F^{(0,1)}(x,0), \qquad (1)$$

"
- the Nielson type interpolation operators given by [6]:
"

$$(N_1F)(x,y) = yF(x,f(x)) + + (1-f(x))F(g(y),y), \qquad (2)$$
$$(N_2F)(x,y) = F(0,y) + F(x,0) - F(0,0).$$

"

"Using the natural condition that the roof stays on its support, i.e., $F_{|\Gamma_3} = 0$ and $F_{|V_2M} = 0$, we get that

$$(L_1^y F)(x,y) = \frac{f(x)-y}{f(x)} F(x,0), \qquad (3)$$

$$(L_2^x F)(x,y) = \frac{(2x-h+y)(x-g(y))}{(h-y)g(y)} F(0,y).$$

For obtaining the first class of surfaces, we consider the Boolean sum of the Lagrange type operators L_1^y and L_2^x, namely,

$$S_L = \left(L_1^y \oplus L_2^x F\right)(x,y) = \frac{f(x)-y}{f(x)} F(x,0) + \frac{(2x-h+y)(x-g(y))}{(h-y)g(y)} F(0,y) + \frac{f(x)-y}{f(x)} \cdot \frac{(2x-h)(x-h)}{h^2} F(0,0) \qquad (4)$$

" (see [4])

Theorem 1. [7] "If $F_{|\Gamma_3} = 0$, $F_{|\Gamma_3} = 0$, then we have the following properties of the operator S_L:

$(S_L F)(x,0) = F(x,0)$,
$(S_L F)(0,y) = F(0,y)$,
$(S_L F)(x,f(x)) = 0.$
"

Proof. The proof follows directly by the expression of S_L from relation (4).
In the second level of approximation we use the Birkhoff interpolation operators given in(1), taking into account the conditions $F_{|\Gamma_3} = 0$ and $F_{|V_2M} = 0$,i.e.,

$$(B_3^x F)(x,y) = [x-g(y)]F^{(1,0)}(0,y),$$
$$(B_3^y F)(x,y) = [y-f(x)]F^{(0,1)}(x,0). \qquad (5)$$

We apply the following approximations:
$F(x,0) \approx (B_3^x F)(x,0) = (x-h)F^{(1,0)}(0,0),$
$F(0,y) \approx (B_3^y F)(0,y) = (y-h)F^{(0,1)}(0,0),$

and by (4), we obtain the following class of surfaces:

$$(S_1 F)(x,y) = \frac{f(x)-y}{f(x)} (B_3^x F)(x,0) + \frac{(2x-h+y)(x-g(y))}{(h-y)g(y)} (B_3^y F)(0,y) - \frac{f(x)-y}{f(x)} \cdot \frac{(2x-h)(x-h)}{h^2} F(0,0),$$

i.e.,

$$(S_1 F)(x,y) = \frac{f(x)-y}{f(x)} (x-h)F^{(1,0)}(0,0) - \frac{(2x-h+y)(x-g(y))}{g(y)} F^{(0,1)}(0,0) - \frac{f(x)-y}{f(x)} \cdot \frac{(2x-h)(x-h)}{h^2} F(0,0).$$

"For obtaining the second class of surfaces, we have the Boolean sum of Nielson type operators N_1 and N_2, obtained for the condition that $F_{|\Gamma_3} = 0$ (see [6]):

$$(S_n) = yF(0,f(x)) + (1-y)F(x,0)$$
$$+[h+y-1-f(x)]F(0,0) \qquad (6)$$
$$-[h-1-f(x)]F(0,y)$$
$$+[f(x)-h]F(g(y),0).$$

"

"In the second level we use the Hermite interpolation operator H_3^y and H_2^x, taking into account the condition $F_{|\Gamma_3} = 0$, i.e.,

$$(H_{31}^y F)(x,y) = \frac{[f(x)-y]^2[f(x)+2y]}{f^3(x)} F(x,0)$$
$$+\frac{y[f(x)-y]^2}{f^2(x)} F^{(0,1)}(x,0)$$
$$+\frac{y^2[y-f(x)]}{f^2(x)} F^{(0,1)}(x,f(x)),$$

$$(H_{21}^x F)(x,y) =$$
$$\frac{(2x-h+y)^2[x-g(y)]^2}{(y-h)^3 g^3(y)} \cdot$$
$$[g(y)(y-h) + 2x(y-h-2g(y))] \cdot F(0,y)$$
$$+\frac{x(2x-h+y)^2[x-g(y)]^2}{(y-h)^2 g^2(y)} F^{(1,0)}(0,y)$$
$$+\frac{8x^2[x-g(y)]^2(2x-h+y)}{(h-y)^2[y-h-2g(y)]^2} \cdot$$
$$F^{(1,0)}\left(\frac{h-y}{2}, y\right) + \frac{x^2[x-g(y)](2x-h+y)^2}{g^2(y)[2g(y)-h+y]^2} \cdot$$
$$F^{(1,0)}(g(y),y).$$

We apply the following approximations:

$$F(0,y) := (H_{31}^y F)(0,y)$$
$$= \frac{(h-y)^2(h+2y)}{h^3} F(0,0)$$
$$+\frac{y(h-y)^2}{h^2} F^{(0,1)}(0,0) + \frac{y^2(y-h)}{h^2} F^{(0,1)}(0,h)$$

$$F(x,0) := (H_{21}^x F)(x,0)$$
$$\frac{(2x-h)^2(x-h)^2(h+6x)}{h^5} \cdot F(0,0)$$
$$+\frac{x(2x-h)^2(x-h]^2}{h^4} F^{(1,0)}(0,0)$$
$$+\frac{8x^2(x-h)^2(2x-h)}{h^4} \cdot F^{(1,0)}\left(\frac{h}{2},0\right)$$
$$+\frac{x^2(x-h)(2x-h^2}{h^4} \cdot F^{(1,0)}(h,0).$$

" (see [4])

and using the form of S_n, we obtain the following class of surfaces:

$$(S_2F)(x,y) = y(H_{31}^y F)(0,f(x)) + (1-y)(H_{21}^x F)(x,0)$$
$$+[h+y-1-f(x)]F(0,0)$$
$$-[h-1-f(x)](H_{31}^y F)(0,y)$$
$$+[f(x)-h](H_{21}^x F)(g(y),0).$$

i.e.,

$$(S_2F)(x,y) = \left\{ \frac{-y[2f(x)+h][f(x)-h]^2}{h^3} \right.$$
$$+\frac{(1-y)(2x-h)^2(x-h)^2(h+6x)}{h^5}$$
$$+[h+y-1-f(x)]$$
$$+\frac{[h-1-f(x)](2y+h)(y-h)^2}{h^3}$$
$$\left. +\frac{[f(x)-h][2g(y)-h]^2[g(y)-h]^2[h+6g(y)]}{h^5} \right\}$$
$$F(0,0) + \left\{ \frac{-yf(x)[h-f(x)]^2}{h^2} \right.$$
$$\left. -\frac{y[h-1-f(x)](h-y)^2}{h^2} \right\} F^{(0,1)}(0,0)$$
$$+\left\{ \frac{-yf^2(x)[f(x)-h]}{h^2} \right.$$
$$\left. -\frac{y^2[h-1-f(x)](y-h)}{h^2} \right\} F^{(0,1)}(0,h)$$
$$+\left\{ \frac{(1-y)x(2x-h)^2(x-h)^2}{h^4} \right.$$
$$\left. +\frac{[f(x)-h]g(y)[2g(y)-h]^2[g(y)-h]^2}{h^4} \right\}$$

$$F^{(1,0)}(0,0)+\left\{\frac{(1-y)8x^2(2x-h)(x-h)^2}{h^4}\right.$$

$$\left.+\frac{[f(x)-h]8g^2(y)[2g(y)-h][g(y)-h]^2}{h^4}\right\}$$

$$F^{(1,0)}\left(\frac{h}{2},0\right)+\left\{\frac{(1-y)x^2(2x-h)^2(x-h)}{h^4}\right.$$

$$\left.+\frac{[f(x)-h]g^2(y)[2g(y)-h]^2[g(y)-h]}{h^4}\right\}$$

$$F^{(1,0)}(h,0).$$

Other classes of surfaces may be obtained using the conditions

$$F_{|\Gamma_3}=F^{(0,1)}{}_{|\Gamma_3}=F^{(1,0)}{}_{|\Gamma_3}=0, \tag{7}$$

$$F_{|V_2M}=F^{(1,0)}{}_{|V_2M}=0.$$

We consider the boolean sum of the Nielson type operators given in (6), taking into account the conditions (7).

In the second level we use the Hermite interpolation operators H_3^y and H_2^x, taking into account the conditions (7), so we have

$$(H_{32}^yF)(x,y)=\frac{[f(x)-y]^2[f(x)+2y]}{f^3(x)}F(x,0)$$

$$+\frac{y[f(x)-y]^2}{f^2(x)}F^{(0,1)}(x,0),$$

$$(H_{22}^xF)(x,y)=$$

$$\frac{(2x-h+y)^2[x-g(y)]^2}{(y-h)^3g^3(y)}\cdot$$

$$[g(y)(y-h)+2x(y-h-2g(y))]\cdot F(0,y)$$

$$+\frac{x(2x-h+y)^2[x-g(y)]^2}{(y-h)^2g^2(y)}F^{(1,0)}(0,y).$$

Using the following approximation:

$$F(0,y):=(H_{32}^yF)(0,y)$$

$$=\frac{(h-y)^2(h+2y)}{h^3}F(0,0)$$

$$+\frac{y(h-y)^2}{h^2}F^{(0,1)}(0,0)$$

$$F(x,0):=(H_{22}^xF)(x,0)$$

$$\frac{(2x-h)^2(x-h)^2(h+6x)}{h^5}\cdot F(0,0)$$

$$+\frac{x(2x-h)^2(x-h]^2}{h^4}F^{(1,0)}(0,0).$$

We obtain the following class of surfaces:

$$(S_3F)(x,y)=y(H_{32}^yF)(0,f(x))+(1-y)(H_{22}^xF)(x,0)$$

$$+[h+y-1-f(x)]F(0,0)$$

$$-[h-1-f(x)](H_{32}^yF)(0,y)$$

$$+[f(x)-h](H_{22}^xF)(g(y),0),$$

further given as

$$(S_3F)(x,y)=\left\{\frac{-y[2f(x)+h][f(x)-h]^2}{h^3}\right.$$

$$+\frac{(1-y)(2x-h)^2(x-h)^2(h+6x)}{h^5}$$

$$+[h+y-1-f(x)]$$

$$+\frac{[h-1-f(x)](2y+h)(y-h)^2}{h^3}$$

$$\left.+\frac{[f(x)-h][2g(y)-h]^2[g(y)-h]^2[h+6g(y)]}{h^5}\right\}$$

$$F(0,0)+\left\{\frac{-yf(x)[h-f(x)]^2}{h^2}\right.$$

$$- \frac{y[h-1-f(x)](h-y)^2}{h^2} \Bigg\} F^{(0,1)}(0,0)$$

$$\Bigg\{ \frac{(1-y)x(2x-h)^2(x-h)^2}{h^4}$$

$$+ \frac{[f(x)-h]g(y)[2g(y)-h]^2[g(y)-h]^2}{h^4} \Bigg\}$$

$$F^{(1,0)}(0,0).$$

3. Numerical examples

Example 1. "Consider $F : \widetilde{T}_h \to R$,

$$F(x,y) = \frac{(x^2+y^2-h^2)^2}{x^2+y^2+1}$$

and $f(x) = \sqrt{h^2+x^2}, h = 4$",(see [7]).

In figure 2 we plot the graph of surface $S_1 F$.

Example 2. "Consider the function $f(x) = \sqrt{h^2+x^2}, h = 1$ and $F : \widetilde{T}_h \to R$.

In Figure 3 we plot the graphs of surface $S_2 F$ assigning to the data

$$\left(F(0,0), F^{(1,0)}(0,0), F^{(1,0)}\left(\frac{h}{2},0\right), F^{(1,0)}(h,0), F^{(0,1)}(0,0), F^{(0,1)}(0,h) \right)$$

the values (-1,-1,1,1,-1,1)", (see [7]).

Example 3. "Consider the function $f(x) = \sqrt{h^2+x^2}, h = 1$ and $F : \widetilde{T}_h \to R$.

In Figure 4 we plot the graphs of surface $S_3 F$ assigning to the data $\left(F(0,0), F^{(1,0)}(0,0), F^{(0,1)}(0,0) \right)$ the values (1,-1,1)", (see [7]).

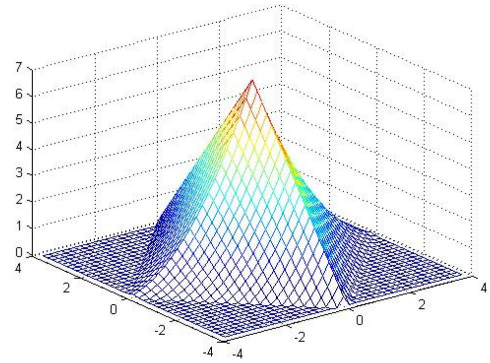

Figure 3:The surface S_1

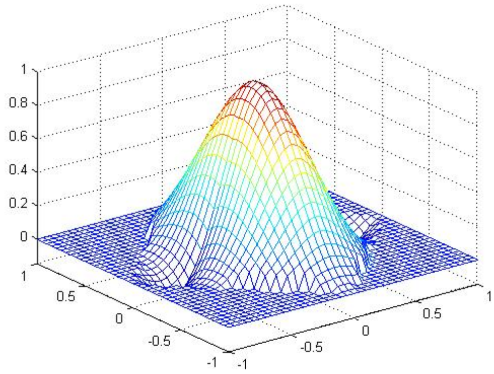

Figure 3:The surface S_2

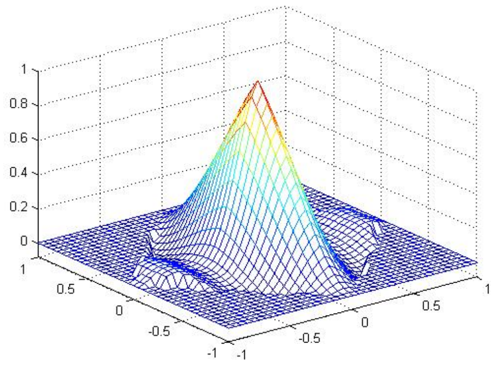

Figure 4:The surface S_3

References

[1] A.Baboş, *Some interpolation operators on triangle*, The 16th International Conference The Knowledge - Based Organization, Applied Technical Sciences and Advanced Military Technologies, Conference Proceedings 3, pp.28-34, 2010.

[2] A.Baboş, *Interpolation operators on a triangle with one curved side*, General Mathematics, Vol.22, No.1, pp.125-131, 2014.

[3] A.Baboş, *Applications of interpolation*, Buletin ştiinţific, Nr.2 (40), pp.121-125, 2015.

[4] A.Baboş,*Surface generated by blending interpolation on a triangle*,Scientific Studies andResearch, SeriesMathematicsandInformatics, Vol 25, Nr.2, Bacău, pp. 5-14, 2015.

[5] T.Cătinaş, *Some classe of surfaces generated by Nielson and Marshall type operators on the triangle with one curved side*, Studia Univ. Babeş Bolyai Math 61 (3), pp.305-314, 2016.

[6] T.Cătinaş, *Extension of some particular interpolation operators to a triangle with one curved side*, Appl. Math. Comput., 315, pp. 286-297, 2017.

[7] T.Cătinaş, P.Blaga, Gh.Coman, *Surfaces generation by blending interpolation on a triangle with one curved side*, Results in Mathematics, Vol.64, pp.343-355, 2013.

[8] Gh.Coman, T.Cătinaş, *Interpolation operators on a triangle with one curved side*, BIT Numer Math 47, 2010.