

# Analysis of the unsaturated hydraulic properties of rocks using multiple laboratory methods

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**Abstract:** Proper characterization of the unsaturated hydraulic properties in rocks is significant for predicting fluid flow in soil, hydrogeologic, and petroleum science and engineering problems. In this study, we contributed rigorous analysis of the unsaturated hydraulic properties of three reservoir rock samples (Berea Sandstone, Guelph Dolomite, and Indiana Limestone). An improved version of the standard evaporation method (HYPROP) was developed to cater specifically to rock samples. The improved HYPROP setup enables measurements of local water pressures within rock samples without disturbing the upper portion of the samples. The obtained results were compared with those obtained using the conventional pressure plate method and a state-of-the-art nuclear magnetic resonance (NMR) method. Observed data were analyzed in terms of four different unimodal and bimodal hydraulic functions. The HYPROP data were found to be relatively close to the pressure plate data of two carbonate rocks. The NMR-based data were reasonably consistent with the HYPROP data, with differences likely due in part to the fact that they were obtained using two different 5-cm long plugs taken from the same core. Heterogeneity along the rock cores from which the samples were taken could be a major reason for the observed differences, and hence should be considered in reservoir analyses.

**Keywords:** Hydraulic properties; Pressure plate; HYPROP; WP4C; NMR.

## INTRODUCTION

The unsaturated hydraulic properties of rock formations are crucial to understanding fluid flow and contaminant transport processes in many vadose zone hydrogeologic, geotechnical, and petroleum science and engineering problems, among other applications. The capillary pressure-saturation and relative permeability functions are the two most important properties of rocks. Several laboratory techniques (such as pressure plate, centrifuge, and mercury intrusion methods) are commonly used to quantify capillary pressure as a function of fluid saturation of rock samples (Shikhov and Arns, 2015). Pressure plate methods are often assumed to give accurate data without a saturation gradient along the samples. Unfortunately, pressure plate methods are generally very time-consuming, often requiring up to several months. Mercury intrusion and centrifuge methods on the other hand are relatively fast (Turturro et al., 2022). With mercury injection capillary pressure (MICP) methods it is possible to measure a broad range of pore throat sizes, including the distribution and connectivity of pores, but samples are contaminated by mercury and generally cannot be used anymore (Liu et al., 2019; Yao and Liu, 2012). Centrifuge methods allow one to estimate the drainage and imbibition capillary pressure curves by applying a centrifugal acceleration on saturated samples with stepwise increasing rotation speeds (Albuquerque et al., 2018; Chen and Balcom, 2005). A drawback of the methods is inaccurate average saturation estimation at each rotation speed (Caputo and Nimmo, 2005), although more dynamic methods have now become also available (Bui and Mori, 2021; van den Berg et al., 2009).

Various methods are also available to measure the relative permeability of reservoirs rocks. Generally, they can be divided into steady-state and unsteady-state flow based methods (Favvas et al., 2009). Regarding steady-state methods, fluid saturation, flow rate, and pressure data are obtained stepwise at equilibrium conditions after fluid injection at each step. Relative permeability values can then be calculated using the Darcy-Buckingham law. These methods can be time-consuming because of the need to reach steady state for each mean pressure. By comparison, unsteady-state laboratory methods are relatively fast, leading potentially to relative permeability and saturation data that can be obtained in a few hours depending upon sample size and permeability. However, many uncertainties still influence the accuracy of the unsteady-state methods, such as non-equilibrium capillary effects and viscous fingering (Kumahor et al., 2015; Zhuang et al., 2017a, 2017b). Almost all of these methods exclusively measure pressures within the reservoir without capturing internal pressures within the rock samples themselves.

We note that reliable hydraulic property measurements are equally important for applications involving unconsolidated materials, including soils. Many approaches have been developed over the years to measure capillary pressure and relative permeability data of soils (e.g., Dane and Hopmans, 2002; Schelle et al., 2013; Zhuang et al., 2017a). Among these, the evaporation method has become especially popular (Schindler, 1980; Šimůnek et al., 1998; Wendroth et al., 1993), including the HYPROP semi-automated evaporation system designed by the METER group (METER, 2015). HYPROP is designed to continuously collect tensiometer readings at two

depths from the evaporation surface as well as corresponding sample weights during the full evaporation process (Schindler et al., 2010), from which the hydraulic properties can be estimated. Lipovetsky et al. (2020) first applied the HYPROP methodology to reservoir rocks; as far as we know no subsequent studies have been reported.

For our studies we improved the HYPROP system used by Lipovetsky et al. (2020) to measure the unsaturated hydraulic properties of reservoir rock samples without disturbing the upper portion of the samples. Different from traditional methods, our approach can measure local fluid pressure inside samples. To validate our approach, we furthermore compared the measured data obtained using the traditional pressure plate method and an updated methodology using nuclear magnetic resonance (NMR) data.

## MATERIALS AND METHODS

### Rock samples

Three different rock outcrop samples, obtained from Kocurek Industries, were used for our studies. One was a sandstone

sample (Berea Sandstone, BS) and two were carbonate rock samples (Guelph Dolomite, GD, and Indiana Limestone, IL). Prior to the HYPROP experiments, the plugs were cleaned using methanol and toluene to remove any salt or impurities from the pore system. After cleaning, the samples were dried for the routine core analyses. The samples for the HYPROP experiments were 2 inches (5.08 cm) in diameter and approximately 5 cm in length. The NMR sample had a diameter of 1.5 inches (4.3 cm) and a length of approximately 3 cm. Petrophysical properties of the samples are shown in Table 1.

Permeability values of the different rock samples were obtained each from the same but longer rock core. The porosities of the two Indiana Limestone plugs were nearly identical. However, it is worth noting that the measured porosity of one Indiana Limestone plug was greater than that of the other one taken from the same core. This variation in porosity may suggest potential heterogeneity within a longer rock core (Manjunath, 2022; Manjunath et al., 2023). Microscope images (Vega 3, TESCAN, Czech Republic) of the selected samples (Guelph Dolomite and Indiana Limestone) are shown in Fig. 1. Mineral compositions of the GD and IL samples are listed in Table 2.

**Table 1.** Porosities and permeabilities of the rock samples used in this study.

| Sample | Porosity (%) | Permeability Klinkenberg (mD) | Length (cm) | Diameter (cm) | Rock type         | Experimental procedure |
|--------|--------------|-------------------------------|-------------|---------------|-------------------|------------------------|
| BS1    | 19.12        | 171.6                         | 5.0         | 5.1           | Berea Sandstone   | HYPROP                 |
| GD1    | 8.01         | 1.18                          | 4.9         | 5.1           | Guelph Dolomite   | HYPROP                 |
| GD2    | 11.20        | 4.37                          | 3.02        | 4.3           | Guelph Dolomite   | NMR                    |
| IL1    | 16.24        | 38.6                          | 4.9         | 5.1           | Indiana Limestone | HYPROP                 |
| IL2    | 16.48        | 68.1                          | 3.03        | 4.3           | Indiana Limestone | NMR                    |

**Table 2.** Mineral composition of the selected rock samples.

| Sample            | Calcite | Magnesian Calcite | Dolomite | K-feldspar | Quartz |
|-------------------|---------|-------------------|----------|------------|--------|
| Guelph Dolomite   | –       | 0.5               | 99.3     | 0.1        | 0.1    |
| Indiana Limestone | 99.3    | –                 | –        | –          | 0.7    |

Total mineralogical composition for rock sample in mass %

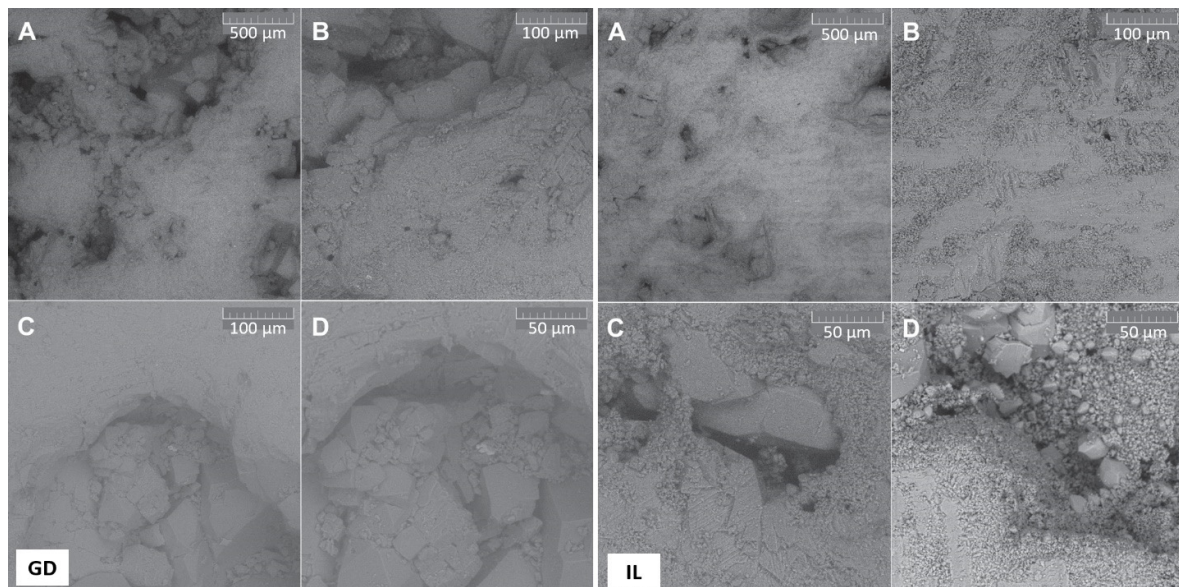


**Fig. 1.** Microscopic images of the Guelph Dolomite (GD) and Indiana Limestone (IL) samples.

## Evaporation method

The standard HYPROP system (Meter Group AG, München, Germany) was previously modified by Lipovetsky et al. (2020) to enable evaporation experiments of rock samples. The standard HYPROP system is often employed to measure water retention and hydraulic conductivity of soils. The soil samples are then saturated and exposed at the top for evaporation. During the process, soil water pressure head readings from the tensiometers at two depths (1.25 and 3.75 cm) and weight losses of the samples are collected. Geometric means of the pressure heads and saturations are used to obtain water retention data, while unsaturated hydraulic conductivities are estimated from the weight loss and pressure head differences between the two tensiometers (Peters and Durner, 2008). Due to the air-entry value of tensiometer cups, the HYPROP evaporation method is able to obtain retention data in the relatively wet and intermediate range to about 1,000 cm capillary pressure head (Schindler et al., 2010). Details about the standard HYPROP system can be found in METER (2015, 2021).

The standard sample sizes of the HYPROP system are 5 or 8 cm in diameter and 5 cm in height, which is not always suitable for rock samples. In our work, sample rings for the rock samples were made using a 3-D printer, with the exact inner size being the same as the diameter of the rock samples. Silicon grease was applied to the inner wall of the sample rings to avoid preferential flow along the wall. In the system used by Lipovetsky et al. (2020), openings for the tensiometers were drilled completely through the samples, with two plugs subsequently used to seal the holes above the tensiometers after they were installed. Their setup potentially creates preferential flow paths between the plugs and the sample. In our approach we drilled two holes with the same dimensions as the two tensiometers from the bottom of the samples only to the correct positions of the two tensiometers (Fig. 2), thereby minimizing the risk of preferential flow through any gaps. Ground material from drilled-out rock parts was used to fill gaps between the tensiometers and the rock samples. The samples were saturated with deaired water from the bottom overnight, with air allowing to escape from the top of the sample. We emphasize that our approach hence differed from the one used by Lipovetsky et al. (2020), who drilled the vertical holes through the entire rock sample and backfilled the upper parts of the holes above the tensiometers with drilled-out 0.5-cm diameter plugs from a neighboring sample of the same rock core. Our approach prevents uneven or preferential flow of moisture between the tensiometers and the top boundary of the samples.

## Pressure plate method

We conducted pressure plate experiments using the SWC-150 Fredlund SWCC device, developed by GCTS Testing Systems

(Arizona, USA). Saturated rock samples were placed in a sealed container for this purpose. The air pressure around the samples was increased step by step for the primary drying process. The bottom of the samples was in contact with a saturated ceramic porous plate. At each step, the air pressure was increased to a certain value to allow water to move out from the sample through the ceramic plate at the bottom. A burette was connected to the reservoir below the ceramic plate to collect outflowing water and measure changes in the water volume. For each pressure step, the water volume was monitored to ensure that equilibrium was reached. The initial water content was calculated based on the difference between the weights of saturated rock samples and oven-dried rock samples. The rock samples and the ceramic plates were saturated prior to the experiments. The maximum capillary pressure head  $h$  could be extended to about  $1.5 \times 10^4$  cm ( $pF = \log(h) = 4.2$ ) before desaturating the ceramic plate. All experiments were conducted in a constant-temperature room at  $21 \pm 0.5$  °C. Distilled and degassed water was used in the experiments.

## Chilled-mirror dewpoint method

The chilled-mirror dewpoint method using WP4C psychrometers (Meter Group AG, München, Germany) was used to obtain water retention data in the relatively dry moisture range (Gee et al., 1992). The method measures the sum of the matric and osmotic (negligible in our study) potentials during the equilibration process between liquid-phase water and vapour-phase water in a sealed container (Bittelli and Flury, 2009; Solone et al., 2012). Neighbouring rock samples were cut into pieces with dimensions of 1 cm in height and 4 cm in diameter, which is the exact inner size of the WP4C sample container. The pieces were saturated by evacuation prior to the measurements. The saturated samples were subsequently put into the WP4C container with the top surface exposed to evaporation. The samples were under primary drying conditions, which is consistent with the measurement conditions of the HYPROP system. During evaporation, water pressure heads, of the samples were measured every 15 min, and water contents gravimetrically, until there were almost no changes in the sample weights. The WP4C measurements were reliable from around  $pF = 4.0$  (about 10,000 cm capillary pressure head) to  $pF = 6.5$  (about 3,100,000 cm capillary pressure head), where  $pF = \log(h)$ .

## NMR method

Nuclear magnetic resonance (NMR) is now widely used to image core body structures to predict capillary pressure and relative permeability curves of reservoir rocks in petroleum engineering (see e.g., Dias et al., 2023; Hosseinzadeh et al., 2020; Lima et al., 2022), but increasingly also in the soils

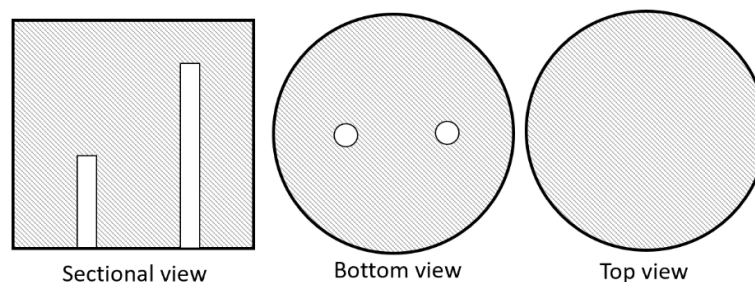


Fig. 2. Schematic of the rock sample with drilled holes for the two tensiometers.

literature (e.g., Rahmati et al., 2019). The method uses the response to hydrogen protons in cores. Petrophysical properties are evaluated based on different decay rates of the NMR signal. Capillary pressure and relative permeability curves can then be obtained from the images using an advanced interpretation method of NMR. NMR measurements were carried out only on the Guelph Dolomite and Indiana Limestone samples. These samples, taken from the same core as the HYPROP plugs, were saturated with NaCl (sodium chloride) brine having a salinity of 30,000 ppm. For this, the plugs were submitted to a vacuum chamber, which was then invaded with brine and subsequently pressurized to 2,000 psi for 24 hours. The samples were weighted regularly during saturation to verify the effectiveness of the procedure. A saturation of 100% is usually expected, but an error of  $\pm 5\%$  is generally thought to be acceptable (API RP 40, 1998). The Guelph Dolomite sample had a saturation of 100% and the Indiana Limestone sample 97%.

The samples were wrapped by Teflon tape to keep them saturated and avoid fluid losses. The NMR measurements were done in a Geospec 2 MHz device (Oxford Instruments, UK) using a Carr-Purcell-Meiboom-Gill (CPMG) pulse sequence (Carr and Purcell, 1954). The CPMG consists of a  $90^\circ$  radiofrequency pulse succeeded by a  $180^\circ$  pulse train lagged at  $\pi/2$  of the first pulse (Lima et al., 2020). To acquire the time distribution curves of the transverse relaxation ( $T_2$ ), the inverse Laplace transform was used with the LithoMetrix™ software from Green Instruments Technology (Fredericton, NB, Canada). The  $T_2$  signal is directly proportional to the pore surface-volume ratio, which is a measure of pore size when the formation is fully saturated with a single and well-behaved fluid, such as brine or light oil. In this way  $T_2$  can be associated to the pore-size distribution, porosity, and fluid type at the pore space (Coates et al., 1998). Smaller pores are related with shorter relaxation times and larger pores with longer relaxation time.

In our studies we used NMR techniques to obtain transverse relaxation times ( $T_2$ ) of the Guelph Dolomite and Indiana Limestone samples. From the acquired data, we estimated the partitioned porosity by applying the superficial relativity for each type of rock (Chen, 2014; Souza et al., 2013) and converted the  $T_2$  values to equivalent pore radii (Hoerlle et al., 2018). The approach allows one to obtain the pore-size distribution of samples with visual heterogeneities in their pore system. Fig. 3

shows NMR derived  $T_2$  relaxation time, which were converted to pore radii using surface relaxations of 8 for the IL sample and 13 for the GD sample. The pore radii distributions in turn were converted to equivalent capillary pressure – saturation curves using the approach of Lima et al. (2020) based on the Young-Laplace equation. The  $T_2$  curves we obtained for Guelph Dolomite and Indiana Limestone (Fig. 3) showed two distinct pore size distribution regions (meso- and macro-pores), which in terms of the unsaturated hydraulic properties can be represented by dual porosity functions. We used the RETC optimization program (van Genuchten et al., 1991) to estimate the hydraulic parameters of various uni- and bimodal hydraulic models for the two samples.

### Hydraulic functions

The water retention and unsaturated hydraulic conductivity data were analyzed in terms of several hydraulic models. We mostly used van Genuchten (1980) type models in our analyses. For unimodal data we used the standard van Genuchten-Mualem (VG) equations given by:

$$S_e(h) = \frac{S(h) - S_r}{1 - S_r} = \frac{1}{\left[1 + (\alpha h)^n\right]^m} \quad (1)$$

$$K(S_e) = K_s S_e^l \left[1 - (1 - S_e^{1/m})^m\right]^2 \quad (2)$$

where  $S$  and  $S_r$  denote water saturation and residual water saturation, respectively,  $S_e$  is effective water saturation,  $h$  is the capillary pressure head,  $K_s$  is the saturated hydraulic conductivity, and  $\alpha$ ,  $n$ ,  $m$  ( $m = 1 + 1/n$ ), and  $l$  are mostly semi-empirical fitting parameters. Here the values of  $K_s$  were fixed at the corresponding values of  $\rho\mu/g$  times the permeability shown in Table 1, where  $\rho$  denotes water density,  $\mu$  is viscosity, and  $g$  represents gravitational acceleration. Bimodal functions were further employed to account for the dual-porosity nature of several samples (Durner, 1994; Priesack and Durner, 2006):

$$S_e(h) = \frac{S(h) - S_r}{1 - S_r} = \frac{w_1}{\left[1 + (\alpha_1 h)^{n_1}\right]^{m_1}} + \frac{w_2}{\left[1 + (\alpha_2 h)^{n_2}\right]^{m_2}} \quad (3)$$

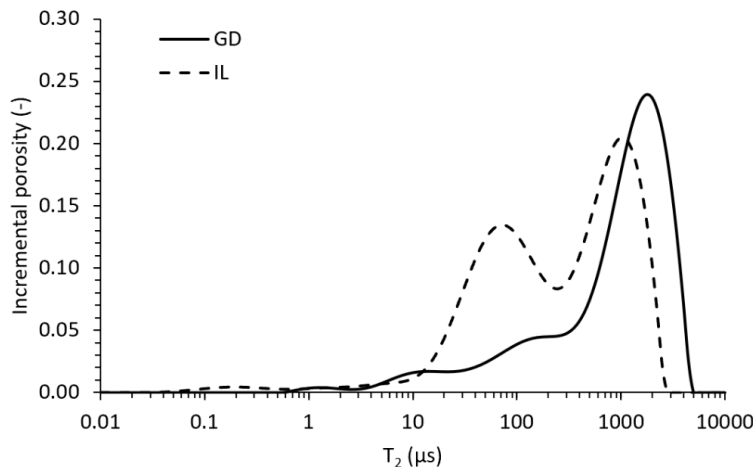


Fig. 3.  $T_2$  relaxation times for the Guelph dolomite (GD) and Indiana limestone (IL) rock samples.

$$K(S_e) = K_s \frac{(w_1 S_1 + w_2 S_2)^l \left\{ w_1 \alpha_1 \left[ 1 - (1 - S_e^{1/m_1})^{m_1} \right] + w_2 \alpha_2 \left[ 1 - (1 - S_e^{1/m_2})^{m_2} \right] \right\}^2}{(w_1 \alpha_1 + w_2 \alpha_2)^2} \quad (4)$$

where  $\alpha_i$ ,  $n_i$ , and  $m_i$  ( $i = 1, 2$ ) are the same parameters as those in Eqs. (1) and (2) but for the macropore and micropore regions, respectively, while  $w_i$  ( $i = 1, 2$ ) denotes the weight factors of the two regions, with  $w_1 + w_2 = 1$ . The van Genuchten (VG) model was additionally extended with the Peters-Durner-Iden formulation (the VG-PDI model). In the VG-PDI formulation, water inside pores is divided into capillary water and adsorbed water, with capillary water being described using the VG model and adsorbed water using the PDI approach. The adsorbed water mainly refers to film flow, angular flow, and isothermal vapour flow in relatively dry regions (Peters et al., 2015). Details about the VG-PDI formulation are given by Peters et al. (2015) and the HYPROP user manual (METER, 2015).

The HYPROP-FIT software (METER, 2021) was used to analyse the measured data. The software used the root mean square error (RMSE) to evaluate deviations between the measured and fitted water retention and unsaturated hydraulic conductivity values. The software also provides values of the corrected Akaike information criterion value (AICc), which is always a negative value. Large absolute AICc values reflect relatively close agreement of the model with the data. The RMSE and AICc values are respectively given by:

$$RMSE = \sqrt{\sum_{i=1}^b \frac{1}{b} (y_i^c - y_i)^2} \quad (5)$$

and

$$AICc = \frac{2k - 2L}{b} + \left[ \frac{2k(k+1)}{b-k-1} \right] \quad (6)$$

in which  $b$  is the number of measured data,  $k$  is the number of parameters in the fitted model,  $L$  is logarithmic likelihood, and  $y_i$  and  $y_i^c$  are the measured and calculated values, respectively.

## RESULTS AND DISCUSSION

The measured water retention data obtained using the three experimental methods are shown in Fig. 4. The HYPROP measurements for each sample lasted about 10 days when the

capillary pressure head reached a maximum of approximately 1000 cm. To also cover the relatively low moisture range, we complemented the HYPROP data with WP4C measurements above capillary pressure heads of about 5000 cm. The retention curve obtained from the HYPROP and WP4C measurements of the BS sample showed a remarkably steep structure with weak bimodal features. The steep structure suggests a relatively uniform pore size distribution within the sample. The HYPROP and WP4C retention data of Guelph Dolomite distributed relatively evenly over the entire moisture range (Fig. 4b), while Indiana Limestone showed more bimodal distribution (Fig. 4c).

The retention data obtained using the pressure plate (PP) method are shown as rectangles in Fig. 4. They took around one month or more for the drying branches of each sample. Extremely long equilibrated times were especially needed for the measurements at low moisture. The pressure plate data of Berea Sandstone (BS) were shifted to the right compared to the HYPROP data. One reason could be uneven water distribution within the rock sample caused by a non-uniform pore size distribution, but imperfect equilibration of the moisture regime appears more likely (which is often the case with PP measurements). Far fewer deviations were observed for Guelph Dolomite. The deviations between the pressure plate and the WP4C data around 10000 cm of the Indiana Limestone sample could also be due to the measured pressure plate data not reaching equilibrium. It took more than one week for each Guelph Dolomite pressure plate measurement at relatively low moisture to reach equilibrium. Because of its apparent shortcomings, we will not further consider the pressure plate data.

The NMR retention data for the GD and IL samples (shown as crosses in Fig. 4) were slightly steeper than the HYPROP and WP4C data. The NMR and HYPROP measurements were conducted using different plugs taken from the same rock core. Slightly different porosity values of the two rock plugs could cause differences in the retention data (see Table 1). Overall, of the three rocks, only Indiana Limestone showed a clear bimodal form of its retention curve.

The combined HYPROP and WP4C retention and unsaturated conductivity data were fitted using four hydraulic functions, i.e., the original unimodal van Genuchten-Mualem (VG) model, the bimodal van Genuchten (BiVG) model,

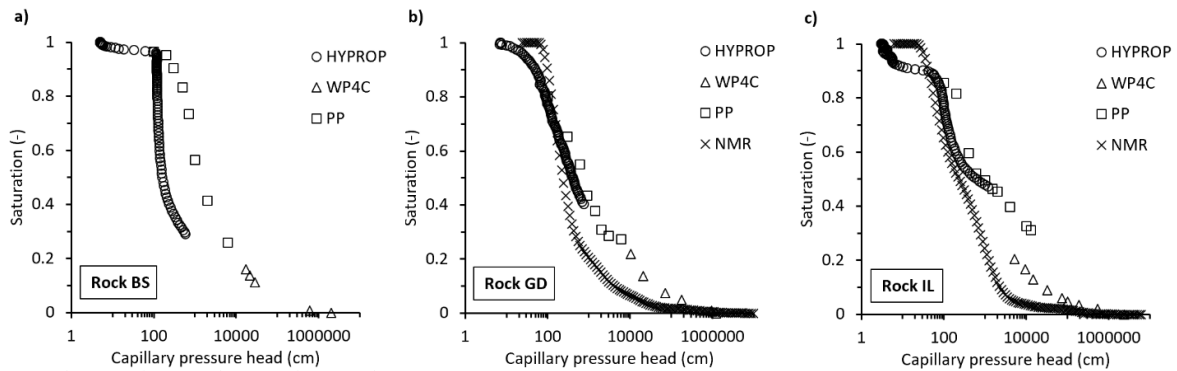


**Fig. 4.** Water retention data of the three rock samples (BS, GD, IL) obtained using the evaporation method (HYPROP, circles), the chilled-mirror dewpoint method (WP4C, triangles), the pressure plate method (PP, rectangles). Also shown as cross marks are retention data based on NMR measurements of the two carbonate rocks (GD and IL).

the van Genuchten-PDI (VGPDI) model, and the bimodal van Genuchten-PDI (BiVGPDI) model. The saturated hydraulic conductivities were fixed at their calculated values derived from the measured permeabilities shown in Table 1.

The measured and fitted retention curves for Berea Sandstone are shown in Fig. 5a, with the optimized hydraulic parameter values and statistical results listed in Table 3. The fitted unimodal curve VG function deviated from the measurements over the full moisture range. However, the BiVG, VGPDI, and BiVGPDI functions closely matched the measurements, with only minor differences in the relatively dry range. Fig. 5b further shows the measured and fitted hydraulic conductivity curves for the Berea Sandstone. Results obtained using the unimodal VG and VGPDI functions deviated slightly from the available unsaturated conductivity data. The optimized bimodal BiVG and BiVGPDI curves almost exactly overlapped the measurements. As expected, the BiVGPDI function, with more fitting parameters, described the observations best among the four models, which is seen also from the statistical analyses (Table 4). However, the BiVG results were also acceptable with fewer fitting parameters. The calculations reflected the only slightly

bimodal pore structure of the Berea Sandstone rock sample. Please note that  $p_{\text{Fdry}}$ ,  $\omega$  and  $A$  in Table 4 are VGPDI parameters (METER, 2021), with  $p_{\text{Fdry}}$  representing the  $p_{\text{F}}$  value where relative saturation in the very dry region becomes zero (oven dryness).

Observed and calculated retention and unsaturated hydraulic conductivity data of Guelph Dolomite are shown in Fig. 6. The water retention results obtained with the four hydraulic functions were almost identical over the full moisture range. The optimized hydraulic conductivity models almost overlapped with each other over the available data range, with excellent descriptions of the HYPROP data. The resulting fitted parameter values are listed in Table 5, with Table 6 listing the corresponding accuracy parameters. The VGPDI model described the measurements minimally better based on the accuracy analyses, although the other functions also matched the experiments over the available data range. The deviating line of the hydraulic conductivity near saturation obtained with VGPDI is an irregularity and could be avoided by fixing  $p_{\text{Fdry}}$  at 4.8 as is generally assumed following Schneider and Goss (2012) among others. In our analyses we optimized the value of  $p_{\text{Fdry}}$ .

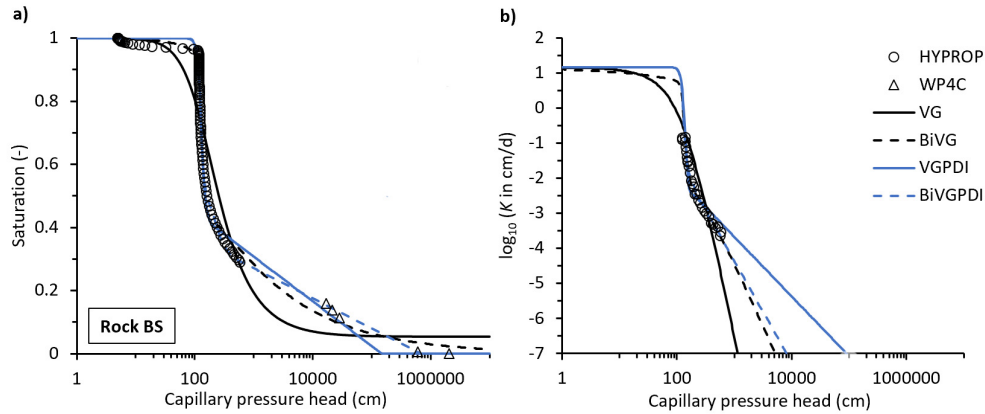


**Fig. 5.** Measured and fitted water retention (a) and unsaturated hydraulic conductivity (b) curves of Berea Sandstone (BS) using the VG model (black solid lines), the bimodal VG model (black dashed lines), the VG-PDI model (blue solid lines), and the bimodal VG-PDI model (blue dashed lines).

**Table 3.** Fitted hydraulic parameter values of the unimodal and bimodal models for Berea Sandstone.

| Parameter (unit)  | VG model | BiVG model | VGPDI model          | BiVGPDI model        |
|-------------------|----------|------------|----------------------|----------------------|
| $\alpha$ (1/cm)   | 0.0077   | 0.0077     | 0.0077               | 0.0045               |
| $n$ (-)           | 1.93     | 15.00      | 12.66                | 6.22                 |
| $S_r$ (-)         | 0.05     | 0.00       | 0.42                 | 0.37                 |
| $K_s$ (cm/d)      | 14.63    | 14.63      | 14.63                | 14.63                |
| $l$ (-)           | 4.38     | 3.04       | 1.28                 | 2.70                 |
| $w_2$ (-)         | —        | 0.51       | —                    | 0.75                 |
| $a_2$ (1/cm)      | —        | 0.0054     | —                    | 0.0079               |
| $m_2$ (-)         | —        | 1.33       | —                    | 15.00                |
| $p_{\text{Fdry}}$ | —        | —          | 5.17                 | 5.84                 |
| $\omega$ (-)      | —        | —          | $4.7 \times 10^{-4}$ | $1.2 \times 10^{-3}$ |
| $A$ (-)           | —        | —          | -1.70                | -2.90                |

**Table 4.** Statistical results of the unimodal and bimodal models for Berea Sandstone.

| Statistical criterion | VG model | BiVG model | VGPDI model | BiVGPDI model |
|-----------------------|----------|------------|-------------|---------------|
| RMSES                 | 0.1047   | 0.0368     | 0.0363      | 0.0305        |
| RMSElogK              | 0.6126   | 0.3140     | 0.4581      | 0.3318        |
| AICc                  | -1027    | -1269      | -1230       | -1286         |

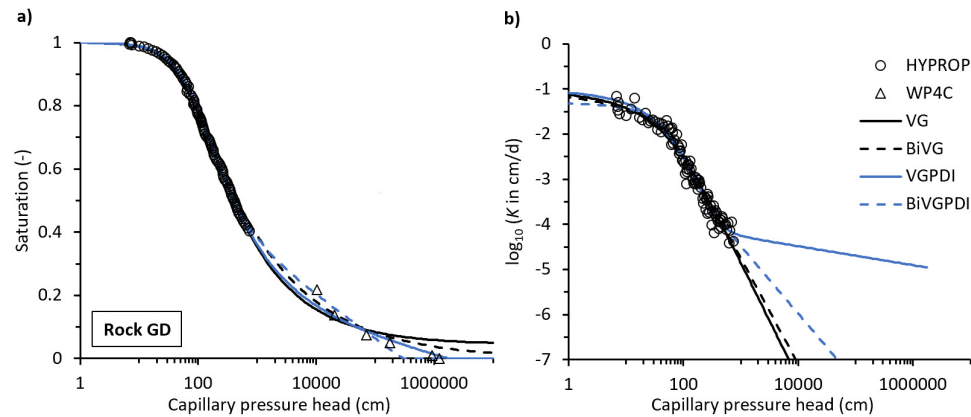


**Fig. 6.** Measured and fitted water retention (a) and unsaturated hydraulic conductivity (b) curves of Guelph Dolomite (GD) using the VG model (black solid lines), the bimodal VG model (black dashed lines), the VG-PDI model (blue solid lines), and the bimodal VG-PDI model (blue dashed lines).

**Table 5.** Fitted parameter values for different unimodal and bimodal models of rock sample GD.

| Parameter<br>(unit) | HYPROP+WP4C |               |                       |                       | NMR         |               |                |                  |
|---------------------|-------------|---------------|-----------------------|-----------------------|-------------|---------------|----------------|------------------|
|                     | VG<br>model | BiVG<br>model | VGPDI<br>model        | BiVGPDI<br>model      | VG<br>model | BiVG<br>model | VGPDI<br>model | BiVGPDI<br>model |
| $\alpha$ (1/cm)     | 0.0124      | 0.0124        | 0.0121                | 0.0448                | 0.0071      | 0.0014        | 0.0067         | 0.0067           |
| $n$ (-)             | 1.45        | 7.60          | 1.55                  | 1.01                  | 1.93        | 1.56          | 2.78           | 3.21             |
| $S_r$ (-)           | 0.00        | 0.00          | 0.25                  | 0.50                  | 0.00        | 0.00          | 0.03           | 0.00             |
| $K_s$ (cm/d)        | 0.10        | 0.10          | 0.10                  | 0.10                  | —           | —             | —              | —                |
| $l$ (-)             | -0.69       | -0.92         | 0.35                  | 1.49                  | —           | —             | —              | —                |
| $w_2$ (-)           | —           | 0.89          | —                     | 0.89                  | —           | 0.73          | —              | 0.27             |
| $\alpha_2$ (1/cm)   | —           | 0.0112        | —                     | 0.0096                | —           | 0.0067        | —              | 0.0015           |
| $n_2$ (-)           | —           | 1.34          | —                     | 1.97                  | —           | 3.20          | —              | 1.56             |
| pFdry               | —           | —             | 6.24                  | 5.47                  | —           | —             | 6.80           | 6.82             |
| $\omega$ (-)        | —           | —             | $8.94 \times 10^{-4}$ | $1.00 \times 10^{-4}$ | —           | —             | —              | —                |
| $A$ (-)             | —           | —             | -0.21                 | -1.51                 | —           | —             | —              | —                |

**Table 6.** Statistical analysis for different unimodal and bimodal models of rock sample GD.

| Statistical<br>criterion | HYPROP+WP4C |               |                |                  | NMR         |               |                |                  |
|--------------------------|-------------|---------------|----------------|------------------|-------------|---------------|----------------|------------------|
|                          | VG<br>model | BiVG<br>model | VGPDI<br>model | BiVGPDI<br>model | VG<br>model | BiVG<br>model | VGPDI<br>model | BiVGPDI<br>model |
| RMSES                    | 0.1713      | 0.1713        | 0.1713         | 0.1713           | 0.0042      | 0.0014        | 0.0016         | 0.0007           |
| RMSElogK                 | 0.1765      | 0.1499        | 0.1515         | 0.1456           | —           | —             | —              | —                |
| AICc                     | -1869       | -1873         | -1874          | -1871            | -1309       | -1733         | -1544          | -1732            |

The results for Indiana Limestone obtained with the HYPROP and WP4C techniques are shown in Fig. 7, with the corresponding parameter values listed in Table 7. For the retention curves, the VG and BiVG models described the entry part reasonably well but not the lower moisture part. In contrast, the PDI functions gave better agreement in the low moisture range. Regarding the unsaturated hydraulic conductivity, the results of all hydraulic functions were almost identical over the available data range. But they deviated significantly from each other between saturation and the first data points. Based on the statistical analyses data (Table 8), the VGPDI and BiVGPDI functions described the measurements better in the low moisture range.

Figs. 4b and 4c show that the NMR-derived water retention curves of Guelph Dolomite and Indiana Limestone deviated from the HYPROP and WP4C data, with the NMR curves being

steeper than those shown by the full HYPROP and WP4C data. Still, the NMR-based retention curves are in the general area of the measured data and hence provide a useful initial indication of the hydraulic properties of the rock samples. For completeness, we also analysed the NMR-based curves in terms of the various hydraulic models. The NMR derived and optimized retention curves of Guelph Dolomite and Indiana Limestone are shown in Fig. 8. The VG model deviated from the NMR curves at relatively high and low moisture contents, while the other models gave almost identical descriptions with generally excellent agreement with the NMR based curve. Based on the accuracy analyses provided in Tables 6 and 8, the BiVG and BiVGPDI models described the NMR curves minimally better than the other models, with only weak hints of having bimodal distributions.

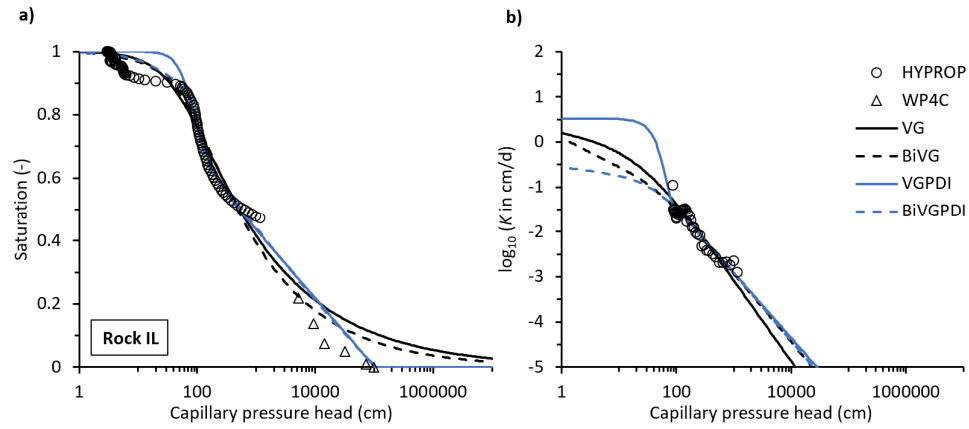


**Fig. 7.** Measured and fitted HYPROP and WP4C water retention (a) and unsaturated hydraulic conductivity (b) curves of Indiana Limestone (IL) using the VG model (black solid lines), the bimodal VG model (black dashed lines), the VG-PDI model (blue solid lines), and the bimodal VG-PDI model (blue dashed lines).

**Table 7.** Fitted parameter values for different unimodal and bimodal models of rock sample IL.

| Parameter<br>(unit) | HYPROP+WP4C |               |                       |                       | NMR         |               |                |                   |
|---------------------|-------------|---------------|-----------------------|-----------------------|-------------|---------------|----------------|-------------------|
|                     | VG<br>model | BiVG<br>model | VGPDI<br>model        | BiVG-PDI<br>model     | VG<br>model | BiVG<br>model | VGPDI<br>model | BiVG-PDI<br>model |
| $\alpha$ (1/cm)     | 0.0193      | 0.5000        | 0.0137                | 0.0282                | 0.0130      | 0.0173        | 0.0135         | 0.0169            |
| $n$ (-)             | 1.30        | 1.64          | 3.23                  | 1.05                  | 1.59        | 3.25          | 1.55           | 3.07              |
| $S_r$ (-)           | 0.00        | 0.00          | 0.69                  | 0.00                  | 0.00        | 0.00          | 0.03           | 0.01              |
| $K_s$ (cm/d)        | 3.29        | 3.29          | 3.29                  | 3.29                  | —           | —             | —              | —                 |
| $l$ (-)             | -2.69       | -3.39         | 5.002                 | 4.23                  | —           | —             | —              | —                 |
| $w_2$ (-)           | —           | 0.98          | —                     | 0.06                  | —           | 0.53          | —              | 0.48              |
| $a_2$ (1/cm)        | —           | 0.0136        | —                     | 0.0093                | —           | 0.0019        | —              | 0.0016            |
| $n_2$ (-)           | —           | 1.36          | —                     | 8.33                  | —           | 2.08          | —              | 2.38              |
| pFdry               | —           | —             | 5.01                  | 5.03                  | —           | —             | 5.04           | 6.09              |
| $\omega$ (-)        | —           | —             | $1.62 \times 10^{-2}$ | $5.43 \times 10^{-2}$ | —           | —             | —              | —                 |
| $A$ (-)             | —           | —             | -1.44                 | -1.50                 | —           | —             | —              | —                 |

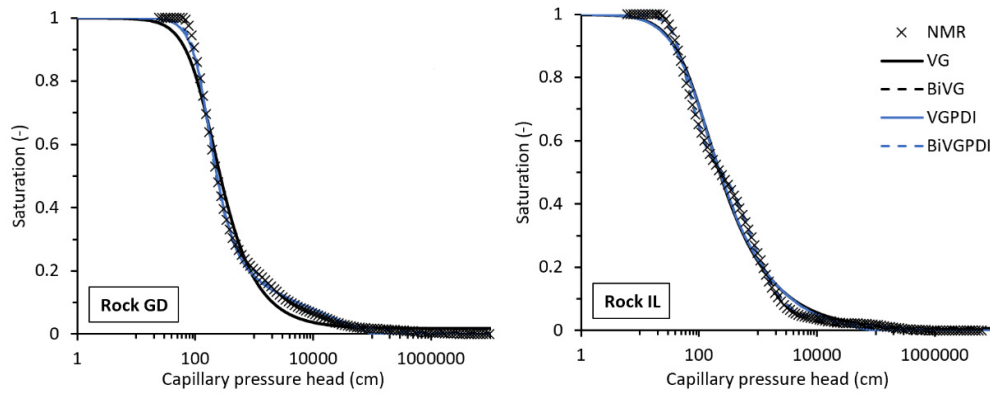


**Fig. 8.** Measured and fitted NMR-based water retention curves of Guelph Dolomite (GD) and Indiana Limestone (IL) samples using the VG model (black solid lines), the bimodal VG model (black dashed lines), the VG-PDI model (blue solid lines), and the bimodal VG-PDI model (blue dashed lines).

**Table 8.** Statistical analysis for different unimodal and bimodal models of rock sample IL.

| Statistical<br>criterion | HYPROP+WP4C |               |                |                   | NMR         |               |                |                   |
|--------------------------|-------------|---------------|----------------|-------------------|-------------|---------------|----------------|-------------------|
|                          | VG<br>model | BiVG<br>model | VGPDI<br>model | BiVG-PDI<br>model | VG<br>model | BiVG<br>model | VGPDI<br>model | BiVG-PDI<br>model |
| RMSES                    | 0.0369      | 0.0345        | 0.0306         | 0.0219            | 0.0044      | 0.0012        | 0.0042         | 0.0008            |
| RMSElogK                 | 0.1713      | 0.1420        | 0.1386         | 0.1417            | —           | —             | —              | —                 |
| AICc                     | -1538       | -1559         | -1592          | -1651             | -1298       | -1609         | -1303          | -1683             |

## CONCLUSIONS

In this work, we measured the unsaturated hydraulic properties of three reservoir rock samples (Berea Sandstone, Guelph Dolomite, and Indiana Limestone) using an improved HYPROP experimental setup. The improved HYPROP was developed to cater specifically to rock samples. This improved HYPROP setup enabled measurements of the local water pressures within the rock samples without disturbing the upper portion of the samples. The WP4C (chilled-mirror dewpoint) method was employed to extend the capillary pressure data to the dry region. We compared our results with conventional pressure plate data and nuclear magnetic resonance (NMR) based data. We have found the pressure plate data shifted slightly to the right of the HYPROP data, while no such clear deviation was observed for Guelph Dolomite and Indiana Limestone. By comparison, the NMR retention data were steeper than the HYPROP and WP4C data. Only a slight bimodal nature was observed for Indiana Limestone.

Four hydraulic functions (i.e., the original van Genuchten (VG), the bimodal van Genuchten (BiVG), the van Genuchten-PDI (VGPDI), and the bimodal van Genuchten-PDI (BiVGPDI) functions), were employed to describe the HYPROP and WP4C retention and unsaturated hydraulic conductivity data, as well as the NMR-based retention curve. For the HYPROP and WP4C measurements, the bimodal models were found to reproduce the data of Berea Sandstone relatively well. The PDI functions matched the data of Guelph Dolomite and Indiana Limestone. All models closely described the NMR measurements. Some of the deviations between the pressure plate, HYPROP, WP4C and NMR-based hydraulic properties may have been due to heterogeneity along the core from which the respective plugs were taken. This possible heterogeneity within the cores may need to be recognized in reservoir analyses in the future.

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