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GRADUS AD PARNASSUM: INTERNATIONAL ONLINE TRAINING FOR MICROTONAL SINGING WITH 22 PITCHES WITHIN THE OCTAVE

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ABSTRACT
In the musical cultures of the world, including the oral traditions of European peoples, a wide variety of musical scales have been and still are used. In contemporary Western music, growing awareness of other musical cultures, as well a certain saturation with the sounds of the 12-tone scale have led to experimentation with alternative scale systems. Applying a microtonal scale that divides the octave into 22 equal parts (22-EDO), this article discusses possible strategies for training musically educated Western adults. According to experiences from the research and teaching/learning process, the article introduces the music theoretical fundamentals of the 22-EDO scale, knowledge of which is an integral part of a potential training method, the creation of teaching materials and musical compositions. This knowledge was also the basis of my practice and performance at concerts in 2022 and 2023, and helps with the evaluation of the results based on direct experience and audio recordings. The analysis focuses on the online Gradus ad Parnassum course, which was initiated during the Covid-19 pandemic to teach microtonal singing.

The findings indicate that the level of familiarity with the intervals (compared to those of the 12-tone scale) had a significant influence on learning outcomes, sometimes facilitating, but sometimes hindering, the process. Although the online format offered a number of advantages, such as connecting participants who would otherwise not have the opportunity to meet, it turned out that in-person training allowed for much faster progress due to the absence of physical distance and technical limitations.

KEYWORDS: microtonality • xenharmony • 22-EDO • group singing • music theory • online singing during Covid

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INTRODUCTION

As a composer the author is interested in finding new ways of creating music, widening the possibilities of musical expression. One specific way is to find alternatives to the traditional Western organisation of pitches as 12 equally distant pitches within the octave. While the free use of the human hearable pitch continuum seems not always aesthetically satisfactory, other distinct pitch organisation systems and principles offer more interesting features for creative musical work. As there have been several attempts to expand the established tonal system in Western music history, this topic, widely referred to as microtonality, became of increasing interest to music theorists and composers during the 20th century.

In the musical cultures of the world, including older music and the oral traditions of European peoples, a wide variety of scales have been and still are used. For the Western ear the music of non-European traditional music sounds in some way 'microtonal', as there are for instance Arabian/Turkish Maqam or Persian Dastgāh music, traditional and classical Indian music, the various African traditional musical cultures and many others. But also older traditional European music cultures might have musical intervals and scales beyond those used in modern Western mainstream music. In Estonia, microtonality can be found in older songs of the Seto culture with the one-three-semitone mode (Pärtlas 1997; Ambrazevičius and Pärtlas 2011; Pärtlas and Oras 2012), in some other older runo songs (for example a swinging song with a 3/4-tone interval between first and second scale degree, see ERA, Pl. 28 B2[1] and Sildoja 2010 [2005]), or the 11th partial tone in a newer, Scandinavian-originated Estonian traditional violin tune (ERA, Pl. 6 B2;[2] Oras and Sarv 2016).

In Western contemporary music, despite a remarkable extension of the set of commonly used instruments during the last 70 years, mainly in terms of percussion instruments, on the side of pitched instruments classical instruments still dominate. This causes several restrictions for composers and difficulties for the performers of microtonal music. Instruments with fixed-tuned pitches are often technically not easy or quite impossible to retune. For piano or organ specialised craftsmen are needed, and even then it will be restricted to 12 or fewer distinct pitches within the octave, differently tuned, possibly as a choice of a scale with more than 12 pitches per octave. Classical woodwind instruments have the finger holes and keywork systems designed to the 12-tone equal tempered scale and only complicated fingerings and changes in the embouchure can provide microtonal pitches, with constraints due to faster playing and the timbral qualities of the sound. The majority of brass instruments are equipped with valves setting the bypass tubes to 12-tone equal tempered intervals.

In contrast, the human voice and instruments with a continuously adjustable pitch like the common bowed string instruments and trombone are not technically restricted to produce any arbitrary pitch within the pitch continuum of their ranges, which should be ideal for microtonal music performance. Unfortunately, it turns out that these musicians are strongly trained to the Western 12-tone equal temperament and it is not easy to perform precisely pitches in between the steps of this given grid.

Often a composer newly interested in microtonality is faced with the same problems as a performer with a continuously adjustable pitch instrument or voice. As long as the composer might be satisfied with a purely technical or mathematical composition

process, these problems rely only on the performer; or these problems wouldn't exist when composing pieces as electronic realisations (MIDI technology and its extensions and alternatives). But as long as the composer desires to develop a mental representation, a question arises: How can one imagine differently organised scales of pitches and their related musical intervals? And later we might come to the question of how to train musicians to intone microtonally?

Microtonally tuned synthesisers can assist first contact with these unfamiliar scales and intervals. Computer-based synthesisers are easy to use, flexible in changing between different microtonal scales and systems and have precise pitch production. They can be mapped to standard MIDI keyboards. More advanced and useful for scales with much more than 12 pitches per octave are isomorphic hexagonal keyboards, as since 2020 the Lumatone controller (Lumatone n.d.), based on a prototype by Siemen Terpstra from 1996, has gained popularity within the microtonal community.

Returning to the essential questions: Why is it difficult for adult Western-trained musicians to sing microtonal intervals or perform them on instruments with continuously adjustable pitch? How can musicians be trained to overcome these obstacles and learn new, or new forms of, musical interval? To answer these questions this article provides a general overview of Western microtonality, followed by an overview of known approaches to microtonal singing and ear training.

As a possible approach, a specific method of online singing training developed by the author and his colleagues will be demonstrated. This arose during the Covid-19 pandemic and the very reduced physical and personal contacts of that period. The method is based on a scale with 22 equally-spaced pitches within the octave, and was developed considering findings on musical intervals from modern psychoacoustics and the personal experiences of the author while learning to sing and intone microtonal intervals on the violin. The 22-step scale was brought into play in England in the 19th century, inspired by Indian music.

The overview includes descriptions of technical learning aids such as synthesisers, audio files, and real-time online audio communication, along with structural exercises that allow success to be evaluated through familiar intervals. The results of the training process are analysed based on the recordings of three concerts, where specially composed songs and pieces in this scale were performed.

Even though quite a large number of audio recordings has been stored from the learning sessions and the concerts, the author refrains from a quantitative pitch analysis, which could theoretically assess the level of precision achieved by the participants. However, reliable pitch measurement is not feasible due to various factors, including the general unstable behaviour of the human voice, technical quality of the recordings (online sessions, the not always ideal acoustics of the concerts, etc.), multiple singers/performers per part, synthesiser support for the singers, polyphony (software voice separation is still not easy, even trained musicians find this difficult), and other practical technical difficulties. Additionally, the learning sessions and concerts happening in a creative environment, with changes in participants from session to session or semester to semester, did not give the fully comparable conditions necessary for reliable quantitative research.

For these reasons the results of the present study will be evaluated based on the experiences of the author only. The evaluation focuses on how well the microtonal

intervals, scales and musical pieces were performed, considering both characteristic intervals and overall impression. However, due to the vast amount of material, this evaluation is restricted to some samples subjectively chosen by the author. This study, needless to say, could serve as a starting point for a more controlled and quantitative study in learning microtonal singing in the future.

MICROTONALITY

The term microtonality intends to describe musical intervals smaller than a semitone, but basically refers more to concepts of pitch organisation beyond the 12-tone equal temperament tuning. This system has dominated and unified tuning principle in Western serious music for roughly the last 300 years and is still widely used today. Apart from *microtonality* several alternative conceptions and definitions have been offered, amongst others *xenharmonic*, a term offered by Ivor Darreg (1963) for new, unusual harmony, derived from the Greek *xenos* 'foreign', 'strange' in order to describe unfamiliar sounding melodies, chords and chord progressions.

Apart from the traditional cultures mentioned above, microtonality has a certain tradition in Europe through the centuries, for example the ancient Greek tone system with the enharmonic tetrachord (containing an interval approximately the size of a quarter tone), and experimental music instruments such as Nicola Vicentino's (1511–1576) archicembalo from 1555 intended for 31 pitches per octave. With the beginning of the 20th century certain composers discovered microtonal possibilities for their creative work, for example the Czech composer Alois Hába (1893–1973), the Russian French resident Ivan Wyschnegradsky (1893–1979), the Mexican Julián Carrillo (1875–1965) and the US-American Harry Partch (1901–1974). Some of them recognised the need to modify musical instruments for microtonality or to create completely new ones. Carrillo designed and patented 15 pianos for various microtonal tunings (Cannon-Brown 2024), Hába and Wyschnegradsky favoured the quarter-tone piano, and Partch developed his own specific set of instruments (Partch 1974 [1949]: 195–357).

Inspired by Partch, a relatively small but very specialised scene of microtonal composers that emerged in the USA, and which is active today. In particular, the development of new and original theories of alternative pitch organisation played an important role, in which theorists like Partch, Erv Wilson (1928–2016), Georg Secor (1943–2020), Gene Ward Smith (1947–2021), Dave Keenan (b. 1959), Paul Erlich (b. 1972), Douglas Blumeyer (b. 1984) and others were and are involved.

Generally speaking there are two different approaches: first, Just Intonation (JI) uses rational number relationships to organise the fundamental frequencies of the pitches used, such as 3/2 or 7/5. Ratios composed from lower integer numbers achieve a maximal consonance impression when using sounds with harmonic spectra, evoking a continuum from consonant to dissonant intervals including unfamiliar ones, often when the numbers 7 and 11 are involved. This method has the disadvantage that it can quickly generate a huge number of pitches. For example, transposing chords or melodic material on a just scale changes their relative interval structure due to the typically unequal steps of just scales, while preserving the interval structures means more pitches must immediately be added to the given scale. Without doubt, the latter increases the

complexity in practical composition as well as the difficulties for performing musicians. The second approach is based on arbitrary equal temperament beyond the traditional Western temperament. Instead of dividing the octave into twelve equal steps, it will be now divided into another number of equal steps, which is abbreviated as n–EDO (Equal Division of the Octave, n is the number of steps) (Monzo 2021a; EDO n.d.). The octave can also be replaced by any other interval, but this is rarely done (Equal-step tuning n.d.). This approach has the advantage that one only has to operate with a relatively limited number of tones, although consonant intervals can only be approximated more or less well.

MICROTONAL SINGING AND EAR TRAINING: TECHNOLOGY AND KNOWN APPROACHES

Microtonal singing and ear training (or solfège) has been practiced and developed by different musicians over the last few decades with very different methodical and technical approaches being proposed. Therefore technical aids have played an important role, comparable in Western music history to how keyboard instruments (organ, piano) have been used to support or train singers. In the contemporary Western microtonal world computer-based electronic keyboard instruments have taken this role, as they are flexible in retuning to whatever microtonal system is needed.

More technical, and with no direct historical prototype, is the use of so-called guide tracks. These are electronic realisations of the musical material played back in real-time as audio files in headphones to the rehearsing or performing musician. This gives the musician a precise model for learning intonation directly from musical material. Later it can act as a template when practising, or as support during studio recordings and concerts even if, ideally, the musician should be able to intone correctly without a guide track after training intensely. Guide tracks were used to record Toby Twining's "Crysalid Requiem" (Twining 2011; 2016; Scheurich 2019) and during the Xenharmonic Summer Camp in West Virginia, USA in 2019.

Another method is the use of tuner devices, either as hardware or as software for a computer or smartphone. The musician can read in real-time information about the produced pitch. Commonly these use classical pitch letters with deviation in cents[3] or frequencies, but today tuner apps with microtonal scales are available (such as Entonal Tuner). The Kepler Quartet has used the support of a tuner device for their recordings of all microtonal string quartets by composer Ben Johnston (Segnitz 2016; Scheurich 2019). As the tuner devices can be attached via contact microphones directly to the bodies of string instruments very good channel separation is provided for ensemble practise and performance. This method is also useful for compositions where the composer demands exact frequencies, often for precise beating[4] effects, as was done for Fakhrtabatabaie's piece "Den Homogenizer" (two violins and viola) during the Xenharmonic Summer Camp 2019. The composer later released the recording of the final version for string quartet with the new title "Trans Cultural Diffusion" (for a performance, see Tabatabaie 2022).

As discussed before, microtonal approaches are more or less divided into Just Intonation approaches and numerous equal temperaments, with the exception of the tradi-

tional 12 per octave. Ear training and singing exercises are also based either on one or the another. Composer Andrew Heathwaite has provided a series of JI exercises, calling them Singtervals (Heathwaite and Hale 2020a; 2020b; 2020c; 2020d). There are provided a soundtrack in order for students to sing along, where the melodic line is supported by the fundamental tones in the bass register. A version with drum kit accompaniment is available. It seems that methodically it is meant to support learning with joy rather than serving as a highly concentrated drill. Casey Hale created scores for Heathwaite's exercises using sagittal accidentals (Heathwaite and Hale 2020c; 2020d). Sagittal notation is a system of arrow-shaped accidentals designed for both JI and EDO systems and can even be read relatively intuitively: the broader the arrow's shape the more the deviation there is from the pitch indicated by the position of the notehead. Sagittal notation was developed by Secor and Keenan with a major early contribution from Smith and has been recently supplemented by Blumeyer (Keenan 2004; Secor and Keenan 2006; Sagittal n.d.).

The composer, improvisor and multi-instrumentalist Jacob Barton has been working with microtonality since the 2000s. Since 2008 he has conducted xenharmonic camps and workshops and has also gained extensive methodological experience in training microtonal intonation, both instrumentally and vocally. He recommends for the training of instrumentalists "first to hear, then to sing, then to play unfamiliar microtonal intervals",[5] and he has used this approach during several workshops. Barton regularly uses so-called spiral chord progressions as microtonal singing exercises. Such chord progressions were discovered by Giovanni Battista Benedetti in the 16th century (Benedetti 1585: 279–280; Duffin n.d.). They pump a certain comma[6] in such a way that when repeated correctly, the chord progression becomes a small step higher or lower, and the more often the chord progression is repeated, the further away it gets from the starting pitch.

As an outcome of the xenharmonic camps Barton collected 48 short songs for a wide variety of microtonal systems and published them in 2013 as *The Sagittal Songbook*. All songs were composed between 2007 and 2012 by 15 composers (Barton 2013: i–iii, xv–xvi). Barton follows a clear methodical approach with the selection of short songs:

> As to the Songbook, I can comment that I took great care in creating the sequence of pieces in the book, from start to finish, that it would progress from easy to difficult systems, and that there would be meaningful links between adjacent pieces. I never documented that explicitly, but that was the intent.[7]

Apart from the JI and xenharmonic approaches there are also several ear training singing exercises based on subdivisions of the twelve tone equal temperament. Dividing the 12-EDO semitone into two equal parts we get quarter tones (24-EDO), into three equal parts we get sixth tones (36-EDO) and finally into six equal parts twelfth tones (72-EDO) where the latter also includes the three previous divisions. This 72-EDO approach will use the familiarity of the 12-EDO system dividing it into smaller and smaller parts. This approach has been taken by the Boston Microtonal Society, the used accidentals were developed by composer Ezra Sims. Julia Werntz, composer and professor at the New England Conservatory developed, based on his predecessor Joseph Maneri, a series of interval ear training exercises, followed by one and two voice composition exercises. The ear training exercises are only interval exercises accompanied by audio examples played by Mat Maneri (Werntz 2015).

Composer and choir conductor Robert Lopez-Hanshaw (2020a; 2020b; 2020c) applies his exercises directly to choir practice using very different methods. He directs attention to the horizontal and vertical aspects during every pitch change, calling it a 'bimodal strategy'. According to this strategy, 'horizontal' means focusing on the intonation of consecutive, melodic intervals, while 'vertical' refers to the intonation of simultaneous sounding intervals or chords. Additionally, he focuses on a target-based approach for microtonal stepwise movement, applies Just Intonation, quantifies comma shifts and practices characteristic chords. But he also favours 12-EDO subdivisions like splitting a semitone into two quarter tones and splitting a whole tone into three third tones, which leads to the 72-EDO scale, similar to the aforementioned Boston-based approach.

A very interesting approach to train singers in accurate microtonal intonation was carried out by Jonathan Wild, recreating Vicentino's vocal music based on 31-EDO. Recording the vocal ensemble with all voices together first, this recording will be played back to each singer separately through headphones, and each voice now recorded individually on a separate track. Then Wild used the software Melodyne to analyse and correct all pitches part by part to the target values, mixing them together, which achieves a naturally sounding choir soundtrack with, probably, Vicentino's intended intonation (Wild and Schubert 2008; Wild 2014). The same technique can be used for contemporary microtonal compositions by preparing guide tracks for singers using their own voices.

PSYCHOACOUSTIC FINDINGS ABOUT MUSICAL INTERVALS

Training the ability to perform microtonal music mainly goes back to the ability to hear musical intervals, to sing them or play them on an instrument with continuously adjustable pitch. In this context it would be useful to take into account contemporary psychoacoustic understanding of musical intervals.

As a wide consensus the recognition and production of musical intervals is based on categorical perception, with these categories not relating to certain fixed physical measurable values, although certain tolerances appear which can be higher or lower, depending on acoustic and musical context. Richard Parncutt (2012: 84–85) explains:

> In categorical perception, different shades of red are perceived as red, different intonations of C♯ are perceived as C♯, and different tunings of a major third interval are perceived as major thirds. Categorical perception allows listeners to accommodate mistunings in musical performances of as much as a quartertone or even a semitone, depending on context.

There are numerous separate psychoacoustic studies to interval recognition and production. In their article "A Psychocultural Theory of Musical Interval: Bye Bye Pythagoras" Parncutt and Graham Hair (2018) not only provide a comprehensive overview of the various interval studies, but also consider historical understandings from ancient Greek up to Helmholtz and today. They come to the conclusion that psychoacoustic studies cannot support an idealistic Pythagorean-based view and consider the use of ratios to be inappropriate:

> Musical intervals are not ratios, nor are they magical mathematical entities. They are learned, approximate, perceptual distances. They emerge from a multigenera-

> tional perceptual-historical process, mediated by the physical properties of musical tones and the physiological and psychological properties and limitations of the human auditory system. (Ibid.: 492)

As tolerances might be, depending on the musical situation, really high, this seems to contradict the idea of playing and singing microtonal intervals at all. But as all the underlying studies probably deal with 12-EDO-tuned intervals and their subjects are traditionally Western trained musicians, we cannot be sure how relevant these findings are for microtonal music. Often when contemporary microtonal music plays around specific higher harmonies as involving the 7th and the 11th partial tone, the composers prefer a consciously slower tempo giving the musician time to produce these pitches precise and contemplation time for the listener. It seems that the learning process should not be underestimated. Parncutt and Hair (ibid.: 487) summarise a study by Franz Loosen: "Loosen (1995) asked musicians to evaluate the tuning of simple rising and falling scales that had been tuned to Just, Pythagorean, and 12-EDO. Violinists preferred something approaching Pythagorean, pianists preferred 12-EDO, and non-musicians had no preference." Indeed microtonal skilled musicians haven't been asked, probably they would be able even to distinguish between these three.

The author agrees that intervals in the psychoacoustic sense are mental and perceptive categories which cannot be clearly nailed to a certain frequency ratio or a potentially irrational value of a certain tempered interval. These intervals are mental categories with their acceptable tolerances depending on various factors: whether the interval is sung or played sequentially or simultaneously; whether both pitches have similar loudness; whether the intervals are in the most comfortable middle range or in a quite low or high register; whether the spectra are harmonic or exhibit certain inharmonicities or is completely inharmonic. The tolerance even depends on some of the intervals themselves: obviously an octave, fifth and fourth have less intonation tolerances than other intervals.

But the author of this article does not agree with Parncutt and Hair (ibid.) refusing idealistic conceptions of understanding musical intervals during the composition process at all. The composition process often is idealistic in itself, not only for the pitch domain. Often not all aspects of a score are recognisable or memorisable by ear, (specially not for one time listening), but they still influence the composition. The frequency ratio approach, which was historically measured as string length ratios on instruments, provides a quite good method for assessing consonance and dissonance, as the timbres involved are harmonic or nearly harmonic.

Based on the psychoacoustic categoriality of musical intervals the author reasons that it must be possible to train musicians to learn new interval categories or subcategories which would enable them to sing microtonal intervals or play them on instruments with continuous pitch. Simply the existence of intervals unfamiliar to the Western ear in other musical cultures and their musicians' ability to produce them clearly and stably leads to the consideration that the musical ear is capable of learning interval categories beyond the twelve 12-EDO related ones.

EXPERIENCES FROM THE XENHARMONIC SUMMER CAMP 2019

The author arrived at questions regarding the methodological possibilities of teaching microtonal singing in the Xenharmonic Summer Camp in 2019, organised by the UnTwelve society (see UnTwelve n.d.), which took place at The Gesundheit Institute in Hillsboro (West Virginia, USA). The aim was to rehearse, record and perform the microtonal music of the participating composers in concert. The main method was the use of guide tracks, i.e. electronically generated audio files played along in real time via headphones for instruments and singing in order to learn the corresponding microtonal intonation. This method enabled a certain liberation from a purely music notation reading approach, moving closer to a more listening-based learning process. In particular, Lilian Hearne's á cappella choral work "Mass" in 22-EDO gave the author the opportunity to learn microtonal singing himself and to overcome difficulties in producing unusual intervals. "The Mass" consisted of parts in different 22-EDO subscales that were referred to with a name and number of steps in the brackets: Sanctus in Porcupine[7], Agnus Dei in Pajara[10] and Kyrie in Machine[6] (subscale naming conventions are further explained). Additionally, we sang together many songs from *The Sagittal Songbook* (Barton 2013). The author of this article also took part as a violin player and played in the following compositions: Nicholas E. M. Osborn's "Mirage for String Trio" (31-EDO) (see Osborn 2019), Stephen Weigel's "Zones of Lasting Novelty for 2 Violins, Trombone and Synthesizer" (9-EDO) (see Weigel 2019), Fakhrtabatabaie's "Den Homogenizer for 2 Violins and Viola" (frequency-based, quarter tones), and Hans-Gunter Lock's "Deep Square No. 4 for BP-Recorder, BP-Clarinet, Violin, Viola and Violoncello" (Bohlen Pierce (BP) scale).

For the author, participation in the Summer Camp in 2019 was very important as he learned essential methods of microtonal music practice through his own experience. Without this, it would not been possible to establish the Gradus ad Parnassum microtonal singing initiative (GP 2023) described and analysed as the subject of this article. I planned to take part in the summer camp again the following year in New Hampshire. Unfortunately, the global Covid-19 pandemic prevented this, but this encouraged alternative projects in the microtonal and xenharmonic community such as the online training presented here.

TOWARDS A MICROTONAL EAR TRAINING, SINGING, COMPOSING AND PERFORMANCE PROJECT

Based on experiences from the Xenharmonic Summer Camp 2019 the author was thinking about how to continue his own microtonal activities in Estonia. Being introduced to so many different microtonal scale systems performed by human musicians at the summer camp was very inspiring. However, the author came to the conclusion that it would be useful to work for a longer time on one specific scale system. But which system should be chosen?

Just Intonation supports a maximum of consonant intervals which are often easy to tune by ear, which would support the learning process. But as explained before, constructing scales from ratio-based intervals leads to a high number of pitches, which seemed not ideal for the planned vocal training project.

The author's first experiences in microtonality came from the Bohlen-Pierce (BP) scale, a scale with neither good approximations to the octave nor to the fifth but repeated over the interval of the twelfth (ratio 3:1, an octave plus fifth, named the tritave), dividing this large interval into 13 equal steps (Bohlen 1978). This gives very interesting and characteristic harmonic features as it closely approximates the odd just ratios of 5:3 (major sixth) and 7:5 (a tritone) combined as a triad 3:5:7. The BP scale seems to be ideal for instrumental music, as it requires only a small number of intervals to be trained and is easy to learn on bowed string instruments when the open strings are tuned to the wider fifth (731 cents). However, as we can no longer rely on the octave identity of pitches and must instead use tritave identity, this makes it too complicated for singing exercises. The scale's range is large and the usual singing practice in octave distance between higher and lower voices cannot be applied here.

The choice would be then some non-twelve EDO scale. Over the rather short history of xenharmonic practice, mainly in the US, and partly supported by older Western historical music theoretical approaches, certain n-EDO scales have been preferred over others due to their beneficial approximations to just intervals (for example 17-EDO, 19-EDO, 22-EDO, 31-EDO, 41-EDO, 53-EDO). The author's choice finally fell on the 22-EDO scale for several reasons. The number of steps to learn is relatively small (compared with 31-, 41- or 53-EDO), and it provides subscales that are significantly different from traditional Western musical experience while not tempering out the 81:80 syntonic comma (unlike historical meantone scales and 12-EDO). Moreover, 22-EDO offers fairly good approximations of some harmonic ratio intervals up to the prime number 11, and includes approximations of four distinct types of third: 1) 7:6 (septimal minor third), 2) 6:5 (common minor third), 3) 5:4 (common major third) and 4) 9:7 (septimal major third). Each of the four thirds allows four different triads to be constituted, albeit with a less accurate approximation of the 3:2 fifth (with 709 cents which is not as precise as the 12-EDO equivalent). The scale supports numerous 7th chords, thereunder those with a fairly good approximation of the 7:4 harmonic seventh. Last but not least, singing Hearne's "Mass" in 22-EDO during the Summer Camp convinced the author of the advantages of this scale.

In creating singing exercises the author made several decisions:

- a 22-EDO theoretical framework is needed, musical notation, interval names and subscales must be specified;
- a dedicated interval exercise is needed but stronger drill should be avoided;
- focus should be to go directly into a musical context with short pieces featuring some special aspects of 22-EDO and related intonation problems;
- as the exercises are directed to composers, participants are invited to compose little exercises or pieces.

THE 22-EDO SCALE AND ITS SUBSCALES

Dividing the octave into 22 equal steps causes an interval with 54.5 cents as its smallest step. In comparison with other higher number equal temperaments like 31-EDO this scale has a rather short history in Western theoretical considerations, beginning only in the second half of the 19th century. Inspired by the unequal and just-based division of

the octave in Indian music theory, of the 22 shruti, the English scientist and music theorist Robert Holford Macdowall Bosanquet (1878) offered a division of the octave into 22 equal steps. James Murray Barbour (1951: 116),[8] as does José Würschmidt, considers the 22-tone equal temperament as the next possibility after the 19-ton equal temperament (see 22edo n.d.). Recently one of the most important microtonal music theorists of our time, Paul Erlich (2001; 2002 [1998]), focused his intense theoretical research on the 22-EDO scale, developing a 10 step out of 22 symmetrical decatonic scale which is today known as Pajara[10]. Erlich also gives an overview of the history and refers to research by David Kraehenbuehl and Christopher Schmidt (1962) that classifies, in regard to Joseph Yasser (1932), the 22-equal tempered scale as 'hyperchromatic'.

In contrast to the classical Western 12-EDO tuning system the 22-EDO scale differentiates between a lesser and greater semitone and doesn't temper out the Syntonic comma 81/80 ≈ 21.5 cents but widens it to 54.5 cents. As the Western 12-EDO chromatic scale contains subscales like the 7-step diatonic scale, the 6-step whole tone scale and many others, we also have a quite large number of subscales within the 22-EDO framework, which are different from subscales possible in 12-EDO. In 22-EDO a 7-step subscale appears named Superpyth[7] containing, similar to the classical diatonic scale, five larger steps and two smaller steps. But the five larger steps are here larger (a greater whole tone instead of the 200-cent whole tone) and the two smaller steps are shrunk to quarter tones (instead of 100-cent semitones). Superpyth[7] can be visualised as a hypothetical keyboard (see Figure 1).

22-EDO Superpyth Keyboard

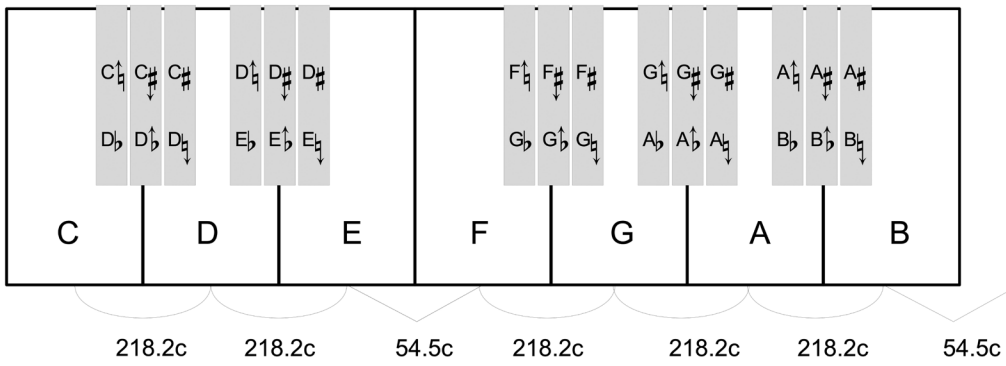

Figure 1. Hypothetical keyboard based on Superpyth[7] scale at the white keys.

In practice, using the Superpyth[7] subscale as the basis for note names and music notation conventions in 22-EDO keeps the intervals of fourth and fifth visually correct, but brings some reading difficulties. Most of the other intervals common with 12-EDO show up with non-standard microtonal accidentals (arrows), and uncommon 22-EDO specific intervals often look like regular intervals (without arrows), which needs a certain amount of learning. This can be demonstrated on the 22-EDO circle of fifths (slightly raised to 709 cents), and results in three levels of alteration, which are expressed by accidentals marked with arrows, deviating upward (up, sharp-down, sharp) and downward (down, flat-up, flat) (see Figures 2 and 3).

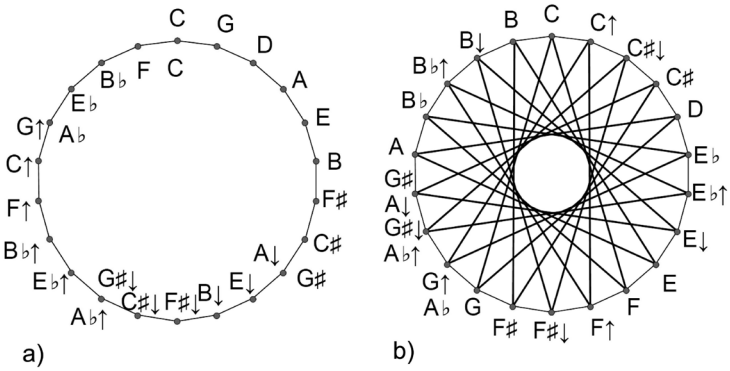

Figure 2. 22-EDO up/down notation, a) circle of fifths, b) pitches in 'chromatic' order. Accidental spelling: +1 step = arrow up, +2 steps = sharp arrow down, +3 steps = sharp; -1 step = arrow down, -2 steps = flat arrow up, -3 steps = flat. (Source: 22 Equal Temperament n.d.)

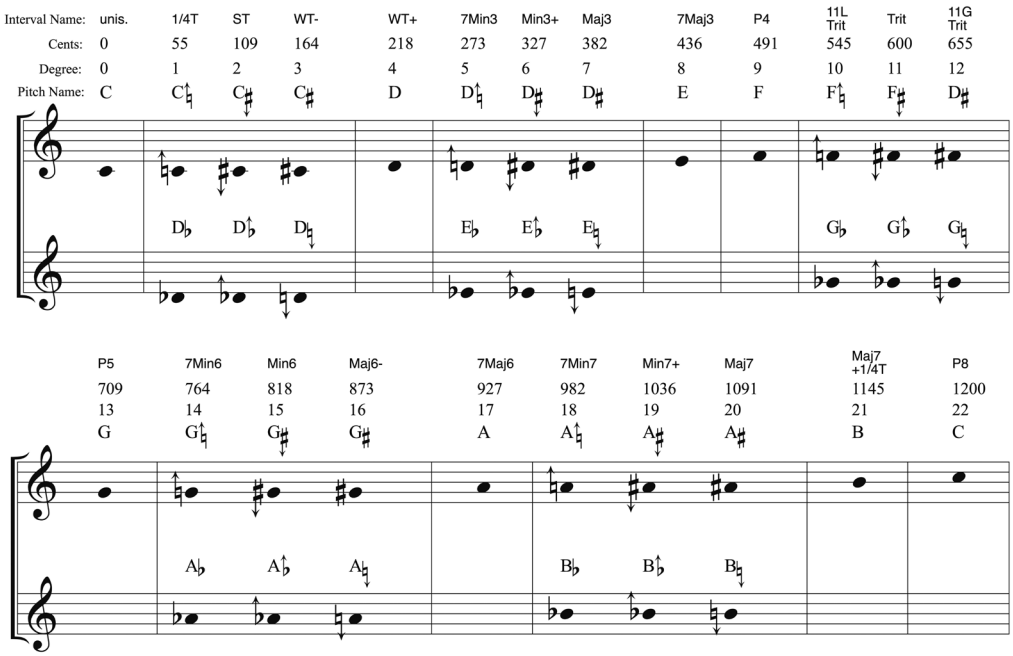

Figure 3. Up/down notation with enharmonic equivalents.

In order to be able to communicate musically and technically, it is essential to give names to the intervals. As there is no common standard yet developed, interval names that refer to their approximated just intoned intervals were used (see Table 1), since in intonational practice Just Intonation intervals are generally easier to sing or play (and they are often intended by composers). The goal was to apply interval names which are meaningful, short and not totally arbitrary. Thus, the tempered 22 system (22-EDO) forms a framework to limit the number of pitches used in musical practice, within which the corresponding just intervals can be intoned. In most cases, absolutely precise

intonation when singing or playing instruments with flexible pitches is not possible anyway, which is why certain tolerances are absolutely permissible.

Table 1. 22-EDO interval names as used for Gradus ad Parnassum.

Degree	Interval Name	Abbreviation
0	unison	unis.
1	quarter tone	1/4T
2	semitone	ST
3	lesser wholetone	WT-
4	greater wholetone	WT+
5	septimal minor third	7Min3
6	greater minor third	Min3+
7	major third	Maj3
8	septimal major third	7Maj3
9	slightly narrower pure fourth	P4
10	lesser undecimal tritone	11L_Trit
11	(12-EDO) tritone	Trit
12	greater undecimal tritone	11G_Trit
13	slightly wider pure fifth	P5
14	septimal minor sixth	7Min6
15	minor sixth	Min6
16	lesser major sixth	Maj6-
17	septimal major sixth	7Maj6
18	septimal minor seventh	7Min7
19	greater minor seventh	Min7+
20	major seventh	Maj7
21	major seventh plus quarter tone	Maj7+1/4T
23	pure octave	P8

The naming convention with the number of steps in brackets refers to so-called Moment of Symmetry scales (term offered in Wilson 1975, abbreviated MOS) which have only two different adjacent step sizes: smaller and larger steps that are distributed as evenly as possible throughout the scale. One and the same MOS type, indicated by a name, covers several MOS scales with different numbers of scale steps while fitting a lower step number scale into a scale with a higher number of steps. For example the classical pentatonic scale, containing three whole tones and two minor thirds, specified within this convention as Meantone[5], is included in the traditional diatonic scale with two semitones and five whole tones, now named Meantone[7]. Both fulfil MOS conditions, with the identical name showing that they belong to the same 'Meantone' MOS type. With the exception of the Enharmonic Scale, all other 22-EDO subscales discussed here are Moment of Symmetry scales (Wilson 1975; MOS Scale n.d.; Grady et al. 2005).

In addition to the notation-inherent Superpyth[7] subscale 22-EDO contains much more other subscales such as Porcupine[7] and [8], Orwell[9], Pajara[10] and others, but also the ancient Greek-based enharmonic scale is fairly well represented (see Figure 4). The number 22 can be divided by two, enabling the representation of the 11-EDO scale, which includes subscales such as Machine[6] and Orgone[7]. This great variety of different subscales, most of which differ significantly from Western listening habits, offers very interest-

ing possibilities for composers. All subscales can appear in different modes (or rotations) if beginning them from a different scale degree, while Figure 4 shows only one mode of each scale. The choice or preference of particular modes depends on the composer but often scale-structural features can cause certain preferences. For example, the Porcupine[7] mode with the greater whole tone between scale degree IV and V (notes F and G in Figure 4 c) has often been preferred because it is the only Porcupine[7] mode containing an interval of a fifth to the root evoking melodic similarities to the traditional minor scale.

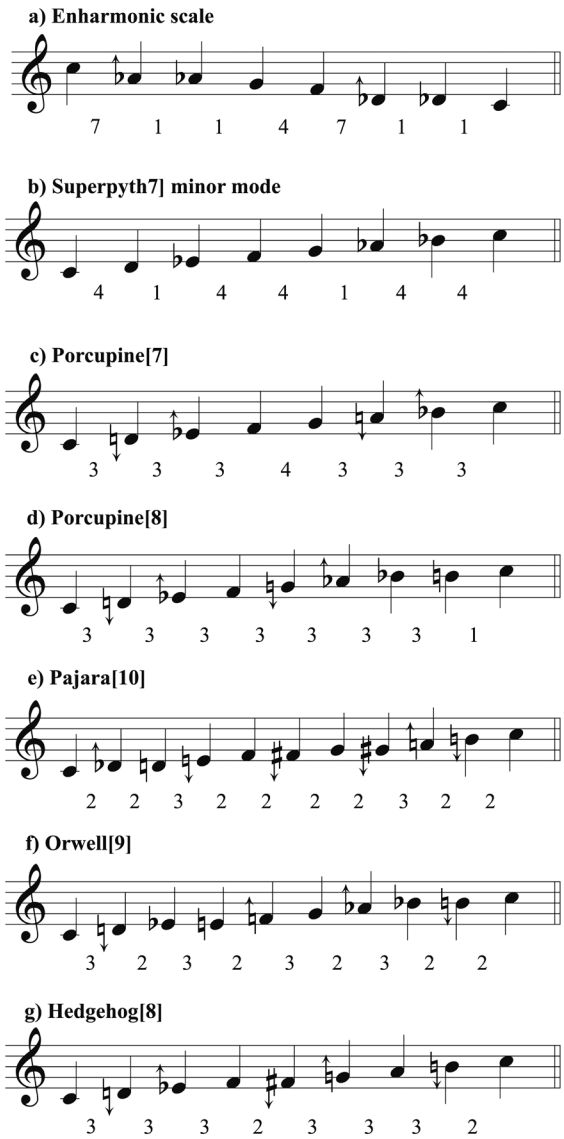

Figure 4. 22-EDO subscales: a) enharmonic scale (written downward as is usual for ancient Greek scales; b) Superpyth[7] minor; c) Porcupine[7] with fifth from root; d) Porcupine[8] with a quarter tone to the last scale degrees; e) Pajara[10] scale; f) Orwell[9] scale; g) Hedgehog[8] scale. The numbers show the interval size between consecutive scale degrees in multiples of the smallest 22-EDO step with 54.5 cents (see Table 1).

PRACTISING MICROTONAL SINGING AT PÄRNU CONTEMPORARY MUSIC DAYS

Pärnu Contemporary Music Days (PNP; in Estonian *Pärnu Nüüdismuusika Päevad*) is an annual festival that takes place in the southwest Estonian city of the same name. PNP was founded in 1988 by composer and conductor Andrus Kallastu and has been organised by the Estonian Arnold Schönberg Society since 1993 (see EASS n.d.; Kallastu 2025). Since 2008 it has mainly consisted of a five to seven day workshop and a symposium. The aim is to bring new knowledge on contemporary musical and artistic practice to Estonia. A wide variety of topics have already been the focus of the festival, such as futurism, performativity and composition models, with the same or similar topics spread over several years.

The plan for PNP2021 was to deal with microtonality and also invite speakers from the USA. The decision was made to focus on the 22-EDO scale as part of the workshop. However, the global pandemic forced us to organise the entire festival purely online. The workshop, for which we had planned to train singing in 22-EDO and its subscales, also had to be carried out online. Our aim was to sing in an unusual scale with unusual intervals, primarily as training for composers who want to understand this scale through practice, rather than as a training for professional singers.

We wanted to explore the special compositional properties of the 22-EDO scale, particularly in terms of harmonic relationships, voice leading and counterpoint. Above all, this last principle, experimenting with medieval and Renaissance composition techniques in our singing exercises and small vocal pieces, led to the naming of this training after Johann Joseph Fux's legendary counterpoint treatise *Gradus ad Parnassum* ('Steps to Parnassus') (see Fux 1725). Indeed, Fux's book is iconic and has served as a role model for later counterpoint study books. Werntz (2015: 61) refers to it, calling her own exercises Steps to the Sea as an English rendering of a modified Fux title *Gradus ad Mare*.

In terms of methodology, however, we took a more modern approach: a microtonal synthesiser played live by the author was used for repeated listening and in order to sing along.

GRADUS AD PARNASSUM ONLINE TRAINING: ORGANISATIONAL AND TECHNICAL BACKGROUND

The workshop during PNP2021 gave rise to the idea of continuing such online training, even against the background of the pandemic, where physical social contact was practically impossible, especially not long-distance travel. We offered our approximately three-hour training session on two Sundays a month, advertised to our target group via Facebook (see Figure 5).

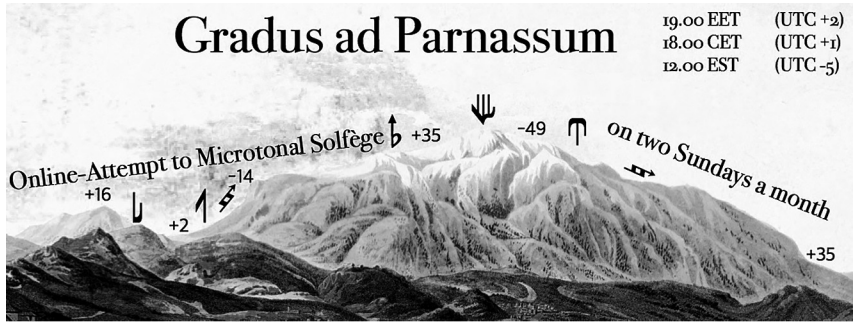

Figure 5. Illustration for Gradus ad Parnassum online training by the author.

As already tried out in the PNP2021 workshop, we came up with Jitsi as free browser-based video conference online software (see Figure 6), which was good for general communication and for displaying visual content such as sheet music, but was unfortunately completely unsuitable for real-time online singing together. For this we found the purely audio-based online application Cleanfeed, which enabled acceptable and reasonably low-latency transmission.

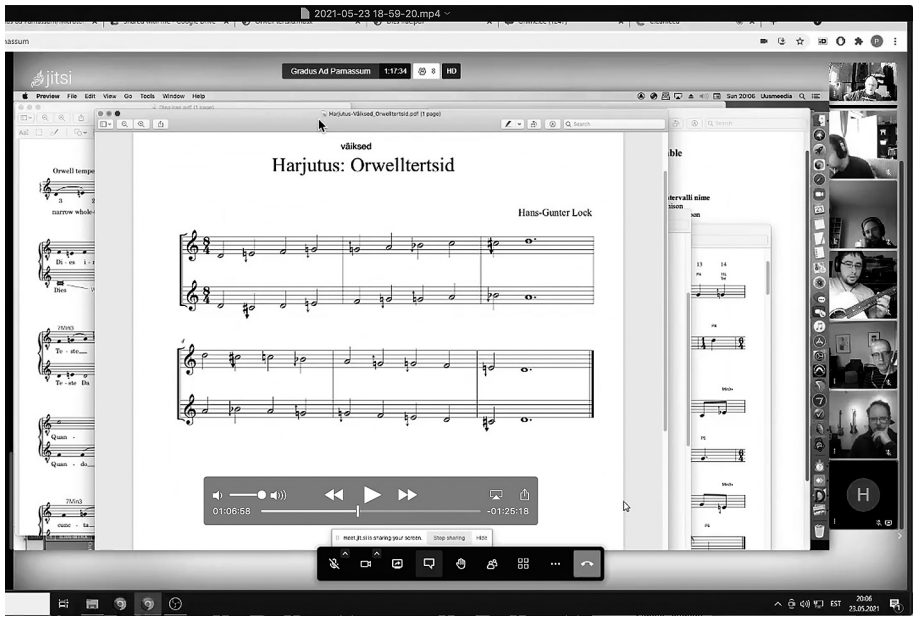

Figure 6. A typical view of an online session in Jitsi.

As a synthesiser solution, both for real-time training and as a simple self-learning solution, we have found browser-based applications that simply copy HTML links to the corresponding microtonal scales. On the one hand, we used Scale Workshop by Sean Archibald, or Sevish, a comprehensive program for creating microtonal scales. On the other hand, Kyam Allami's and Counterpoint's[9] tools proved to be practical for subscales up to twelve tones, as these could be very easily mapped to a common (Halberstadt-) MIDI-keyboard (Figures 7 and 8).

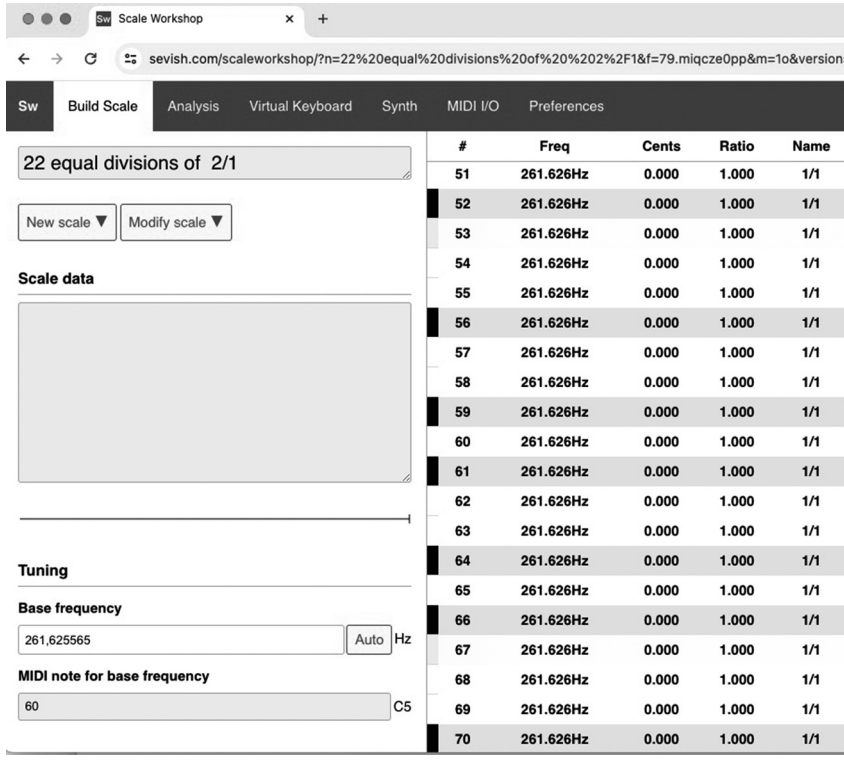

Figure 7. Screenshot of Sevish Scale Workshop. The whole 22-EDO scale is applied. (Sevish n.d.)

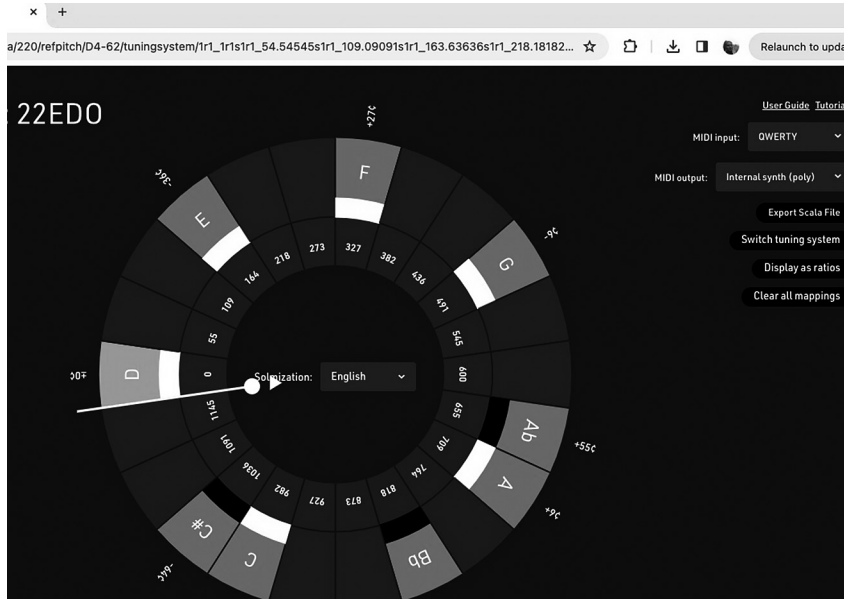

Figure 8. Screenshot of Allami's Leimma applications as used for simple and shareable microtonal synthesisers. A Porcupine[8] scale with one extra note is applied. (Allami and Counterpoint n.d.)

The Gradus ad Parnassum online training took place in three terms: spring 2021, autumn 2021 and spring 2022. Audio material, theoretical explanations and music examples were shared online and are still available. All training sessions were documented using video and audio and can be viewed or listened to at any time (GP 2023). The theoretical materials (explanations and online synthesiser links), the exercises and small vocal pieces have been summarised in the PDF publication *Codex Pærnuensis* (Lock 2025).

Our vocal-practical part at the PNP2023 festival might be considered the fourth term of Gradus ad Parnassum. However, this term did not take place online but physically in Pärnu with the composers and musicians present. In addition to the many instrumental contributions to this festival, almost all Gradus ad Parnassum vocal pieces created during the three online terms were performed in concerts in Pärnu and Tallinn. PNP2023 can therefore be viewed as the preliminary conclusion of Gradus ad Parnassum.

GRADUS AD PARNASSUM METHODS

With the aforementioned technical basis we were able to offer online ear training sessions. Every session started with the 22-EDO interval exercise. Initially we spent longer on the 13 intervals from the quarter tone to the fifths, and later also the remaining larger intervals up to the octave (see Figure 9). Every interval was performed three times: first only listening to the synthesiser, the second and third times singing with the synthesiser. We were singing in two groups: one group sang the root note and the other group sang the moving voice to the respective interval, with the groups changing.

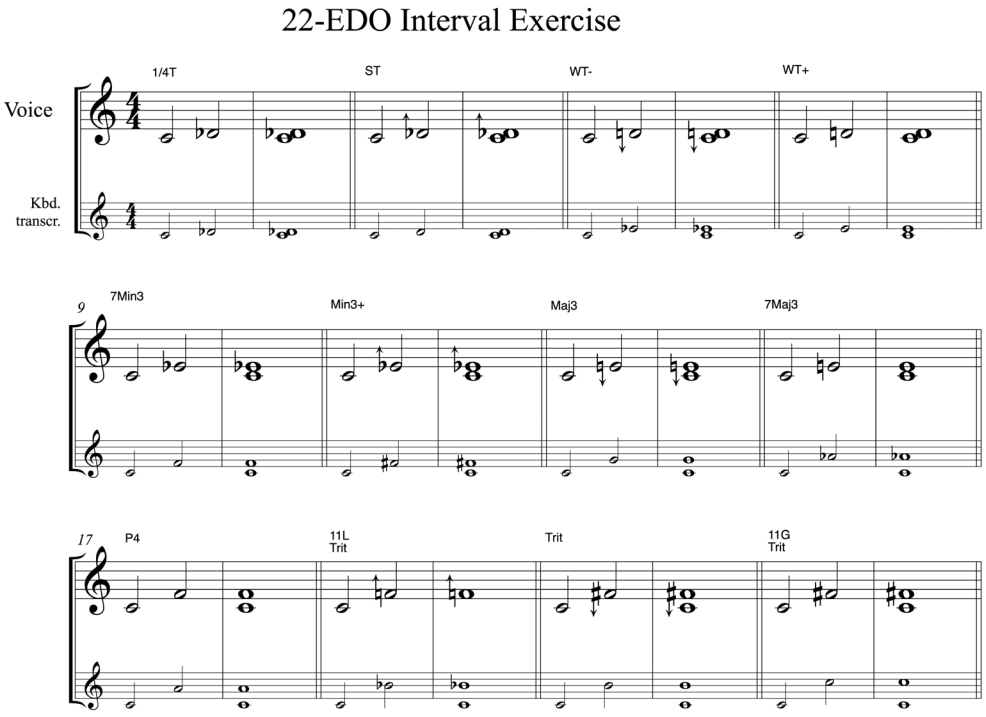

Figure 9. Beginning of the 22-EDO Interval Exercise (Lock 2025: 38).

Generally we trained by singing along with the synthesiser, as the strategy was to hear the correct intonation over and over. We did not force ourselves to sing the intervals, exercises and vocal pieces á cappella. This was also because listening to each other over the internet was not as good as being together in a physical space.

Singing and also playing continuous pitch instruments can generally not be performed absolutely exactly as depending on the musical context certain deviations from the ideal version of how it sounds on a synthesiser must be accepted. In addition, when learning simultaneous intervals, minimal beating (see Note 4) and the perceived roughness[10] of smaller number JI ratio-based intervals can be used as learning tools, especially for intervals up to the fifth. Those 22-EDO intervals have been named for Gradus ad Parnassum regarding to their approximated JI counterparts (see Tables 1 and 2).

Table 2. Just Intonation intervals approximated in 22-EDO.

Intervals up to the tritone					**Interval inversions to the octave**			
JI ratio	**JI to cents**	**22EDO (cents)**	**dev. from JI**	**Interval Name**	**JI ratio**	**JI to cents**	**22EDO (cents)**	**dev. from JI**
8/7	231	218	-13	septimal major second / greater whole tone	7/4	969	982	+13
7/6	267	273	+6	septimal minor third	12/7	933	927	-6
6/5	316	327	+11	minor third	5/3	884	873	-11
5/4	386	382	-4	major third	8/5	814	818	+4
9/7	435	436	+1	septimal major third	14/9	765	764	-1
4/3	498	491	-7	fourth	3/2	702	709	+7
11/8	551	545	-6	undecimal lesser tritone	16/11	649	655	+6

An important part has been the subscale exercises (Enharmonic, Porcupine[7], Superpyth[7], Pajara[10], Orwell[9], Hedgehog[8] and others, see Figure 4), also singing them in two groups with one group holding the root pitch as a drone and the other ascending the scale until reaching the octave. Then the scale was sung in descending order (as reading the musical notation backward in Figure 4) by one group while the other group held the highest note (the octave) as a drone. For Pajara[10] and Orwell[9], we switched the drone in the middle of the scale to the tritone, which allows the moving group to listen only to smaller intervals. During PNP2021 we only trained intervals and scale exercises due to the lack of vocal pieces, which were created later. As we had more composed pieces we didn't practice scale exercises for the sake of if but rehearsed these scales related to the piece we planned to learn. For this purpose most of the pieces composed for Gradus ad Parnassum show the related scale before the beginning.

A very practical tool has been one-voice exercises that contain several unfamiliar intervals but reach out to a familiar interval (see Figure 10). During these 'arrival exercises' a chain of intervals must be intoned properly to reach the right pitch or interval. These exercises are based on the structural features of the subscales involved and their specific combinations of familiar and unfamiliar intervals, offering ideal opportunities to practice microtonal singing. Finnish composer Juhani Nuorvala's Porcupine exercise contains three consecutive minor thirds (327 cents), each a bit larger than the 12-tone tempered one, meaning musicians must reach the 22-EDO representation of the septimal minor seventh (982 cents). From this it must be possible to go back to the start-

ing pitch over two (slightly smaller) fourths (GP 2021a: 54:55–59:16; Lock 2025: 17). In learning to intone the enharmonic tetrachord it has been useful to sing first the familiar intervals up and down (the major third and the fourth, see Figure 6b, first half) and then continue singing the complete tetrachord downwards while inserting the unfamiliarly sounding note A𝄭, between the notes of the semitone A𝄲 – G resulting two quarter tones (Figure 10b, second half). If the unfamiliar septimal major third has already been sufficiently memorised as its simultaneously sounding form, listening to the upper note C held by the synthesiser provides the possibility of intonational orientation for the note A𝄭 when the quarter tone step A𝄲 – A𝄭 has been taken (GP 2021b: 59:00–1:03:24).

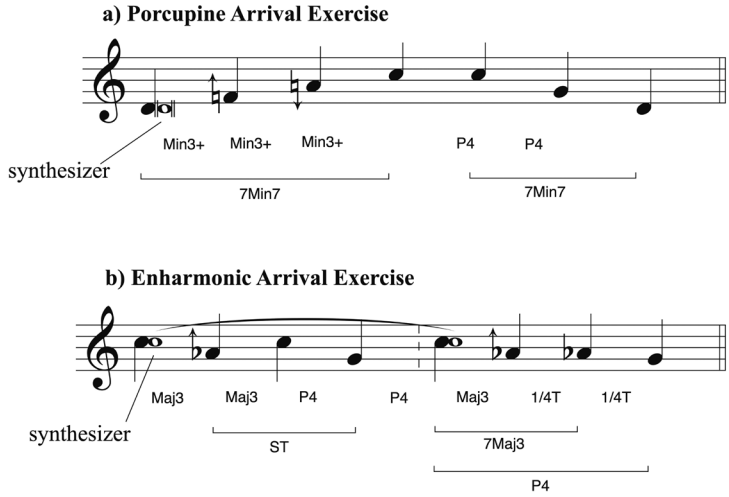

Figure 10. 'Arrival' exercises: a) Nuorvala's Porcupine exercise and b) Enharmonic exercise (GP 2023).

Because the 22-EDO intonational realm was new for all the participants, the exercises and pieces were mainly practiced slowly, and the online situation often forced us to be even slower, sometimes even extremely slow because of latency. For polyphonic compositions (mainly 2 or 3 voices) every voice was initially sung together by all participants. Later we split into groups and the groups rotated so that every group practiced each voice once.

EXERCISES AND PIECES COMPOSED FOR GRADUS AD PARNASSUM

Even though we didn't have any restrictions on our online sessions, no demand of skill and no participation fee was asked, although most regular participants were Western-trained musicians with the ability to read and perform from music notation. Most of them were composers and submitted both exercises and pieces for Gradus ad Parnassum and took part regularly in singing. Without a doubt, Nuorvala delivered most of the pieces in all three terms, contributing the majority of compositions to the Pärnu Codex for Voices and Synthesiser suite (Nuorvala 2021–2023). His pieces are often based on

historical references such as the ancient Greek enharmonic scale or the organum composition technique. Usually, these pieces focus on certain intonational problems and subscales but they are nevertheless not just pure exercises but also fully fledged small musical works with astonishing musical qualities (see Figure 11).

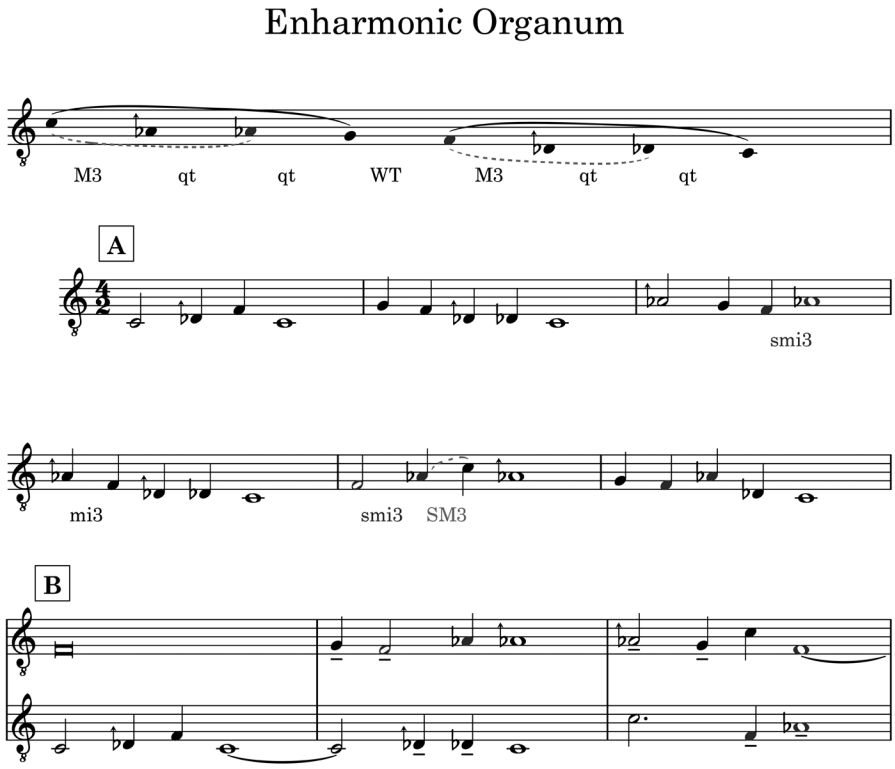

Figure 11. Excerpt from "Enharmonic Organum" by Nuorvala (2021–2023: 19–20; Lock 2025: 44). Before the piece begins the scale information is given with abbreviations for the intervals. Important intervals and pitches are additionally depicted as coloured notes and slurs, which should help people learn to sing unfamiliar microtonal intervals and melodies.

Particularly during the third semester in spring 2022, the California-based composer and microtonal music theorist Joseph Monzo contributed to the creation of new vocal pieces. These include a small vocal work inspired by Bach chorales in the harmonically difficult Hedgehog[8] subscale (see Figure 12), and a surprisingly harmonious miniature vocal piece with tetrads based on the Hedgehog[14] subscale. Monzo's 22-EDO oeuvre also includes a number of instrumental pieces that were performed at the PNP2022 and PNP2023 festivals, with the composer physically present at PNP2023.

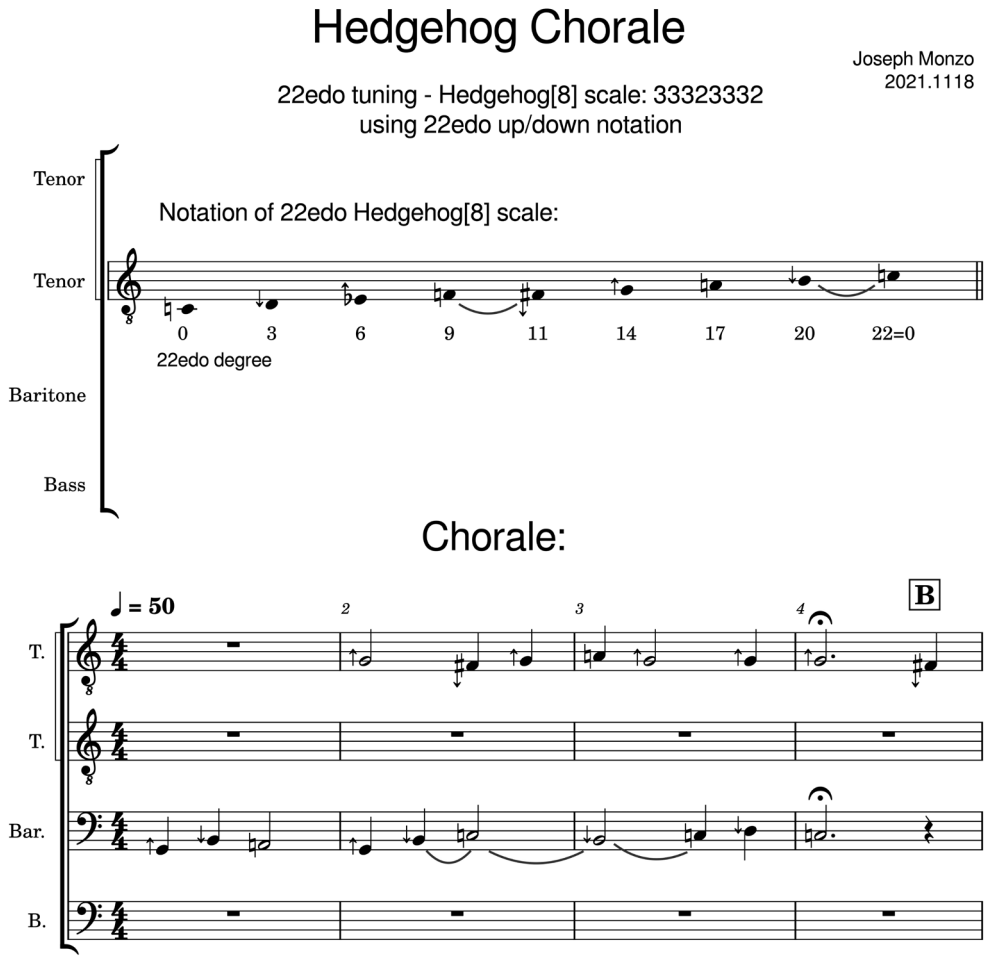

Figure 12. Excerpt from Monzo's "Hedgehog Chorale" (Monzo 2021b; Lock 2025: 54–55). Before the piece begins the scale information is given with the 22-EDO pitch-class numbers. Hedgehog[8] contains six lesser whole tones (3-step interval) and two semitones (2-step interval). For easier reading the author marked the semitone interval with a slur.

The author of this article has of course also had the opportunity to enrich Gradus ad Parnassum with his own pieces. Starting with pure subscale exercises for the PNP2021 workshop (Lock 2025: 2–14), he offered smaller exercises in Orwell[9] (ibid.: 31) and Pajara[10] (ibid.: 46–47). His Credo for a three-part male choir has been developed from the latter (ibid.: 65–70). The fact that the Porcupine[7] scale can be transformed into a minor-Superpyth[7] scale within the framework of a fourth led to his four-part Christmas chorale "O Magnum Mysterium" (ibid.: 59–64).

However, there were also other interesting contributions, for example Gerhard Lock's Kite-Dart models and exercises (ibid.: 48–50) and "Pajara Cycle – Kites and Dart" (ibid.: 82–83) for variable ensemble. The composer mapped the pitches of the Pajara[10] subscale on the geometric shapes of Roger Penrose's (1974) tilings. After the initial notated part of the piece the musicians have the opportunity to improvise both vocally

and instrumentally based on the given motivic models. In the third term, Andrus Kallastu contributed demanding singing exercises with combinatorically organised countless 22-EDO cadence models in two type series, which he supplemented with a third type for PNP2023. Smaller exercises came from the American composer Barton (Lock 2025: 29) and the Finland-based Romanian composer Sebastian Dumitrescu (ibid.: 24).

In general, the methodological intention was to provide training with neither detailed control of the exercise results, nor a specific learning goal. The latter wasn't even possible to plan from the beginning because the exercises were composed more or less at the same time and we didn't know, at least at the beginning, how far we would go. We always sang together with the synthesiser in order to become more or less intuitively familiar with the intervals of the 22-EDO scale through long-term practise. Nevertheless, from term to term we were able to undertake more difficult tasks and a lot of progress was noticeable. Apart from the technical exercises with intervals and scales most of the pieces composed for Gradus ad Parnassum are fully fledged pieces of music, some of them shorter, some longer. This gave the opportunity to apply the technically learned objectives directly in a real musical context and led us to be able to perform a selection of these musical works in our workshop concerts during PNP2022 and PNP2023.

RESULTS

The Gradus ad Parnassum course gave the possibility for musically professional and semi-professional trained people to meet to understand and practise singing within the 22-EDO microtonal framework and its dedicated subscales. Although it is difficult to give a comprehensive assessment of the result due to the complexity of the task, there is no doubt that all the activities led to positive results as a new framework of melodic and harmonic possibilities was introduced to the participants. Selected aspects of the author's experience and listening analysis are presented here in more detail with the auditory assessment of intonation accuracy, the intervals themselves and their combinations in the appropriate melodic and harmonic context are evaluated.

The focus is on the three concerts, recordings of which are available online (see Pärnu 2022; Pärnu 2023; Tallinn 2023). Due to the relatively good comparability of sound recordings from the three concerts, the intonation of selected small vocal works by Nuorvala and the author, composed during Gradus ad Parnassum, will be analysed and compared here. The selected works are based on the enharmonic scale, Porcupine and Superpyth subscales.

The 22-EDO interval exercises were practiced most during Gradus ad Parnassum as every session started with them (Lock 2025: 38). In summary, the exercise practise has shown that intervals close to familiar are the easiest to sing, while unfamiliar intervals that are different from or with no close counterpart in Western music are the most complicated to learn. These are intervals between two familiar categories, such as the quarter tone with 55 cents between unison and the familiar 100 cent semitone, or the greater undecimal tritone with 655 cents. The latter is particularly challenging as it is close to the pure fifth (3/2, 702 cents) and sounding it simultaneously, the ear hears a high amount of roughness, meaning it can be considered a highly dissonant interval.

The author has observed similar difficulties when a known category splits into two new ones, as with the lesser whole tone and the greater whole tone. The lesser whole tone (164 cents) falls between the familiar 100-cent semitone and the 200-cent whole tone, while the greater whole tone (218 cents) is noticeably larger. Both these microtonal intervals needed certain concentration during the learning process establishing two new categories instead of a familiar category between them.

In 22-EDO, both the major and minor thirds come in two different versions. Interestingly the author recognised that the thirds, even though there are two categories instead of one familiar one, were much easier to learn than the lesser and greater whole tone. It might be that in our Western musical practice (or maybe for the human ear as such) thirds have generally larger tolerance while singing and continuous pitch instruments have the ability to tune these intervals (if sounding simultaneously and if the musical context allows) with minimal beatings, giving sounding experiences of these intervals different from 12-EDO (if correctly) tuned keyboard instruments. It also helps that all four thirds are relatively close to Just Intonation thirds with small numbers in their ratios (7/6, 6/5, 5/4, 9/7) and beating minima can be used fairly well as a learning tool.

As the tritone is known as difficult to intone in Western music, a certain instability has also been recognised in its intonation in 22-EDO. Apart from the octave, the familiar 600 cent tritone is the only interval shared with 12-EDO as it divides the octave in two equal parts, as with all even-numbered EDOs. Around this interval there are two more tritones: one a quarter tone smaller than the undecimal lesser tritone, which was not so hard to learn, and the other a quarter tone larger, the undecimal greater tritone, with its difficulties as previously described.

In the framework of Gradus ad Parnassum we eventually rehearsed in physical contact and performed the vocal pieces in three workshop concerts. The first concert took place in January 2022 in Pärnu (see Pärnu 2022), due to pandemic restrictions we were only three singers (Kallastu, G. Lock and the author). Next year more participants were able to attend (Kallastu, Kristjan Kannukene, G. Lock, Monzo, Gonzalo Muruaga Olguin, James Robinson, Matej Sloboda and the author) to make a 22-EDO choir performing in two concerts, one in Pärnu (see Pärnu 2023) and the other, the last of our performances, in Tallinn (see Tallinn 2023). None of us is a professional singer and these days none of us sings regularly in choirs, we are instrumentalists, composers, improvisors and musicologists. Here, some pieces will be evaluated with regard to the quality of microtonal intonation. The evaluation of the intonation from these concert recordings has its limitations, as often only the upper voice is clearly listenable and so as it was sung to the accompaniment of the preproduced synthesiser track, only melodic intervals are assessed here.

It turned out that the enharmonic scale seemed to be the easiest 22-EDO subscale to learn, as the enharmonic tetrachord contains downwards a major third followed by two quarter tones ending up in a fourth from the initial pitch. The performance of Nuorvala's "Enharmonic Organum" (Nuorvala 2021–2023: 19–20) in (Pärnu 2022: 16:30–19:17) was very good, but with some instability of the quarter tones, sometimes too narrow as tending to the unison, and sometimes too wide and tending almost to the semitone. The same problems with the quarter tones were encountered in Tallinn (Tallinn 2023: 9:55–12:06) and the best performance of this piece was in Pärnu (Pärnu 2023: 10:44–12:52). Difficulties like these are not surprising, as it is not necessarily easy to learn to intonate

new interval categories stably and with sufficient confidence, particularly in a concert situation.

The seven-note Porcupine scale is quite often used by composers. Nuorvala's "Organum: Halleluja" (Nuorvala 2021–2023: 1–2) uses the mode with a fourth and fifth upward from the root pitch. While this mode is very useful for the composer to transform a traditional minor scale into Porcupine, it is still problematic to intone because performers often fall back into the minor scale habit. Two of Nuorvala's "Halleluja" perfomances were very indifferent in intonation, the Porcupine characteristics were not really listenable (Pärnu 2022: 13:31–14:50; Tallinn 2023: 6:58–7:58). The performance in Pärnu in 2023 has a quite good beginning but during the performance it deviates to the traditional minor mode (Pärnu 2023: 7:55–8:42). Similar problems also occurred during the rehearsals of Nuorvala's Porcupine adaptation of the medieval "L'Homme armée" ('The Armed Man') (Nuorvala 2021–2023: 25–26), but apparently due to reliable instrumental support from the ensemble, the performance in Tallinn (Tallinn 2023: 1:05:41–1:06:50) turned out quite well.

In contrast, the singing of Nuorvala's piece "Porcupine Progression" (Nuorvala 2021–2023: 23–24) didn't show the aforementioned problems. The eight tone mode with all consecutive lesser whole tones and a quarter tone remaining to the octave (see Figure 4 d) does not resemble a traditional scale so much and the structure of the piece is more focused on harmony. The first performance was very good (Pärnu 2022: 22:00–24:18). Later the quarter tone sometimes tended to unison (Pärnu 2023: 15:14–17:32), but it was better in Tallinn (Tallinn 2023: 14:32–16:51). The probably reasons for the problem with quarter tones are similar to those pointed out in the example of the "Enharmonic Organum" performed in Pärnu in 2022.

A quite challenging task has been the author's piece "O Magnum Mysterium" ('O Great Mystery') (Lock 2025: 59–64), which uses both the Porcupine scale and the Superpyth scale. While the Porcupine scale is like a flattened version of a minor scale levelling out the whole tones and the semitone, the Superpyth minor scale is just the opposite, sharpening the semitone into a quarter tone and the whole tones into greater whole tones. Performance of this piece should be focused on these melodic contrasts. The first performance observed was quite good although sometimes the contrast between Porcupine and Superpyth minor isn't recognisable (Pärnu 2022: 40:32–43:35). It was much better performed in 2023 (Pärnu 2023: 29:03–32:06). Due to the more problematic acoustics the Tallinn performance (Tallinn 2023: 26:32–29:35) is hard to evaluate, but the intonation quality seems to be comparable with performance in Pärnu in 2023.

Listening to the recordings of the vocal "Study in Pajara" by Nuorvala (Nuorvala 2021–2023: 3–4) the evaluation of the intonation is not as easy, as with the pieces in other subscales, because it also appears in 12-EDO with only minor differences. Therefore Pajara[10] in 22-EDO consists of semitone and lesser whole tone steps, which have a 2:3 relationship. However, in the same scale established in 12-EDO, the semitone and whole tone differ more from each other as they have a relationship of 1:2. For both pieces – Nuorvala's "Study in Pajara" and the "Credo" composed by the author (Lock 2025: 65–70) – the best performance in terms of intonation was in Pärnu (Pärnu 2023: 13:10–14:55; 32:22–36:15). In Pajara it is important to focus specially on different places in the scale where the lesser whole tone and greater whole tone intervals appear. The latter is the result of two consecutive semitones as consecutive semitones dominate the

Pajara scale and all four types of third are possible. During the first performance of both pieces (Pärnu 2022: 19:35–21:44; 43:58–47:51) the intonation of the different whole tones was often not correct, and similar problems occurred in Tallinn (Tallinn 2023: 12:30–14:17; 29:53–33:45). Nuorvala's "Study in Pajara" ends with the whole scale downwards in the upper voice, which was often very challenging to sing and to get in tune with the harmonies of the other two voices, but in both performances in 2023 this was a success (Pärnu 2023: 14:35–14:55; Tallinn 2023: 13:56–14:17).

CONCLUSIONS

There is no question that the main impetus for creating an online training course for microtonal ear training and singing came from the completely unexpected Covid-19 pandemic with its drastic global impact on all areas of life, but especially on public life and on people's mobility. Musicians depend on social life, whether it is the opportunity to rehearse together or to perform in public. Singing together in particular was restricted at the beginning of the pandemic because of increased infections during choir rehearsals reported in the US and elsewhere (Hamner et al. 2020; Charlotte 2023). The summer camp planned by the UnTwelve Society for 2020 in the USA with a focus on microtonal singing had to be cancelled, not to mention all the other cancelled social activities during this time. While schools and universities had to switch suddenly to online learning, choirs had to rehearse remotely via software like Cleanfeed, just as we did. In this period online offerings like Gradus ad Parnassum were gratefully accepted. A little later, bassoonist Johnny Reinhardt established the Microtonal University (starting in September 2021), as his New York-based American Festival for Microtonal Music AFMM could no longer take place (Reinhardt n.d.; 2021). All in all, in addition to our positive experiences with online singing during this difficult time, similar results are found in a randomised, controlled, psychological pilot study (Schäfer 2023).

The aim of this research was to show how adults socialised in Western music can be trained to sing microtonally with unusual intervals and scale structures. This was demonstrated using the example of the Gradus ad Parnassum online initiative, and the methodological approaches developed within this framework. Despite the problems posed by analysis of the selected performances using the concert recordings, the learning methodology worked, as evidenced by the clear progress made during online training and the 2023 concerts compared to those of 2022.

In more detail, the analysis above demonstrates two tendencies. First, the 22-EDO subscales closer to the familiar 12-EDO possible structures at the first glance seem to be easier to learn. However, they bear the inherent risk of falling back into habitual intonation insofar as the concentration of the singer might decrease. This has been considered specially for Pajara[10] (Figure 4 e), Superpyth[7] (Figure 4 b) and for this Porcupine[7] mode, which preserves the pure fifth to the fundamental and is close to the traditional minor scale (Figure 4 c). Second, these 22-EDO subscales, which have no direct 12-EDO equivalent, are clearly harder to intone correctly. However, since they clearly differ from the usual learned intonation patterns falling back into the common habit isn't quite possible as the given musical structures (melodic movements or simultaneous intervals) will diverge substantially and be clearly recognised as incorrect and far from the struc-

ture intended by the musical notation. Subscales with this feature are the Enharmonic scale (Figure 4 a) and the Porcupine[7] mode with six consecutive lesser whole tones and a greater whole tone to the octave (Figure 4 d without the B-natural note).

The training situation of Gradus ad Parnassum was distinctive because it took place in an online format due to Covid-19. First of all, it brought together musicians who physically wouldn't have had the chance to meet so many times. It challenged composers to create exercises and shorter vocal pieces that could immediately be tested. Ultimately it showed that effective microtonal vocal training is possible online for purely intonation training, which can be recommended when physical meetings are not so easily possible.

As disadvantages, it should be emphasised that online training often needed some patience in terms of technical internet explanations to participants, and due to latency, especially with participants from overseas. This resulted in a much slower tempo of rehearsal for the pieces and exercises, which wasn't so much a problem in a purely intonational training situation, but limited or made impossible the work on musical expression. Especially when compared to the physical meetings held during the PNP2023 Festival, the slowness and inadequacies of online training became apparent. It should also be mentioned that the technical requirements for the instructor were quite complicated, as the microphone for speaking and singing was mixed using the microtonal synthesiser, which was made possible thanks to the sound studio of the New Media Department at the Estonian Academy of Arts. Under the circumstances of the pandemic, it was the best that could be done to realise such ideas and keep microtonal activities going. All in all, we can say that the positive factors still outweigh the negative.

NOTES

1 A swinging song "The Swing Wants Mittens" (*Kiik tahab kindaid*). ERA, Pl. 28 B2 < Jõelähtme parish, Viimsi community, Randvere village. – Recorded by August Pulst and Herbert Tampere in 1936 at the Estonian Broadcasting from Mari Kilu, aged 84, Liisu Tamp, aged 65.

2 "Father's Swan Piece" (*Isa luige lugu*). ERA, Pl. 6 B2. < Tori parish and community. – Recorded by August Pulst and Herbert Tampere in 1936 at the Estonian Broadcasting from Mart Männimets (violin), aged 64.; Mihkel Toom (violin), aged 63.

3 A cent is the 1,200th part of an octave divided logarithmically on base 2. The common 12 tone equal tempered intervals appear as multiples of 100, for example 100 cents the semitone, 200 cents the whole tone, 300 cents the minor third. With exception of the octave 2/1 just intoned and other microtonal intervals have cent values not divisible by 100, for example the 5/4 just major third with approximately 386.3 cents or the smallest step in the 22-EDO scale with approx. 54.5 cents.

4 Beating is a phenomenon that occurs when two sound waves with frequencies close to each other create an interference pattern, periodically amplifying and canceling each other out. The frequency of the beating is equal to the difference between the initial frequencies (Greated 2001).

5 Barton in his email to Hans-Gunter Lock on July 11, 2024.

6 A comma is a small JI interval between two pitches, which are both derived from the initial 1/1 ratio and arrive at nearly the same pitch through different ways of combining just interval ratios, for example four pure fifths 3/2 and two octaves down (81/64) is not the same as the JI major third 5/4. The interval difference (hence the quotient of the two ratios) is with 81/80 the syntonic comma. Temperaments will achieve a compromise between these two interval versions and the corresponding comma is then 'tempered out'. One and the same comma can be tempered out in many different ways.

7 Barton in his email to the author on July 11, 2024.

8 But Barbour (ibid.) is wrong when he cites Friedrich Wilhelm Opelt as a source for treating the 22-equal tempered scale 25 years before Bosanquet because Opelt's enharmonic scale is a JI scale with 22 tones, counting both the fundamental and the octave. Thus, this scale has only 21 steps (Opelt 1852: 19, 78–80). In addition, Barbour is mistaken in the chapter of Opelt's textbook: this scale is described in the third chapter instead of the fourth.

9 Counterpoint is the collaborative practice of Samuel Diggins and Tero Parviainen.

10 Roughness describes a psychoacoustic perception phenomenon experienced when listening to complex sounds, intervals or chords. It is related to the musical consonance–dissonance phenomenon, while avoiding historical implications such as voice leading and other musical contexts. The concept of "sensory consonance and dissonance" (Sethares 2005) goes back at least to Hermann Ludwig von Helmholtz' groundbreaking work *On the Sensation of Tone* (1863, Eng. trans. 1875) where he proposed that beating rates up to 30 to 40 Hz are perceived as maximally rough. This has been refined by Reinier Plomp and Willem Johannes Maria Levelt (1965) and William Sethares (2005), and in comparison with historical understandings it has been categorised as Consonance-Dissonance Conception number 5 (CDC-5) by James Tenney (1988). Roughness can either be rated or mathematically modelled. A contemporary overview of this topic can be found in Masina and Lo Presti 2024.

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