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Fractal codes with locality and availability

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Codes with locality and availability are used in distributed storage for data recovery. Locality λ enables the recovery of lost data on each server by exploiting data from only λ remaining intact servers. Availability α means that an α subset of servers providing localities are available for data recovery for each server. The secondary benefit of these codes in distributed storage is that they allow access to hot data in times of high demand, exploiting locality and availability. In this paper two families of codes with locality availability and scalability are presented which can be represented as fractals.

Keywords: data recovery, distributed storage, codes with locality and availability, simplex code, fractal

1 Introduction

At the present time and also in the foreseeable future, many ICT applications will rely on clouds which are based on distributed storage systems [1, 2]. Nowadays, such storage systems are composed of many servers, which as a rule, fail sometimes [3]. This invokes the necessity to introduce some redundancy in such storage systems. Simple replication of data necessitated a high number of additional servers and therefore was not cost efficient. Therefore, it was gradually replaced by more sophisticated systems based on error control codes. Reed Solomon (RS) codes were considered as a viable candidate because they are Maximum Distance Separable (MDS) codes and they were previously used in RAID systems [4, 5].

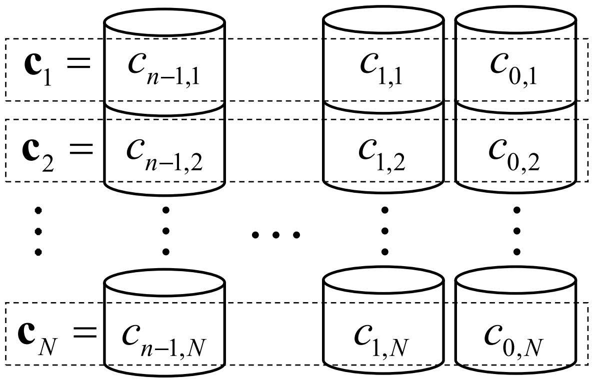

Fig. 1. Illustration of how symbols of N codewords which protect information could be stored in n servers (nodes or codeword symbols).

Note 1 The server content could be mapped onto the codeword symbol as illustrated in Fig. 1. As a result, a unit of protected information in each server can be represented by a separate codeword symbol distinct from all other codeword symbols.

Note 2 An erased codeword symbol is one whose value (contents) is lost but whose position inside the codeword is known.

The MDS property makes the RS codes so called minimum-storage regenerating (MSR) codes, from the point of view of data repair in distributed storages. In other words, if $s = n - k$ from n servers (codeword symbols) each containing the same volume of payload information is lost (deleted) only $n - k$ redundant servers with the same storage capacity are needed for the data repair, where n is the codeword length of the RS code. In this case the data in any $s = n - k$ servers from the n servers participating in this codeword could be regenerated. In other words, RS codes with codeword length n and dimension (number of information symbols) k possess code distance d_m equal to $n - k + 1$.

Note 3 Linear block codes are often denoted as $[n,\ k,\ d_m]$ - codes.

However, in distributed data storage the overhead costs are invoked not only by the number of redundant servers [3]. The other parameter contributing to costs is the repair bandwidth which characterizes the volume of information which is necessary to be communicated for a data repair of one server. The repair bandwidth is inversely proportional to the number of nodes λ which participate in the data repair.

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Therefore, codes with locality have recently become popular because they allow for recovering the lost data by procedures which need to download data only from a restricted number of local nodes [6-18]. Further improvement is possible if the codes used behind the locality also have another property denoted as availability [19-29]. Availability means that there are more disjoint subsets of nodes providing the locality. In practice the availability is important not only for data recovery but also for decreasing the access times to so called hot data if the demand is high.

In [29] the graph of [7, 3, 4] simplex code inspired a scalable family of codes with locality and availability. In the following sections it is shown that the same code can form the basis for families of codes with locality and availability which can be interpreted as fractals [30-31].

2 First family of codes with locality and availability forming a fractal

The graph of [7, 3, 4] simplex code C with codeword

$$\boldsymbol{c} = (c_6, c_5, c_4, c_3, c_2, c_1, c_0) \tag{1}$$

is depicted in Fig. 2.

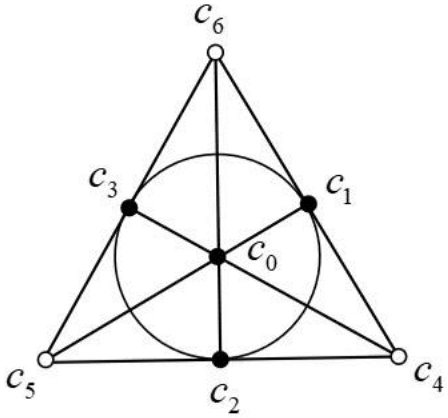

Fig. 2. Graph of the [7, 3, 4] simplex code

Let us denote $\boldsymbol{m} = (m_1, m_2, m_3)$ the information vector and

$$c_6 = m_1 \tag{2}$$

$$c_5 = m_2 \tag{3}$$

$$c_4 = m_3 \tag{4}$$

Then the following generator matrix $\boldsymbol{G}_1$

$$\boldsymbol{G}_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{5}$$

can be used for its definition and encoding. In other words, each codeword $\boldsymbol{c} \in C$ can be obtained as a linear combination of the rows in $\boldsymbol{G}_1$. For a particular information vector $\boldsymbol{m} = (m_1, m_2, m_3)$ the code-word is calculated as follows:

$$\boldsymbol{c} = \boldsymbol{m}\boldsymbol{G}_1 \tag{6}$$

It is well known [22] that this [7, 3, 4] code has locality $\lambda_1 = 2$ and availability $\alpha_1 = 3$.

Definition

The i-th code C_i from the family of codes with locality and availability forming a fractal can be represented by a generator matrix $\boldsymbol{G}_i$ obtained from the matrix of the code C_{i-1} from the same family with generator matrix $\boldsymbol{G}_{i-1}$ using a Kronecker product as follows:

$$\boldsymbol{G}_i = \boldsymbol{G}_1 \otimes \boldsymbol{G}_{i-1} \tag{7}$$

Therefore:

$$\boldsymbol{G}_{i+1} = \begin{bmatrix} \boldsymbol{G}_i & 0 & 0 & \boldsymbol{G}_i & 0 & \boldsymbol{G}_i & \boldsymbol{G}_i \\ 0 & \boldsymbol{G}_i & 0 & \boldsymbol{G}_i & \boldsymbol{G}_i & 0 & \boldsymbol{G}_i \\ 0 & 0 & \boldsymbol{G}_i & 0 & \boldsymbol{G}_i & \boldsymbol{G}_i & \boldsymbol{G}_i \end{bmatrix} \tag{8}$$

For example:

$$\boldsymbol{G}_2 = \begin{bmatrix} \boldsymbol{G}_1 & 0 & 0 & \boldsymbol{G}_1 & 0 & \boldsymbol{G}_1 & \boldsymbol{G}_1 \\ 0 & \boldsymbol{G}_1 & 0 & \boldsymbol{G}_1 & \boldsymbol{G}_1 & 0 & \boldsymbol{G}_1 \\ 0 & 0 & \boldsymbol{G}_1 & 0 & \boldsymbol{G}_1 & \boldsymbol{G}_1 & \boldsymbol{G}_1 \end{bmatrix} \tag{9}$$

The graph of C_2 with generator matrix (9) is illustrated in Fig. 3. From this illustration it is obvious that the proposed family of codes with locality and availability is forming a fractal. It remains to analyze the basic parameters of the codes and analyze their properties. The code C_2 is a [49, 9, 16] linear binary block code. The code distance of C_2 was confirmed by calculating the weight spectrum of the code. The weight spectrum is defined as the number w_j of codewords with specific Hamming weight j in the code.

For C_2 it is: $w_0 = 1, w_1 = 0, w_2 = 0, \ldots w_{15} = 0$, $w_{16} = 49,\ w_{17} = 0,\ w_{23} = 0,\ w_{24} = 294$, $w_{25} = 0,\ w_{27} = 0,\ w_{28} = 168,\ w_{29} = 0, \ldots$ $w_{49} = 0$.

The code C_2 has locality $\lambda_2 = 2$ and its availability is $\alpha_2 = 6$. This availability can be deduced observing Fig. 3 but it was also confirmed by analyzing the $\boldsymbol{G}_2$ matrix. It was confirmed that each symbol in its codeword can be obtained as a sum of 2 of the other 6 pairs of symbols which are disjoint. These pairs are presented in Tab. 1 for each symbol represented by its index.

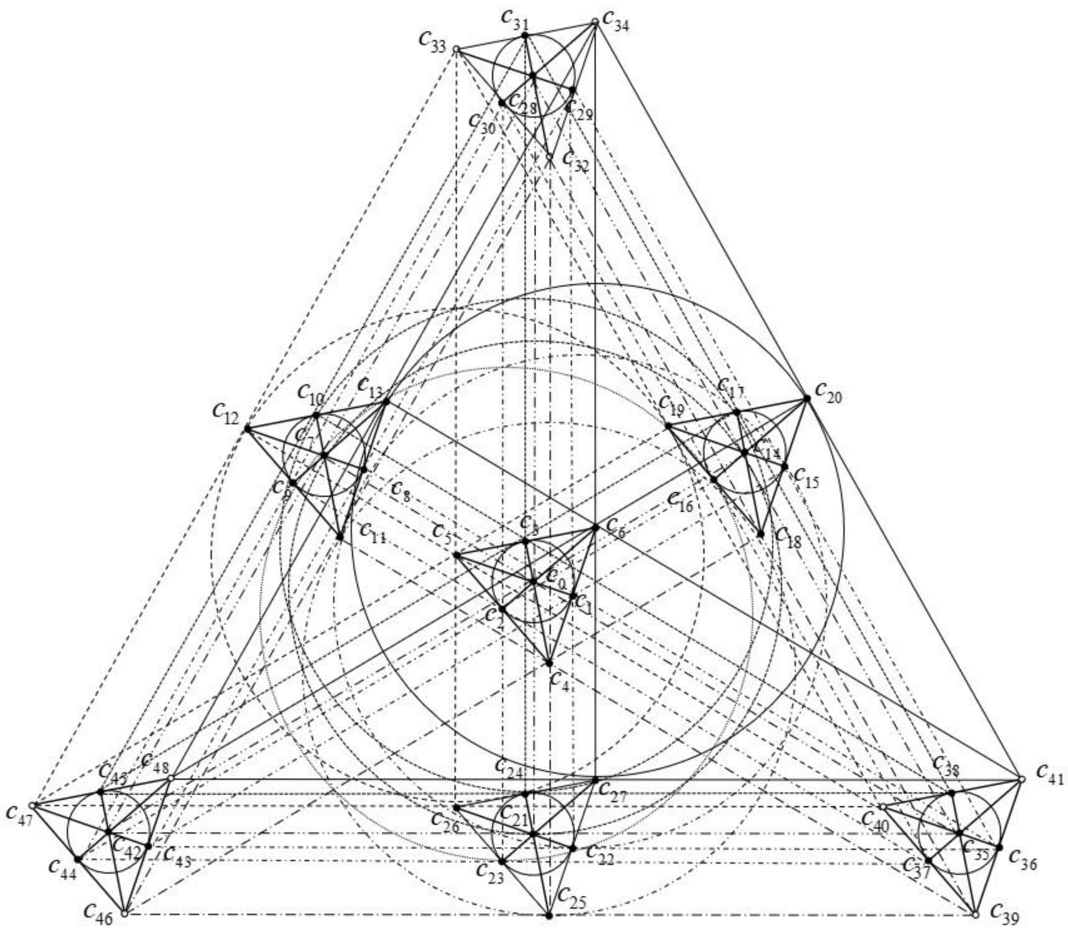

Fig. 3. Graph of the [49, 9, 16] code defined by (9)

Code C_3 is a [343, 27, 64] code with locality $\lambda_3 = 2$ and availability $\alpha_6 = 9$.

Its weight spectrum is:
$w_0 = 1,\ w_{64} = 343,\ w_{96} = 6174,$
$w_{112} = 40572,\ w_{128} = 240786,\ w_{136} = 592704,$
$w_{144} = 1115436,\ w_{148} = 790272, w_{160} = 7705152,$
$w_{164} = 11854080,\ w_{168} = 19954368,$
$w_{172} = 25552128,\ \ w_{176} = 24449040,$
$w_{180} = 20547072,\ w_{184} = 10273536,$
$w_{188} = 2370816,\ w_{192} = 395136,\ w_{196} = 32256.$

The i-th code C_i is $[7^i, 3^i, 4^i]$ from the proposed infinite family of codes which can be represented by a graph which forms a fractal is a linear binary code with locality $\lambda_i = 2$ and availability $\alpha_i = 3i$.

Table 1

Symbol index	Locality two groups in [49, 9, 16] code given by pairs of symbol indexes
0	{1, 5}, {3, 4}, {2, 6}, {7, 35}, {14, 42}, {21, 28}
1	{4, 6}, {0, 5}, {2, 3}, {8, 36}, {15, 43}, {22,29}
2	{4, 5}, {0, 6}, {1, 3}, {9, 37}, {16, 44}, {23, 30}
3	{5, 6}, {0, 4}, {1, 2}, {10, 38}, {17, 45}, {24, 31}
4	{2, 5}, {0, 3}, {1, 6}, {11, 39}, {10, 46}, {25, 32}
5	{3, 6}, {0, 1}, {2, 4}, {12, 40}, {19, 47}, {26, 33}
6	{3, 5}, {0, 2}, {1, 4}, {13, 41}, {20, 48}, {27, 34}
7	{8, 12}, {9, 13}, {10, 11}, {28, 42}, {0, 35}, {14, 21}
8	{11, 13}, {7, 12}, {9, 10}, {29, 43}, {1, 36}, {15, 22}
9	{11, 12}, {7, 13}, {8, 10}, {30, 44}, {2, 37}, {16, 23}
10	{12, 13}, {7, 11}, {8, 9}, {31, 45}, {3, 38}, {17, 24}
11	{9, 12}, {7, 10}, {8, 13}, {32, 46}, {4, 39}, {18, 25}
12	{10, 13}, {7, 8}, {9, 11}, {33, 47}, {5, 40}, {19, 26}
13	{10, 12}, {7, 9}, {8, 11}, {34, 48}, {6, 41}, {20, 27}
14	{15, 19}, {17, 18}, {16, 20}, {28, 35}, {0, 42}, {7, 21}
15	{18, 20}, {14, 19}, {16, 17}, {29, 36}, {1, 43}, {8, 22}
16	{18, 19}, {14, 20}, {15, 17}, {30, 37}, {2, 44}, {9, 23}
17	{19, 20}, {14, 18}, {15, 16}, {31, 38}, {3, 45}, {10, 24}
18	{16, 19}, {14, 17}, {15, 20}, {32, 39}, {4, 46}, {11, 25}
19	{17, 20}, {14, 15}, {16, 18}, {33, 40}, {5, 47}, {12, 26}
20	{17, 19}, {14, 16}, {15, 18}, {34, 41}, {6, 48}, {13, 27}
21	{22, 26}, {23, 27}, {24, 25}, {35, 42}, {0, 28}, {7, 14}
22	{25, 27}, {21, 26}, {23, 24}, {36, 43}, {1, 29}, {8, 15}
23	{25, 26}, {21, 27}, {22, 24}, {37, 44}, {2, 30}, {9, 16}
24	{26, 27}, {21, 25}, {22, 23}, {38, 45}, {3, 31}, {10, 17}
25	{23, 26}, {21, 24}, {22, 27}, {39, 46}, {4, 32}, {11, 18}
26	{24, 27}, {21, 22}, {23, 25}, {40, 47}, {5, 33}, {12, 19}
27	{24, 26}, {21, 23}, {22, 25}, {41, 48}, {6, 34}, {13, 20}
28	{31, 32}, {29, 33}, {30, 34}, {7, 42}, {0, 21}, {14, 35}
29	{32, 34}, {28, 33}, {30, 31}, {8, 43}, {1, 22}, {15, 36}
30	{32, 33}, {28, 34}, {29, 31}, {9, 44}, {2, 23}, {16, 37}
31	{33, 34}, {28, 32}, {29, 30}, {10, 45}, {3, 24}, {17, 38}
32	{30, 33}, {28, 31}, {29, 34}, {11, 46}, {4, 25}, {18, 39}
33	{31, 34}, {28, 29}, {30, 32}, {12, 47}, {5, 26}, {19, 40}
34	{31, 33}, {28, 30}, {29, 32}, {13, 48}, {6, 27}, {20, 41}
35	{37, 41}, {36, 40}, {38, 39}, {21, 42}, {0, 7}, {14, 28}
36	{39, 41}, {35, 40}, {37, 38}, {22, 43}, {1, 8}, {15, 29}
37	{39, 40}, {35, 41}, {36, 38}, {23, 44}, {2, 9}, {16, 30}
38	{40, 41}, {35, 39}, {36, 37}, {24, 45}, {3, 10}, {17, 31}
39	{37, 40}, {35, 38}, {36, 41}, {25, 46}, {4, 11}, {18, 32}
40	{38, 41}, {35, 36}, {37, 39}, {26, 47}, {5, 12}, {19, 33}
41	{38, 40}, {35, 37}, {36, 39}, {27, 48}, {6, 13}, {20, 34}
42	{43, 47}, {45, 46}, {44, 48}, {21, 35}, {0, 14}, {7, 28}
43	{46, 48}, {42, 47}, {44, 45}, {22, 36}, {1, 15}, {8, 29}
44	{46, 47}, {42, 48}, {43, 45}, {23, 37}, {2, 16}, {9, 30}
45	{47, 48}, {42, 46}, {43, 44}, {24, 38}, {3, 17}, {10, 31}
46	{44, 47}, {42, 45}, {43, 48}, {25, 39}, {4, 18}, {11, 32}
47	{45, 48}, {42, 43}, {44, 46}, {26, 40}, {5, 19}, {12, 33}
48	{45, 47}, {42, 44}, {43, 46}, {27, 41}, {6, 20}, {13, 34}

3 Second family of codes with locality and availability forming a fractal

The second family of codes which is proposed in this paper can be represented by a 3-dimensional fractal. The basic element from which the construction can start is a $[14, 4, 7]$ code $C_1^{'}$ from [32]. Its generator matrix is

$$\boldsymbol{G}_1' = \begin{bmatrix} 1&0&0&0&1&1&0&1&0&0&0&1&1&1 \\ 0&1&0&0&0&1&1&1&1&0&1&0&1&0 \\ 0&0&1&0&1&0&1&1&0&1&1&0&0&1 \\ 0&0&0&1&0&0&0&0&1&1&1&1&1&1 \end{bmatrix} \quad (10)$$

The generator matrix of any code from the second family can be obtained using a Kronecker product

$$\boldsymbol{G}_i' = \boldsymbol{G}_1' \otimes \boldsymbol{G}_{i-1}' \quad (11)$$

and it is a $[14^i, 4^i, 7^i]$ code over $GF(2)$ with locality $\lambda_i' = 2$ and availability $\alpha_i' = 6i$.

For example, the C_2' is a $[196, 16, 64]$ code with locality $\lambda_2' = 2$ and availability $\alpha_2' = 12$.

Its weight spectrum is: $w_0 = 1,\ w_{49} = 64,$ $w_{56} = 112, w_{64} = 49,\ w_{86} = 1568,\ w_{87} = 3136,$ $w_{88} = 2352,\ w_{96} = 14406,\ w_{97} = 18816,$ $w_{104} = 14112,\ \ w_{105} = 10752,\ w_{112} = 168.$

4 Conclusion

Two infinite families of linear binary block codes were presented which can be represented by graphs which form fractals. The i-th code C_i from the first family is a $[7^i, 3^i, 4^i]$ linear binary block code with locality $\lambda_i = 2$ and availability $\alpha_i = 3i$. The i-th code C_i' from the second family is a $[14^i, 4^i, 7^i]$ linear binary block code with locality $\lambda_2' = 2$ and availability $\alpha_2' = 6i$.

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