Journal of ELECTRICAL ENGINEERING, VOL 73(2022), NO3, 209–214

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PAPERS

Robust boundary of stability and design of robust controller in uncertain polytopic linear systems

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This paper is devoted to design of a robust controller for an uncertain continuous-time linear polytopic system. The design procedure will take place in two steps. In the first step, we calculate the robust stability boundary of a closed loop system. Then on the basis of the result obtained in this way, we select a robust controller design method that accepts the calculated stability boundary. This procedure allows better implementation of the second step of the designed robust controller. Two examples of application of the proposed method follow finally, which illustrate its effectiveness.

K e y w o r d s: robust control, uncertain polytopic system, output feedback, PID controller, regional pole

1 Introduction

Robust control belongs to challenging control theory and practice topics. More decades were necessary to develop it. While applications in real life had to cope with uncertainties of system, we can say that control practice itself necessitated such a robust control, whose essence reflect the needs of such practice. This kind of control should keep the required performance within a domain of uncertainty that has been specified before. Either when designing robust control or when analyzing stability of a robust control, designers developed a whole spectrum of approaches. There are excellent books and papers on the robust control theory fundamentals. The linear (bilinear) matrix inequalities LMI (BMI) play an important role in the time domain robust controller design procedures [1]. They played a particular role in developing the theory of robust controllers design, [2–10], and others. In the frequency domain, to robust (decentralized) controller design, interesting results are obtained in the following papers [11–15], and others. [16] brings an overview of some important methods of designing robust controllers either in the time domain or in the frequency one in time-invariant linear systems. This e-publication is accessible for free to the authors.

Two-steps method of a robust controller design is described here. Boundary of the closed-loop system robust stability being calculated in the first step. The results obtained in this way will be the basis of selection of the method that will be used when designing the robust controller accepting the boundary of the stability that has been calculated in this way. This procedure will allow more precise and better implementation of the step No. 2 of the design of the robust controller.

Section 2 of this paper provides preliminary results and formulates the problem. Main results in Section 3 give theoretical bases to calculate the robust boundary stability for the uncertain polytopic system. Two examples are used in this paper to compare two control methods, as follows: H_2 and the approach of regional pole placement. Section 4 brings some examples showing its effectiveness. Another advantages are summarized in the Conclusion that is the last section of this paper.

Let us define notation conditions that are used here. Let $P = P^\top \in R^{n\times n}$ be a symmetric matrix. Then, the positive (negative) definiteness of the matrix shall be denoted by the inequality $P > 0$, $(P < 0)$. We use I_n and 0_n to denote respectively the identity matrix and the zero one (of course, their corresponding dimensions are assumed).

2 Uncertain polytopic system – mathematical model

The polytopic continuous time-invariant system is given in the form

$$\dot{x} = A(\xi)x + B(\xi)u; \quad y = Cx, \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R_i^l$ are state, control input, controlled output respectively and system matrices $(A(\xi), B(\xi)) = \sum_{i=1}^N (A_i, B_i)\xi_i$ belong to the convex set a polytope with N vertices. The following definition applies for the uncertainty $\xi \in \Omega_\xi$

$$\Omega_\xi = \Big\{\xi_i \geq 0, i = 1, 2, \ldots, N, \sum_{i=1}^N \xi_i = 1, \sum_{i=1}^N \dot{\xi}_i = 0\Big\}. \tag{2}$$

Let us assume that matrices A_i, B_i, C have constant entries of corresponding dimensions. The uncertainties ξ_i, i=1,2, … ,N are unknown parameters that are either

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https://doi.org/10.2478/jee-2022-0028, Print (till 2015) ISSN 1335-3632, On-line ISSN 1339-309X

constant or time varying. The plant state should be expanded, so that the plant output integral feedback shall be obtained for the robust PI-D controller design. See [8] for more detail. Let the controller integral part of the output feedback be as follows: $\dot{z} = y = Cx$. Let us define the plant new state as follows

$$\dot{x}_n = \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = An(\xi)x_n + Bn(\xi)u; \quad y_n = Cnx_n, \tag{3}$$

where

$$An(\xi) = \begin{bmatrix} A(\xi) & 0 \\ C & 0 \end{bmatrix},\ Bn = \begin{bmatrix} B(\xi) \\ 0 \end{bmatrix},\ Cn = \begin{bmatrix} C & 0 \\ 0 & I_z \end{bmatrix}.$$

The following problem is studied: Calculate both the robust stability boundary and design the robust controller PI-D (4), which ensure the properties of robustness of the system with closed-loop, as well as the quadratic stability that depends on parameters, assuming that all eigenvalues of the closed loop are within the LMI region as prescribed, the control PI-D algorithm being as follows

$$\begin{aligned} u = k_p y + k_i z + k_d \dot{y} &= [k_p C \ \ k_i]x_n + [k_d Cd \ \ 0]\dot{x}_n \\ &= Kx_n + K_d \dot{x}_n \end{aligned} \tag{4}$$

where Cd is the derivative output feedback matrix.

LEMMA 1 [11]. *Let us assume that the equality*

$$G = cH + \varrho I_n, \tag{5}$$

holds for scalars $c, \varrho \in R$ *and the identity matrix* $I_n \in R^{n\times n}$. *Then the eigenvalues of the matrix* G *are given as follows:* $\lambda_k = c\gamma_k + \varrho$ *where* γ_k *is the eigenvalue of the matrix* $H, k = 1, 2, \dots, n$

LEMMA 2. *The sum of two matrices* $G + H \in R^{n\times n}$ *is stable if and only if there exists a positive definite Lyapunov matrix* $P > 0$ *such that the Lyapunov inequality*

$$(G + H)^\top P + P(G + H) \le 0, \tag{6}$$

holds.

LEMMA 3. *Assume that the uncertain system is described by* (1) *and gain's static output feedback controller matrix is given as* K. *Then the following necessary and sufficient condition for the existence of a robust controller with gain* K *such that the closed-loop system is asymptotically stable applies*

$$\lambda(A(\xi), B(\xi), C, K) \in C^-,$$

where C^- *is the left hand side of the complex plane.*

3 Main results

Robust stability boundary as well as the procedure of design of the PI-D controller are obtained in this section. The PI-D control algorithm ensures quadratic stability (PDQS) that depends on parameters, of the closed-loop system, which applies to generalized LMI regions, which are defined by the following characteristic function (13). First, the robust stability boundary will be obtained for uncertain polytopic systems. Let us split uncertain polytopic system (3) into the following form

$$\dot{x}_n = An(\xi)x_n + Bn(\xi)u; y_n = Cnx_n, \tag{7}$$

or for i-th vertex of polytope

$$\dot{x}_n = An_i x_n + Bn_i u; y_n = Cnx_n, i = 1, 2, \dots, N. \tag{8}$$

THEOREM 1. *Let us have the matrix* An_i *with constant entries. The sum of* $D_i = An_i + I\alpha$ *is stable for some* α *if positive definite matrix* $P_i \in R^{n\times n}$ *exists, such that the following inequality holds*

$$(An_i)^\top P_i + P_i(An_i) + 2\alpha P_i \le 0. \tag{9}$$

P r o o f . Due to the Lyapunov stability condition, Lemma 1, the following inequality is obtained

$$(An_i + I\alpha)^T P_i + P_i(An_i + I\alpha) \le 0. \tag{10}$$

Let us make a minor manipulation. We get (9). In this way, the sufficient condition of stability is proven. If assuming stability of the matrix D_i, we can use Lyapunov stability theory to prove existence of a positive definite matrix P_i, while holding the inequality (10). In this way, the theorem is proven.

Consider an uncertain polytopic system in the form $\dot{x}_n = A(\xi)x = \sum_{i=0}^{N}((An_i + I\alpha)\xi_i)x_n$. Making minor modification of [2], the following Lemma gives as the stability of such complex system that is uncertain

LEMMA 4. *The linear uncertain polytopic system* $An(\xi)$ *is stable if matrices* $G, H \in R^{n\times n}$, *parameter* α *and symmetric positive definite matrix* $P_i, i = 1, 2, \dots, N$ *exist, such that the following inequality holds*

$$\begin{bmatrix} D_i^\top H^\top + H(An_i + I\alpha) & * \\ P_i - H^\top + G^\top(An_i + I\alpha) & -G - G^\top \end{bmatrix} \le 0, \quad i = 1, 2, \dots, N. \tag{11}$$

Parameter α could be obtained from (9) or (11). If $\alpha \ge 0$, then the uncertain system is stable. The design of the robust controller should hold the following inequality in this case, dominant eigenvalues of the system with closed loop

$$\lambda_k(An_i + B_i * controller) \le \lambda_k(An_i + I\alpha), \quad k = 1, 2, \dots, d_s;\ i = 1, 2, \dots, N,$$

where d_s is the number of the dominant eigenvalues for which the closed-loop system becomes stable, obviously

$d_s = 1$. When $\alpha < 0$, the dynamic of the corresponding system of the robust controller that is being designed should not be worse than that of the system with open loop.

Negative α indicates unstability of the system. The robust controller designer should make an observation about robust controller design methods to choose such method for which the designed controller would be able to move to the left system dominant eigenvalues by α in the complex plane. Assume, that the "ideal" controller is obtained, where the closed-loop system in the i-th vertex is given as $D_i = An_i + I\alpha$, $i = 1, 2, \ldots, N$. Then, the boundary of robust stability of the system with closed-loop is as follows:

$$b_s = \alpha + \min_i(\max(real(eig(An_i)))),\ i = 1, 2, \ldots, N\,.$$

The above observation implies that one of the best methods of the design of the robust controller is that of the regional LMI pole placement approach.

Let us recall some LMI region results from [2], and [3].

DEFINITION 1. An LMI region is any subset D of the complex plane C_o that can be defined by its characteristic function

$$D = \{z \in C_o : L + zM + z^\top M^\top < 0\}\,. \tag{12}$$

where L, M are real matrices such that $L = L^\top$ is a symmetric matrix and $M \in R^{d\times d}$.

LEMMA 6. *The closed-loop uncertain system* (1)+(3) *given by matrix $A_c(\xi)$ is robustly D-stable if a symmetric matrix $P(\xi)$ exists such that*

$$L \otimes P(\xi) + M \otimes (P(\xi)A_c(\xi)) + M^\top \otimes (A_c(\xi)^\top P(\xi)) < 0 \tag{13}$$

for $P(\xi) > 0$.

The inequality (13) is the extended time derivative Lyapunov function [3]. After modification of (13) for the time varying system $\dot{\xi}_j \in \Omega_\xi$ it can be rewritten as follows [4]

$$v_1^\top \begin{bmatrix} L \otimes P(\xi) + L \otimes P(\dot{\xi}) & M \otimes P(\xi) \\ M^\top \otimes P(\xi) & 0 \end{bmatrix} v_1 < 0, \tag{14}$$

where

$$v_1^\top = [(1_d \otimes x_n)^\top \quad (1_d \otimes \dot{x}_n)^\top]$$

Inequality (14) with (3) and (4) represents the time derivative of the extended Lyapunov function for the general LMI region given by characteristic function (12). Let us assume that in (14) and (13) the Lyapunov function is given as

$$V(\xi) = x_n^\top P(\xi) x_n, \quad P(\xi) = \sum_{i=1}^{N} P_j \xi_j\,. \tag{15}$$

The time derivative of the Lyapunov function is given as follows

$$\begin{aligned}\frac{dV(\xi)}{dt} &= \dot{x}_n^T P(\xi) x_n + x_n^T P(\dot{\xi}) x_n + x_n^T P(\xi)\dot{x}_n, \\ &= v^\top \begin{bmatrix} P(\dot{\xi}) & P(\xi) & 0 \\ P(\xi) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v < 0\end{aligned} \tag{16}$$

where

$$v^\top = [x_n^\top \quad \dot{x}_n^\top \quad u^\top]$$

$$P(\dot{\xi}) = \sum_{j=1}^{N} P_j \dot{\xi}_j \le \sum_{j=1}^{N} P_j \overline{\dot{\xi}_j}\,.$$

Quadratic cost function can be used for the purposes of assessment of the quality of the closed-loop performance with LMI regions, in the frame of H_2 in the following form

$$J_c = \int_{t_0}^{\infty} J(x_n, \dot{x}_n, u)\mathrm{d}t, \tag{17}$$

where

$$J(\cdot) = x_n^\top Q x_n + \dot{x}_n^\top S \dot{x}_n + u^\top R u = v^\top \begin{bmatrix} Q & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & R \end{bmatrix} v\,,$$

Q, S are positive definite/semidefinite matrices and R is positive definite matrix. Now, the conservatives should be decreased. Let us begin with splitting the system matrices from the Lyapunov matrices. Then, we should ensure convexity of the matrix $A_c(\xi)$ of the system with closed loop, in the context with the uncertainty ξ. Auxiliary matrices shall be introduced now, to the first derivative of the extended Lyapunov function $N_1, N_2 \in R^{n\times n}$, $N_3 \in R^{n\times m}$, $N_4, N_5 \in R^{m\times n}$, $N_6 \in R^{m\times m}$ as follows

$$v^\top \begin{bmatrix} 2N_1^\top \\ 2N_2^\top \\ 2N_3^\top \end{bmatrix} [-An(\xi) \quad I \quad -Bn(\xi)]v = 0, \tag{18}$$

and

$$v^\top \begin{bmatrix} 2N_4^\top \\ 2N_5^\top \\ 2N_6^\top \end{bmatrix} [-K \quad -K_d \quad I_u]\, v = 0\,. \tag{19}$$

Summarizing (18), (19), and (17) and splitting the obtained results concerning uncertainty, the time derivative can be obtained, of the extended Lyapunov function. After substituting the results obtained with (17) with the Bellman-Lyapunov equation [5], we get the following equation

$$B_e = \frac{\mathrm{d}V_{ext}(\xi)}{\mathrm{d}t} + J(.) = \sum_{i=1}^{N} v_2^\top W_i v_2 \xi_j < 0, \tag{20}$$

where $W_i = \{w_{ikl}\}_{3\times 3},\ i = 1,2,\dots,N$,

$$v_2^\top = \begin{bmatrix}(1_d \otimes x_n)^\top & (1_d \otimes \dot{x}_n)^\top & (1_d \otimes u)^\top\end{bmatrix}$$

and

$$w_{i11} = L \otimes P_i + L \otimes \sum_{i=1}^{N} P_i \dot{\xi}_i - N_1^{\rightarrow;p}(I_d \otimes A_i) - (I_d \otimes A_i)^\top N_1 - N_4^\top (I_d \otimes K - (I_d \otimes K)^\top N_4 + Q\,,$$

$$w_{i12} = M^\top \otimes P_i + N_1^\top - (I_d \otimes A_i) N_2 - N_4^\top (I_d \otimes K_d) - (I_d \otimes K)^\top N_5\,,$$

$$w_{i13} = -(I_d \otimes A_i)^\top N_3 - N_1^\top (I_d \otimes B_i) + N_4^\top - (I_d \otimes K)^\top N_6\,,$$

$$w_{i23} = -N_2^\top (I_d \otimes B_i) + N_3 + N_5^\top - (I_d \otimes K_d)^\top N_6\,,$$

$$w_{i33} = -N_3^\top (I_d \otimes B_i) - (I_d \otimes B_i)^\top N_3 + N_6 + N_6^\top + R\,,$$

$$w_{i22} = N_2^\top + N_2 - N_5^\top (I_d \otimes K_d) - (I_d \otimes K_d)^\top N_5 + S\,.$$

Please, note, that the new extended variables $N_i, i = 1,2,\dots,6$ and Q,S,R in (20) are calculated as follows: $N_i = N_{inew} = I_d \otimes N_{iold}$, $(Q,S,R)_{new} = I_d \otimes (Q,S,R)_{old}$. Now, let us continue without any attempt to change the denotation. This chapter will be closed with summary of its main results.

THEOREM 2. *The uncertain system* (3) *with control algorithm* (4) *is PDQS in the* $D - LMI$ *region and simultaneously it minimizes the* H_2 *performance* (17) *if there exist positive definite matrix* $P(\xi)$ (15), *auxiliary matrices* $N_1,\dots,N_6$ *gains* K, K_d *and performance matrices* Q,S,R *such that* (20) *hold for all* $i = 1,2,\dots,N$, $\dot{\xi}_i \in \Omega_\xi$.

Proof of the theorem sufficient stability condition immediately follows from the above discussion.

Assuming (20), the robust controller that is being desinged ensures the parameter dependent quadratic stability, the eigenvalues of the closed loop being in the prescribed LMI region for all given uncertainties.

4 Examples

The first example clarifies the basic properties of the proposal of robust controllers. First, the boundary of the robust stability of the polytopic system was calculated. This auxiliary variable allows selection the most suitable robust controller design from among all existing methods. In our case, the polytopic system is a third order one, $n = 3$, with one control variable, $m = 1$, and two vertices of the polytope, $N = 2$. We aim to make a design of robust PID controller, while guaranteeing the robust, parameter dependent quadratic stability and minimal value of quadratic criteria function in the form of (17) and the following data: $S = sI,\ s = 0.1;\ Q = qI, q = 0.5;\ R = rI_r,\ r = 1$.

Another constraint applies to all Lyapunov matrices $0 < P < \rho * I, \rho = 10^8$. The uncertain system with the I part of the controller in the two vertices is as follows $i = 1$

$$An_1 = \begin{bmatrix} -0.95 & 0.2 & 1 & 0 \\ 0.15 & -0.86 & 1.2 & 0 \\ 0.2 & 0.75 & -1.03 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}; Bn_1 = \begin{bmatrix} 1 \\ .1 \\ 0 \\ 0 \end{bmatrix}$$

$i = 2$

$$An_2 = \begin{bmatrix} -0.75 & 0.3 & 0.9 & 0 \\ 0.2 & -0.72 & 1.1 & 0 \\ 0.3 & 0.8 & -1.35 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}; Bn_2 = \begin{bmatrix} 0.95 \\ 0.05 \\ 0 \\ 0 \end{bmatrix}$$

$$C_n = C_D = [1 \quad 0 \quad 0 \quad 0]\,.$$

At first, the robust stability boundary is calculated. Due to (11) and Lemma 5, the robust stability boundary is as follows

$$b_s = \alpha + \min_i(\max(real(eig(An_i)))) = -0.045\,.$$

The uncertain system is unstable. The following three robust controller design procedures will be used: H_2 and the design of PID robust controller; regional pole placement approach with PID robust controller; and regional pole placement approach with PI-D robust controller. The last case leads to the following derivative output feedback matrix

$$C_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Case H_2

If no extended Lyapunov function (20) is used in the robust controller design H_2, then robust design procedure is obtained. Due to the robust stability boundary b_s, the system matrix An_i is changed as follows: $Anew_i = An_i + abs(b_s) * I$.

For the above performance parameters and robust controller design procedure we get a robust PID controller as follows (20)

$$R(s) = -582.093 - \frac{33.8911}{s} - 603.6796s$$

with the following closed-loop eigenvalues $E_{\mathrm{CL},i}$

$$E_{\mathrm{CL},1} = \{-1.866, -0.90, .-0.06 - 0.0448\},$$

$$E_{\mathrm{CL},2} = \{-2.01, -0.902, -0.0657 - 0.072\}\,.$$

Robust stability of the system with closed loop is ensured now, with better dynamics than that in the previous case.

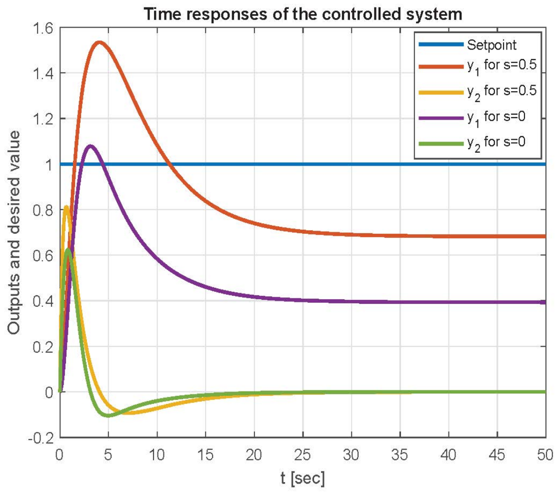

Fig. 1. Controlled system time response

Case PI-D reg. pole place.

Two states derivative feedback

Accelerometers can be used to open new ways of stabilization of control systems with closed loop. Output and state derivative feedback are used to this end. States derivative feedback can be found if we want to stabilite a real plant. It can be found (without limitation) in the following systems: vibration control; suppression of mechanical vibration; power system stabilization (PSS). This example uses two state variables as the feedback given by the output matrix C_D. (20) brings a procedure of designing a robust PI-D controller, using the regional pole placement. For the case of two states derivative feedback and $\alpha_s = 0.1$, the following parameters of the robust PI-D controller are obtained

$$R(s) = -1.267 - \frac{0.8916}{s} - 2.1836\dot{x}_2 - 4.8637\dot{x}_3 .$$

The closed-loop eigenvalues for two vertices are

$$E_{\mathrm{CL1}} = \{-3.294, \dots, -0.10002\},$$

$$E_{\mathrm{CL2}} = \{-3.765, \dots, -0.101 \pm 0.096i\}.$$

If taking all states derivative feedback for the same Case $\alpha_s = 0.1$, the following robust controller parameters will be obtained

$$R(s) = -1.9591 - \frac{0.8509}{s} - 3.075\dot{x}_1 - 1.27\dot{x}_2 - 3.65\dot{x}_3 .$$

The closed-loop eigenvalues are

$$E_{\mathrm{CL1}} = \{-1.9615, \dots, -0.10013\},$$

$$E_{\mathrm{CL2}} = \{-2.177, \dots, -0.1002 \pm 0.0984i\}.$$

For both cases of the state derivative feedback, the system with closed loop is robustly stable, with slightly better dynamic in the first case (*ie* two-states feedback).

The second example is from [6]. It is a case of a design of a cost-guaranteed robust controller for the purposes of lateral axis dynamics stabilization in the case of the aircraft L-1011. Now, let us have the following system matrices

$$A = \begin{bmatrix} -2.98 & q_1 & 0 & -0.034 \\ -0.99 & -0.21 & 0.035 & -0.0011 \\ 0 & 0 & 0 & 1 \\ 0.39 & -5.555 & 0 & -1.89 \end{bmatrix}, B = \begin{bmatrix} -0.032 \\ 0 \\ 0 \\ -1.6 \end{bmatrix}.$$

Output matrix

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with parameter bound $-0.57 < q_1 < 2.43$. Polytopic system could be obtained if putting $A_1 = \{a_{1ij}\}$, $a_{112} = -0.57$; $A_2 = \{a_{2ij}, a_{212} = 2.43\}$. Finding a static output feedback matrix is the problem that is restated in the original paper. In the new formulation, the solution of a set of Lyapunov inequalities that are inversely coupled are replaced with the simultaneous solution of a Lyapunov inequality and an algebraic Riccati inequality. Using the algorithm, the given degree of stability may be prescribed in such a way that the static output feedback gain remains small.

H_2 approach.

This paper, designs the two PD controllers using the method of regional pole placement approach with H_2 performance (20) with extended quadratic performance criterion (17). First, the robust stability boundary

$$b_s = \alpha + \min_i(\max(real(eig(An_i)))),\ i = 1, 2, \dots, N$$

is calculated. The obtained value $b_s = -0.1096$, that is, the necessary condition of the guarantee of robust stability of a an closed-loop uncertain system, is that all its real part of eigenvalues need to be less than b_s. The system matrix An_i is changed as follows: $Anew_i = An_i + abs(b_s) * I$, the performance parameters (17) being $S = sI$, $s = 0.5$, $Q = qI$, $q = 0.5$, $R = rI_u$, $r = 1$. After some modification of (20) for the purposes of the robust controller H_2 design procedure, the following two robust PD controllers are obtained

$$R_1 = 0.9764 + 3.3961s; R_2 = -1.9033 + 0.7664s$$

Note that we have two outputs in this example and only one input. The two vertices have the following closed-loop eigenvalues

$$E_{\mathrm{CL},1} = \{-0.244 \pm 0.153i, -1.457, -3.169\},$$

$$E_{\mathrm{CL},2} = \{-0.641, -1.302, -1.586 \pm 0.648i\}.$$

Now, let us experimentally change the performance parameters as follows: $S = sI$, $s = 0.0$, $Q = qI$, $q = 0.5$, $R = rI_u$, $r = 1$. The following robust PD controllers are obtained

$$R_1 = 0.9323 + 2.0661s; \quad R_2 = -0.7417 + 0.7371s.$$

Closed-loop eigenvalues $(E_{\mathrm{CL},i})$ are

$$E_{\mathrm{CL},1} = \{-0.251 \pm 0.163i, -1.36, -3.16\},$$

$$E_{\mathrm{CL},2} = \{-0.735, -1.08, -1.604 \pm 0.65i\}.$$

Regional pole placement approach $+H_2$.

Robust PID controller design algorithm is given by (20). Having the following parameters of quadratic cost functions (17) $S = sI$, $s = 0.5$, $Q = qI$, $q = 0.5$, $R = rI_u$, $r = 1$ and the demanded degree of the closed-loop stability being $\alpha = 0.1$, the following two robust PD controller parameters are contained

$$R_1 = 0.0996 - 4.6451s\,; \;\; R_2 = 3.6485 - 0.6133s\,.$$

the closed-loop eigenvalues being

$$E_{\mathrm{CL},1} = \{-0.288 \pm 0.84i, -2.726 - 15.21\}\,,$$

$$E_{\mathrm{CL},2} = \{-0.972 \pm 1.32i, -1.358, -15, 21\}\,.$$

The proposed method is pretty effective, as proven by the presented examples. They demonstrate different approaches to the PID controller design. The methods are chosen by the designer. In the example "Case PI-D", two simulation result were graphically compared (Fig. 1) for weighting factors $s = 0.5$ and $s = 0.0$. Impact of the weighting factor of the criterion function is higher for the output y_1. The designer can affect the quality of the control by choosing suitable weighting coefficients.

5 Conclusion

This paper brings a derived method of calculation of the boundary of robust stability of the uncertain polytopic system. These data help to select a robust controllers design method and thus increases calculation efficiency. Implementation of the calculation of the stability boundary using the proposed method is illustrated using two examples. In the end, in the case of H_2, the effect has been experimentally detected, of the derivative component in the quadratic criterion function on the quality of the transition process.

Acknowledgment

This work has been supported by Grant VEGA 1/0049/20 of the Slovak Scientific Grant Agency.

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Received 3 April 2022