
Chiral bright, dark and singular Solitons for the Nonlinear Schrödinger Equation in (1+2) Dimensions

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Abstract

In this study, we employ two efficient analytical techniques, the simplest equation approach and the tanh-coth method to investigate the (1+2) dimensional nonlinear chiral Schrödinger's equation. In condensed matter physics and nonlinear optics, this equation is essential, describing the propagation of light in materials with significant nonlinearities and chiral properties. By employing the simplest equation method, we derive exact solutions that reveal the complex relationship between nonlinearity and chirality. Additionally, the tanh-coth method yields exact soliton solutions, enhancing our understanding of soliton dynamics in chiral media. Our results provide new insights into the behavior of nonlinear waves in (1+2) dimensions and demonstrate the effectiveness of these methods in solving complex nonlinear equations. We present analytical solutions for bright and dark solitons. We also discuss the variables of the system in the following section and conditions enabling various solutions, including singular and periodic solitons.

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Keywords: Simplest equation method; nonlinear chiral Schrödinger's equation; tanh-coth method.

1. INTRODUCTION

Exploring nonlinear wave phenomena is foundational in modern physics, offering profound insights into the diverse and complex dynamics of various physical systems. Among the numerous nonlinear wave equations, a basic model appears in the form of the (1+2) dimensional nonlinear chiral Schrödinger equation, capturing the complex relationship between nonlinearity and chirality in wave propagation phenomena. This equation represents an extension of the standard Schrödinger equation to incorporate chirality, a fundamental property observed in numerous physical systems, including molecular structures, condensed matter systems, and optical media. Consequently, understanding the dynamics controlled by the chiral Schrödinger equation in (1+2) dimensions, which is nonlinear, holds paramount importance in explaining the behavior of waves in chiral media and guiding experimental investigations across various disciplines including nonlinear optics, spintronics, and topological photonics.

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Previous studies have extensively explored the properties and solutions of the (1+2) dimensional chiral Schrödinger equation by employing various analytical and numerical techniques. Notable contributions by researchers such as [1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12] have investigated the existence, stability, and propagation characteristics of solitons, breathers, and other coherent structures in chiral media. While these efforts have provided valuable insights into the behavior of chiral waves, unexplored territory remains regarding the application of systematic analytical methods such as the simplest equation method and the tanh/coth approach to this particular equation. The main research gap is that earlier studies of the (1 + 2)-dimensional chiral nonlinear Schrödinger equation mainly emphasized trial-solution techniques, stability issues, or particular classes of localized waves, whereas a unified treatment based on the simplest equation method and the tanh-coth method for this specific higher-dimensional chiral model was still missing. In particular, the literature lacked an explicit analytical framework that simultaneously identifies several families of exact solutions together with their parameter restrictions for the same (1 + 2)-dimensional chiral equation. Driven by the goal of developing new analytical methods for solving complex nonlinear wave equations and enhancing our understanding of chiral wave dynamics, this study presents a comprehensive analysis of the (1+2)-dimensional Chiral Schrödinger equation. Central to our investigation are two powerful methods: the tanh/coth method and the simplest equation method, which provide techniques for precisely deriving analytical solutions to nonlinear partial differential equations. By employing these methods, we aim to comprehend the complex dynamics of chiral waves in nonlinear media, shedding light on the underlying mechanisms governing wave propagation phenomena in chiral systems. Our study builds upon previous research [13; 14; 15] that has explored the nonlinear Schrödinger equation's solution using the simplest equation method, deriving soliton solutions. The novelty of the present work is twofold. First, we apply two complementary analytical schemes—the simplest equation method and the tanh-coth method—to the same (1 + 2)-dimensional chiral nonlinear Schrödinger model, allowing direct comparison of the solution structures obtained by each method. Second, unlike earlier studies on more general nonlinear Schrödinger-type equations, the present work derives for this chiral higher-dimensional model a broad family of exact waves including bright, secondary bright, dark, secondary dark, singular, and periodic solutions, together with the accompanying parameter constraints that govern their existence. However, our focus

lies in investigating a version of this equation dealing with two spatial dimensions and time, enabling a deeper understanding of chiral wave dynamics in a broader context. Through this investigation, we aim to expand the existing body of knowledge on the nonlinear chiral Schrödinger equation in (1+2) dimensions by offering new insights into chiral wave behavior and demonstrating the effectiveness of the simplest equation method and the tanh/coth method in solving complex nonlinear equations. Our findings hold implications for a wide range of physical systems exhibiting chiral phenomena, offering paths for further exploration and experimental validation. Optical solitons, which maintain their shape, speed, and amplitude as they travel through a nonlinear medium, have been extensively researched [16; 17; 18; 19; 20; 21; 22; 23]. Explaining the procedure through numerical simulations or analytical approximations of the equation for solving the NLSE with wave solutions is a crucial area of study in optical fiber communication systems [16; 17; 18; 19; 20; 21; 22; 23; 24]. These solutions include both bright and dark solitons, appearing in different dispersion regimes. Different versions of the NLSE, featuring varied modified and non-linear terms, are used to explain physical phenomena, medical imaging, also optical effects [18; 19; 20; 21; 22; 23; 24; 25]. Research in this field significantly impacts optical communication and computing technologies. Different methods have been employed by researchers to explore these Schrödinger models and discover new optical solutions [26; 27]. The need to come up with models with more than one dimension is caused by the fact that the complex structures, which are noticeable in the real world, are only clearly seen in multi-dimensional models across the physical and biological systems. To strengthen the connection with current nonlinear optics, we also note recent studies on optical solitons and modern nonlinear Schrödinger models. For example, Rabie et al. investigated optical solitons and stability analysis for a nonlinear Schrödinger equation with nonlocal nonlinearity and nonlinear chromatic dispersion [34], while Ullah et al. studied optical soliton solutions for the perturbed nonlinear Schrödinger equation using the tanh-coth method together with related analytical tools [35]. These references support the continued relevance of exact-wave analysis in present-day optical applications. Since the availability of ultrashort pulsed lasers, many different authors have invested considerable effort in the study of the nonlinear interaction phenomena, employing more and more refined algorithms as we will see in next sections. Many reliable numerical and analytical approaches have been used to find soliton solutions, and this paper also includes them [19; 20; 21; 22; 23; 24; 25;

26; 27]. In the following sections, we will apply different techniques to find solutions to this extended equation [28].

The document begins by introducing the nonlinear Schrödinger's equation in chiral (1+2) dimensions, providing a review of existing literature relevant to this topic. Following this, the main equation is presented in the second section. The third section outlines the methodologies employed to solve this equation, including the simplest equation method and the tanh/coth method. After that, we found a variety of solutions. Results are explained with the help of graphical representations in the following part, demonstrating the adaptability and efficacy of the approaches covered. Ultimately, the fifth section concludes the discussion, providing a summary of the discoveries and advancements in the field of nonlinear wave equations.

2. MAIN EQUATION

The chiral nonlinear Schrödinger's equation (1+2) dimensions [10] is as follows:

$$iq_t + a(q_{xx} + q_{yy}) + i\{b_1(qq_x^* - q^*q_x) + b_2(qq_y^* - q^*q_y)\}q = 0, \quad (1)$$

where a denotes the dispersion coefficient, b_1 and b_2 the coefficients of the nonlinear coupling terms, and the first term in Eq. (1) indicates the evolutionary component; it cannot be integrated using the inverse scattering transform approach. It is significant to notice that the equation described is not preserved under the Galilean transformation. Physically, the parameters b_1 and b_2 measure the strength of the derivative-type chiral nonlinear coupling along the x - and y -directions, respectively. Hence, they control how the local amplitude interacts with directional phase gradients, and they directly influence the balance between dispersion and nonlinearity. This is visible in the reduced ordinary differential equation through the combination $(\kappa_1 b_1 + \kappa_2 b_2)$ and in the bright/dark soliton constraints through expressions such as $(B_1 b_1 + B_2 b_2)$, showing that b_1 and b_2 affect amplitude, width, phase evolution, and propagation speed. The following transformation is introduced to solve this equation.

$$q(t, x, y) = u(\psi) e^{i\phi(x, y, t)}, \quad \text{where } \psi = B_1 x + B_2 y - vt, \quad (2)$$

$$\phi(x, y, t) = \kappa_1 x + \kappa_2 y + \omega t + \theta, \quad (3)$$

where ω is the soliton frequency and θ is the phase constant in Eq. (3). The frequencies in the x and y directions are, respectively, κ_1 and κ_2 . Eq. (2) is substituted

into Eq. (1), and the real and imaginary components result in:

$$a(B_1^2 + B_2^2) \frac{d^2 u}{d\psi} - (a(\kappa_1^2 + \kappa_2^2) + \omega)u + 2(\kappa_1 b_1 + \kappa_2 b_2)u^3 = 0, \quad (4)$$

$$(-v + 2a(\kappa_1 B_1 + \kappa_2 B_2)) \frac{du}{d\psi} = 0, \quad (5)$$

A positive integer $m = 1$ is obtained by using homogeneous balance method incorporating the leading nonlinear term u^3 and the leading derivative term u'' . In the next subsections, we will use Eq. (4) to determine soliton solutions through the use of the simplest equation approach. The traveling-wave transformation in Eqs. (2)–(3) is selected because it converts the original $(2 + 1)$ -dimensional partial differential equation into an ordinary differential equation in a single traveling variable, while also separating the oscillatory phase from the real amplitude profile. This makes the problem compatible with the simplest equation and tanh-coth schemes, both of which are naturally formulated for reduced ODEs. Alternative reductions, such as similarity transformations, bilinear/Hirota transformations, or symmetry-based reductions, could also be considered in principle; however, the present choice is especially suitable for extracting localized traveling structures with transparent parameter constraints.

3. ORGANIZED PROCEDURE

Consider the generalized PDE:

$$M(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, u_{xt}, u_{xy}, u_{ty}, \dots) = 0. \quad (6)$$

The first step is to use a traveling wave transformation with traveling wave variable, $C(\zeta)$, where, $\zeta = x + y + \omega t$. The goal of this transformation is to change the supplied PDE into an ODE. Then, to continue with the solution procedure, we utilize the idea of the simplest equation technique.

$$N(C, C', C'', C''', \dots) = 0. \quad (7)$$

—Simplest Equation Method

We will use the simplest equation technique concept [29; 30; 31; 32]. We find the exact solution of Eq. (7) in the form [33].

$$u(\psi) = R(\psi)^{\frac{2(n-m)}{m}} \frac{dR}{d\psi}, \quad (8)$$

the function $R(\psi)$ represents the solution of the following ODE [14]:

$$R_{\psi}^2 = R^2 - \mu R^4, \quad (9)$$

Using Eqs. (8) and (9), we obtain a set of algebraic equations that are represented in terms of the function $R(\psi)$. The system is then solved for the unknowns. Then, we replace $R(\psi)$ in Eq. (8) with the following form to obtain the solution of the PDE.

$$R(\psi) = \frac{4a}{4a^2 e^{\psi} + \mu e^{-\psi}}, \quad (10)$$

where μ is constant.

For $a = 1$ we have

$$R(\psi) = \frac{4}{4e^{\psi} + \mu e^{-\psi}}. \quad (11)$$

—Method of Tanh Coth

(i) To start, have a look at the nonlinear equation in its general form:

$$B(v, v_{\psi}, v_{\phi}, v_{\phi\phi}, \dots) = 0. \quad (12)$$

(ii) The wave variable $\tau = \phi - Q\psi$ is introduced in order to get the traveling wave solution of Eq. (12) so that

$$v(\phi, \psi) = K(\mu\tau). \quad (13)$$

Based on this, one can use the following transformations.

$$\begin{aligned} \frac{\partial}{\partial \phi} &= -Q \frac{d}{d\tau}, \\ \frac{\partial}{\partial \psi} &= \mu \frac{d}{d\tau}, \\ \frac{\partial^2}{\partial \psi^2} &= \mu^2 \frac{d^2}{d\tau^2}, \\ \frac{\partial^3}{\partial \psi^3} &= \mu^3 \frac{d^3}{d\tau^3}, \end{aligned} \quad (14)$$

and so on for other derivatives. Using (14) transforms the PDE (12) into an ODE.

$$L(V, V', V'', \dots) = 0. \quad (15)$$

(iii) A simplified ODE can be obtained by integrating this equation and taking the integration constant to zero if all of the terms in the resulting ODE have derivatives in τ .

(iv) An additional independent variable

$$Z = \tanh(\mu \tau), \quad (16)$$

is presented with the derivatives as follows:

$$\begin{aligned} \frac{d}{d\tau} &= \mu (1 - Z^2) \frac{d}{dZ}, \\ \frac{d^2}{d\tau^2} &= -2\mu^2 Z (1 - Z^2) \frac{d}{dZ} + \mu^2 (1 - Z^2)^2 \frac{d^2}{dZ^2}, \\ \frac{d^3}{d\tau^3} &= 2\mu^3 (1 - Z^2) (3Z^2 - 1) \frac{d}{dZ} - 6\mu^3 Z (1 - Z^2)^2 \frac{d^2}{dZ^2} + \mu^3 (1 - Z^2)^3 \frac{d^3}{dZ^3}, \end{aligned} \quad (17)$$

where it is possible to deduce other derivatives similarly.

(v) The form's ansatz

$$K(\mu \tau) = O(Z) = \sum_{i=0}^J a_i Z^i + \sum_{i=1}^J b_i Y^{-i}, \quad (18)$$

is presented, where J is often a positive integer that will be found. To get around this problem, a transformation formula is applied if J is not an integer. After completing ODE (15) with (17) and (18), an equation in powers of Z is obtained.

(vi) The highest order linear terms in the resulting equation with the highest order nonlinear terms are balanced to find the parameter J . After determining J , all powers of Z in the resulting equation are collected, and their coefficients are set to zero. This process yields a system of algebraic equations involving a_i and b_i ($i = 0, \dots, J$), Q , and μ . Once these parameters are determined, equation (18) can be used to obtain a closed-form analytical solution, where J is typically taken as a positive integer.

4. APPLICATION OF METHOD OF THE SIMPLEST EQUATION

Using the traveling wave variable in Eq. (1), we get the following ODEs:

$$(2K_1 b_1 + 2K_2 b_2)u(\psi)^3 + (-aK_1^2 - aK_2^2 - \omega)u(\psi) + (aB_1^2 + aB_2^2)u_{\psi\psi} = 0, \quad (19)$$

$$(2aB_1 K_1 + 2aB_2 K_2 - v)u_{\psi} = 0. \quad (20)$$

We next continue by replacing the derivatives of the function $R(\psi)$ in Eq. (8). These derivatives could be shown as follows in this context:

$$u_\psi = 2R(\psi)(R(\psi)^2 - \mu R(\psi)^4) + \frac{1}{2} \frac{R(\psi)^2 \left(2R(\psi)\sqrt{R(\psi)^2 - \mu R(\psi)^4} - 4\mu R(\psi)^3 \sqrt{R(\psi)^2 - \mu R(\psi)^4} \right)}{\sqrt{R(\psi)^2 - \mu R(\psi)^4}}. \quad (21)$$

The expression for $u(\psi)$ in Eq. (19) is being replaced with the following one:

$$u(\psi) = R^2 R_\psi. \quad (22)$$

We obtain an algebraic system of equations in terms of R when we substitute Eq. (19) and (20) into the ODE. Next, the system is solved in order to find the values of the unknowns ω , v , μ , and K_1 .

$$v = 2aB_1K_1 + 2aB_2K_2, \quad \mu = 0, \quad \omega = \frac{a(9B_1^2b_1^2 + 9B_2^2b_1^2 - K_2^2b_1^2 - K_2^2b_2^2)}{b_1^2}, \quad K_1 = -\frac{K_2b_2}{b_1}.$$

The following is a description of the (1+2) dimensional non-linear chiral Schrodinger equation's solution.

$$q(t, x, y) = e^{3vt - 3B_1x - 3B_2y + i(\omega t + K_1x + K_2y + \theta)}. \quad (23)$$

In terms of real-valued constants, we have a 3D graph of $q(t, x, y)$.

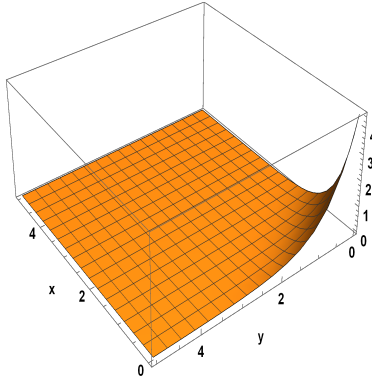


Fig.1. The graph of the solution for $|q(t, x, y)|$, where $B_1 = 0.8$, $B_2 = 0.5$, $v = 0.5$.

5. APPLICATION OF TANH COTH METHOD

As mentioned earlier in Eq. (1), we will get this transformation (24) in the form of ODE:

$$(2K_1b_1 + 2K_2b_2)(u(\psi))^3 + (-aK_1^2 - aK_2^2 - \omega)u(\psi) + (aB_1^2 + aB_2^2)\frac{d^2}{d\psi^2}u(\psi) = 0, \quad (24)$$

by comparing the terms u'' with u^3 we get

$$J = 1, \quad (25)$$

which leads to,

$$u(\psi) = a_0 + a_1Y(\psi) + \frac{c_1}{Y(\psi)}, \quad (26)$$

three sets of solutions are found when we substitute Eq. (26) into Eq. (24) collect the coefficients of Y , and solve the resulting system.

(1) The initial set of parameters is as follows

$$\begin{aligned} \omega &= \frac{-a^3\mu^4B_1^4 - 2a^3\mu^4B_1^2B_2^2 - a^3\mu^4B_2^4 - 2a\mu^2B_1^2b_1^2c_1^4 - 2a\mu^2B_2^2b_1^2c_1^4}{b_1^2c_1^4} \\ &\quad - \frac{2a^2\mu^2B_1^2K_2b_2c_1^2 - 2a^2\mu^2B_2^2K_2b_2c_1^2 - a^2K_2^2b_1^2c_1^4 - a^2K_2^2b_2^2c_1^4}{b_1^2c_1^4}, \\ K_1 &= -\frac{a\mu^2B_1^2 + a\mu^2B_2^2 + K_2b_2c_1^2}{b_1c_1^2}, \end{aligned} \quad (27)$$

$$a_0 = 0,$$

$$a_1 = 0.$$

Consequently,

$$q_1(t, x, y) = -\frac{c_1 e^{i(\omega t + xK_1 + yK_2 + \theta)}}{\tanh(\mu(2atB_1K_1 + 2atB_2K_2 - B_1x - B_2y))}. \quad (28)$$

To represent $q_1(t, x, y)$ in terms of real-valued constants, we have a 3D graph.

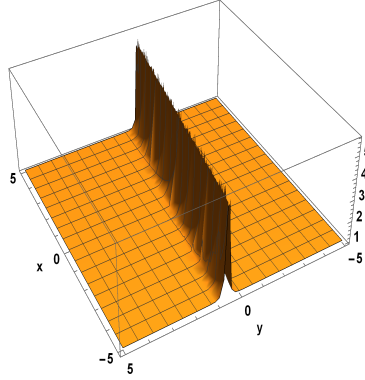


Fig.2. The solution for $|q_1(t, x, y)|$ where
 $c_1 = 1, \mu = 0.3, a = 0.8,$
 $t = 1.5, B_1 = 2, B_2 = 1, K_1 = -5.80, K_2 = 2.$

(2) The subsequent set of parameters is as follows

$$\begin{aligned} \omega &= \frac{-a^3\mu^4B_1^4 - 2a^3\mu^4B_1^2B_2^2 - a^3\mu^4B_2^4 - 8a\mu^2B_1^2b_1^2c_1^4 - 8a\mu^2B_2^2b_1^2c_1^4}{b_1^2c_1^4} \\ &\quad - \frac{2a^2\mu^2B_1^2K_2b_2c_1^2 - 2a^2\mu^2B_2^2K_2b_2c_1^2 - aK_2^2b_1^2c_1^4 - aK_2^2b_2^2c_1^4}{b_1^2c_1^4}, \\ K_1 &= -\frac{a\mu^2B_1^2 + a\mu^2B_2^2 + K_2b_2c_1^2}{b_1c_1^2}, \\ a_0 &= 0, \\ a_1 &= c_1. \end{aligned} \tag{29}$$

Consequently,

$$q_2(t, x, y) = -\frac{c_1 e^{i(\omega t + xK_1 + yK_2 + \theta)} \left((\tanh(\mu(2atB_1K_1 + 2atB_2K_2 - B_1x - B_2y)))^2 + 1 \right)}{\tanh(\mu(2atB_1K_1 + 2atB_2K_2 - B_1x - B_2y))}. \tag{30}$$

To represent $q_2(t, x, y)$ in terms of real-valued constants, we have a 3D graph.

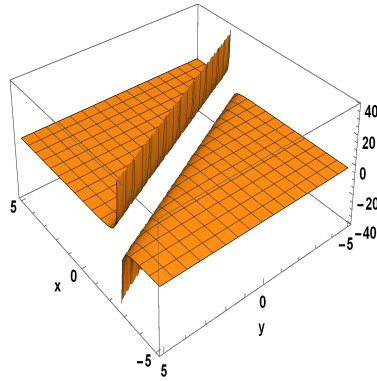


Fig.3. The solution for $|q_2(t, x, y)|$ where
 $c_1 = 1, \mu = 0.3, a = 0.8,$
 $t = 1.5, B_1 = 2, B_2 = 1, K_1 = -5.80, K_2 = 2.$

6. BRIGHT SOLITON SOLUTIONS

We employ the following transformation to obtain bright soliton solutions (BESSs) for the NLSE (1).

$$q(t, x, y) = R \operatorname{sech}(B_1 x + B_2 y - \gamma t) e^{i(-kt + B_1 x + B_2 y)}, \quad (31)$$

Thus, throughout the entire manuscript $R, k, B_1, B_2,$ and γ are free, non-zero constants, Eq. (31) is inserted into Eq. (1), and following a few basic arithmetic operations, we obtain:

$$\begin{aligned} k &= 0, \\ 2aB_1^2 + 2aB_2^2 - \gamma &= 0, \\ 2R^2B_1b_1 + 2R^2B_2b_2 - 2aB_1^2 - 2aB_2^2 &= 0, \end{aligned} \quad (32)$$

when system (32) is solved,

$$\begin{aligned} \gamma &= 2R^2(B_1b_1 + B_2b_2), \\ a &= \frac{R^2(B_1b_1 + B_2b_2)}{B_1^2 + B_2^2}, \\ k &= 0, \end{aligned} \quad (33)$$

where the free parameters are non-zero (R , B_1 , and B_2). These values allow us to ultimately obtain the BESSs, which are as follows:

$$q(t, x, y) = R \operatorname{sech}(2R^2 B_1 b_1 t + 2R^2 B_2 b_2 t - x B_1 - y B_2) e^{i(x B_1 + y B_2)} \quad (34)$$

only applicable under the earlier-obtained conditions found in (33).

To represent $q(t, x, y)$ in terms of real-valued constants, we have a 3D graph.

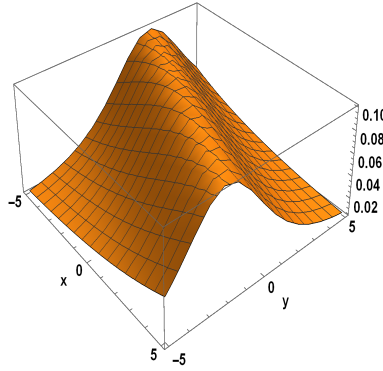


Fig.4. The solution for $|q(t, x, y)|$ where
 $R = 0.1$, $B_1 = 0.2$, $B_2 = 0.5$,
 $b_1 = 0.4$, $b_2 = 0.5$.

For the bright soliton displayed in Fig.4, the parameter R acts as the amplitude scale, so increasing R increases the peak height of the pulse. The parameters B_1 and B_2 determine the spatial localization and orientation of the soliton through the argument $B_1 x + B_2 y$; larger magnitudes of B_1 or B_2 produce stronger localization, that is, a narrower profile, along the corresponding effective propagation direction. In addition, the propagation factor $\gamma = 2R^2(B_1 b_1 + B_2 b_2)$ shows that R , B_1 , and B_2 also influence the translational speed of the bright pulse, while the combination $B_1^2 + B_2^2$ enters the dispersion balance through Eq.(33).

6.1. Secondary Bright Soliton Solutions.

It is assumed that in order to discover a second set of BESSs to Eq. (1), the following ansatz,

$$q(t, x, y) = R \operatorname{sech}(B_1 x + B_2 y) e^{i(-kt + B_1 x + B_2 y)}, \quad (35)$$

Using the same method as earlier and plugging Eq. (35) into Eq.(1), we find

$$\begin{aligned} k &= 0, \\ -2aB_1^2 - 2aB_2^2 &= 0, \\ 2R^2B_1b_1 + 2R^2B_2b_2 - 2aB_1^2 - 2aB_2^2 &= 0, \end{aligned} \quad (36)$$

once the simultaneous system (36) is solved, we have

$$\begin{aligned} B_1 &= -\frac{B_2b_2}{b_1}, \\ k &= 0, \end{aligned} \quad (37)$$

where the free parameters are (R, B_1) . Thus, the second BESSs are found.

$$q(t, x, y) = R \operatorname{sech} \left(\frac{B_2(xb_2 - yb_1)}{b_1} \right) e^{\frac{-iB_2(xb_2 - yb_1)}{b_1}}. \quad (38)$$

It is only applicable under the earlier-obtained conditions found in (37). To represent $q(t, x, y)$ in terms of real-valued constants, we have a 3D graph.

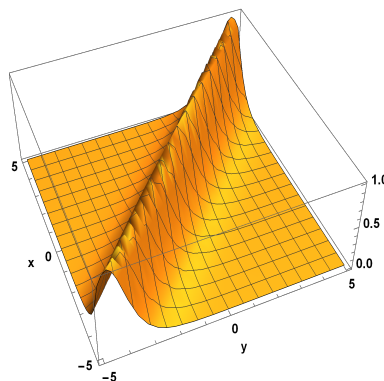


Fig.5. The solution for $|q(t, x, y)|$ when
 $R = 1, B_2 = 1.5,$
 $b_1 = 1.3, b_2 = 1.4.$

7. SINGULAR SOLUTIONS

Considering the following ansatz, the singular solutions to the extended NLSE (1) are found.

$$q(t, x, y) = R \operatorname{csch}(B_1x + B_2y) e^{i(-kt + B_1x + B_2y)}, \quad (39)$$

where the non-zero free constants are a, b, R, k, α , and β . By putting (39) into (1) and going by the preceding discussion, we get,

$$B_1 = -\frac{B_2 b_2}{b_1}, \quad (40)$$

$$k = 0,$$

where the free parameters are (R, B_1) . Given this, the singular equation that follows is derived.

$$q(t, x, y) = R \operatorname{csch} \left(\frac{B_2 (xb_2 - yb_1)}{b_1} \right) e^{\frac{-iB_2 (xb_2 - yb_1)}{b_1}}. \quad (41)$$

This approach is only applicable in the given requirements (40). In terms of real-valued constants, we have a 3D graph of $u(t, x, y)$.

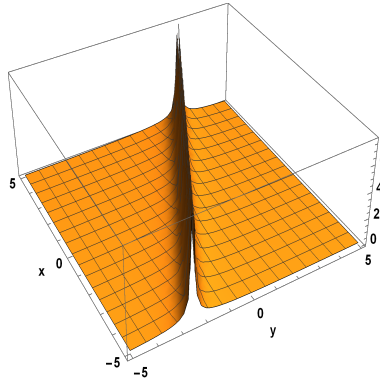


Fig.6. The solution for $|q(t, x, y)|$ when
 $R = 0.2, B_2 = 0.3,$
 $b_1 = 0.5, b_2 = 1.$

8. SOLUTIONS WITH PERIODICITY

We shall demonstrate here that periodic solutions of the following kind can be admitted by the NLSE (1).

$$q(t, x, y) = R \sec(-vt + B_1 x + B_2 y) e^{i(-kt + B_1 x + B_2 y)}, \quad (42)$$

after performing a few basic arithmetic operations and inserting ansatz (42) into equation (1), we get

$$\begin{aligned} k &= -2R^2(B_1b_1 + B_2b_2), \\ a &= -\frac{R^2(B_1b_1 + B_2b_2)}{B_1^2 + B_2^2}, \\ v &= -2R^2(B_1b_1 + B_2b_2), \end{aligned} \quad (43)$$

where the free parameters are (R, B_1, B_2) . Consequently, we ultimately get the subsequent solutions with periodicity.

$$q(t, x, y) = R \sec(2R^2tB_1b_1 + 2R^2tB_2b_2 + xB_1 + yB_2) e^{i(2R^2tB_1b_1 + 2R^2tB_2b_2 + xB_1 + yB_2)}. \quad (44)$$

Keep in mind that the conditions found before in Eq. (43) only apply to this solution. In terms of real-valued constants, we have a 3D graph of $q(t, x, y)$.

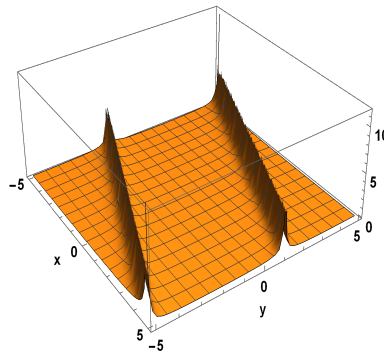


Fig.7. The solution for $|q(t, x, y)|$ when
 $R = 0.1, B_1 = 0.33, B_2 = 0.53,$
 $b_1 = 0.41, b_2 = 0.52.$

9. SOLUTIONS FOR DARK ENVELOPE SOLITON

The following ansatz is given to identify dark envelope soliton solutions (DESSs) to the NLSE (1).

$$q(t, x, y) = -R \tanh(-vt + B_1x + B_2y) e^{i(-kt + B_1x + B_2y)}, \quad (45)$$

by inserting ansatz (45) into equation (1) and using the previously described steps, we obtain

$$\begin{aligned} 2R^2B_1b_1 + 2R^2B_2b_2 - aB_1^2 - aB_2^2 + k &= 0, \\ -2aB_1^2 - 2aB_2^2 + v &= 0, \\ -2R^2B_1b_1 - 2R^2B_2b_2 - 2aB_1^2 - 2aB_2^2 &= 0, \end{aligned} \quad (46)$$

the solution of system (46) gives us

$$\begin{aligned} a &= -\frac{R^2(B_1b_1 + B_2b_2)}{B_1^2 + B_2^2}, \\ k &= -3R^2(B_1b_1 + B_2b_2), \\ v &= -2R^2(B_1b_1 + B_2b_2), \end{aligned} \quad (47)$$

thus, the DESSs are found.

$$q(t, x, y) = R \tanh(2R^2tB_1b_1 + 2R^2tB_2b_2 + xB_1 + yB_2) e^{i(3R^2tB_1b_1 + 3R^2tB_2b_2 + xB_1 + yB_2)}. \quad (48)$$

Remember that this solution is valid only under the conditions (47). We have a three-dimensional graph of real-valued constants called $q(t, x, y)$.

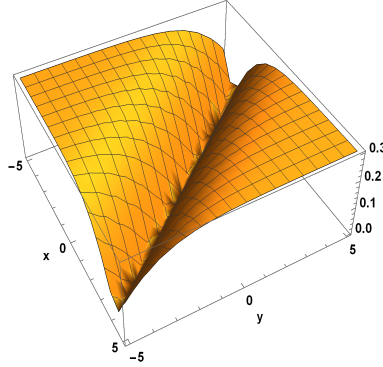


Fig.8. The solution for $|q(t, x, y)|$ when
 $R = 0.3, B_1 = 0.5, B_2 = 0.4,$
 $b_1 = 0.7, b_2 = 0.6.$

9.1. Secondary Dark Soliton Solutions.

The following ansatz is expected in order to identify a second set of DESSs to the NLSE (1).

$$q(t, x, y) = R \tanh(B_1 x + B_2 y) e^{i(B_1 x + B_2 y - kt)}, \quad (49)$$

by entering ansatz (49) into equation (1) and following the steps outlined above, we obtain.

$$\begin{aligned} 2R^2 B_1 b_1 + 2R^2 B_2 b_2 - aB_1^2 - aB_2^2 + k &= 0, \\ 2aB_1^2 + 2aB_2^2 &= 0, \\ -2R^2 B_1 b_1 - 2R^2 B_2 b_2 - 2aB_1^2 - 2aB_2^2 &= 0, \end{aligned} \quad (50)$$

by solving the system (50), we have

$$\begin{aligned} B_1 &= -\frac{B_2 b_2}{b_1}, \\ k &= 0, \end{aligned}$$

thus, the second group of DESSs appears as

$$q(t, x, y) = -R \tanh\left(\frac{B_2 (xb_2 - yb_1)}{b_1}\right) e^{\frac{-iB_2 (xb_2 - yb_1)}{b_1}}. \quad (51)$$

We have a three-dimensional graph of real-valued constants called $q(t, x, y)$.

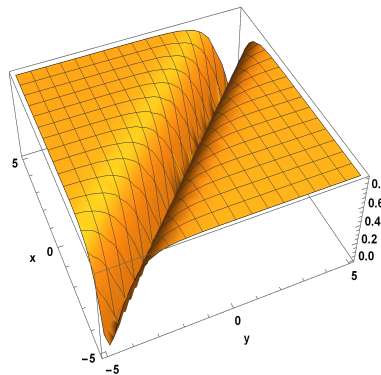


Fig.9. The solution for $|q(t, x, y)|$ when
 $R = 0.9, B_2 = 0.88,$
 $b_1 = 0.86, b_2 = 0.9.$

9.2. Secondary Solutions with Periodicity.

In order to identify an additional set of periodic solutions for the NLSE (1), the subsequent ansatz is examined.

$$q(t, x, y) = R \tan(B_1 x + B_2 y) e^{iB_1 x + B_2 y - kt}, \quad (52)$$

The following results are obtained by inserting ansatz (52) into equation (1) and continuing as previously.

$$\begin{aligned} -2R^2 B_1 b_1 - 2R^2 B_2 b_2 - aB_1^2 - aB_2^2 + k &= 0, \\ 2aB_1^2 + 2aB_2^2 &= 0, \\ 2R^2 B_1 b_1 + 2R^2 B_2 b_2 + 2aB_1^2 + 2aB_2^2 &= 0, \end{aligned} \quad (53)$$

follow immediately. This leads to

$$\begin{aligned} B_1 &= -\frac{B_2 b_2}{b_1}, \\ k &= 0, \end{aligned}$$

Consequently, given the criteria (53), the further periodic solutions are obtained as,

$$q(t, x, y) = -R \tan\left(\frac{B_2 (xb_2 - yb_1)}{b_1}\right) e^{\frac{-iB_2 (xb_2 - yb_1)}{b_1}}. \quad (54)$$

For real-valued constants, we have a three-dimensional graph of $q(t, x, y)$.

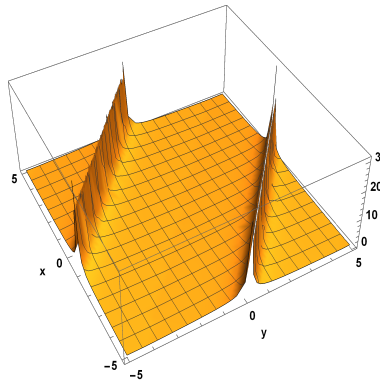


Fig.10. The solution for $|q(t, x, y)|$ when
 $R = 0.4$, $B_2 = 0.3$,
 $b_1 = 0.5$, $b_2 = 0.6$.

10. RESULTS AND DISCUSSIONS:

This section contains the findings from our investigation of the tanh/coth approach and the simplest equation method for the (1+2) dimensional non-linear chiral Schrödinger equation. We obtained precise analytical answers using these analytical methods, which offer important new perspectives on the dynamics of chiral waves in nonlinear media. We thoroughly examined the nonlinear chiral Schrödinger equation. Two robust methods such as the simplest equation technique and the tanh/coth approach were utilized. The simplest equation technique uncovered a new solution that followed an exponential pattern which is portrayed in 3D in figure (1). The tanh/coth method identified solitons—special wave patterns that change based on certain factors. Figure (2) illustrates these solitons, showing their distinct shapes. Next, a bright envelope soliton solution is shown in figure (4). These graphs show us bright wave packets with a peak in the middle. Then, a secondary bright soliton solution is depicted in fig (5), which is different from the first set. After that, a singular solution in fig (6), which highlights points like jumps. Subsequently, a periodic, and two sets of dark solitons are portrayed respectively in figs (7), (8) and (9). Finally, we have another set of periodic solutions in fig (10).

11. CONCLUSION

Finally, we conclude that by applying the simplest equation approach and the tanh/coth method to analyze the (1+2) dimensional non-linear chiral Schrödinger equation, we have obtained exact analytical solutions that expand our knowledge of chiral wave dynamics in nonlinear media. Along with other solutions including single and periodic solutions, we obtained new exact and dark envelope soliton solutions. Additionally, the prerequisites for the associated parameters required for the creation of these various solutions have been discovered. A basis for future theoretical and experimental studies in chiral phenomena is laid by the obtained results and visual representations, which provide insightful information about the behavior of soliton solutions. These results have significance for a broad range of applications, such as materials science, photonics, and optical communication, and they further the field of nonlinear wave theory. Many scientific phenomena in nonlinear optics can be explained by the found solutions, since the NLS model is relevant to optical applications.

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