

A COMPARATIVE ANALYSIS OF THE LATERAL-TORSIONAL BUCKLING RESISTANCE OF THE HOT-ROLLED AND WELDED SECTIONS

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ABSTRACT:

This paper presents a comparative study of several design approaches for evaluating the lateral-torsional buckling resistance of steel girders with doubly symmetric I and H cross-sections, both hot-rolled and welded. The investigated methods include the general method and alternative approaches specified in EN 1993-1-1:2006 and EN 1993-1-1:2022, as well as simplified procedures that avoid the explicit evaluation of the elastic critical moment M_{cr} . A numerical application on an IPE 400 beam is used to quantify the influence of the chosen method on the resulting non-dimensional slenderness, reduction factors and design bending resistances. The results highlight the difference between methods, show that the direct simplified procedure leads to the most conservative resistance, and underline the conditions under which simplified approaches may become unconservative for lateral-torsional buckling verification.

1. INTRODUCTION

Beams that are not restrained along their length and are bent about their strong axis are susceptible to lateral-torsional buckling. The critical buckling load of such beams depends on the unbraced length, i.e. the distance between points where lateral deflection is prevented, and on the lateral bending stiffness (EI_z). End and intermediate restraints that inhibit the development of the buckled shape increase the global stability of the beam; in terms of buckling deformations, this corresponds to in-plane rotational restraint about the z-axis.

Bracing may be used to improve the resistance of a beam susceptible to lateral-torsional instability, provided that the bracing is sufficiently stiff to restrain lateral displacements at the braced points and sufficiently strong to carry the forces transmitted from the main member. A direct application of the full lateral-torsional instability theory in design is generally inappropriate because the procedures are too complex for routine practice. Moreover, significant differences exist between the assumptions underlying the theoretical models and the actual behaviour of real beams.

This paper compares five methods for determining the lateral-torsional buckling resistance of hot-rolled or welded I and H sections, based on the theoretical elastic critical moments in

accordance with EN 1993-1-1 and various technical documents. A dedicated computer program is also used to evaluate the lateral-torsional buckling resistance of plate girders; in the numerical example a hot-rolled IPE 400 section is considered. To verify the lateral-torsional buckling resistance of a section, its non-dimensional effective slenderness $\bar{\lambda}_{LT}$ is used. Variations in lateral support conditions or in the shape of the bending moment diagram are accounted for in the design process by coefficients k and C , which modify the basic slenderness $\bar{\lambda}_{LT}$ or the elastic critical moment M_{cr} .

2. METHODS FOR DETERMINING THE LATERAL-TORSIONAL BUCKLING RESISTANCE

2.1 General method according to EN 1993-1-1:2005 and EN 1993-1-1:2022 (M1)

The design buckling resistance moment of a laterally unrestrained beam is given by (EN 1993-1-1:2005) and (EN 1993-1-1:2022):

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}} \quad (1)$$

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where: $W_y = \begin{cases} W_{pl,y} & \text{for Class 1 or 2 crosssections} \\ W_{el,y} & \text{for Class 3 cross - sections} \\ W_{eff,y} & \text{for Class 4 cross - sections} \end{cases}$
 χ_{LT} is the reduction factor for lateral-torsional buckling

For steel beams with constant cross-section, the reduction factor χ_{LT} is expressed as a function of the non-dimensional slenderness $\bar{\lambda}_{LT}$ and the imperfection factor α_{LT} , with limits and buckling curves given in Table 1 and Table 2 of the paper (Moga, 2026).

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \leq 1 \quad (2)$$

Where: $\phi_{LT} = 0.5 \left[1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right]$;
 $\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$; M_{cr} is the elastic critical moment for lateral- torsional buckling.

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0.21	0.34	0.49	0.76

Table 1. Imperfection factors

For general cases recommendations for buckling curves are given in Table 2.

Cross-section	Limits	Buckling curve
Rolled I - sections	$h/b \leq 2$	a
	$h/b > 2$	b
Welded I - sections	$h/b \leq 2$	c
	$h/b > 2$	d
Other cross - sections	-	d

Table 2. Buckling curves – General cases

2.2 Alternative method for hot-rolled or equivalent welded sections according to SR EN 1993-1-1:2006 (M2)

According to SR EN 1993-1-1:2006, the reduction factor, χ_{LT} , for hot-rolled or equivalent welded sections should be taken from (SR EN 1993-1-1:2006):

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2}}; \text{but } \chi_{LT} \leq \begin{cases} 1.0 \\ 1/\bar{\lambda}_{LT}^2 \end{cases} \quad (3)$$

where: $\bar{\lambda}_{LT} > \bar{\lambda}_{LT,0}$;
 $\phi_{LT} = 0.5 \left[1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \cdot \bar{\lambda}_{LT}^2 \right]$;
 $\bar{\lambda}_{LT,0} = 0.4$ (maximum value);
 $\beta = 0.75$ (minimum value).

In this method, the moment distribution between the lateral restraints (fork boundary conditions) of members is taken into account by using a modified reduction factor, $\chi_{LT,mod}$, evaluated with the expression (Boissonnade, et al. 2006):

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f}; \chi_{LT,mod} \leq 1.0 \quad (4)$$

The recommended value of the parameter f is given by:

$$f = 1 - 0.5(1 - k_c) \left[1 - 2.0 (\bar{\lambda}_{LT} - 0.8)^2 \right]; f \leq 1.0 \quad (5)$$

The correction factor k_c is given in Table 6.6 of EN 1993-1-1:2006 and depends on the bending moment distribution diagram.

For hot-rolled or equivalent welded sections, recommendations for buckling curves are given in Table 3.

Cross-section	Limits	Buckling curve
Rolled I - sections	$h/b \leq 2$	b
	$h/b > 2$	c
Welded I - sections	$h/b \leq 2$	c
	$h/b > 2$	d

Table 3. Buckling curves for hot-rolled or welded sections

2.3 Alternative method for hot-rolled or equivalent welded sections according to EN 1993-1-1:2022 (M3)

For doubly symmetric I and H sections, the lateral-torsional buckling reduction factor χ_{LT} may be taken as (SR EN 1993-1-1:2023) and (EN 1993-1-1:2022):

$$\chi_{LT} = \frac{f_M}{\phi_{LT} + \sqrt{\phi_{LT}^2 - f_M \bar{\lambda}_{LT}^2}}, \quad \chi_{LT} \leq 1 \quad (6)$$

Where:

$$\phi_{LT} = 0.5 \left[1 + f_M \left(\left(\frac{\bar{\lambda}_{LT}}{\bar{\lambda}_z} \right)^2 \alpha_{LT} (\bar{\lambda}_z - 0.2) + \bar{\lambda}_{LT}^2 \right) \right] \quad (7)$$

α_{LT} is the imperfection factor taken from Table 8.5 of EN 1993-1-1:2022;

The non-dimensional slenderness $\bar{\lambda}_{LT}$ relative to lateral buckling is given by:

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{Rk}}{M_{cr}}} = \sqrt{\frac{W_y f_y}{M_{cr}}} \quad (8)$$

The corresponding relative slenderness for weak flexural buckling $\bar{\lambda}_z$ is defined with the buckling length taken as the distance between the discrete lateral restraints and is given by:

$$\bar{\lambda}_z = \sqrt{\frac{N_{Rk}}{N_{cr,z}}} = \sqrt{\frac{A \cdot f_y}{N_{cr,z}}} \quad (9)$$

The factor f_M takes into account the effect of the bending moment distribution between discrete lateral restraints, Table 8.6 of EN 1993-1-1:2022. It may conservatively be taken as 1.0.

2.4 Simplified method without determination of M_{cr} for hot-rolled or welded I and H sections (M4)

NCCI presents a method for determining the non-dimensional slenderness of I and H sections without the explicit determination of M_{cr} . It is a conservative method and it can be refined to take into account section geometry and bending moment distribution. When the loading is not destabilizing, $\bar{\lambda}_{LT}$ is given by (NCCI, 2006a):

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UV \frac{\bar{\lambda}_z}{\lambda_1} \sqrt{\beta_w} = \frac{1}{\sqrt{C_1}} UV \bar{\lambda}_z \sqrt{\beta_w} \quad (10)$$

where: C_1 - is a parameter depending on the shape of bending moment diagram (Conservatively, $C_1=1$ is used in the simplified method);
 V is a parameter related to the slenderness;
 U is a parameter depending on the geometry of the section, given by:

$$U = \sqrt{\frac{W_{pl,y} g}{A} \sqrt{\frac{I_z}{I_w}}}; \text{ Conservatively, } U = 0.9 \quad (11)$$

If the loading is not destabilizing, V may be taken as:

$$V = \begin{cases} 1.0 & \text{— mono — symmetric sections} \\ \frac{1}{\sqrt[4]{1 + \frac{1}{20} \left(\frac{\lambda_z}{h/t_f} \right)^2}} & \text{— doubly — symmetric sections} \end{cases} \quad (12)$$

The exact value of V is given by:

$$V = \frac{1}{\sqrt[4]{\left(\frac{k}{k_w} \right)^2 + \frac{(\lambda_z)^2}{\frac{\pi^2 E A I_w}{G I_t I_z}}}} \quad (13)$$

Knowing the non-dimensional slenderness $\bar{\lambda}_{LT}$, the reduction coefficient χ_{LT} can be determined using equation (2) (Moga, 2018).

2.5 Simplified direct method (M5)

For straight segments of hot-rolled I and H sections with no destabilizing loads, the value $\bar{\lambda}_{LT}$ may conservatively be taken from Table 4. These values are derived from equation (7), taking $C_1=1$, $U=0.9$ and $\sqrt{\beta_w} = 1.0$. (NCCI, 2006a) and (Moga, 2013b)

S 235	S 275	S 355	S 420	S 460
$\bar{\lambda}_{LT}$	$\bar{\lambda}_{LT}$	$\bar{\lambda}_{LT}$	$\bar{\lambda}_{LT}$	$\bar{\lambda}_{LT}$
$= \frac{L/i_z}{104}$	$= \frac{L/i_z}{96}$	$= \frac{L/i_z}{85}$	$= \frac{L/i_z}{78}$	$= \frac{L/i_z}{75}$

Table 4. Parameters $\bar{\lambda}_{LT}$

3. DETERMINATION OF THE ELASTIC CRITICAL BUCKLING MOMENT

For a double-T cross-section (I-section), the elastic critical buckling moment for lateral-torsional buckling is given by the expression derived from buckling theory (NCCI, 2006b), (Seminar, 1993) and (Seminar, 1995):

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2 g} \sqrt{\left(\left(\frac{k}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} + (C_2 z_g - C_3 z_j)^2 - (C_2 z_g - C_3 z_j) \right)} \quad (14)$$

For doubly symmetric I-sections, $z_j = 0$, and equation (14) becomes:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2 g} \sqrt{\left(\left(\frac{k}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} + (C_2 z_g)^2 - (C_2 z_g) \right)} \quad (15)$$

where: C_1 , C_2 and C_3 are coefficients depending on the loading (bending moment diagram), section properties and support conditions;

$$g = \sqrt{\left(1 - \frac{I_z}{I_y} \right)} \approx 1.0.$$

If there are no destabilizing loads $z_g = 0$ and M_{cr} is:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2} \sqrt{\left(\left(\frac{k}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} \right)} \quad (16)$$

In the case of I and H doubly symmetric cross-sections, the following approximation is used:

$$\frac{\pi^2 E A I_w}{G I_t I_z} \approx 20 \left(\frac{h}{t_f} \right)^2 \quad (17)$$

$$z_j = z_s - \frac{1}{2 I_y} \int_A z (y^2 + z^2) \cdot dA = \begin{cases} 0.4 \cdot h_s (2\beta_f - 1) & \text{— when } \beta_f > 0.5 \\ 0.5 \cdot h_s (2\beta_f - 1) & \text{— when } \beta_f < 0.5 \end{cases} \quad (18)$$

$$\beta_f = \frac{b_{fc}^3 t_{fc}}{b_{ft}^3 t_{ft} + b_{fc}^3 t_{fc}} \quad (19)$$

Figure 5 presents the sign convention for the geometrical parameters.

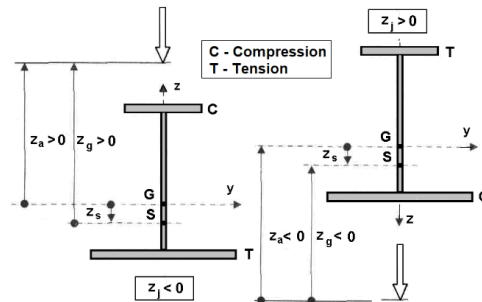


Figure 5. Sign convention for the geometrical parameters

4. NUMERICAL ANALYSIS

The lateral-torsional buckling resistance for a simply supported steel girder made up of an IPE 400 hot-rolled section is determined. The design data are as follows (Guțiu, 2014):

- Statical scheme (simple supported beam) and loading, Figure 6;
- Steel: S355 – $f_y = 355 \text{ N/mm}^2$;
- Loadings: $g = 1.5 \text{ kN/m}$, $Q = 35 \text{ kN}$;
- Loading safety coefficients: $\gamma_G = 1.35$; $\gamma_Q = 1.50$;
- Cross-section characteristics, Figure 7;
- The girder is laterally restrained of forks type at the end supports and in the points of concentrated forces.

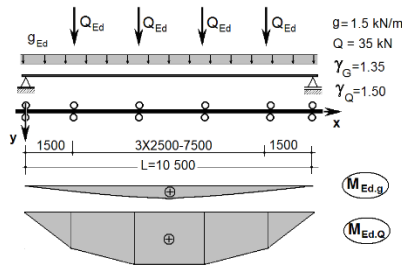


Figure 6. Static scheme and loading

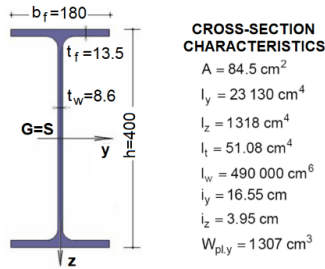


Figure 7. Hot-rolled IPE 400 section. Cross-section characteristics

Observations:

- The central panel, where the bending moment is maximum, is analysed;
- In this case there are no destabilizing loads ($z_g=0$), because in the loading points the girder is laterally restrained;
- In the central panel the bending moment is constant and, consequently, $C_2=0$;
- The girder section is of Class 1 cross-section for bending;
- In this paper have been taken some design data from (Moga, 2018), (Belaid et al 2023), (Piotrowski, 2015) and (Moga, 2013a).

4.1 General method (M1)

For doubly-symmetric I sections, with no destabilizing loads ($C_2=0$), the elastic critical moment is given (Guțiu, 2013):

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2} \left[\sqrt{\left(\frac{k}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z}} \right] =$$

$$= 1.0 \frac{\pi^2 2.1 \cdot 10^6 \cdot 1318}{250^2} \sqrt{\frac{490\,000}{1318} + \frac{250^2 \cdot 0.81 \cdot 10^6 \cdot 51.08}{\pi^2 2.1 \cdot 10^6 1318}} 10^{-4}$$

$$= 943 \text{ kNm}$$

The non-dimensional slenderness:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{1307 \cdot 3550}{943 \cdot 10^4}} = 0.70$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right]$$

$$= 0.5 \left[1 + 0.34 (0.7 - 0.2) + 0.70^2 \right] = 0.83$$

$\alpha_{LT} = 0.34$ – curve “b” when $h/b > 2$ (Table 2)

The reduction factor for lateral-torsional buckling:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.83 + \sqrt{0.83^2 - 0.70^2}} = 0.78$$

4.2 Alternative method according to SR EN 1993-1-1:2006 (M2)

The reduction factor for lateral-torsional buckling (Moga, 2013b):

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2}} = \frac{1}{0.76 + \sqrt{0.76^2 - 0.75 \cdot 0.70^2}}$$

$$= 0.82$$

The condition is verified: $\chi_{LT} \leq \begin{cases} 1.0 \\ 1/\bar{\lambda}_{LT}^2 = 1/0.70^2 = 2.04 \end{cases}$

where:

$$\bar{\lambda}_{LT} = 0.70 > \bar{\lambda}_{LT,0} = 0.4;$$

$$\phi_{LT} = 0.5 \left[1 + 0.49 (0.70 - 0.40) + 0.75 \cdot 0.70^2 \right] = 0.76$$

$$\bar{\lambda}_{LT,0} = 0.4 \text{ (max. value) and } \beta = 0.75 \text{ (min. value);}$$

$$\alpha_{LT} = 0.49 \text{ – curve “c” when } h/b > 2 \text{ (Table 3).}$$

The modified reduction factor:

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} = \chi_{LT} = 0.82$$

Parameter $f = 1 - 0.5 (1 - k_c) \left[1 - 2.0 (\bar{\lambda}_{LT} - 0.8) \right]^2 = 1$;
for $k_c = 1$.

4.3 Alternative method according to EN 1993-1-1:2022 (M3)

For $h/b > 1.2$ and $t_f < 40$ mm:

$$\alpha_{LT} = 0.12 \sqrt{\frac{W_{el,y}}{W_{el,z}}} = 0.12 \sqrt{\frac{1160}{146}} = 0.34; \quad f_M = 1.0$$

$$N_{z,cr} = \frac{\pi^2 EI_z}{l_z^2} = \frac{\pi^2 \cdot 2.1 \cdot 10^6 \cdot 1318}{250^2} = 436.63 \text{ daN}$$

$$\bar{\lambda}_z = \sqrt{\frac{N_{Rk}}{N_{cr,z}}} = \sqrt{\frac{A \cdot f_y}{N_{cr,z}}} = \sqrt{\frac{84.5 \cdot 3550}{436630}} = 0.83$$

$$\phi_{LT} = 0.5 \left[1 + f_M \left(\left(\frac{\bar{\lambda}_{LT}}{\bar{\lambda}_z} \right)^2 \alpha_{LT} (\bar{\lambda}_z - 0.2) + \bar{\lambda}_{LT}^2 \right) \right] =$$

$$= 0.5 \left[1 + 1.0 \left(\left(\frac{0.7}{0.83} \right)^2 0.34 (0.83 - 0.2) + 0.70^2 \right) \right] = 0.82$$

The reduction factor for lateral-torsional buckling:

$$\chi_{LT} = \frac{f_M}{\phi_{LT} + \sqrt{\phi_{LT}^2 - f_M \bar{\lambda}_{LT}^2}} = \frac{1}{0.82 + \sqrt{0.82^2 - 1.0 \cdot 0.7^2}}$$

$$= 0.80$$

4.4 Simplified method without determination of M_{cr} (M4)

When the loading is not destabilizing, $\bar{\lambda}_{LT}$ is given by:

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UV \frac{\lambda_z}{\lambda_1} \sqrt{\beta_w} = \frac{1}{\sqrt{C_1}} UV \bar{\lambda}_z \sqrt{\beta_w}$$

$$= 1.0 \cdot 0.88 \cdot 0.98 \frac{63.29}{76.1} \sqrt{1.0} = 0.72$$

where:

$$1/\sqrt{C_1} = 1.0 \text{ – for } k = 1;$$

$$U = \sqrt{\frac{W_{pl,y} g}{A} \sqrt{\frac{I_z}{I_\omega}}} = \sqrt{\frac{1307 \cdot 0.97}{84.5} \sqrt{\frac{1318}{490\,000}}} = 0.88$$

$$g = \sqrt{\left(1 - \frac{I_z}{I_y}\right)} = \sqrt{\left(1 - \frac{1318}{23130}\right)} = 0.97$$

$$V = \frac{1}{\sqrt[4]{1 + \frac{1}{20} \left(\frac{\lambda_z}{h/t_f}\right)^2}} = \frac{1}{\sqrt[4]{1 + \frac{1}{20} \left(\frac{250/3.95}{60/1.35}\right)^2}} = 0.98$$

Parameters:

$$\lambda_z = \frac{kL}{i_z} = \frac{250}{3.95} = 63.29$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = 93.9 \cdot \varepsilon = 93.9 \cdot 0.81 = 76.1$$

$$\beta_w = \frac{W_y}{W_{pl,y}} = 1.0 \text{ (Class 1)}$$

$$\alpha_{LT} = 0.49 - \text{curve „c” for } h/b > 2, \text{ (see Table 1 and Table 3)}$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 0.5 [1 + 0.49(0.72 - 0.2) + 0.72^2] = 0.89$$

It results:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.89 + \sqrt{0.89^2 - 0.72^2}} = 0.71$$

4.5 Simplified method to determine relative slenderness $\bar{\lambda}_{LT}$ (M5)

$$\bar{\lambda}_{LT} = \frac{L/i_z}{85} = \frac{250/3.95}{85} = 0.74$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 0.5 [1 + 0.49(0.74 - 0.2) + 0.74^2] = 0.91$$

$$\alpha_{LT} = 0.49 - \text{curve „c” when } h/b > 2 \text{ (Table 3).}$$

It results:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = \frac{1}{0.91 + \sqrt{0.91^2 - 0.74^2}} = 0.69$$

4.6 Analysis using computing design

Using the computing program LT BEAM, the value $M_{cr} = 943.7 \text{ kNm}$ has been obtained. Using the computing program AXIS VM X8, taking $M_{cr} = 943.7 \text{ kNm}$, the value $\chi_{LT} = 0.825$ has been obtained, which is close to that obtained by analytical calculation using Method M2. If a pre-calculated value is not entered for M_{cr} , the following results are obtained:

$$M_{cr} = 943.7 \text{ kNm and } \chi_{LT} = 0.885$$

The deformed shape of the beam in the central panel is shown in Figure 8.

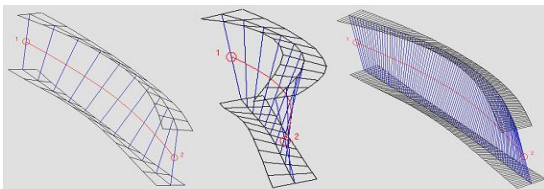


Figure 8. The deformed shape of the beam on the central section

5. CENTRALIZATION OF RESULTS

The lateral-torsional buckling resistance may be determined with the formula:

$$M_{b,Rd} = \chi_{LT} \cdot W_y \frac{f_y}{\gamma_{M1}} = \chi_{LT} \cdot W_{pl,y} \frac{f_y}{\gamma_{M1}} = \chi_{LT} \cdot 1307 \frac{3550}{1.10} 10^{-4} = \chi_{LT} \cdot 421.80 \text{ kN} \cdot \text{m}$$

The plastic bending resistance without taking into consideration the lateral torsional bending resistance ($\bar{\lambda}_{LT} = 1.0$) is given by:

$$M_{pl,Rd} = W_y \frac{f_y}{\gamma_{M1}} = W_{pl,y} \frac{f_y}{\gamma_{M1}} = 1307 \frac{3550}{1.10} 10^{-4} = 421.80 \text{ kNm}$$

The design bending moment:

$$M_{Ed} = \frac{g_{Ed} \cdot L^2}{8} + (2Q_{Ed} \cdot 4.0 - Q_{Ed} \cdot 2.5) = \frac{g_{Ed} \cdot L^2}{8} + 5.5 \cdot Q_{Ed} = 1.35 \frac{1.5 \cdot 10.5^2}{8} + 1.50 \cdot 5.5 \cdot 35 = 316.65 \text{ kNm}$$

The analysis results are centralized in the Table 9 and Table 10.

	Method of analysis	Parameter		M _{b,Rd} [kNm]
		$\bar{\lambda}_{LT}$	χ_{LT}	
M1	General method according to EN 1993-1-1:2005 and SR EN 1993-1-1:2006	0.70	0.78	329.00
M2	Alternative method according to EN 1993-1-1:2005 and SR EN 1993-1-1:2006	0.70	0.82	346.88
M3	Alternative method according to EN 1993-1-1:2022 and SR EN 1993-1-1:2023	0.70	0.80	337.44
M4	Simplified method without determination of M _{cr}	0.72	0.71	299.48
M5	Simplified direct method	0.74	0.69	291.04

Table 9. Centralization of results. Factors $\bar{\lambda}_{LT}$ and χ_{LT}

	Method of analysis	M _{b,Rd} [kN m]	100x $\frac{M_{b,Rd}}{M_{Ed}}$	100x $\frac{M_{b,Rd}}{M_{pl,Rd}}$
M1	General method according to EN 1993-1-1:2005 and SR EN 1993-1-1:2006	329.00	104%	78%
M2	Alternative method according to EN 1993-1-1:2005 and SR EN 1993-1-1:2006	346.88	110%	82%
M3	Alternative method according to EN 1993-1-1:2022 and SR EN 1993-1-1:2023	337.44	107%	80%
M4	Simplified method without determination of M _{cr}	299.48	95%	71%
M5	Simplified direct method	291.04	92%	69%

Table 10. Centralization of results. Results comparison

It is found that, for the considered IPE 400 girder, the maximum lateral-torsional buckling resistance is obtained with the general method, while the minimum resistance is provided by the simplified direct method, the latter being therefore the most conservative approach.

When methods M4 and M5 are used, the requirement for lateral-torsional buckling resistance is not satisfied for the investigated loading and support conditions. By computing program, results are close to the maximum values obtained by analytical calculation (Method 2).

Figure 11 presents in graphical form the results given in Table 10.

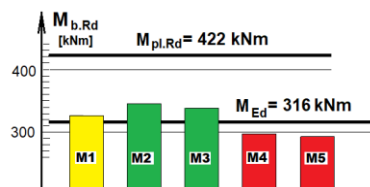


Figure 11. Lateral-torsional buckling resistance

6. CONCLUSIONS

The comparative application of the five design approaches (general method, two alternative methods, simplified method without M_{cr} and simplified direct method) to a hot-rolled IPE 400 girder shows significant differences in the evaluated lateral-torsional buckling resistance, with $M_{b,Rd}$ varying between approximately 291 kNm and 347 kNm. The highest lateral-torsional buckling resistance is obtained with the general method (M1), while the simplified direct method (M5) gives the lowest resistance, confirming its conservative character for the investigated range of slenderness and loading.

For the studied example, methods M1–M3 lead to safety ratios $M_{b,Rd}/M_{Ed}$ larger than 1.0, whereas M4 and M5 provide ratio below unity, indicating that the simplified procedures may predict an unsafe condition. The comparison between $M_{b,Rd}$ and the plastic resistance $M_{pl,Rd}$ highlights that lateral-torsional buckling reduces the usable bending capacity of the IPE 400 section to roughly 69–82% of the plastic moment, depending on the selected method, underlining the need to explicitly check stability even for compact (Class 1) sections.

Numerical results obtained with the LT BEAM and AXIS VM X8 programs are close to the upper bound of the analytically derived values, especially those from Method M2, which supports the adequacy of the Eurocode-based analytical formulations for the considered structural system and boundary conditions.

From a practical design point of view, the general and alternative methods (M1–M3) are recommended when accurate evaluation of the lateral-torsional buckling resistance is required, especially for girders with discrete lateral restraints and non-uniform moment diagrams, while the simplified methods (M4–M5) should be restricted to preliminary design or to cases that strictly satisfy their underlying assumptions.

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