

INFLUENCE OF STRUCTURAL BRACING ON STABILITY AND TORSIONAL RESISTANCE OF TRUSS FOOTBRIDGES

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ABSTRACT:

Structures composed of lattice beam subassemblies are commonly used for constructing pedestrian bridges with medium spans. These structures are typically equipped with wind bracing at the top. Another scenario where lattice walls are employed is in box girders with concrete decks. During the installation phase, these girders are fitted with upper flange bracing to ensure overall structural stability. For composite decks or those with orthotropic steel plating, lattice bracing may also be arranged at the bottom. Truss walls can be transformed into plate beams using the transformed beam method, which simplifies efficiency analysis, particularly for torsional stresses. This case study presents an example of equating a truss structure with a plate structure under torsional loading. The efficiency of closed sections compared to open sections for pure torsional resistance is analysed.

1. INTRODUCTION

Structures made of lattice beam subassemblies are widely used for medium-span pedestrian bridges. When the construction depth is sufficient, wind bracing is installed at the top. Secondary bracing ensures the stability of the upper flanges in the horizontal plane, enhances the structure's bending rigidity in both vertical and horizontal planes, and improves dynamic parameters, contributing to pedestrian comfort.

Lattice walls can be converted into plate beams with equivalent web thickness using the transformed beam method. This approach simplifies efficiency analysis, especially for torsional stresses. Another application of lattice wall is in box girders with concrete decks. During construction, bracing is added to the upper flanges to maintain stability until the concrete hardens. For composite or orthotropic steel decks, lattice bracing at the bottom can be equated to a plate beam to analyse torsional effects.

This case study demonstrates the equivalence of a truss structure to a plate structure under torsional loading, comparing the efficiency of closed versus open sections for pure torsion.

Similar theoretical models and approach for determining torsional stiffness and general behaviour analysis of a transformed section were treated in (Zhang, 2022), (Sabzehzar, 2024), (Smyrniaos, 2015) and (Vayas, 2011).

2. EVALUATION OF EQUIVALENT THICKNESS OF A TRUSS SYSTEM

The torsional stiffness of a box girder, governed by Saint-Venant effects, can be 100–1000 times greater than that of an equivalent I-section. This stiffness depends on the material's shear modulus (G) and the torsional constant (I_r), which is tied to the cross-section's geometry.

The torsional resistance of a cross-section is given by:

$$I_r = I_t + \frac{\Omega^2}{\oint \frac{ds}{g}} \approx K_T = \frac{\Omega^2}{\sum \frac{b_j}{g_j}} \quad (1)$$

$$\text{where: } I_t = \frac{1}{3} \int_0^a g^3 ds = \frac{1}{3} \sum_{j=1}^a a_j \cdot g_j^3$$
$$\Omega = \oint r \cdot ds = 2 \cdot A_m = 2 \cdot bh$$

Figure 1 illustrates the shear flow in a box girder under Saint-Venant torsion.

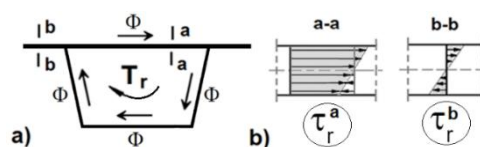


Figure 1. Shear flow in box girder due to Saint-Venant torsion

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There are situations when one of the walls of a tubular or a box section is a truss girder, with varied lattice geometry. In the case of metal bridges, this occurs in composite steel-concrete box sections. These are provided with superior bracing to ensure the stability of the flanges during the erection phase, until the concrete slab is cast (Figure 2). During construction, a steel box with a top flange lateral truss is often referred to as a quasi-closed box. The primary purpose of the top truss is to simulate the stiffness of a closed box system.

Quasi-closed sections, fitted with a bracing of bars (lattice beam), arranged at the bottom of the box girder, also shall be used, (Helwig, et al. 2007).

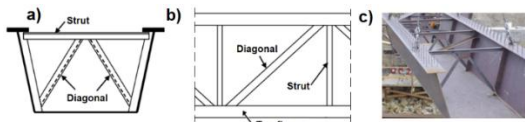


Figure 2. Box girder bracing system: a) internal cross frame; b) top bracing; c) photo image of a box girder in construction stage

The shear flow acting on the fictitious plate is then transformed into diagonal member forces in the lateral truss, as demonstrated in Figure 3. (Helwig, et al. 2007)

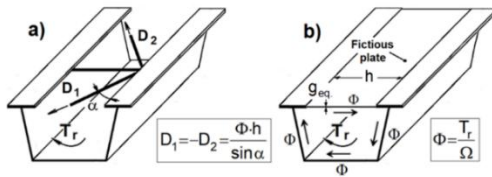


Figure 3. Diagonal brace forces due to torsion and the equivalent plate girder model

Although both Saint-Venant and warping torsion are developed in box girder sections, Saint-Venant torsion often dominates the stiffness of the closed cross-section. The longitudinal normal stresses due to warping torsion are usually negligible. (Helwig, et al. 2007)

The relative proportion of each type of torsion present in a member depends on both its length and cross section. The parameter R is used to determine whether pure or warping torsion predominates. This parameter is related to the member length as well as the ratio of the pure torsional rigidity GK_T and the warping torsional rigidity El_ω , (Chen, et al. 2005)

$$R = L \cdot \sqrt{\frac{GK_T}{El_\omega}} \quad (2)$$

Table 4 classifies torsional predominance based on the value of R . (Chen, et al. 2005):

Torsional predominance	R
Pure warping	<0.3
Dominating warping	0.3-0.2
Mixed	2-5
Dominating pure torsion	5-10
Pure torsion	>10

Table 4. Pure and Warping Torsional Predominance

For torsion calculations, the truss system can be replaced by a continuous plate web of equivalent thickness, g_{eq} . The equivalent thickness of the plate can be evaluated from the equal deformation energy of the lattice panel (L_E) and the deformation energy (L_T) of the continuously equivalent panel with thickness g_{eq} . (Helwig, et al. 2007), (Chen, et al. 2005) and (Helwig, 2015)

$$L_E = L_T \quad (3)$$

The mechanical model is shown in Figure 5, consisting in separating the truss girder from the rest of the plate girder.

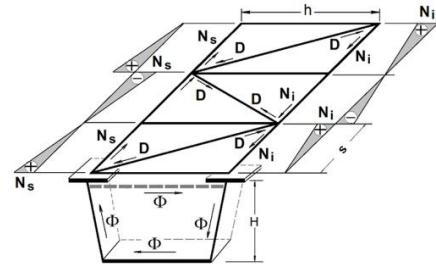


Figure 5. Mechanical model for determining equivalent plate thickness

Applying shear stress duality principles, τ , tangential stress flows generate concentrated nodal forces producing axial forces (N). The hypothesis of linear variation of axial efforts N from zero to $N_s = -N_i$ on the length of each length panel λ is made, where:

$$N_s = -N_i = \Phi \cdot s \quad (4)$$

Knowing the result of shear stresses τ_r on the truss, $\Phi \cdot h$, the axial force D in the diagonal will result:

$$D = \pm \frac{\Phi \cdot h}{\sin \alpha} = \pm \Phi \cdot d \quad (5)$$

The expression of deformation energy for a latticed panel, L_E , can be written in the following form:

$$L_E = L_{Ed} + L_{Es} + L_{Ei} \quad (6)$$

where:

$$L_{Ed} = \frac{D \cdot \Delta d}{2} = \frac{\Phi^2 \cdot d^3}{2 \cdot E \cdot A_d} \quad \text{- deformation energy of the diagonal;}$$

$$L_{Es} = \frac{1}{3} \frac{N_s \cdot \Delta s}{2} = \frac{\Phi^2 \cdot s^3}{6 \cdot E \cdot A_s} \quad \text{- deformation energy of the top chord;}$$

$$L_{Ei} = \frac{1}{3} \frac{N_i \cdot \Delta s}{2} = \frac{\Phi^2 \cdot s^3}{6 \cdot E \cdot A_i} \quad \text{- deformation energy of the bottom chord;}$$

A_d, A_s, A_i - cross-section areas of bars (Table 6)

The deformation energy for the continuous equivalent panel, L_T , is:

$$L_T = \frac{s}{2} \iint_A \tau_r \cdot \gamma_0 \cdot dA = \frac{s}{2 \cdot G} \iint_A \tau_r^2 \cdot dA$$

$$= \frac{s}{2 \cdot G} \int_0^h \left(\frac{\Phi}{g_{eq}} \right)^2 \cdot g_{eq} \cdot dl \quad (7)$$

It results:

$$L_T = \frac{\Phi^2}{2 \cdot G} \cdot \frac{h}{g_{eq.}} \cdot s \quad (8)$$

Through appropriate substitutions, the truss girder's equivalent thickness $g_{eq.}$ is calculated as:

$$g_{eq.} = \frac{E}{G} \cdot \frac{s \cdot h}{\frac{d^3}{A_d} + \frac{s^3}{3} \left(\frac{1}{A_s} + \frac{1}{A_i} \right)} \quad (9)$$

Table 6 provides comprehensive formulas for equivalent wall thickness determination across various truss geometries. (Chen, et al. 2005) and (Helwig, 2015)

	Truss configuration	Equivalent plate thicknesses $g_{eq.}$
1		$g_{eq.} = \frac{E}{G} \cdot \frac{s \cdot h}{\frac{d^3}{A_d} + \frac{s^3}{3} \left(\frac{2}{A_f} \right)}$
2		$g_{eq.} = \frac{E}{G} \cdot \frac{s \cdot h}{\frac{2d^3}{A_d} + \frac{h^3}{4A_m} + \frac{s^3}{12} \left(\frac{2}{A_f} \right)}$
3		$g_{eq.} = \frac{E}{G} \cdot \frac{s \cdot h}{\frac{d^3}{2A_d} + \frac{s^3}{12} \left(\frac{2}{A_f} \right)}$
4		$g_{eq.} = \frac{E}{G} \cdot \frac{s \cdot h}{\frac{d^3}{A_d} + \frac{h^3}{A_m} + \frac{s^3}{12} \left(\frac{2}{A_f} \right)}$
Notations: s - panel length; h - width between flanges; d - diagonal length; A_f - area of flanges; A_d; A_m - areas of diagonals and of struts; E - modulus elasticity; G - shear modulus: $G = E/[2(1 + \mu)]$. Observation: If the flanges have different cross-section areas (A_s and A_i), the bracket $\left(\frac{2}{A_f} \right)$ shall be replaced by $\left(\frac{1}{A_s} + \frac{1}{A_i} \right)$.		

Table 6. The equivalent plate thicknesses

The conversion of a three-dimensional prismatic truss system to an equivalent box girder follows the same concept of converting the lateral bracing of an open box section to an equivalent plate and forming a closed box.

Each of the main trusses and lateral bracing trusses is transformed into an equivalent plate with its thickness, Figure 7.

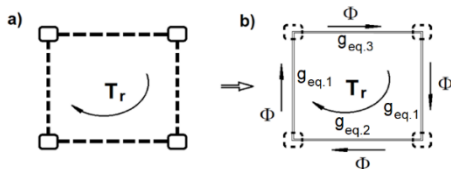


Figure 7. Prismatic truss system and the equivalent box girder

For an open box section with lateral bracing, the equivalent closed box retains its flanges and webs. The stresses and deflections of the braced open box under load can be computed with sufficient accuracy using the equivalent closed box section. In the case of a prismatic truss system, all sides of the prism are converted to equivalent plates.

The main bending components (chord members) are retained in the cross-section. (Helwig, et al. 2007), (Chen, et al. 2005) and (Helwig, 2015).

Similar approaches and another case of lattice configuration can be found in (Vayas, 2013).

3. NUMERICAL APPLICATION FOR A STRUCTURAL ANALYSIS OF A TRUSS FOOTBRIDGE

3.1 Equivalent plate thickness of a truss main girder

The equivalent thickness for the main truss girders of a pedestrian bridge with the design characteristics shown in Table 8 is assessed. The truss panel length is $s = 150$ cm.

Cross-section	Member	Section area [cm ²]
	Chords: 2L 150x150x10	48.4
	Diagonals: 2L 70x70x6	16.26
	Struts: 4L 70x70x6	32.52

Table 8. Design data for the main truss girder

The results are summarized in Table 9.

	Variant 1	Variant 2	Variant 3	Variant 4
$g_{eq.}$ [mm]	0.50	0.84	1.0	0.4

Table 9. Synthetical results of equivalent thicknesses

3.2 Pure torsion of a truss footbridge

The torsion parameters for a footbridge with truss girders are evaluated.

The design characteristics are:

- material S355; span of the footbridge $L=30.00$ m; distance between the main girders $b=4.00$ m; the main truss girders are Variant 3 (Table 6);
- the lower brace is a truss of Variant 2 (Table 6), having diagonals made of 2L 70x70x6, and the role of the struts is fulfilled by cross girders of the deck, made of steel rolled IPE 200;
- the upper brace of the closed section truss girder is of Variant 2 (Table 6) type, having the diagonals and struts made of 2L 70x70x6.

The design cross-section for the constructive solution is shown in Figure 10.

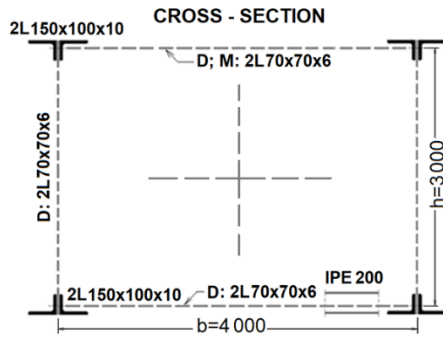


Figure 10. Footbridges cross-section

The equivalent plate thicknesses are illustrated in Figure 11.

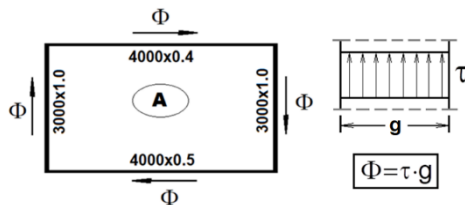


Figure 11. Equivalent plate of cross-sections

3.3 Efficiency degree to torsion resistance of closed section vs. open section

Torsion moment of inertia of the footbridge structure:

$$I_t = \frac{1}{3} \int_0^a g^3 ds = \frac{1}{3} \sum_{j=1}^a a_j \cdot g_j^3 = \frac{1}{3} (2 \cdot 300 \cdot 0.1^3 + 400 \cdot 0.05^3 + 400 \cdot 0.04^3) = 0.23 \text{ cm}^4$$

$$I_r = I_t + \frac{\Omega^2}{\oint \frac{ds}{g}} = 0.23 + \frac{4 \cdot (300 \cdot 400)^2}{\frac{400}{0.05} + \frac{400}{0.04} + 2 \cdot \frac{300}{0.1}} = 2.4 \cdot 10^6 \text{ cm}^4$$

$$\Omega = 2 \cdot A_m = 2 \cdot (300 \cdot 400) = 2.4 \cdot 10^5 \text{ cm}^2$$

The maximum admissible values (design stresses) of the shear stresses are:

- for open cross-section

$$\tau_{T.max} = \frac{T_{Rd}^{open} \cdot g_{max}^{open}}{I_t^{open}} = f_y / \sqrt{3} \quad (10.a)$$

- for closed cross-section

$$\tau_{T.max} = T_{Rd}^{clos.} \left(\frac{1}{g_{min}^{clos.} \cdot \Omega} + \frac{g_{min}^{clos.}}{I_r} \right) = f_y / \sqrt{3} \quad (10.b)$$

The torsion resistance of a footbridge with a closed cross-section is:

$$T_{Rd}^{clos.} = \frac{f_y / \sqrt{3}}{\frac{1}{g_{min}^{clos.} \cdot \Omega} + \frac{g_{min}^{clos.}}{I_r}} = 1970 \text{ kN} \cdot \text{m}$$

The required thickness of the open section wall can be determined so that the two sections to have the same torsion resistance (Moga, 2016):

$$T_{Rd}^{open} = \frac{I_t^{open}}{g_{max}} f_y / \sqrt{3} = T_{Rd}^{clos.} = \frac{f_y / \sqrt{3}}{\frac{1}{g_{min}^{clos.} \cdot \Omega} + \frac{g_{min}^{clos.}}{I_r}} \Rightarrow \frac{I_t^{open}}{g_{max}} = \frac{1}{\frac{1}{g_{min}^{clos.} \cdot \Omega} + \frac{g_{min}^{clos.}}{I_r}}$$

It is obtained:

$$g_{eq.} = \sqrt{\frac{3}{(\sum b_j^{open}) \cdot \left(\frac{1}{g_{min}^{clos.}} + \frac{g_{min}^{clos.}}{I_r} \right)}} \quad (11)$$

In this case it is obtained: $g = 54$ mm, and $T_{Rd}^A = T_{Rd}^{B1}$, Figure 12.

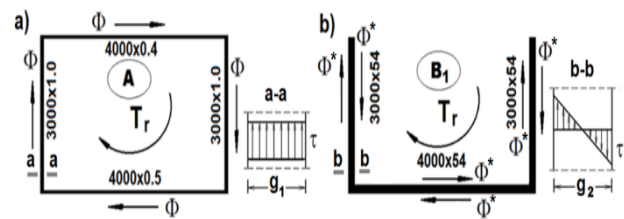


Figure 12. Equivalent in torsion resistance of a closed cross-section vs. an open cross-section

The same procedure can be used to obtain the equivalent thickness of the open cross-section to have the same rigidity with the closed cross-section, but in this case the obtained equivalent thickness value is much higher.

For comparison, closed cross sections can often have torsional stiffnesses thousands of times greater than similar open sections.

The case presented in Figure 13 is analysed.

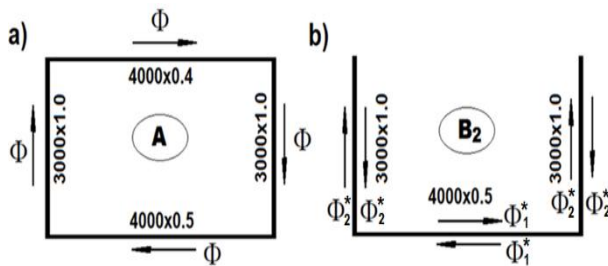


Figure 13. Close cross-sections vs. open cross-section

By imposing the condition that the maximum torsion shear stress is equal to the closed section with that for the open section, the efficiency of the closed tube compared to the open tube, noted E, (Moga, 2018), the following equation is obtained:

$$E = \frac{T_{Rd}^{clos.}}{T_{Rd}^{open}} = \left(\frac{\frac{g_{max}^{open}}{I_t^{open}}}{\frac{1}{g_{min}^{clos.} \cdot \Omega} + \frac{g_{min}^{clos.}}{I_r}} \right) \approx \frac{\Omega}{I_t^{open}} (g_{max}^{open} \cdot g_{min}^{clos.}) \quad (12)$$

The parameter E can be named efficiency degree to torsion of closed section vs. open section, (Moga, 2016).

In the case of Figure 13 shall be obtained: E=4370.

Additional relevant research addressing similar technical challenges to those examined in this study can be found in the following publications: (Moga C, et al. 2020a), (Gutiu, et al. 2014), (Moga C, et al. 2021a), (Moga C, 2021b) and (Moga C, et al. 2020b).

4. CONCLUSIONS

For box sections, the equivalence between a braced lattice wall and a plate beam proves particularly useful when analysing torsional effects on stress distribution within the structure, as well as when evaluating the dynamic response of load-bearing elements.

In steel bridge decks, secondary bracing systems offer significant advantages for maintaining overall stability, particularly for primary truss girders.

Simultaneously, these bracing systems substantially enhance the structure's dynamic performance by dramatically increasing torsional stiffness and favourably modifying vibration parameters.

Closed cross-sections demonstrate considerable advantages over open sections in terms of torsional shear stress resistance.

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