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# SIMPLIFIED CALCULATION OF DEFLECTION IN FLAT SLABS MADE OF RC AND PC USING A MODIFIED STRIP METHOD

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**KEY WORDS:** deformation calculation, simplified method, prestressed concrete, reinforced concrete, post-tensioned flat slab

**ABSTRACT:**

In this article, the topic of reinforced and prestressed concrete (RC and PC) flat slabs deflection is addressed, focusing on a new method that facilitates a simplified calculation. The proposed calculation method is based on the strip method, and for its validation the results from a parametric case study are compared with those obtained by other calculation methods suitable for estimating the deflection of flat slabs. To be more practical, the proposed method is transformed into a coefficient method (closed form formula) and allows the calculation of the deflection also for external, corner and isolated slab panels, unlike the existing closed form formula approaches which are only available either for the internal or corner slab panels. In addition, the difference between the deflection values obtained considering the uncracked versus cracked concrete section, respectively between the instantaneous (short-term) deflection and the long-term deflection values of the RC and PC slabs is addressed.

## 1. INTRODUCTION

### 1.1 General aspects

The limitation of the slab deflection is a key aspect in the structural design. The main reasons for the need of this limitation are (ADAPT Corporation, Bijan Aalami, 2011):

- the non-structural elements: a slab must have sufficient bending stiffness (outside its plane) to prevent the degradation of non-structural elements that come into contact with it (e.g. partition walls, exterior walls, the finishing applied to the floor, etc.);
- the aesthetics: the deflection of the slab must be small enough not to be perceived by the occupants (e.g. visible deformations of the ceiling / bottom side of the slab, cracks with reduced width, etc.);
- the functionality: the deflection of the slab must not affect the drainage of water nor lead to the formation of puddles (e.g. roof decking), respectively it must not affect the proper functioning of the equipment (e.g. floors).

In Romania, the use of reinforced concrete (RC) flat slabs ceased after the earthquake of March 4, 1977, when the central body of the Computing Centre (Centrul de calcul) of the Ministry of Transport near the CFR Palace in Bucharest collapsed (www.agerpres.ro, 2022). The Computing Centre had 3 levels, it was segmented into 3 parts, the lateral parts had a structural system composed of RC frames, while the central part had a structural system made of flat slabs (with hollow boxes) with columns (without structural walls). The central part of the building had 4 square bays with a 12 m span and 3 m span cantilevers on the perimeter, while the RC columns had capitals (Figure 1). Due to the high flexibility and low resistance to lateral forces, during the earthquake the structure collapsed through formation of plastic hinges at the ends of the columns. The return of the use of RC flat slabs occurred slowly, marked by the appearance of the preliminary drafting of the guide for the structural design of flat slabs CR 2-01-A in 1998 (Universitatea Tehnică de Construcții București, 1998). After the publication of the new code for seismic design of structures, P100-1/2006, the structural designers in Romania have designed more often structures having RC flat slabs. In the past 10 to15 years, in cities such as Cluj-Napoca approx. 50% of the residential and office buildings were built with RC flat slabs, and more recently the prestressed concrete (PC) flat slabs with post-tensioned tendons are becoming popular. In 2012, the first edition of the Guide for structural design of flat slabs GP 118 (Universitatea Tehnică de Construcții București, 2012) was pre-published as a draft version. In 2019, the mandatory use of perimeter beams for structures having flat slabs and categorized as importance classes 1 and 2 was introduced in the seismic design code P100-1/2013 (revised in 2019) together with the restriction to use structures with columns and flat slabs (without structural walls) only for buildings with a maximum height of 2 floors, regardless the importance class.

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For now, in Romania there is no technical standard in force that regulates in detail the structural design of flat slabs, due to the fact that the codes CR 2-01-A (Technical University of Construction Bucharest, 1998) and GP 118 (Technical University of Construction Bucharest, 2012) have remained at the preliminary drafting phase, a final version was never published in the Official Monitor of Romania. Moreover, the topic of flat slabs is only briefly addressed in the European design standards (Eurocodes), reference to the design of flat slabs being made only in EC2-1-1 (CEN_EC2-1-1, 2004), in a condensed manner on several pages.

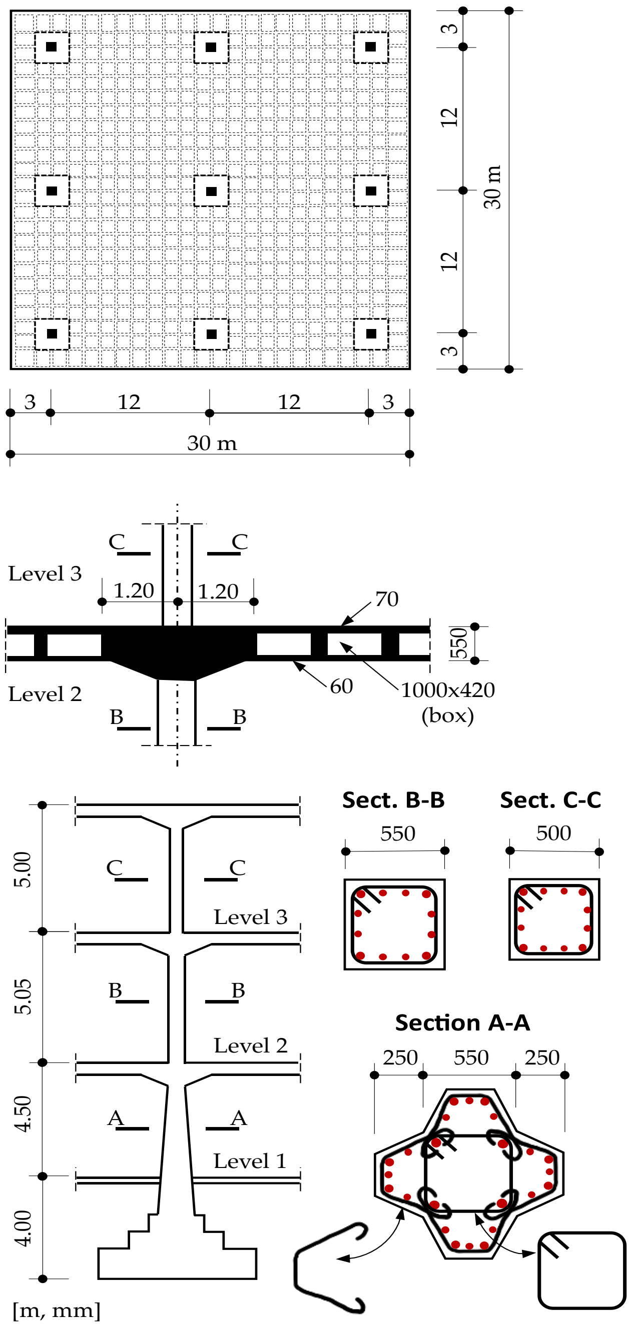



Figure 1. Palatul CFR – load bearing structure of central building (ICCPDC, 2022) (INCERC, 2022) (Ministerul_Transporturilor, 2020) collapsed during the 4th of March 1977 earthquake, in Bucharest, Romania

In the context of current legislation, a considerable number of complaints have been filed regarding buildings with RC flat slab floors, particularly in the residential sector, in Cluj-Napoca. It is not uncommon for these buildings to display cracks of varying widths in partitioning and closing masonry walls within the first few years of use. In some cases, these cracks persist to grow and extend even after the first three years after the construction was completed, despite the implementation of successive repairs to the finishes. One potential cause of these degradation phenomena is the excessive deflection of the slabs.

### 1.2 Pre-dimensioning

The process of pre-dimensioning a slab requires the estimation of the required slab thickness to ensure that the structure meets the necessary standards for strength and stability under both design loads (e.g. self-weight, weight of finishes, live load, technological load, etc.) and exceptional (e.g. fire, earthquake, explosion, etc.) loads. It is necessary to ensure that the final thickness of the slab provides the necessary bearing capacity in order to satisfy the requirements of the ultimate limit state (ULS), as well as to guarantee that deflections and crack openings related to the serviceability limit state (SLS) are within the permitted maximum limits set out in the design norms (e.g. (CEN_EC2-1-1, 2004), (CEN_EC0, 2002), (CEN_EC1-1-1, 2002) etc.) and/or in the project (e.g. design theme, technical report, structural design notes, etc.). The required slab thickness (Table 1) is primarily determined by the maximum span value of the slab panel, the presence or absence of beams on the slab perimeter, and the magnitude of loads applied to the slab. Secondary factors include the tensile strength of the reinforcement and the class of strength of the concrete. The required thickness of a RC slab can be reduced if the concrete is prestressed. The most used monolithic PC slabs are those with post-tensioned reinforcement.

| **Total imposed load** | **Span/depth ratios** ($6m \le l_a \le 13$ m) | |
|---|---|---|
| kN/m$^2$ | RC | PT |
| 2.5 | 30 | 40 |
| 5.0 | 24 | 36 |
| 10.0 | 17 | 30 |

Notes:
1) The total imposed load does not include slab`s self-weight.
2) $l_a$ = the maximum span of the considered flat slab panel.
3) The table addresses multi-span flat slabs.
4) The span/thickness ratio is applicable for all internal panels and for external panels when external beams are used.
5) If no perimeter beams are used, then a thickness of the external slab panels increased with approx. 15% will be considered.
6) For each panel of the flat slab a ratio of $l_a / l_b \le 1.50$ was considered, while for the overall floor was considered $l_{a,max} / l_{a,min} \le 1.2$ and $l_{b,max} / l_{b,min} \le 1.2$.
7) The strength class of the reinforcing steel is B500.

Table 1. Recommended span/depth ratio for solid flat slabs for a maximum deflection of $l_a/250$, based on (Toader, Kiss, Țere, & Puskás, 2021) and (Szydłowski, Łabuzek & Rodacka, 2019)

The characteristic (also called rare) combination of actions related to SLS, according to EC0 (CEN_EC0, 2002), is:

$$q_{SLS\_CAR} = 1.0 \cdot (gp_k + qp_k) + 1.0 \cdot qu_k \quad (1)$$

Where: $q_{SLS\_CAR}$ = the value of the total load acting on the slab calculated for SLS from the characteristic combination of actions;
$gp_k$ = slab`s self-weight, with characteristic value;
$qp_k$ = permanent load (from technological actions, finishes, light partitions, etc.) additional to that of its self-weight acting on the slab, with characteristic value;
$qu_k$ = live load acting on the slab, with characteristic value.

And the group of actions in ULS for the fundamental combination, according to EC0 (CEN_EC0, 2002), is:

$$q_{SLU\_GF} = 1.35 \cdot (gp_k + qp_k) + 1.5 \cdot qu_k \qquad (2)$$

Where: $q_{SLU\_GF}$ = the total load acting on the slab calculated for the ULS from fundamental combination, with design value.

### 1.3 Maximum deflection allowed

The maximum admissible values of the slab deflection can be specified in the design theme or can derive from it. In the absence of such clarifications, the maximum permissible values specified in the technical regulations (Table 2) shall be used.

| Member type | Limit | Deflection to consider |
|---|---|---|
| *ACI 318-19* | | |
| Flat roofs not supporting or attached to nonstructural members likely to be damaged by large deflection | L/180 | Immediate deflection due to total variable actions (with characteristic values) |
| Floors not supporting or attached to nonstructural members likely to be damaged by large deflection | L/360 | Immediate deflection due to total variable actions (with characteristic values) |
| Floors and roofs supporting or attached to nonstructural members likely to be damaged | L/480 | Deflection after attachment of nonstructural members (loads with characteristic values) |
| Floors and roofs supporting or attached to nonstructural members not likely to be damage nonstructural members | L/240 | Deflection after attachment of nonstructural members (loads with characteristic values) |
| *BS 8110* | | |
| Floors, roofs and any structural member | L/250 | Total deflection |
| Floors and roofs supporting brittle nonstructural members | L/500 or 20 mm | Deflection after attachment of nonstructural members |
| Floors and roofs supporting nonbrittle nonstructural members | L/350 or 20 mm | Deflection after attachment of nonstructural members |
| *EC 2-1-1:2004* | | |
| Floors and roofs likely to damage nonstructural members | L/500 | Deflection after construction (usually with quasi-permanent loads for long term deflection) |
| Floors and roofs | L/250 | Total deflection (usually with quasi-permanent loads for long term deflection) |
| *NBN B 03-001:2002* | | |
| Floors and roofs supporting nonbrittle nonstructural members | L/350 | Deflection after attachment of nonstructural members (loads with characteristic values) |
| Floors and roofs supporting brittle nonstructural members without openings | L/500 | Deflection after attachment of nonstructural members (loads with characteristic values) |
| Floors and roofs supporting brittle nonstructural members with openings | L/1000 | Deflection after attachment of nonstructural members (loads with characteristic values) |
| Floors and roofs supporting removable walls | L/250 | Deflection after attachment of nonstructural members (loads with characteristic values) |
| *STAS 10107/0-90* | | |
| Floors and flat roofs | L/350 | Short term deflection due to agglomeration of people |
| Floors and flat roofs supporting or attached to nonstructural members likely to be damaged by large deflection | L/400 | Long term deflection after attachment of nonstructural members |
| Floors and flat roofs not supporting or attached to nonstructural members likely to be damaged by large deflection | L/250 | Long term deflection after attachment of nonstructural members |

Notes:
1) Where L is the shorter span length between to neighbouring vertical members.
2) For cantilevers L is to be considered twice the length of the cantilever.
3) Deflection of roofs should also be checked for assuring drainage, avoiding ponding and proper operation.

Table 2. Deflection limits for floors (American Concrete Institute, 2019) (Aalami, 2014) (ADAPT Corporation, Bijan Aalami, 2011) (CEN_EC2-1-1, 2004) (Institut belge de normalisation (IBN), 2002) (Khan & Williams, 1995) (Institutul român de standardizare, 1990)

### 1.4 Long-term deflection

When subjected to a constant load over time, a RC or PC member will exhibit an increasing deflection. This deflection will eventually stabilize, typically around 60 months after the building is completed, as noted in ACI 318-99 (American Concrete Institute, 2019). The initial increase in deflection is primarily due to concrete shrinkage caused by chemical reactions and the evaporation of water. The subsequent increase occurs due to creep, which refers to the gradual deformation of compressed concrete under long-term loads. There is computational software that can be used to rigorously determine the deflection of RC and PC structural elements, generally this software is based on the Finite Element Method (FEM). To account for the effects of shrinkage and creeping of concrete, it is necessary to calculate some coefficients determined on an analytical or graphical basis using norms (CEN_EC2-1-1, 2004) and pre-norms (fib_bulletin65, 2012) (fib_bulletin66, 2012). Additionally, the reduction in the slab's flexural stiffness due to cracking must be taken into account in cross-sections where the demanding bending moment exceeds the cracking bending moment ($M_{Ed} > M_{cr}$). For these calculations, it is necessary that the slab element has already been dimensioned (its longitudinal reinforcement has been established).

In current design practice, the instantaneous deflection of the slab under service loads (SLS) is calculated and then adjusted (Aalami, 2014) to account for the effects of concrete shrinkage, creep, and flexural cracking. The instantaneous deflection is determined using the secant modulus of elasticity of concrete at 28 days, referred to as $E_{cm}$, in accordance with (CEN_EC2-1-1, 2004). This calculation typically involves performing a linear elastic static analysis that considers the gross concrete section (uncracked) (Aalami, 2014). However, for a more detailed and accurate assessment, it is essential to incorporate the effects of slab cracking and the resulting reduction in flexural stiffness.

The calculation of the deflection of PC slabs is simpler than that of RC slabs when in SLS the slab is working in the uncracked stage, because in this case the moment of inertia of the gross concrete section can be used in the calculation (Khan & Williams, 1995). However, for slabs with bonded tendons the calculation should be performed on the transformed (idealized) cross-section, and for non-bonded tendons on the net cross-section (Khan & Williams, 1995). Currently, the following simplifications can be applied in deflection analysis (Khan & Williams 1995):

- the pretensioning force in the tendon is constant along the entire length;
- the inclination of the tendons is small, consequently the horizontal component of the precompression is constant;
- the variation of the prestressing force in the tendon due to the deflection of the slab can be neglected.

Over the years, numerous experimental tests have been carried out to evaluate the short and long-term deflection of the flat slabs (Gilbert & Guo, 2005) (Rangan, 1975). It has been observed that in RC flat slabs, if the calculation for long-term deflection uses the effective bending rigidity equal to 25% of the flexural stiffness of the gross concrete section (Rangan, 1975), then a conservative estimation of the real deflections is obtained. However, not all authors recommend the same percentage to be conservative, in GP 118-2012 (Universitatea Tehnică de Construcții București, 2012) a value of 30% is indicated, while in the design and expertise practise a 15% is used (Z. Kiss, 2013-2023). The factors contributing to this reduction in flexural stiffness would include the following: concrete shrinkage, creep, and flexural cracking.

It has been observed analytically and experimentally that the long-term deflection (after 60 months) in RC elements can be almost double the value of the short-term deflection due to creep deformation of concrete (American Concrete Institute, 2019), (Ravasini, et al. 2023), (Kilpatrick & Gilbert, 2018). The calculation of the long-term deflection in PC members unlike in RC ones, must include the shortening of the member over time under the action of the prestressing force.

## 2 METHODS OF DEFLECTION CALCULATION

The calculation of slab deflection based on the idealization of orthogonal strips has been a topic of research since the 1960s. Early studies by Rice and Kulka (1960) were followed by further investigations by Rangan (1975) and others. Subsequently, analytical and numerical methods were employed (Eigelaar 2010; Wium and Eigelaar, 2010; Kim, Kang, and Lee 2017). Recently, artificial neural networks have been utilized (Hossain, Ahmed and Siddiquee, 2019). In the case of the strip method, each slab panel is divided into column strips and middle strips, which are treated as beams (frame elements), as illustrated in Figure 2. The complexity of this problem can be reduced to two key aspects: firstly, the determination of the appropriate proportion of load distribution between the equivalent beams (the grid of column and middle strips) and secondly, the identification of an appropriate degree of embedment/fixation at the ends of the equivalent beams.

### 2.1 Closed form formula method

In general, the maximum deflection of a flat slab can be calculated using the following equation from (Aalami, 2014), (Chao & Naaman, 2006) and (Bares, 1971):

$$w_{max} = k \cdot \frac{q \cdot l_a^4}{E_{cm} \cdot h_{pl}^3} \quad (3)$$

where k = coefficient for closed form formula (Table 3)
$l_a$ = the long span of the considered flat slab panel
q = total load value applied to the slab in the SLS: characteristic combination. For RC flat slabs $q = q_{SLS\_CAR}$ in equation (1), respectively for PC flat slabs $q = q_{SLS\_CAR} - q_{bal\_equiv}$ (where $q_{bal\_equiv}$ represents the equivalent balancing load (gravitationally uniform distributed) due to prestressing)
$E_{cm}$ = modulus of elasticity for concrete (CEN_EC2-1-1, 2004)
$h_{pl}$ = slab thickness

In the calculation of flat slab deflection when the joint column–slab is considered to be hinged, the deflections are approximately 33% higher (in the case of panels with equal spans, $l_a = l_b$), approximately 60% higher (in the case of panels with $l_a = 1.5 \times l_b$), and approximately 100% higher (in the case of panels with la $= 2 \times l_b$) than in the case of a rigid joint. It can be observed that as the stiffness of the columns increases, the discrepancies between the two models also increase. It is notable that if the cross-sectional area of the columns is doubled, the deflection of the slab is reduced by up to 30% (Chao & Naaman, 2006). Nevertheless, the real mechanical behaviour of the column-monolithic flat slab joint demonstrates that the connection is somewhere between a hinge and a rigid joint (Chao & Naaman, 2006). This mechanical behaviour is attributed to the cracking of the slab in the immediate vicinity of the columns. The cracking is more pronounced in RC slabs than in PC slabs (Gilbert, Cleland & Long, 1985). As a result, it can be considered a conservative approach to model the column-slab connection as a hinge when calculating the slab deflection.

| **Long span / short span ratio** ($l_a$ / $l_b$)3) | **k coefficient**1),2) for closed form formula | | | |
|---|---|---|---|---|
| | (Bares, 1971) | (Naaman, 2004)4) | (Chao & Naaman, 2006)5) | (Chao & Naaman, 2006)6) |
| source of k values | internal panel (uniform loading) | corner panel (alternate loading) | | |
| 1.0 | 0.0653 | 0.11 | 0.13 | 0.19 |
| 1.1 | 0.0548 | 0.1045 | 0.118 | 0.1727 |
| 1.2 | 0.0481 | 0.099 | 0.106 | 0.1583 |
| 1.3 | 0.0436 | 0.0935 | 0.094 | 0.1461 |
| 1.4 | 0.0403 | 0.088 | 0.082 | 0.1357 |
| 1.5 | 0.0379 | 0.0825 | 0.07 | 0.135 |
| 2.0 | 0.0328 | 0.055 | 0.069 | 0.135 |

Notes:

1) The approach assumes the entire slab is uniform loaded and does not account for pattern loading, which could result in larger deflections (Chao & Naaman, 2006).
2) The values of the k coefficient are given for internal flat slab panels. Are considered floors with panels having identical geometry (slab thickness and spans).
3) Values of $l_a$ (panel`s long span) and $l_b$ (panel`s short span) are corresponding to the rectangular shape panel for which the deflections is to be calculated.
4) $k = 0.11 \cdot (1.5 - 0.5 \cdot l_a/l_b)$, based on (Naaman, 2004) and (Chao & Naaman, 2006).
5) Column - slab rigid connection, equation 9 from (Chao & Naaman, 2006):
- $k = (0.25 - 0.12 \cdot l_a/l_b)$ for $1 \le l_a/l_b\ 1 \le 1.5$
- $k = (0.073 - 0.002 \cdot l_a/l_b)$ for $l_a/l_b \ge 1.5$

6) Column – slab hinge connection, equation 8 from (Chao & Naaman, 2006):
- $k = (0.3 - 0.11 \cdot l_a/l_b)$ for $1 \le l_a/l_b\ 1 \le 1.5$
- $k = 0.135$ for $l_a/l_b \ge 1.5$

Table 3. Deflection coefficients k for two-way flat slabs to be used in closed form formula for calculating the value of the maximum deflection

### 2.2 Deflection of the slab strips modelled as beams

The method of calculating deflection using slab strips (as frame/beam elements) is a well-established technique that has been employed since before the advent of personal computers (Aalami, 2014). The calculation consists in dividing the flat slab into strips along the support line (e.g. column line) and carry the loads that are applied on the tributary width of the slab. Each strip is then isolated and statically analysed (Table 4). To calculate the deflection at the midpoint of the slab panel, the deflections of the orthogonal strips that intersect at the intended point are summed up (Figure 2).

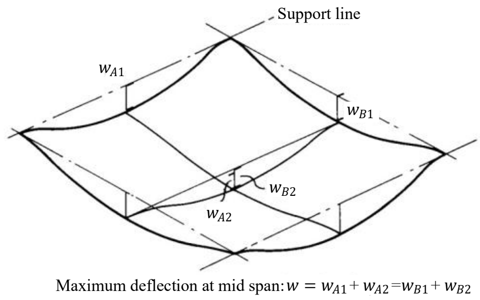



Figure 2. Strip method (Crossing beam) for flat slab deflection calculation, based on (Aalami, 2014) and (Khan & Williams, 1995)

| Static scheme | Bending moment diagram | Deformed shape |
|---|---|---|
| 1 — $q$; $l$ | $0.5 \cdot l$; $M^+_{max} = \frac{q \cdot l^2}{8}$ | $0.5 \cdot l$; $w_{max} = \frac{5}{384} \cdot \frac{q \cdot l^4}{E \cdot I}$ |
| 2 — $F$; $l/2$, $l/2$ | $0.5 \cdot l$; $M^+_{max} = \frac{F \cdot l}{4}$ | $0.5 \cdot l$; $w_{max} = \frac{1}{48} \cdot \frac{F \cdot l^3}{E \cdot I}$ |
| 3 — $M$, $M$; $l$ | $M^+_{max} = M$ | $0.5 \cdot l$; $w_{max} = \frac{1}{8} \cdot \frac{M \cdot l^2}{E \cdot I}$ |
| 4 — $q$; $l$ | $M^-_{max} = -\frac{q \cdot l^2}{8}$; $0.375 \cdot l$; $M^+_{max} = \frac{9 \cdot q \cdot l^2}{128}$ | $w_{l/2} = \frac{1}{192} \cdot \frac{q \cdot l^4}{E \cdot I}$; $0.442 \cdot l$; $w_{max} = \frac{1}{184.6} \cdot \frac{q \cdot l^4}{E \cdot I}$ |
| 5 — $F$; $l/2$, $l/2$ | $M^-_{max} = -\frac{3 \cdot F \cdot l}{16}$; $0.5 \cdot l$; $M^+_{max} = \frac{2.5 \cdot F \cdot l}{16}$ | $w_{l/2} = \frac{7}{768} \cdot \frac{F \cdot l^3}{E \cdot I}$; $0.447 \cdot l$; $w_{max} = \frac{1}{107.3} \cdot \frac{F \cdot l^3}{E \cdot I}$ |
| 6 — $M$; $l$ | $M^+_{max} = M$ | $w_{l/2} = \frac{1}{32} \cdot \frac{M \cdot l^2}{E \cdot I}$; $1/3 \cdot l$; $w_{max} = \frac{1}{27} \cdot \frac{M \cdot l^2}{E \cdot I}$ |
| 7 — $q$; $l$ | $0.5 \cdot l$; $M^-_{max} = -\frac{q \cdot l^2}{12}$; $M^+_{max} = \frac{q \cdot l^2}{24}$ | $0.5 \cdot l$; $w_{max} = \frac{1}{384} \cdot \frac{q \cdot l^4}{E \cdot I}$ |
| 8 — $F$; $l/2$, $l/2$ | $0.5 \cdot l$; $M^-_{max} = -\frac{F \cdot l}{8}$; $M^+_{max} = \frac{F \cdot l}{8}$ | $0.5 \cdot l$; $w_{max} = \frac{1}{192} \cdot \frac{F \cdot l^3}{E \cdot I}$ |
| 9 — $q$; $l$ | $M^-_{max} = -\frac{q \cdot l^2}{2}$ | $w_{max} = \frac{1}{8} \cdot \frac{q \cdot l^4}{E \cdot I}$ |
| 10 — $F$; $l$ | $M^-_{max} = -F \cdot l$ | $w_{max} = \frac{1}{3} \cdot \frac{F \cdot l^3}{E \cdot I}$ |
| 11 — $M$; $l$ | $M^-_{max} = M$ | $w_{max} = -\frac{1}{2} \cdot \frac{M \cdot l^2}{E \cdot I}$ |

Table 4. Deflection and bending moments diagrams for slab strips modelled as linear elements (beams), based on (Schneider, 2014)

### 2.3 Deflection calculation with FEM

Slab deflection calculation using FEM can be done using thin shell elements with 3 (triangle) or 4 (square) nodes. By discretizing the slab, we aim to obtain finite elements with side lengths between $1\cdot h_{pl}$ and $4\cdot h_{pl}$. The types of static analysis that can be used are linear static analysis and nonlinear static analysis. In the case of nonlinear static analysis, it is relevant to consider the material nonlinearity, including the effects of concrete cracking due to bending, creep and shrinkage deformation of concrete, respectively relaxation of reinforcement. It is recommended to carry out the static analysis both on the slab floor of a single isolated level (with support on vertical elements such as columns and walls, or on point and linear supports), as well as on the slab floor as an integral part of the entire structural system of the building. The final structural design has to be based on the most disadvantageous stress and deformation states in both structural models.

### 2.4 Proposed method

The yield line mechanism is a failure pattern often used for calculating the force diagrams at ULS, but not to verify deformation state at SLS (Figure 3). While for SLS the load distribution on the support lines/column strips and on the middle strips has a triangular or trapezoidal shape (for rectangular slab panels), at SLU the loads can be considered as uniformly distributed along the strips. This makes it difficult to enable quick calculation of deflections of the flat slabs, but through the present study the following calculation method is proposed, which shows similarities and differences compared to the classic strips method. Table 5 summarises the proposed method.

| **Parameter** | **Strip method** (classic) | **Modified strip method** (proposed) |
|---|---|---|
| Types of strips | column strips (CS) and middle strips (MS) | CS and MS |
| Width of strips | 0.5·(slab span), for CS<br>0.5·(slab span), for MS<br>*(in EC2-2004, ACI 318-19, GP 118-2012)*<br><br>0.4·(slab span), for CS<br>0.6·(slab span), for MS<br>*(in DIN 1045 and SIA 262, and older version of ACI 318 and BS 8110)* | 1 m, for CS<br>1 m, for MS |
| Distribution of tributary load | triangle and trapezoidal | uniform |
| Tributary load value | $q^{1)}\cdot$(with of the tributary area in each point along the strip) | $q^{1)}\cdot$(1 m) |
| Fixation degree at the end of strips | further static analyse is required, taking into account the torsional stiffness for each orthogonal column strip | hinge - at end support lines<br><br>full continuity - at intermediate support lines |

Notes:
1) For flat slabs made of RC is considered $q = q_{SLS\_CAR}$ in equation (1), respectively for those made of PC is considered $q = q_{SLS\_CAR} - q_{bal\_equiv}$ (where $q_{bal\_equiv}$ represents the equivalent balancing load (gravitationally uniform distributed) due to prestressing)

Table 5. Comparison between strip methods, classic versus proposed, to be used in the calculation of deflection at SLS, being considered $E = E_{cm}$ and the width of the strip b = 1 m

Calculations based on yield line mechanism involve considering all possible failure mechanisms (Figure 3). As a result, in the design of slabs, both the failure mechanism with parallel yield lines oriented in one direction (e.g. Ox direction) and the mechanism with yield lines in the second possible direction (e.g. Oy direction) must be considered. In this way, a conservative design is obtained (Khan & Williams, 1995). The direction of unloading of the slab towards the supports is indicated by its cracking mode. While in the beam slab floor the slab first unloads on the beams and then the beams on the columns, in the flat slab floor the slab first unloads on the column strips and then on the columns (Khan & Williams, 1995) (Aalami, 2014).

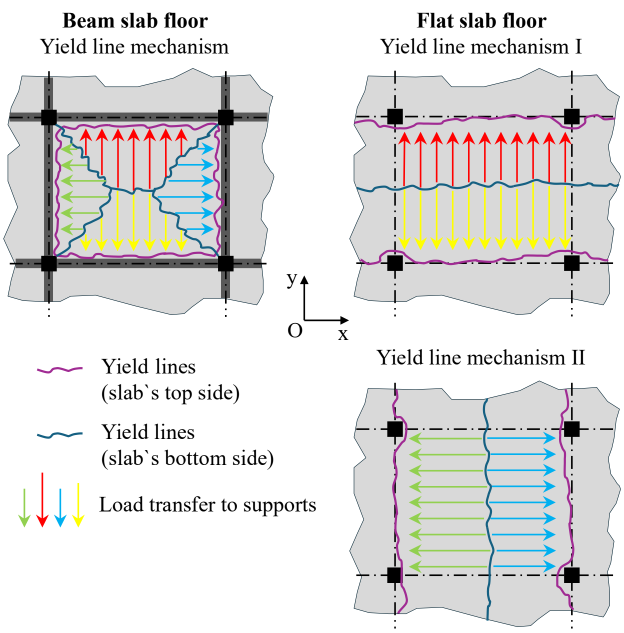



Figure 3. Failure mechanisms of beam slab floor versus flat slab floor, considering gravitational loading and uniformly distributed over the entire surface of the slab, considering (Khan & Williams, 1995)

The cylindrical rigidity of the flat slab panels was neglected, because by comparative calculation it was observed that it has an insignificant contribution to the total bending rigidity of the slab (Khan & Williams, 1995) (Aalami, 2014). The following describes the types of rectangular plate panels, with side ratio ranging from 0.5 to 2.0, for which the deflections in Table 6 were calculated:

1 – isolated slab panel with point supports at corners: This panel experiences unrestrained rotation at the edges (free rotation at edges);
2 – internal panel: This panel has bending moment continuity on all sides (restricts rotation on the edges);
3 – corner panel: This panel has bending moment continuity on two adjacent sides (restricts rotation on those two sides);
4 – marginal panel: This panel has three sides with bending moment continuity, featuring one long free side (4.1), respectively one short side (4.2);
5 – end panel: This panel has bending moment continuity on one of the short sides (5.1), respectively on one of the long sides (5.2);
6 – middle panel: This panel has bending moment continuity on two parallel sides, on the short sides (6.1), respectively on the long sides (6.2).

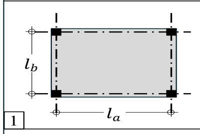
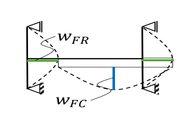
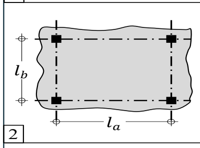
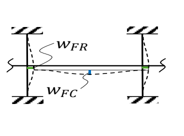
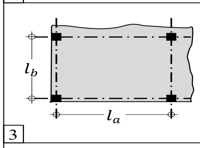
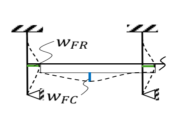
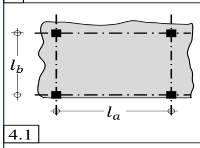
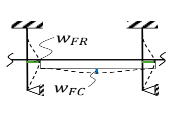
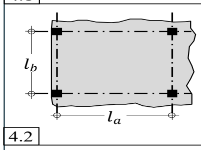
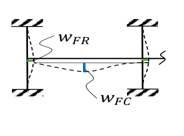
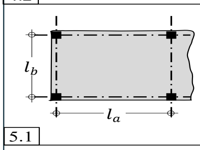
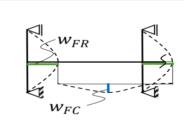
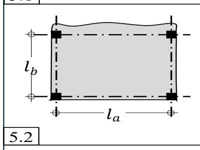
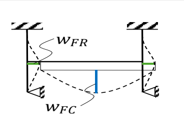
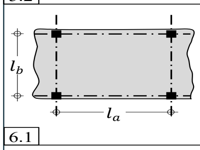
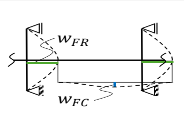
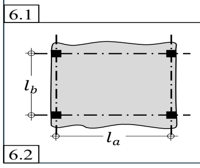
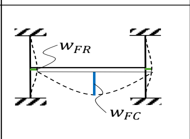
| Slab panel $l_b \le l_a \le 2\cdot l_b$ | Static scheme $I=(1m\cdot h_{pl}^3)/12$ | Maximum deflection $w_{tot} = w_{FR} + w_{FC}$ |
|---|---|---|
|  1 |  | $\frac{5}{384}\cdot\left(1+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{5}{32}\cdot\left(1+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  2 |  | $\frac{1}{384}\cdot\left(1+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{32}\cdot\left(1+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  3 |  | $\frac{1}{192}\cdot\left(1+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{16}\cdot\left(1+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  4.1 |  | $\frac{1}{384}\cdot\left(1+\frac{2\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{32}\cdot\left(1+\frac{2\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  4.2 |  | $\frac{1}{384}\cdot\left(2+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{32}\cdot\left(2+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  5.1 |  | $\frac{1}{384}\cdot\left(2+\frac{5\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{32}\cdot\left(2+\frac{5\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  5.2 |  | $\frac{1}{384}\cdot\left(5+\frac{2\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{32}\cdot\left(5+\frac{2\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  6.1 |  | $\frac{1}{384}\cdot\left(1+\frac{5\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{32}\cdot\left(1+\frac{5\cdot l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |
|  6.2 |  | $\frac{1}{384}\cdot\left(5+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot I} = \frac{1}{32}\cdot\left(5+\frac{l_b^4}{l_a^4}\right)\cdot\frac{q\cdot l_a^4}{E\cdot h_{pl}^3}$ |

Table 6. Values of mid span deflection for 9 types of slab panels formulated with the proposed method

## 3 CASE STUDY, RESULTS AND DISCUSSION

The next stage of the investigation is to conduct a case study, with the parameters being the flat slab thickness (which has 14 distinct values, as set out in Table 7) and floor configurations with 1, 2, 3, 4, 6, 9 and 25 bays. In the static analysis, the column-slab joint was modelled as a hinge, and the concrete members were considered uncracked. A total of 36 finite element (FE) models were used, comprising two models (RC and PC) with nine slab configurations and two modelling approaches each (bending moment continuity modelled as physical continuity, respectively as restricted rotation). The deflection of the flat slabs was extracted from these models, and the equivalent coefficients k were calculated (Figures 4 to 12).

| **Long span** ($l_a$ in m) | **Slab thickness** ($h_{pl}$ in mm) | |
|---|---|---|
| | RC | PT |
| 5.0 | 240 | 160 |
| 5.5 | 265 | 180 |
| 6.0 | 290 | 195 |
| 6.5 | 310 | 210 |
| 7.0 | 335 | 225 |
| 7.5 | 360 | 240 |
| 10.0 | 480 | 320 |

Notes:
1) The total imposed load considered is $qp_k$ (permanent loading due to floor finishing and partition walls) + $qu_k$ (live load due to human activity) = 2.5 + 2.5 = 5.0 kN/m$^2$, and it does not include slab`s self-weight ($gp_k$).
2) A 90% of the PT slab`s self-weight is balanced through prestressing the concrete.
3) $l_a$ = the maximum span of the considered flat slab panel.
4) The short span is kept constant, $l_b$ = 5.0 m.

Table 7. Slab thickness for the slabs analysed, using pre-dimensioning from Table 1

The results of the analysis (depicted in Figures 4-12) demonstrate a highly accurate correlation between the FEM and the modified strip method (SM), which has been proposed for the calculation of maximum slab deflection. The SM provides the same k-equivalent coefficient for internal slab panels as Bares (1971) and a superior approximation for corner slab panels in comparison to Naaman (2004) and Chao & Naaman (2006), as evidenced by the FEA results.

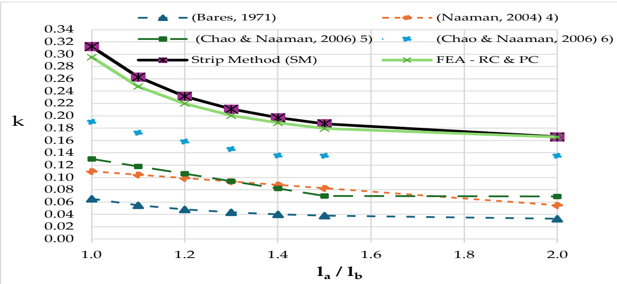



Figure 4. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for an isolated slab panel point supported (type 1, as defined in Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

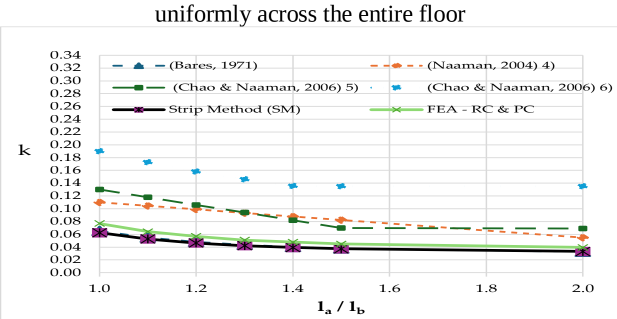



Figure 5. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for an internal panel (type 2 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

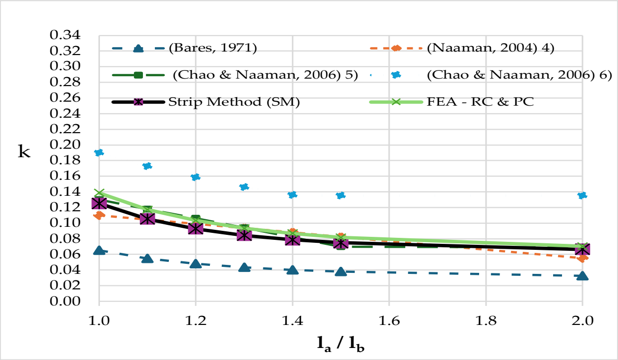



Figure 6. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for a corner panel (type 3 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

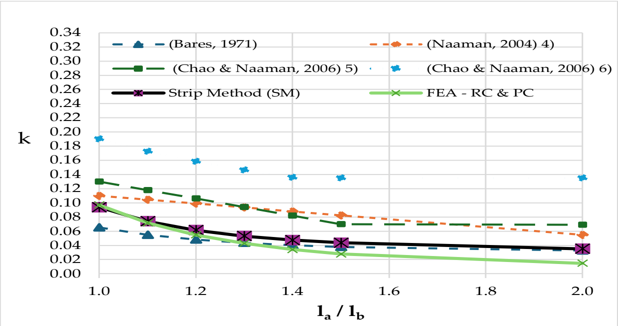



Figure 7. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for a marginal panel (type 4.1 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

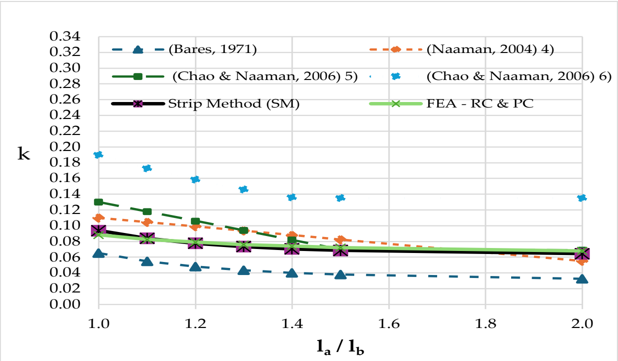



Figure 8. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for a marginal panel (type 4.2 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

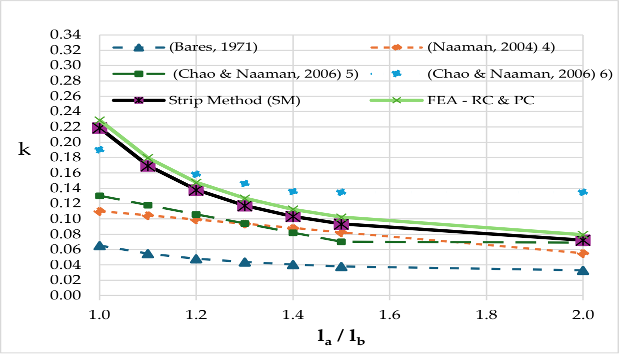



Figure 9. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for an end panel (type 5.1 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

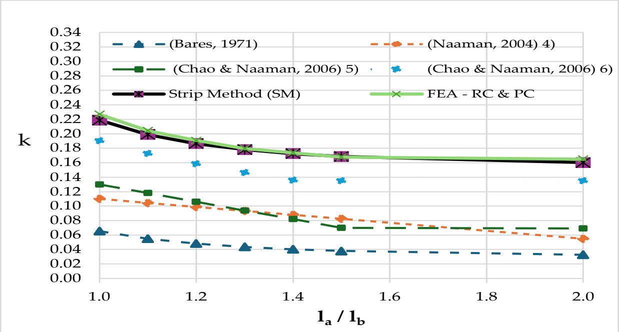



Figure 10. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for an end panel (type 5.2 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

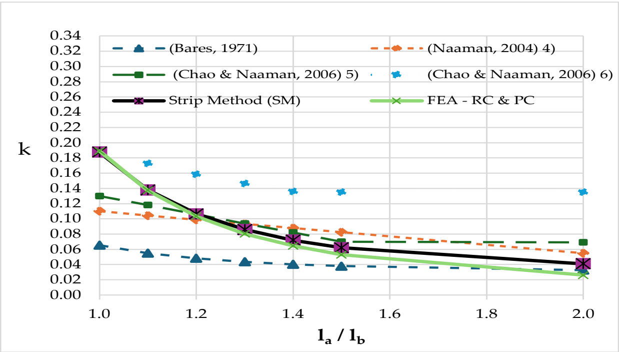



Figure 11. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for a middle panel (type 6.1 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

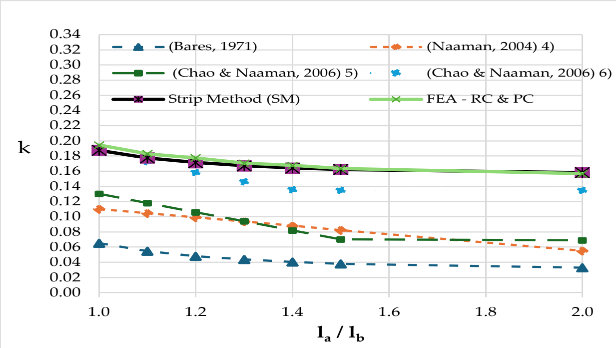



Figure 12. The equivalent k coefficient (calculated from eq. 3 and using Table 6) for a middle panel (type 6.2 - Table 6) is calculated using the proposed modified strip method (SM) and compared with the coefficient method (closed form formula) and FEA, with the load distributed uniformly across the entire floor

## 4 CONCLUSION

The purpose of this study is to formulate simple mathematical equations for calculating the short-term deflection of the flat slabs made of RC and PC. The proposed equations are written considering uniformly distributed loads over the entire surface of the slab, the floor has slab panels with the same geometry, respectively the column – slab joint is considered hinged. The proposed method with simplified calculation of the deflection of flat slabs is obviously less rigorous than others, but nevertheless it provides reasonable estimates of deflections which require a small number of mathematical operations. In contrast with the existent closed form formula approaches that allow only to calculate the deflection of internal and corner flat slab panels (Table 3), the present method gives solutions to calculate deflections also for the external slab panels (placed at the corner or at the edge of the floor) as well as for the isolated ones. Finally, it should be noted that the proposed method of calculation does not consider all possible parameters of a flat slab, such as the presence of column capitals, the use of drop panels, the presence of perimeter floor beams or the rotational fixation of the slabs edge in case of perimeter structural walls. If such elements are present, the deflection value calculated with the proposed method will be higher than the real ones, but still remains useful for rough estimations.

As **future research direction**, the author is focusing on the study of bioinspired self-centring load-bearing structures with viscoelastic damping in members such as prestressed columns, slabs and cable stayed facades, for which the influence of this novel feature upon the deflection of integrated flat slabs is of great importance.

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