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EFFECT OF ASPECT RATIO, DEGREE OF ORTHOTROPY AND COEFFICIENT OF THERMAL EXPANSION ON TRANSVERSE DISPLACEMENT OF LAMINATED BEAMS

S. Kulkarni [a]

[a*] Symbiosis Institute of Technology, Symbiosis International (Deemed University), Lavale, Pune-412115, India,
E-mail: sanjay.kulkarni@sitpune.edu.in

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KEY WORDS: coefficient of thermal expansions, degree of orthotropy, transverse deflection, shear deformation theory

ABSTRACT:

The impact of orthotropy on thermal displacement is useful to optimize thermal management systems in space structures. The thermal response of the beam is greatly affected by its aspect ratio. The expansion or contraction due to difference in coefficient of thermal expansion affects the structural integrity of the beam. An analytical model helps to understand the effect of these parameters on transverse thermal displacement of laminated beam. Hence, in the present work the effect of change of aspect ratio, degree of orthotropy and coefficient of thermal expansion ratio on transverse thermal displacement of orthotropic and cross ply laminated beams has been studied and presented. The trigonometric beam theory is used. The present theory has sine function in terms of thickness coordinate representing the shear deformation effect. The equations of motions are developed by using principle of virtual work. The transverse deflections are obtained by applying Navier's solution. A computer program in FORTRAN-77 is developed to obtain transverse deflection for various aspect ratios, modular ratios and coefficient of thermal expansion ratios.

1. INTRODUCTION

The impact of modular ratio or degree of orthotropy on the transverse thermal displacement of laminated composite beam plays an important role in understanding the bending behaviour of laminated composite beam. The prediction thermal displacements help in preventing thermally induced damages in composite structures. Hence, an analytical model for thermal displacements will help to understand the material efficiency and structural performance of laminated beam.

In the literature, researchers presented thermal analysis of laminated beams by using various beam theories. The thermal buckling of composite laminated beam was presented for the first time in literature by (Bouazza et. al 2018) with the use of a refined hyperbolic shear deformation theory.

A trigonometric beam theory was applied to understand flexural behaviour of laminated composite and sandwich beams by (Sayyad et. al 2015). The displacement and stresses in the composite and sandwich beams were evaluated under mechanical load. A fifth order shear and normal deformation beam theory was presented by (Ghumare et. al 2017) to know the static flexural behaviour and elastic buckling of functionally graded material beams.

The effect of transverse normal was included in the theory by using polynomial shape function. The analysis of beams curved in elevation subjected to mechanical and thermal load by using polynomial beam theory has been presented by (Avhad and Sayyad, 2022). The theory includes the effect of transverse normal with a polynomial function. The stress analysis of isotropic, laminated and sandwich beams was presented by (Ghugal et. al 2023).

A quasi-two-dimensional higher order theory was used for the analysis of these beams. The effect of transverse shear and normal deformations was included in the theory.

The analysis of clamped laminated beams with non-uniform boundary conditions with thermal effect was presented by (Qian et. al, 2022).

The thermal response of laminated beam with clamped non-uniform temperature boundary conditions was presented in this paper. The heat conduction law was used in this paper. The polynomial curved beam theory was presented by (Sayyad and Avhad, 2022).

The theory was applied to laminated composites, sandwich and functionally graded curved beams. The theory includes the transverse normal effect. The linear thermal load is considered in the analysis. The effect of degree of orthotropy on transverse deflection of two-layer composite laminated plate has been studied and presented by (Kulkarni and Ghugal, 2015).

Trigonometric shear deformation theory was used by the authors and the plate was subjected to thermal load. The exact elasticity solution of the composite laminated beams in simply supported

Corresponding Author: Sanjay Kulkarni, Symbiosis Institute of Technology, Symbiosis International (Deemed University), Lavale, Pune- 412115, India, Email: Sanjay.kulkarni@sitpune.edu.in.

condition and subjected to thermo-loads was studied and presented by (Hai et. al 2015).

An analytical method was presented to obtain displacement and stresses in the beam. A new fifth order shear and normal deformation theory was presented by (Naik and Sayyad, 2020) for the analysis of laminated composite under cylindrical bending.

The thermal load was taken into consideration for the analysis of laminated composites and sandwich plates under cylindrical bending. The flexural behaviour of composite laminated beams under combined thermal and transverse mechanical loads by using hyperbolic, fifth order and trigonometric beam theory has been studied and presented by (Ghugal et. al 2023).

In this work nonlinear thermal load is taken into consideration during analysis. Thermal buckling of laminated beams subjected to uniform temperature distribution has been studied and presented by (Khdeir, 2001). Refined beam theory was used to determine critical buckling of laminated beam.

Bending analysis of cross ply laminated plates under combined non-linear thermal and transverse mechanical loads was presented by (Ghugal and Kulkarni, 2013). The analytical model was developed to obtain displacements and stresses in the laminated plates.

A sine-based refined finite element theory was applied by (Vidal and Polit, 2009) to composite laminated beams subjected to thermal and mechanical loadings. A finite element approach was carried out by author using Lagrange and Hermite interpolations. The rectangular laminated beams under thermal and mechanical loads were analysed with three-noded finite element method by (Vidal and Polit, 2006).

The composite laminate beam with tapered orthotropic layers was analysed for inter-laminar stresses by Vilar et. al 2023). The effect of degree of orthotropy on compressive strength of composite laminates was experimentally and computationally studied and presented by (Patel et. al 2014).

The instability of laminated beams and plates due to non-uniform thermal load was presented by (Tylikowski and Hetnarski, 1996). The nonlinear displacements of laminated beams under non-uniform temperature field were presented by (Akbas, 2018).

The cylindrical bending of laminated and sandwich plates under hygrothermal and thermal load was presented by (Naik and Sayyad, 2021) using fifth order beam theory.

The aim of the present work is to understand the effect of degree of orthotropy on transverse thermal displacement of orthotropic and cross ply composite laminated beams. The effect of changing aspect and coefficient of thermal expansion ratio on transverse thermal displacements are investigated and presented in this paper.

2. THE GEOMETRY OF BEAM

The geometry of the straight beam is as shown in the figure 1. The length, width and height (thickness) of the beam are represented by *a*, *b* and *h* along *x*, *y* and *z* axis respectively.

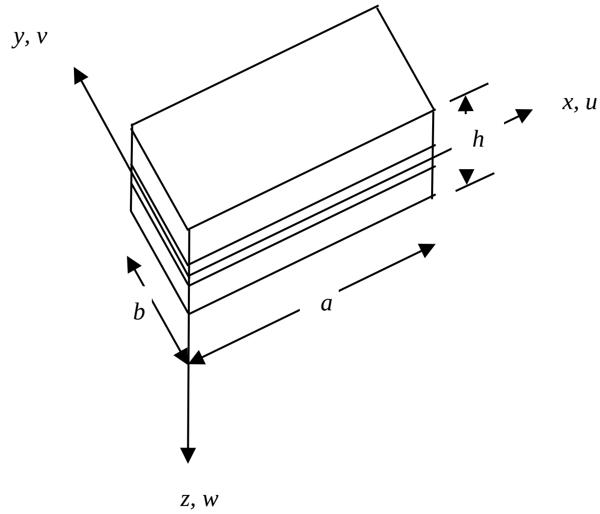

Figure 1. The coordinate system and geometry of beam

The beam has *N* number of orthotropic layers. The width of beam (b) is small in comparison with length of beam (a). The cross section of the beam is rectangular. The z axis in downward direction is considered as positive.

The upper surface of beam $\left(z = -\frac{h}{2}\right)$ is acted upon by thermal load $T(x,z)$. The beam under consideration occupies the region in Cartesian coordinate$(O - x, y, z)$.

$$0 \le x \le a, 0 \le y \le b, -\frac{h}{2} \le z \le \frac{h}{2} \tag{1}$$

The geometry of the laminated beam is in Cartesian coordinate as shown in figure 1 and represented by equation (1).

The axial and transverse displacements in *x* and *z* directions are represented by *u* and *w*. The orthotropic layers are assumed to made up of graphite epoxy material. The material has high modulus ratio and high coefficient of thermal expansion ratio.

The high coefficient of thermal expansion material is used in aerospace engineering for thermal protection systems.

3. THE KINEMATIC OF THE THEORY AND METHODOLOGY

The displacement field of the theory for beam is expressed as follows:

$$u(x,z) = u_0(x) - z\frac{\partial w}{\partial x} + \frac{h}{\pi} sin\frac{\pi z}{h} \varphi(x) \tag{2}$$

$$w(x,z) = w(x) \tag{3}$$

The displacement filed consists of mid-plane stretching, bending and shear deformation parameters (equation 2).

The shear deformation is in the form sinusoidal trigonometric function in terms of thickness coordinate.

Here, $u(x,z)$ is the axial displacement in x direction and $w(x)$ is the transverse displacement in the z direction. The centre line displacement (mid-plane stretching) and shear slope are represented by $u_0(x)$ and φ respectively. The mid-plane stretching and shear slope are the functions of *x* only.

3.1 Development of Governing Equations

The normal strain, shear strains, normal stress and transverse shear stress are obtained by using strain-displacement and constitutive relationship.

$$\varepsilon_x = \frac{\partial u}{\partial x} \tag{4}$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \tag{5}$$

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & 0 \\ 0 & \overline{Q}_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \gamma_{xz} \end{Bmatrix} \tag{6}$$

$$\overline{Q}_{11}^{(k)} = \frac{\overline{E}_1^{(k)}}{1-\mu_{12}^{(k)}\mu_{21}^{(k)}}, \overline{Q}_{22}^{(k)} = \frac{\overline{E}_2^{(k)}}{1-\mu_{12}^{(k)}\mu_{21}^{(k)}}, \overline{Q}_{55}^{(k)} = \overline{G}_{13}^{(k)}, \overline{Q}_{44}^{(k)} = \overline{G}_{23}^{(k)}$$

The linear temperature variation is assumed across the thickness of laminated beam. The temperature profile is as given below.

$$T\,(x,z) = \frac{2}{h} z T_1 \tag{7}$$

In the above temperature profile, the term T_1 represents thermal load and T represents the temperature difference. The terms T and T_1 are the functions of (*x*, *z*) and *x* respectively.

The equations of motion are obtained from the principle of virtual work with the use of integration by parts and following equations of motion are obtained.

$$\delta u_0 : -A_{11}\frac{\partial^2 u_0}{\partial x^2} + B_{11}\frac{\partial^3 w}{\partial x^3} - SB_{11}\frac{\partial^2 \varphi}{\partial x^2} + TB_{11}\frac{\partial T_0}{\partial x} = 0 \tag{8}$$

$$\delta w : D_{11}\frac{\partial^4 w}{\partial x^4} - B_{11}\frac{\partial^3 u_0}{\partial x^3} - S_{11}\frac{\partial^3 \varphi}{\partial x^3} + TD_{11}\frac{\partial^2 T_0}{\partial x^2} = q \tag{9}$$

$$\delta \varphi : S_{11}\frac{\partial^3 w}{\partial x^3} - SS_{11}\frac{\partial^2 \varphi}{\partial x^2} + C_{55}\varphi - SB_{11}\frac{\partial^2 u_0}{\partial x^2} + TS_{11}\frac{\partial T_0}{\partial x} = 0 \tag{10}$$

Where: the stiffness coefficients $A_{ij}, B_{ij}, \ldots.$ etc are defined as follows:

$$(A_{11}, B_{11}, D_{11}) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)}(1, z, z^2)\,dz \tag{11}$$

$$(SB_{11}, S_{11}, SS_{11}) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)} \frac{h}{\pi} sin\frac{\pi z}{h}\left(1, z, \frac{h}{\pi} sin\frac{\pi z}{h}\right) dz \tag{12}$$

$$(TB_{11}, TD_{11}, TS_{11}) = \sum_{k=1}^{N}(\alpha_x)\int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)}\left(z, z^2, \frac{h}{\pi} sin\frac{\pi z}{h}\right) dz \tag{13}$$

$$C_{55} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{55}^{(k)} cos^2\frac{\pi z}{h}\,dz \tag{14}$$

The Navier's solution has been applied further to obtain transverse displacements in the simply supported laminated beam under thermal influence.

A computer program in FORTRAN has been developed to evaluate the thermal transverse displacements for various aspect ratios, various modular ratios and various ratios of coefficient of thermal expansions.

The theory has been applied on orthotropic, two-layer and three-layer laminated beams to understand the effect of degree of orthotropy, effect of change of coefficient of thermal expansion and effect of change of aspect ratio on transverse deflection of laminated beams.

The material properties are as follows:

$$E_L = 172.4\text{GPa}, E_T = 6.895\text{ GPa}, G_{LT} = 3.448\text{ GPa}, G_{TT} = 1.379\text{ GPa}, \mu_{LT} = 0.25, \alpha_T = 1125\alpha_L$$

Where: E_L and E_T are modulus of elasticity in fibre and transverse directions. The modulus of rigidity and Poisson's ratio are represented by *G* and μ respectively. The fibre and transverse directions are represented by *L* and *T* respectively. The coefficient of thermal expansion in fibre and transverse directions are shown by α_L and α_T respectively.

3.1.1 Numerical Results and Discussion: The numerical results of transverse central displacement of cross play laminated beams are presented in following normalized form for the purpose of discussion.

The dimensionless (normalized) transverse displacements are obtained and presented in this paper. The following equation is dimensionless form of the transverse displacement.

$$\overline{w}\left(\frac{a}{2}, 0\right) = \frac{wh}{(\alpha_L T_0 a^2)} \tag{15}$$

The following three examples are considered and transverse displacements are obtained and presented in dimensionless form as given by equation (15).

Example-1: The effect of change of aspect ratio (*a/h*) on transverse displacement of orthotropic and cross play laminated beams (two-layer and three-layer) under sinusoidal thermal load.

Example-2: The effect of degree of orthotropy on transverse displacement of orthotropic (0^0), two layer (0/90) and three layer (0/90/0) laminated beams under sinusoidal thermal load for aspect ratio 4 and 10. ($\alpha_T = 1125\alpha_L$).

Example-3: Transverse displacements in orthotropic, two-layer and three-layer laminated beam under sinusoidal thermal load for various ratios of coefficient of thermal expansion for aspect ratios 4 and 10.

The numerical results of transverse displacement obtained in the above examples by using trigonometric shear deformation beam theory (TSDT) are new in the literature.

Theory	$\frac{a}{h}$	0 $\bar{w}$	0/90 $\bar{w}$	0/90/0 $\bar{w}$
TSDT	100	0.2026	41.2708	0.5518
Sin Ref-c (Vidal and Polite, 2009)	100	-	41.2860	0.5506
Exact (Vidal and Polite, 2009)	100	-	43.1550	2.8197
TSDT	50	0.2026	41.2555	0.5498
TSDT	25	0.2026	41.1941	0.5417
TSDT	20	0.2026	41.1481	0.5356
TSDT	12.5	0.2026	40.9493	0.5092
TSDT	10	0.2026	40.7663	0.5518
Sin Ref-c (Vidal and Polite, 2009)	10	-	45.8820	0.3762
Exact (Vidal and Polite, 2009)	10	-	43.2930	1.8934
TSDT	6.25	0.2026	39.9792	0.3824
TSDT	5	0.2026	39.2611	0.2902
TSDT	4	0.2026	38.1549	0.1508
Sin Ref-c (Vidal and Polite, 2009)	4	-	45.8820	0.2332
Exact (Vidal and Polite, 2009)	4	-	42.8890	3.6156
TSDT	2	0.2026	29.6295	-0.8265

Table 1. Effect of change of aspect ratio on transverse displacement of single layer, two-layer and three-layer composite laminated beams under sinusoidal thermal load. (Example-1). ($\alpha_T = 1125\alpha_L$)

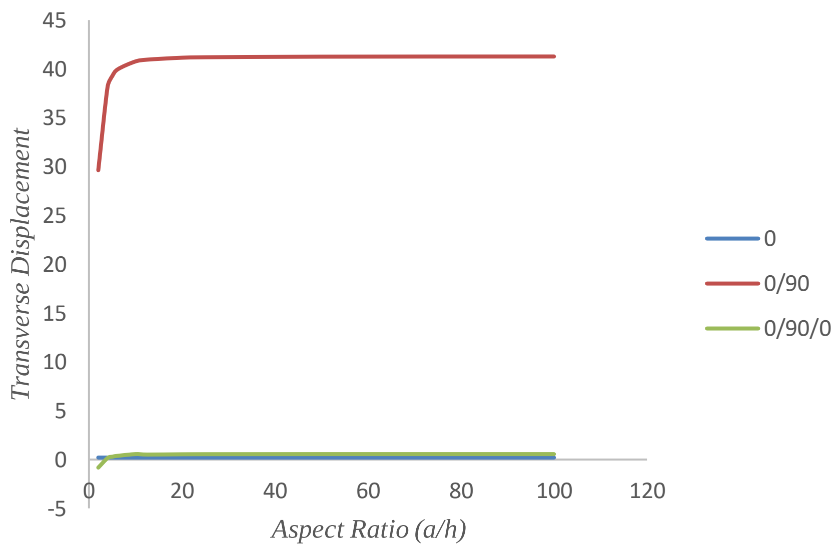

Figure 2. Effect of change of aspect ratio on transverse deflection of orthotropic (0), two layer (0/90) and three layers (0/90/0) laminated composite beams

The results are presented in the Table 1 and figure 2.

The transverse displacements are obtained for various aspect ratios. The aspect ratio from 2 to 100 has been taken into consideration. A considerable change in the values of transverse displacement is noted in two-layer (0/90) laminated beam.

Whereas no effect is seen on transverse displacement of orthotropic beam. In case of three layer beam a slight difference is noted for thick beams and for rest of the aspect ratios from 10 to 100 the transverse deflection nearly remains same.

The effect of change of aspect ratio on transverse deformation is seen in case of bilayer (0/90) beam. This is clearly noted from figure 2.

High aspect ratio refers to thin or slender beam promote bending stiffness which makes the beam less flexible in transverse direction.

On the other hand, low aspect ratio allows more transverse displacement in the beam. It is clear that change of aspect ratio matters in cross-ply laminated beams in thermal environment.

This practical take away is essential to understand while designing an aircraft wing to obtain an efficient structure.

$\frac{E_1}{E_2}$	0 $\bar{w}$	0 $\bar{w}$	0/90 $\bar{w}$	0/90 $\bar{w}$	0/90/0 $\bar{w}$	0/90/0 $\bar{w}$
	$\frac{a}{h}=4$	$\frac{a}{h}=10$	$\frac{a}{h}=4$	$\frac{a}{h}=10$	$\frac{a}{h}=4$	$\frac{a}{h}=10$
5	0.2026	0.2026	84.4701	85.4319	1.5232	1.8738
10	0.2026	0.2026	65.3103	66.9826	0.6607	1.0077
15	0.2026	0.2026	53.0596	55.1694	0.3748	0.7176
20	0.2026	0.2026	44.4918	46.8949	0.2338	0.5723
25	0.2026	0.2026	38.1549	40.7663	0.1508	0.4851
30	0.2026	0.2026	33.2768	36.0421	0.0968	0.4270
35	0.2026	0.2026	29.4056	32.2882	0.0594	0.3855
40	0.2026	0.2026	26.2593	29.2333	0.0323	0.3544

Table 2. Effect of degree of orthotropy on transverse displacement of orthotropic (0^0), two layer (0/90) and three layer (0/90/0) laminated beams under sinusoidal thermal load for aspect ratio 4 and 10. (Example-2) ($\alpha_T = 1125\alpha_L$)

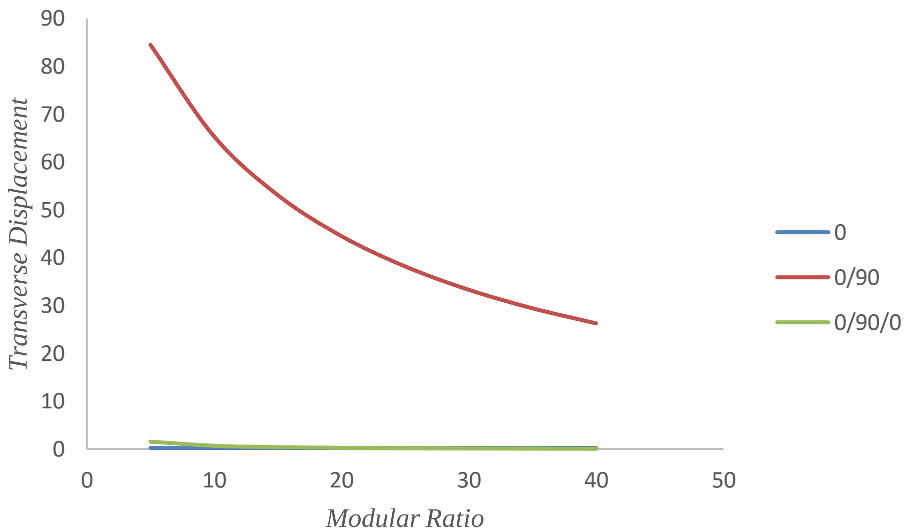

Figure. 3. Effect of change of modular ratio on transverse displacement of single layer, two layer and three layers composite laminated beams for aspect ratio 4

The results are presented in the Table 2 and figure 3.

The numerical results of transverse displacements corresponding to the change in modular ratio are shown in table 2. In single layer orthotropic beam, the transverse displacement obtained is irrespective of modular ratio. There is no change in deflection even when modular ratio changes from 5 to 40. Similarly, in case of three layer beam a small change is noticed in displacement when modular ratio changes.

The effect of change modular ratio on transverse displacement is seen in bilayer (0/90) beam. It is observed that as the modular ratio increases the transvers displacement decreases i.e. the effect of increment in the degree of orthotropy results in reducing the transverse displacement under sinusoidal thermal load in two-layer laminated beam.

The figure 3 shows this effect clearly. The results of transverse displacements in two-layer laminated beam shows that for low modular ratio the transverse displacement is considerably high as compare higher modular ratios.

To reduce the transverse displacement high modular ratio is preferable. These investigations help to select material having minimum thermal displacement.

$\frac{\alpha_2}{\alpha_1}$	0 $\bar{w}$ $\frac{a}{h}=4$	0 $\bar{w}$ $\frac{a}{h}=10$	0/90 $\bar{w}$ $\frac{a}{h}=4$	0/90 $\bar{w}$ $\frac{a}{h}=10$	0/90/0 $\bar{w}$ $\frac{a}{h}=4$	0/90/0 $\bar{w}$ $\frac{a}{h}=10$
2	0.2026	0.2026	0.2364	0.2387	0.2026	0.2029
3	0.2026	0.2026	0.2702	0.2748	0.2026	0.2031
4	0.2026	0.2026	0.3039	0.3109	0.2025	0.2034
5	0.2026	0.2026	0.3377	0.3470	0.2025	0.2036
6	0.2026	0.2026	0.3715	0.3831	0.2024	0.2039
7	0.2026	0.2026	0.4052	0.4192	0.2024	0.2042
8	0.2026	0.2026	0.4390	0.4553	0.2023	0.2044
9	0.2026	0.2026	0.4728	0.4914	0.2023	0.2047
10	0.2026	0.2026	0.5065	0.5274	0.2022	0.2049
11	0.2026	0.2026	0.5403	0.5635	0.2022	0.2052
12	0.2026	0.2026	0.5741	0.5996	0.2021	0.2054
13	0.2026	0.2026	0.6078	0.6357	0.2021	0.2057
14	0.2026	0.2026	0.6416	0.6718	0.2020	0.2059
15	0.2026	0.2026	0.6754	0.7079	0.2020	0.2062
16	0.2026	0.2026	0.7091	0.7440	0.2020	0.2064
17	0.2026	0.2026	0.7429	0.7801	0.2019	0.2067
18	0.2026	0.2026	0.7767	0.8161	0.2019	0.2069
19	0.2026	0.2026	0.8104	0.8522	0.2018	0.2072
20	0.2026	0.2026	0.8442	0.8883	0.2018	0.2074

Table 3. Transverse displacement in orthotropic, two-layer and three-layer laminated beam under sinusoidal thermal load for various ratios of coefficient of thermal expansion for aspect ratios 4 and 10. (Example-3)

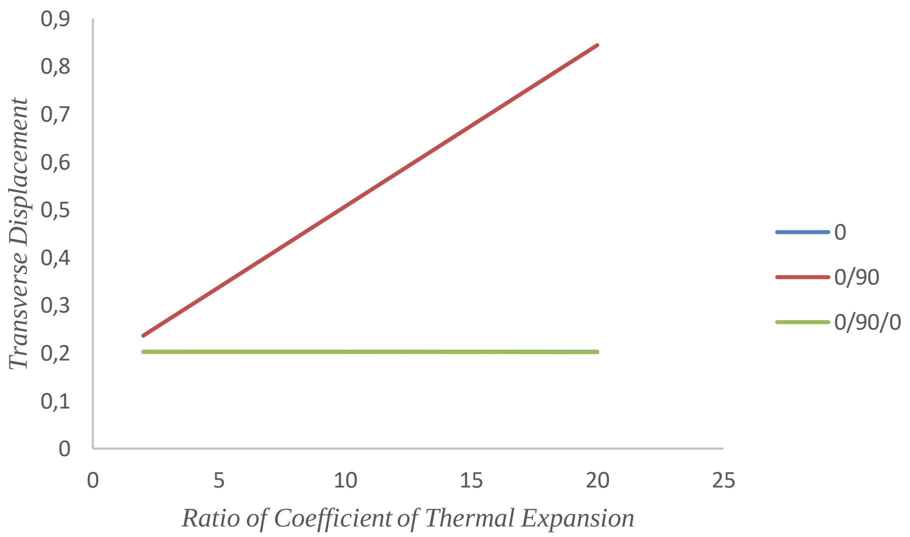

Figure. 4. Effect of change of coefficient of thermal expansion's ratio on transverse displacement of orthotropic, bilayer and three-layer composite laminated beams for aspect ratio 4

The effect of change of coefficient of thermal expansion ratio on transverse displacement is seen only in bilayer (0/90) laminated beam.

These numerical results are presented in table 3 and figure 4. A rise in the results of transverse displacement is noted as the ratio of coefficient of thermal expansion changes from 2 to 20 for aspect ratios 4 and 10. These results are shown in table 3. The figure 4 shows the linear variation in transverse displacement corresponding to the rise in the ratio of coefficient of thermal expansion.

Transverse displacement in bilayer beam (0/90) is directly proportional to the coefficient of thermal expansion ratio. This indicates that to obtain lower transverse displacement low ratio of coefficient of thermal expansion is preferable in bilayer antisymmetric beam. The effect of change of coefficient of thermal expansion ratio on transverse displacement is not seen in orthotropic and three-layer laminated beams as shown in figure 4.

In cross-ply laminated beam understanding of transverse thermal displacements is essential. Researchers can consider these effects while design and manufacturing of space structures.

4. CONCLUSIONS

The effect of change of aspect ratio, degree of material orthotropy and ratio of coefficient of thermal expansion on transverse displacement of orthotropic, two layers antisymmetric (0/90) and three layers symmetric (0/90/0) laminated beam has been studied and presented by using trigonometric beam theory. These three factors have considerable effect on transverse displacement of bilayer beam (0/90).

The considerable change is noted in the results of transverse displacement for thick and thin bilayer beam. The transverse displacements are large for thin bilayer beam. It is clear that for low modular ratio the transverse displacement in bilayer beam is high whereas for high modular ratio transverse displacement is low. Therefore, it is preferable to maintain medium degree of material orthotropy in bilayer beam.

The transverse displacement is directly proportional to the ratio of coefficient of thermal expansions in bilayer beam. These finding are useful during the design and manufacturing of satellite structure with low thermal displacements.

The orthotropic and three-layer laminated beam does not show any significant effect on its transverse displacement even these three factors namely aspect ratio, degree of material orthotropy and change of coefficient of thermal expansion ratio changes. The change in coefficient of thermal expansion ratio has a considerable effect on transverse displacement, since the transverse displacement refers to the movement of beam normal to its length.

The change in the ratio of coefficient of thermal expansion can cause the beam to bend or wrap because of uneven expansion of its layers.

Hence, the present study plays a significant role in understanding the effect material orthotropy and change in coefficient of thermal expansion ratio on transverse displacement of laminated beams.

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References

Akbas, S. D., 2018. Nonlinear thermal displacements of laminated composite beams. *Coupled System Mechanics,* 7 (6), pp. 691-705. https://doi.org/10.12989/csm.2018.7.6.691.

Avhad, P. V., Sayyad, A. S. 2022. On the deformation of laminated composite and sandwich beams. *Curved and Layer, Struct* 9, pp. 1-12.

Bouazza, M., Benseddig, N., Zankour, A.M. 2018. Thermal buckling analysis of laminated composite beams using hyperbolic refined shear deformation theory. *J Therm Stress* 42, pp. 332-340.

Ghugal, Y. M., Kulkarni, S. K. 2013. Flexural analysis of cross play laminated plates subjected to nonlinear thermal and mechanical loadings. *Acta Mechanica*. 224, pp. 675-690.

Ghugal, Y. M., Kulkarni, S. K., Borate, R. D. 2023. Flexural response of cross ply laminated beams subjected to combined thermal and transverse mechanical loads. *Acta Mechanica*. 234, pp. 3725-3735.

Ghugal, Y. M., Ruoccc, E., Gajbhiye, P. D., Bhaiya, V., Yelve, N. P. 2023. Stress analysis of laminated and sandwich beams subjected to concentrated load by using quasi-two-dimensional theory. *Int. J. Computational Materials Science and Engineering* DOI: 10.1142/S204768412350046X.

Ghumare, S. M., Sayyad. A. S. 2017. A new fifth order shear and normal deformation theory for static bending and elastic buckling of P-FGM beams. *Latin American J. Solid and Struct* 14, pp. 1893-1911.

Hai, Q., Ding, Z., Wei-qing, L., Hai, F. 2015. Elasticity solution of laminated beams subjected to thermo-loads. *J. Cent. South Univ.* 22, pp. 2297-2305.

Khdeir, A. A. 2001. Thermal buckling of cross ply laminated composite beams. *Acta Mechanica*. 149, pp. 201-213.

Kulkarni, S. K., Ghugal, Y. M. 2015. Effect of degree of orthotropy on transverse deflection of composites under thermal load. *Advances in Structural Engineering*. DOI 10.1007/978-81-322-2190-6_15.

Naik, N. S., Sayyad, A. S. 2020. 1 D Thermal analysis of laminated composite and sandwich plates using a new fifth order shear and normal deformation theory. *Materials Today* Proceedings. 21, pp. 1084-1088.

Naik, N. S., Sayyad, A. S., 2021. Higher order displacement model for cylindrical bending of laminated and sandwich plates subjected to environmental loads. *Mechanics of Advanced Composite Structures,* 8 (1), pp. 185-201.

Patel, M., Daynes, S., Wang, C. H., 2024. The influence of orthotropy and taper angle on the compressive strength of composite laminates with scarfed holes. *Advanced Material Research,* 891.

Qian, H., Qiu, Y., Lu, C., Yang, Y., Sha, X. 2022. Thermal analysis for clamped laminated beams with non-uniform temperature boundary conditions. *Thin Walled Structures* 179, 109693 DOI: https://doi.org/10.1016/j.tws.2022.109693.

Sayyad, A. S., Avhad, P. V. 2022. Higher order model for the thermal analysis of laminated composite, sandwich and functionally graded curved beams. *J. Thermal Stresses*. 45, pp. 382-400. https://doi.org/10.1080/01495739.2022.2050476.

Sayyad, A. S., Ghugal, Y. M., Naik, N. S. 2015. Bending analysis of laminated composite and sandwich beams according to refined trigonometric beam theory. *Curved and Layer, Struct* 2, pp. 279-289.

Tylikowski, A., Hetnarski, R. B., 1996. Thermally induced Instability of laminated beams and plates. *Journal of Applied Mechanics,* 63 (4), pp. 884-890.

Vidal, P., Polit, O., 2006. A thermomechanical finite element for the analysis of rectangular laminated beams. *Finite Elements in*

Analysis and Design, 42 (10), pp. 868-883 Doi: 10.1016/j.finel.2006.01.005.

Vidal, P., Polit, O., 2009. A refined sine-based finite element with transverse normal deformation for analysis of laminated beams under thermomechanical loads. *Journal of Mechanics of Materials and Structures*, 4 (6), pp. 1127-1155 http://dx.doi.org/10.2140/jomms.2009.4.1127.

Vilar, M. M. S., Masjedi, P. K., Hadjiloizi, D. A., Weaver, P. M., 2023. Analytical interlaminar stresses of composite laminated beams with orthotropic tapered layers. *Composite Structures*, 319 (1), https://doi.org/10.1016/j.compstruct.2023.117063.