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CONTRIBUTING TO A STUDY OF THE INFLUENCE OF SOIL PARAMETER VARIABILITY ON FLOW IN POROUS MEDIA

S. Bir [a, *], H. Ait Aider [a], A. Bouheraoua [a]

[a] Laboratory of Experimental and Numerical Modeling of Materials and Structures in Civil Engineering LaMoMS. Faculty of Construction Engineering. Mouloud MAMMERI University of Tizi-Ouzou, address: Faculty of Construction Engineering. Mouloud Mammeri University B.P.17 RP, 15000 Tizi-Ouzou, ALGERIA, phone/fax: +213 26.10.03.17, E-mails: * birsaid3@gmail.com, h_aitaider@yahoo.fr, abouheraoua2@yahoo.fr

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ABSTRACT:

The paper presents a finite element study of the influence of spatial variations in soil properties on the progress of water flow and solute transport in partially saturated porous media. Four parameters were retained as random variables: the saturation hydraulic conductivity K_{sat} and the saturation water content θ_{sat} given their common interdependence of soil and flow as well as the two parameters of the VAN GENUCHTEN model, α and η. Log-Normal distribution laws were used. The probabilistic study was carried out using the Monte Carlo method. The results of three hundred and fifty (350) flow and solute transport calculations were analyzed in terms of variability of hydraulic loads to show the influence of the structure of soil variability. Three cases of variability were retained:

- The first is statistically homogeneous (150 calculations)
- The second is stratified (150 calculations per horizontal layers)
- The third and last is heterogeneous (50 calculations in both the vertical and horizontal directions).

The reduction in variability of results when the soil also varies in the horizontal direction was shown quantitatively. Comparison with classical deterministic solutions was also carried out. It indicates that the average evolution of flow and transport can be reproduced by a deterministic calculation. The probabilistic treatment cannot therefore explain the existence of a bias in relation to field observations.

1. INTRODUCTION

Predicting flow and transport issues in porous media is an aspect of hydrogeology for which most engineers feel that their calculations are unreliable. Laboratory tests are often considered pessimistic, and it is accepted that permeability and hydraulic conductivity coefficients can be wrong by one or more orders of magnitude. By applying the tools of statistical and probabilistic analysis to flow and transport problems, we can make progress in understanding possible discrepancies between calculations and observations. Publications on the probabilistic analysis of flow and transport issues remained scarce after the 1970s, when the first works by FREEZE (1977), DAVIDSON (1977), SMITH and HEBBERT (1979), and RUSSO and BRESLER (1982) were published. The research conducted by G. AUVINET (1995) at the "Laboratory of Géomécanique - ENSG" VANDŒUVRE - LÈS-NANCY, focused on the potential benefits of the Monte Carlo method for solving permanent flow problems in porous media.

Emphasis was placed on the implementation procedure, which needs to be adapted when the medium is anisotropic and heterogeneous. Application examples were presented, and the problem of the computation time required to obtain a given accuracy was analyzed, as well as the extremely slow convergence of the method, which makes it generally unattractive for current problems. On the other hand, it was emphasized that the method can be of interest for three-dimensional problems with complex geometries. The work of M. SOUTTER and M. MUSY (1997), for their part, focused on the regional assessment of the risks of pesticide contamination of groundwater, for which a computational procedure consisted of combining a stochastic (Monte Carlo) application of deterministic models simulating the local fate of pesticides with geostatistical interpolation techniques, thus also enabling the uncertainties affecting the predictions made to be assessed.

* Corresponding author: Assistant Professor Class "B", Laboratory of Experimental and Numerical Modeling of Materials and Structures in Civil Engineering LaMoMS. Faculty of Construction Engineering. Mouloud MAMMERI University of Tizi-Ouzou, address: Faculty of Construction Engineering. Mouloud Mammeri University B.P.17 RP, 15000 Tizi-Ouzou, ALGERIA, phone/fax: +213 26.10.03.17, E-mail: birsaid3@gmail.com.

Recently, Ali SI SMAIL, Ali BOUHERAOUA and Malek ABDESSELAM (2013) have focused on analyzing the variability of soil parameters on flow and solute transport through hydrodynamic characterization of soils in the upper OUED SÉBAOU valley (Algeria) using a variety of methods: experimental, numerical and analytical.

The work reported in this article focuses on the effect of spatial variability in soil properties on the problem of solute flow and transport in porous media. This question arose because previous attempts at probabilistic modeling of the various flow and transport problems dealt with up to now had ended in failure, due to the existence of a significant bias between predictions and measurements: the improvement of prediction methods could be achieved either by revising the computational models used for the classical deterministic analysis of flow and transport, or by taking better account of the real properties of natural soils, and in particular their variations in space. These two approaches were explored in parallel, and this article describes the results of the probabilistic analysis.

2. THE SOURCES OF ERRORS IN CALCULATING THE FLOW AND TRANSPORT PROBLEM IN POROUS MEDIA

Forecasting hydraulic potentials and flow and transport velocities in porous media remains a complex challenge in which field experience plays a major role. The importance of experience is directly linked to the many sources of error that remain in the calculation procedures used in practice. All untreated effluent is a permanent source of pollution, both for surface water and groundwater, and therefore also for agricultural land. Consequently, it is very important to study infiltration processes, particularly in the unsaturated zone just below the soil surface, as this is the interface through which exchanges between the external environment and the soil take place. Prediction of this type of flow is generally based on the numerical solution of Richards' partial differential equation, which first requires knowledge of the hydrodynamic properties of the flow domain, as well as the initial and boundary conditions that characterize this domain. These hydrodynamic properties are represented both by the water retention curve, which relates the volumetric water content (θ) to the pressure potential in the soil (H), and by the hydraulic conductivity curve, which relates the hydraulic conductivity (K) to (H) or (θ). In practical terms, there are several ways of assessing the hydrodynamic properties of soils, based on field and laboratory measurements. Most of these methods are time-consuming and costly to implement, particularly those for measuring the hydraulic conductivity of fine-textured soils (M.T. VAN GENUCHTEN and D.R. NIELSEN, 1985). It should be noted that these properties often vary in space and time. This can be the result of several factors, such as soil surface heterogeneity, agricultural tillage, swelling and shrinkage of fine soils, particle dispersion, crust formation, and changes in the concentration and chemical composition of the soil solution (ŠIMUNEK J. and M.T. VAN GENUCHTEN, 1996). In recent years, particular interest has been shown in methods for measuring hydrodynamic properties. Studies in this area remain very active, as shown by recent advancements (L. LASSABATERE et al., 2010; X. XU et al., 2012). The disc infiltrometer (K.M. PERROUX and I. WHITE, 1988; R. HAVERKAMP et al., 1994; J. ŠIMUNEK et al., 1999; T.B. RAMOS et al, 2006) is particularly effective in conditions close to saturation (h > -25 cm). It can be used to estimate the effect of macrospores and preferential flows on infiltration processes. It has the advantage of operating in situ, without disturbing the soil, and of being quick to implement. Infiltration flows obtained with the disc infiltrometer are generally analyzed using analytical models, such as that of R.A. WOODING (1968), or a numerical model inverting Richards' equation (J. ŠIMUNEK and M.T. VAN GENUCHTEN, 1996). The principle of this inversion is based on the minimization of an objective function, formulated in terms of the deviation between measured and optimized values. This method is iterative, depending on the initial values of the parameters to be optimized and the desired accuracy. The hydrodynamic characterization envisaged aims to study the evolution of the hydraulic conductivity and moisture content of the valley soils as a function of their pressure potential. In other words, it consists in establishing the characteristic curves K(h) and θ(h). Under unsaturated conditions (h<0), the predominant forces in the soil are capillary and adsorption forces. The latter depend essentially on pore geometry, soil structure and specific surface area of solid particles, or soil texture (A. MUSY and M. SUTTER, 1991). In this regard, the various parameters of the mathematical models giving K(h) and θ(h), essentially depend on the soil texture (percentage of sands, silts and clays), its bulk density and sometimes its organic matter content.

The results of the cumulative infiltration [I(t)] performed in situ using the disk infiltrometer, adopting the multi-potential method of M. D. ANKENY et al. (1991), are combined with the initial θ_i and final θ_f humidities, in order to establish the objective function to be minimized by numerical inversion. The parameters to be optimized are those of the MVG model (Y. MUALEM, 1976, M.T. VAN GENUCHTEN, 1980). However, it remains to define what the representative value of a layer can be based on the values determined at a few points. Probabilistic studies have been undertaken to answer this question.

3. OBJECTIFS ET HYPOTHESES

The aim of the study reported in this article was to quantify the effect of potential spatial variation in the values of soil properties on the course of water flow and solute transport, and to estimate the uncertainty in the result induced by the uncertainty in the data for a constant calculation model. The finite element method was used in this study, in order to be able to vary soil properties in the horizontal direction and not only in the vertical direction, which could have been done using a two - dimensional flow and transport calculation program, as in various previous works cited in this article. As in most numerical models for calculating flow and transport in porous media, the numerical simulation is based on physical assumptions and laws (conceptual model) using the same conceptual model as in the construction phase of the deterministic direct model. The necessary simplifications concern:

- The aquifer or flow has the character of a heterogeneous reservoir. It can however be validly represented at the scale of the selected domain by a flow equation assuming a continuous porous medium (Richards equation) according to the notion of representative element volume REV.
- The mathematical model used to simulate groundwater flow over the entire study area was

created using the COMSOL Multiphysics software for solving the diffusivity PDE (Richards's equation). The simulation model used (Eulerian flow fields) is a rectangular domain, height z = 3m, width r = 2m. A drain 0.4m long and 0.01m thick was placed so that the flow carrying the solute was directed downwards to diffuse the contaminated water. The hydric section is characterized by an initial hydraulic pressure at drain level H_P = 0.01m, so that the initial water level in the soil is at 0.01m, giving a Dirichlet boundary condition in pressure head equal to 0.01m. For the two sides of the rectangular soil section, we assume no flow through them, while at the base we impose a Neumann condition of non-zero flow equal to $N_0 = 0.01K_{sat}$. As for solute transport, this penetrates the drain with water at a concentration of C_0, bearing in mind that the soil is initially taken at zero concentration and has a small leak at its lower base.

The procedure developed in this study for the statistical analysis of flow and transport in porous media has three objectives:

- Take into account the natural variability of calculation parameters in porous media.
- Perform flow calculations in porous media automatically.
- To facilitate analysis of results.

The calculation procedure has three distinct phases:

1. Generation of random values for the four (04) calculation parameters;
2. n calculations (150 for each of the variability cases, statistically homogeneous and stratified, and 50 for the heterogeneous case) required to estimate the solution to the problem;
3. Statistical analysis of results (pressure head: H).

4. ANALYSIS METHOD AND CALCULATION TOOLS:

In order to be able to break down the soil mass subjected to two-dimensional flow into smaller units with different properties in each of the cases of their spatial variability, particularly the last two (stratified, and heterogeneous), we chose to carry out finite element calculations in the COMSOL Multiphysics code, opting for the simulation model of a rectangular domain, having for the mesh represented on (figure 1). The mesh is made up of 15310 nodes and 3744 finite elements. In our study, we are interested in analyzing the effect of soil parameter variability in the calculation of transient flow and solute transport.

The probabilistic analysis was carried out using the Monte Carlo method, which is the only method currently applicable to complex finite element calculations.

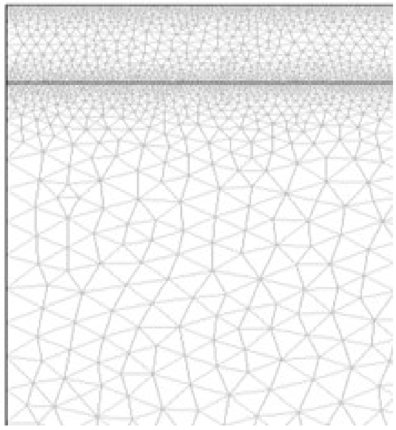

Figure 1- Meshing used to study flow and transport

5. DETERMINISTIC APPROACH

The investigation was carried out using the calculation code for flow and solute transport in partially saturated porous media (COMSOL Multiphysics version 4.2 and 4.3), a finite element software package enabling numerical modeling in 1D, 2D and 3D. This software enables both phenomena to be calculated simultaneously. It includes many other applications in related fields, such as heat transfer, structural mechanics, fluid flow, chemical species transport and many others. In the case of our study, only Richard's Law (Fluid Flow /Porous Media and Subsurface Flow) and Solute Transport (Chemical Species Transport) modules were used, since the conditions are partially saturated soil and solution transport. These two modules are coupled: one calculates the flow of water in the soil, while the second calculates the movement of solutes linked to the movement of water. A brief description of these two modules is given below.

5.1 Flow in porous media with variable saturation

Richard's equation describes the movement of a fluid in a partially saturated porous medium, taking into account changes in hydraulic properties as certain pores become saturated or unsaturated. The Richards Equation interface contains various integrated fluid retention models that can be selected by the user, such as the VAN GENUCHTEN or BROOKS - COREY models. As with the Darcy Law interface, only the pressure is calculated. Richards' equation is non-linear, as hydraulic properties vary with saturation:

$$[C + S_e S]\frac{\partial H_P}{\partial t} + \nabla.[-K_S k_r \nabla(H_P + D)] = Q_S$$

C : retention capacity, it represents the variation of water content in soil per unit variation in matrix load matricielle C(h) = (dθ/dh) ;
S_e : coefficient of effective saturation in soil ;
S : coefficient of water storage in soil ;
H_P(m) : pressure head, t: time (days);
K_S: saturation hydraulic conductivity;
D: coordinate system (x,y);
Q_S: term flow source.

5.2 Solute Transport

This module calculates the movement of a solution in a fluid. In our case, the fluid is water and the solution is salt water. In this module, materials are defined by a dispersivity or dispersion coefficient, effective porosity and sorption and diffusion parameters. Boundary conditions such as constant concentrations or salt mass fluxes can be applied:

$$\frac{\partial}{\partial t}\theta c + \frac{\partial}{\partial t}\rho_b c_p = \nabla.\left[-\theta D_L \nabla c + uc\right] = \sum R_L + \sum R_p + S_c$$

c : dissolved concentration (kg/m);
c_P : mass of contaminant adsorbed per unit dry weight of solid (mg/kg);
θ : volumetric water content;
ρ_b : represents the bulk density (kg/m^3);
D_L : hydrodynamic dispersion tensor (m^2/d);
R_L : represents reactions in water, it is equivalent to reactions involving solutes bound to soil particles (kg/m^3.d);
S_c : solute added per unit volume of soil per unit time (kg/m3.d).

6. THE PROBABILISTIC APPROACH: MONTE CARLO APPROACH

The Monte Carlo approach, which involves running a series of deterministic numerical simulations of the phenomenon under study, using values for the calculation parameters distributed in accordance with the probability laws of the data, was applied by assigning random values to the calculation parameters in each element of the massif.

6.1 Probability laws for calculation data

The soil properties chosen as calculation parameters in our case study are saturation hydraulic conductivity K_{sat} and saturation water content θ_{sat}, given their common interdependence of soil and flow, a choice dictated by the influence of their significant variability highlighted by many authors on the evolution of flow and transport, as well as the two parameters of the VAN GENUCHTEN model, α and η, following a Log-Normal distribution law defined by a mean m and a coefficient of variation (Coef-Var) for each of the four (04) parameters. Statistical analysis of the values retained for each of the parameters yielded the following results shown in Table 1:

Parameters Variable	Log-Normal distribution	
Statistics Moment	Average	Coef-Var
K_{sat}	0.1728 m/day	50%
θ_{sat}	16%	62.5%
α	1.39	25%
η	1.06	30%

Table 1- Data for the finite element calculations (distributions of values produced by the stochastic number generator)

6.2 Calculation procedure

As the statistical approach using the Monte Carlo simulation technique is slow and costly, we've developed an automatic calculation method that significantly reduces computing time and makes it easier to analyze the results. It comprises three distinct parts:

1- Generation of random values for the parameters K_{sat}, θ_{sat}, α and η;
2- Sequential execution of n deterministic flow and transport calculations, with automatic storage of results at different mesh points;
3- Statistics analysis of results.

6.3 Calculation data

We've calculated the pressure head at each time step in a heterogeneous soil mass subjected to an assumed incompressible flow of constant density in the three variability conditions:

6.3.1 Statistically Homogeneous Variability: In this case of probabilistic analysis, the four (04) parameters K_{sat}, θ_{sat}, α, and η vary from case to case in the different 150 calculations performed, their values are drawn randomly in accordance with a Log-Normal law. For each of the calculations, the values of the four (04) parameters K_{sat}, θ_{sat}, α, and η remain constant in each soil mass studied, hence the name statistically homogeneous.

6.3.2 One-dimensional variability called Stratified (homogeneous laminated multilayer's): In this case of probabilistic analysis, we have divided the soil of each case of the 150 calculations carried out into 20 layers, each of which is homogeneous in K_{sat}, θ_{sat}, α and η, we will note the same initial conditions and the same mesh (same number of nodes and elements) as those presented previously.

6.3.3 Two-dimensional heterogeneous variability: In this analysis case, we have subdivided the soil of each case of the 50 calculations performed into 400 subsoil, each of which is homogeneous in K_{sat}, θ_{sat}, α and η, we will also note the same initial conditions and the same mesh (same number of nodes and elements) as those presented previously. The boundary conditions are as follows:

- The hydric section is characterized by an initial pressure head at drain level H_P = 0.01m, so that the initial water level in the soil is at 0.01 m, giving a Dirichlet boundary condition in hydraulic load equal to 0.01m.
- For the two sides of the rectangular soil section, we assume no flow through them, while at the base we impose a Neumann condition of non-zero flow equal to $N_0 = 0.01K_{sat}$.
- As for solute transport, this penetrates the drain with water at a concentration of C_0, bearing in mind that the soil is initially taken at zero concentration and has a small leak at its lower base.

7. CALCULATION HYPOTHESES

The numerical calculation model of flow in a porous medium is a simplified representation of reality. The necessary simplifications concern:

- The porous medium is assumed to be undeformable in the face of attractive flow forces and solute interactions with soil particles;
- Water flow is assumed to be incompressible;
- Water is diffused from the drain and infiltrates into the subsoil;
- The porous medium is initially partially saturated and becomes saturated at a certain time, which explains the use of Richard's equation for a complete unsaturated-saturated model;
- The voids in the porous matrix are occupied by two phases, water and air;
- Air forms a continuous phase with the atmosphere under uniform pressure;
- The water coming from the drain contains a pollutant called "solute" of constant initial concentration C_0;
- The solute infiltrates the soil with the water and moves through it by advection-dispersion;
- In addition, the solute adsorbs to soil particles, reducing aqueous concentrations and slowing solute movement relative to water. Biodegradation also reduces concentrations in both liquid and solid phases;
- Sorption and biodegradation are linearly proportional to aqueous concentrations. The flow contained in the drain is the only pollutant source, and the solute is free to leave the soil column with the flow stream. Initially, the soil is not contaminated by the solute. We will model its transport over a 10-day period.

Only 150 calculations were performed for the first two cases of spatial variability, and 50 for the third and final case, which does not necessarily guarantee the accuracy of the conclusions of the Monte Carlo probabilistic analysis. However, the cumbersome nature of the numerical approach imposed a strict limit on the total number of calculations. The data for the calculations were created using a random number generator (Excel in our case study), based on a Log-Normal distribution, with the following theoretical mean values, and coefficient of variation as shown in Table 1.

Five hundred (500) values of each of the four (04) parameters were created for the case of statistically homogeneous variability, one thousand (1000) for the case of one-dimensional (stratified) variability and two thousand (2000) for the case of two-dimensional (heterogeneous) variability. In addition, we performed a deterministic calculation to compare these results with those obtained in the case of probabilistic analysis: the first uses the arithmetic mean of the four (04) parameters. The parameters averaged are the values of K_{sat}, θ_{sat}, α, and η created by the random number generator (Excel in our cases of study) following the probability laws of each parameter.

8. RESULTS OF PROBABILISTIC CALCULATIONS

We will successively present and compare the main results of statistically homogeneous, one-dimensional and two-dimensional variability calculations.

8.1 Statistically Homogeneous Variability

In this case, the calculation parameters (K_{sat}, θ_{sat}, α, and η) remain constant in each calculation, but vary when moving to another. In this way, all the calculation assumptions and initial conditions as well as the same mesh (same number of nodes and elements) as those presented previously are verified. Once the series of one hundred and fifty (150) calculations had been completed, we determined the averages of the hydraulic load, for each time step, at different positions x along the width of the soil mass, together with their standard deviation and coefficient of variation.

8.1.1 Pressure head: The calculation results are illustrated in Figures 2, 3 and 4, devoted to pressure head. First of all, it can be seen that the evolution of the average pressure head does not begin simultaneously in all the horizontal positions of the soil mass, and that the points furthest from the drain show a delay, to show an evolution only at the end of the 5th day, to mention only the position (x=1.5m). For the two remaining positions (x=1.8, and 2m), it can be seen that the load did not show a significant evolution. As for the first two positions (x=0.5m, and x=0.7m), they showed a clear and rapid evolution of the pressure head from the first day (t=1day). This can be explained by the fact that this zone of soil is in a non-saturation phase and that the pores of the medium are empty, which results in high flow velocities, thus increasing the pressure head. After 10 days, the pressure head is almost stabilized and decreases linearly with the width of the bed. This is due to the fact that the pores in the medium are more or less saturated with water, so the number of empty pores decreases over time, reducing flow velocities and therefore pressure head.

At the following positions: 0.5m, 0.7m, 1m, 1.2m, 1.5m and 1.8m, the pressure head have a standard deviation that does not exceed 0.8m during water flow. This standard deviation passes through a maximum at each position of the soil mass width. The standard deviation, initially greater near the drain boundary, evolves differently over time as a function of the horizontal position x: we can see that as we move away from the abscissa x = 0.5m, the value of the maximum decreases to reach its minimum value (0.05m) at position x = 1.5m, then rises again to the value (1.5m) at position x = 1.8m.

The lowest values of the coefficients of variation of the pressure head at positions x = 0.5m, x = 0.7m, and 1m quickly reached their maximum values of less than 90%, and 65% respectively, and quickly fell below 45%, then quickly fell below 40% for x = 0.5m, and quickly fell below 50% for x = 0.7m, decreasing over time to end below 11%, 31%, and 35% respectively, as the average load increased. As for the remaining positions (x = 1.2m, 1.5m, 1.8m, and 2m), they presented profiles of coefficients of variation that increased with horizontal position x (width of the massif) in more or less saturated zones. These two results, characterized by the difference in the shape of their curves, testify to the regularizing effect of spatial variations in soil properties, since the coefficients of variation of the four (04) random parameters (K_{sat}, θ_{sat}, α, and η) are equal to 50%, 62.5%, 25% and 30% respectively, and demonstrate that it is in the partially saturated zones that it has the greatest impact.

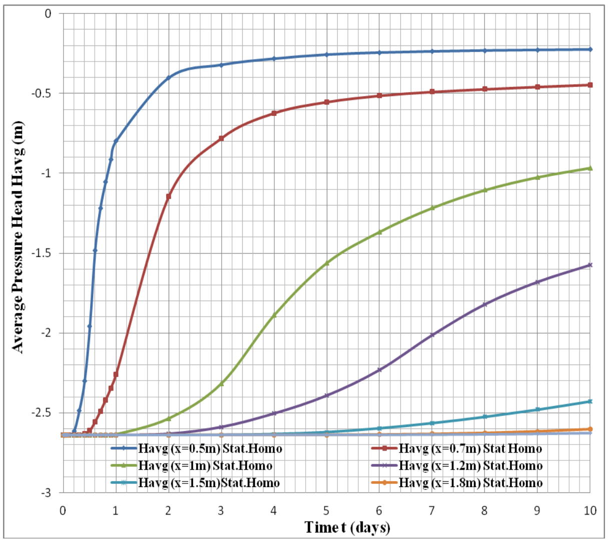

Figure 2- Statistically homogeneous case: Average Pressure Head for different positions x

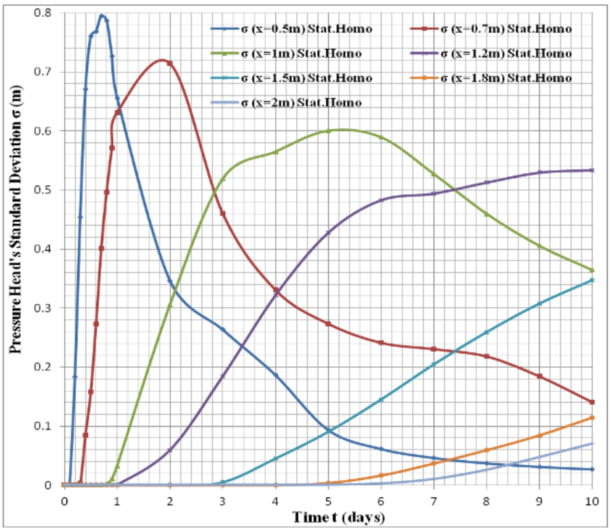

Figure 3- Statistically homogeneous case: Pressure Head's Standard Deviation for different positions x

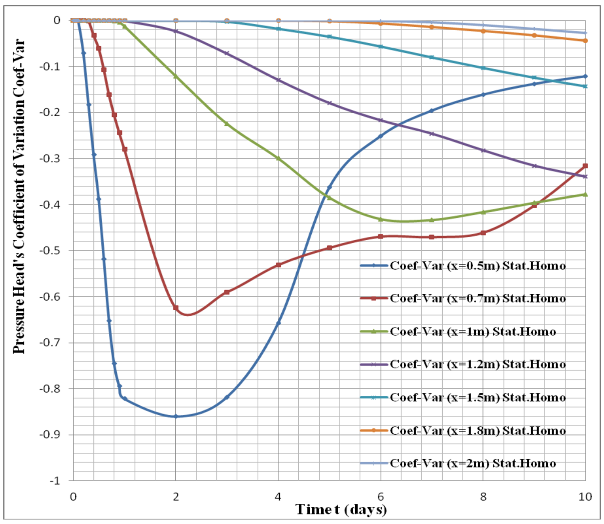

Figure 4- Statistically homogeneous case: Pressure Head's Coefficients of Variation for different positions x.

8.2 One-dimensional variability called Stratified (homogeneous laminated multilayers)

In this case, the calculation parameters (K_{sat}, θ_{sat}, α, and η) vary with depth, but remain constant in each layer over time. Thus, all conditions at the interfaces separating two soil layers of different hydrodynamic nature are verified, which are respectively:

- Continuity of soil pressure: regardless of the sequence of strata, soil water pressure is continuous across the interface, so, $h|_{L-\varepsilon} = h|_{L+\varepsilon}$;
- Continuity of water mass flow for which the flow leaving a layer must be equal to that entering the underlying layer, so, $q|_{L-\varepsilon} = q|_{L+\varepsilon}$; L: interface dimension and ε the infinitely small positive value. Heterogeneities result in discontinuities in the coefficients of the system of equations as they pass from one layer to another. However, certain physical quantities remain continuous: Darcy velocity, pressure and contaminant fluxes. The soil mass is divided into twenty (20) layers of z = 0.15m thickness, which corresponds to the same mesh of the deterministic case of 15310 nodes and 3744 finite elements, shown in figure 1. The points selected for probabilistic analysis will hereafter be designated by their depth from the top surface of the mesh. Once the series of one hundred and fifty (150) calculations had been completed, we determined the averages of the pressure head, for each time step, at different positions x along the width of the soil mass, together with their standard deviation and coefficient of variation.

8.2.1 Pressure head: Figures 5, 6 and 7 respectively examine pressure head values: average (mean) values, standard deviations and coefficients of variation at different positions x of the soil mass width. For the first two elements of the results, the overall trend is the same as in the previous calculation, with a maximum of standard deviations at each position x of the width of the soil mass, while for the coefficients of variation the trend is quite the opposite, with a more or less significant and regular increase for each position x. First of all, we can see that the evolution of the average pressure head does not begin simultaneously in all horizontal positions of the soil mass, and that the points furthest from the drain lag behind, showing an evolution only at the end of the 3rd day. For the three remaining positions (x = 1.5m, 1.8m, and 2m), we can see that the load did not show any significant evolution. As for the first two positions (x = 0.5m, and x = 0.7m), they showed a clear and rapid evolution of the pressure head from the very first day (t = 1day). This can be explained by the fact that in this zone, the soil is in a non-saturation phase and the pores in the medium are empty, resulting in high flow velocities which in turn increase the pressure head. After 10 days, the pressure head is almost stabilized and decreases linearly with the width of the bed. This is due to the fact that the pores in the medium are more or less saturated with water, so the number of empty pores decreases over time, reducing flow velocities and therefore pressure head.

At positions 0.5m, 0.7m, 1m, 1.2m, 1.5m and 1.8m, a pressure head has a standard deviation of no more than 0.9m during water flow. This standard deviation passes through a maximum at each position of the soil mass width. The standard deviation, initially greater near the drain boundary, evolves differently over time as a function of the horizontal position x: we can see that as we move away from the abscissa x = 0.5m, the value of the maximum decreases to reach its minimum value (0.025m) at position x = 1.8m, only to rise again to the value (0.23m) at position x = 1.5m.

The pressure head's coefficients of variation decrease over time, as the average load increases. The coefficients of variation increase with horizontal position x (bed width) in more or less saturated zones. The lowest values, at position x = 0.5m, and x = 0.7m, quickly fall below 70% and 60% respectively, ending up below 94% and 70% respectively. For the other positions, the curves show a regular and more or less increasing trend, with a maximum at the end for each. We see the same result as in the previous case, which bears witness to the regulating effect of spatial variations in soil properties, and also demonstrates that it is at the level of partially saturated zones that it has the greatest impact.

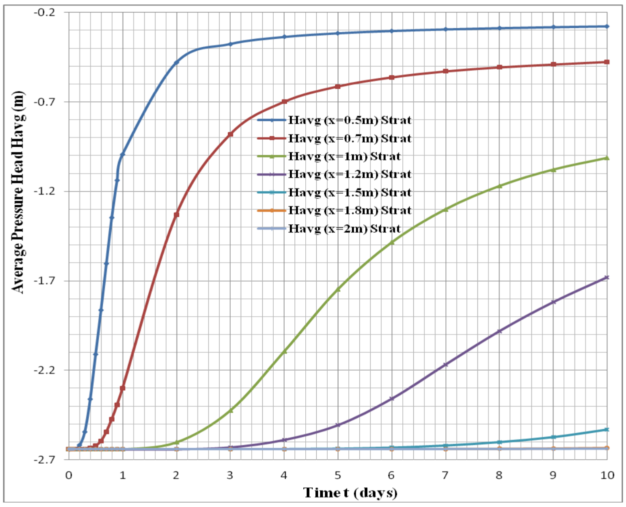

Figure 5- Stratified case: Average Pressure Head for different positions x

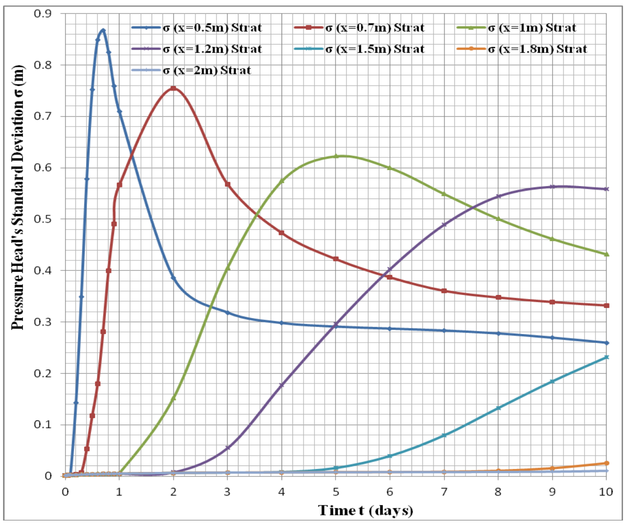

Figure 6- Stratified case: Pressure Head's Standard Deviations for different positions x

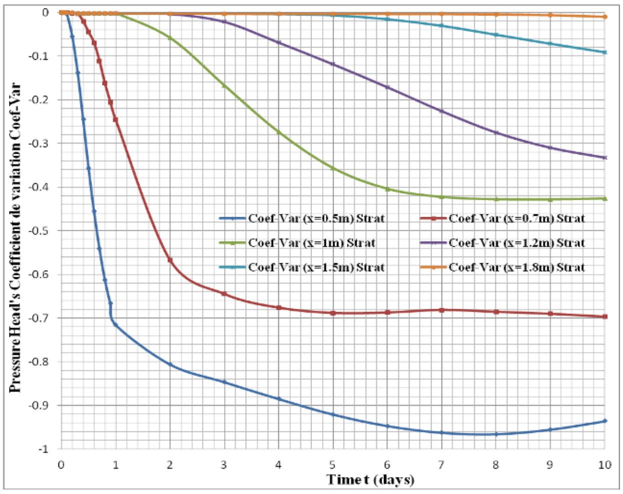

Figure 7- Stratified case: Pressure Head's Coefficient of Variation for different positions x

8.3 Two-dimensional heterogeneous variability

In this case involving probabilistic analysis, we have subdivided the soil in each case of the 50 calculations carried out into 400 subsoils, each of which is homogeneous in K_{sat}, θ_{sat}, α, and η, we will also note the same initial conditions and the same mesh (same number of nodes and elements) as those presented previously. As in the previous case once the series of fifty (50) calculations had been completed, we calculated the mean values, standard deviations and coefficient of variation of the pressure head at different positions (x) of the soil mass width. The results of these statistical calculations are shown in Figures 8, 9 and 10. In this case, the calculation parameters K_{sat}, θ_{sat}, α, and η vary with depth and, within the same horizontal layer, from one element (or group of elements) to another. The massif, 3m thick and 2m wide, is divided into twenty (20) layers of thickness z = 0.15m in the vertical direction and width x = 0.1m in the horizontal direction. The choice of the thickness value is dictated by the need to impose a two-dimensional pore water flow and a non-deformable porous medium for each soil element obtained from the subdivision. We also note the same initial conditions and the same mesh (same number of nodes and elements) as those presented previously.

8.3.1 Pressure head: Figures 8, 9 and 10 analyze the pressure head mean values, standard deviations and coefficients of variation at the various x positions. The pattern of results is not the same overall as in the previous calculation, giving quite the opposite profiles for the average pressure head, with a maximum of standard deviations at some positions and an increasing order at others, and a more or less regular increase in coefficients of variation after an initial period of decrease. Coefficients of variation are of the order of 20%. First of all, it can be seen that the evolution of the average pressure head does not begin simultaneously in all horizontal positions of the soil mass, and that the points closest to the drain show an enormous delay, to show an insignificant evolution only at the end of the 9th day for position (x = 0.5m), and the 8th day for (x = 0.7m) respectively. For the three positions (x = 1m, 1.2m, and 1.5m) we can see that the average load took less time to show increasing profiles over time. As for the first two positions (x = 1.8m, and x = 2m), they showed a clear and rapid evolution of the pressure head from the first day (t = 1day).

This can be explained by the fact that in this zone the soil is in the unsaturated phase and the pores in the medium are empty, resulting in higher flow velocities and a consequent increase in the pressure head. At the end of 10 days, the pressure head is almost stabilized and increases linearly with the width of the bed. This can be explained by the fact that the further we move away from the vertical limit of the drain, the less saturated the pores are with water, so their void number decreases over time, resulting in higher flow velocities which increase pressure head. This result is quite the opposite of the two previous cases, demonstrating the effect of the change in scale (from larger to smaller) of the variability of soil parameters on flow, which is visibly noticeable at the saturation front, i.e. between the interface of the partially saturated and fully saturated zones.

At positions 0.5m, 1m, 1.2m, 1.5m and 1.8m, and 2m, the pressure head have a standard deviation that does not exceed 0.72m during water flow. This standard deviation passes through a maximum at each of the positions, x = 1m, x = 1.2m, x=1.5m and, x=1.8m of the width of the soil mass to decrease at the end to less significant values. It can be seen that for these four positions, the standard deviation, initially greater at x = 1.8m, evolves differently over time as a function of the horizontal position x: it can be seen that the further away from the abscissa x = 1m the value of the maximum tends to fall more rapidly, reaching its minimum value (0.027m) at position x = 1.8m. For the remaining two positions, we note that at x = 0.5m, the standard deviation initially shows a regular growth pattern which is maintained over time to reach its maximum below (0.38m) at the end. For the last position, x = 0.7m, which is a particular feature of this final case of variability, the standard deviation initially takes on its minimum value of less than 0.59m, rising very rapidly over time to reach its maximum of no more than 2.69m at the end. This result is different from the two previous cases, and shows us, through the x = 0.7m position, the extent to which the effect of change of scale (passage from a larger scale to a smaller one) of variability of soil parameters can impact the course of flow in porous media, and that it is always at the partially saturated zone (x = 0.7m: wet zone and close to the vertical limit of the drain) that this change in the effect of variability visibly results.

The pressure head's coefficients of variation for the positions x = 1m, x = 1.2m, x = 1.5, and x = 1.8m, initially decrease over time, due to the increase in average load, to reach its minimum at x = 1.8m and then rise again to reach its maximum at the end at position x = 1.2m. Coefficients of variation increase with horizontal position x (width of bed) in more or less saturated zones. For the other positions x = 0.5m, and x = 0.7m, the curves show a steep downward trend, with a maximum at the end for each, while at the final position x = 2m, the coefficient of variation shows no change, as the average pressure head is at its maximum. These two results, characterized by the difference in the shape of their curves, testify to the regularizing effect of spatial variations in soil properties, since the coefficients of variation of the four (04) random parameters (K_{sat}, θ_{sat}, α, and η) are respectively equal to 50%, 62. 5%, 25% and 30% respectively, demonstrating that it is in the more to less saturated zones that it has the greatest impact, which is quite the opposite of the two previous cases where this finding was made in the partially saturated zones, This demonstrates once again the effect of changing the scale of variability of soil parameters on the prediction of the flow path in partially saturated porous media, which is indeed our case of so-called complete modeling: saturated and unsaturated.

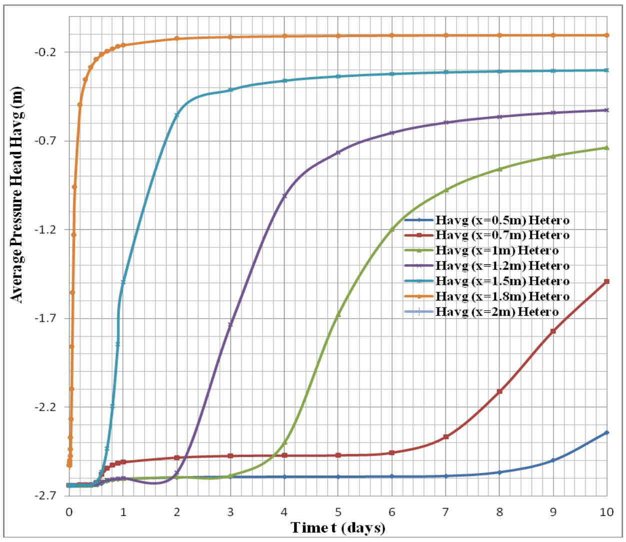

Figure 8- Heterogeneous case: Average Pressure Head for different positions x

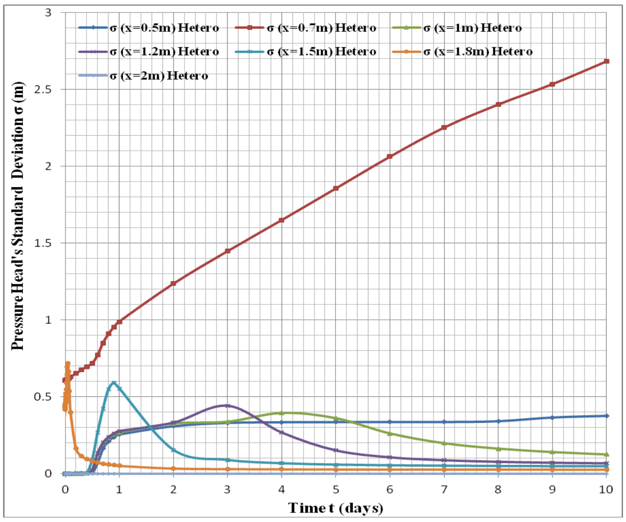

Figure 9- Heterogeneous case: Pressure Head's Standard Deviation for different positions x

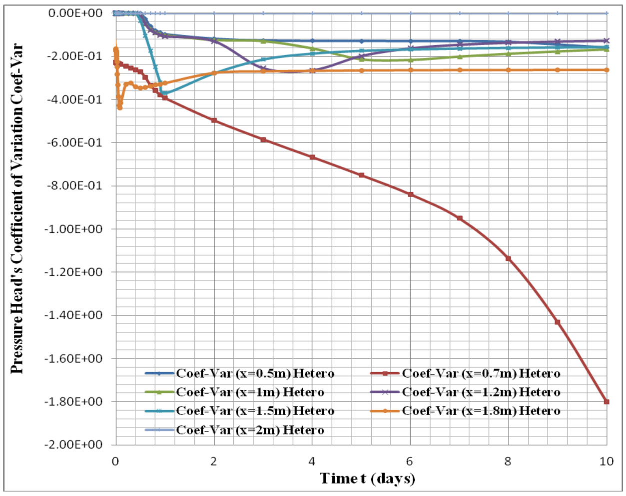

Figure 10- Heterogeneous Case: Pressure Head's Coefficients of variation for different positions x

9. COMPARISON OF PROBABILISTIC AND DETERMINISTIC CALCULATIONS

9.1 Comparison of different probabilistic calculations

9.1.1 Pressure head: The first two probabilistic calculations (statistically homogeneous and stratified) gave more or less equal trends in mean hydraulic load values for any position along the width of the soil mass. As for the third probabilistic case (heterogeneous), it gave evolutions little different from those of the first two cases of calculation of the average values of the pressure head at position x = 1 m. As for the rest of the positions, the evolution of the mean values of the pressure heads was very different and far from that of the first two probabilistic calculations, for which we note the following observations:

- ❖ Pressure heads (heterogeneous calculation) much lower and lower than those of the first two calculations respectively for x = 0.5m, and, x = 0.7m.
- ❖ Pressure heads (heterogeneous calculation) higher and much higher than those of the first two calculations respectively for x = 1.2m, and, x = 1.5m, x = 1.8m, x=2m.

The pressure head's standard deviations of the first two calculations (statistically homogeneous, and stratified) show little difference for all positions except for the last position x = 2m, where the similar evolution of the pressure head's standard deviations of the last two calculations (stratified, and heterogeneous) is very low compared to that of the first calculation (statistically homogeneous). The pressure head's standard deviations of the last calculation (heterogeneous) were clearly lower than those of the first calculations (statistically homogeneous, and stratified) at positions x = 1m, x = 1.2m, x = 1.5m, and x = 2m. The pressure head's standard deviation of the last calculation is respectively, initially lower than those of the first calculations to end up higher at x = 0.5m, and initially clearly higher than those of the first calculations to end up globally little different at x = 1.8m. As for the x = 0.7m position, the standard deviation of the last calculation (heterogeneous) is significantly higher than that of the first two calculations (statistically homogeneous and stratified). As for the pressure head's coefficients of variation of the first two probabilistic calculations (statistically homogeneous and stratified), they show little difference for all positions except for the last position x = 2m, where the similar evolutions for the last two calculations (stratified and heterogeneous) are greater than those of the first calculation (statistically homogeneous). The pressure head's coefficients of variation of the last (heterogeneous) calculation are respectively much higher and lower than those of the first calculations (statistically homogeneous, and stratified) at the positions x = 0.5m, x = 1m, x = 2m, and x = 0.7m, x = 1.5m, x = 1.8m. The pressure head's coefficient of variation of the last calculation is initially lower than those of the first calculations, and ends up being higher at the x = 1.2m position.

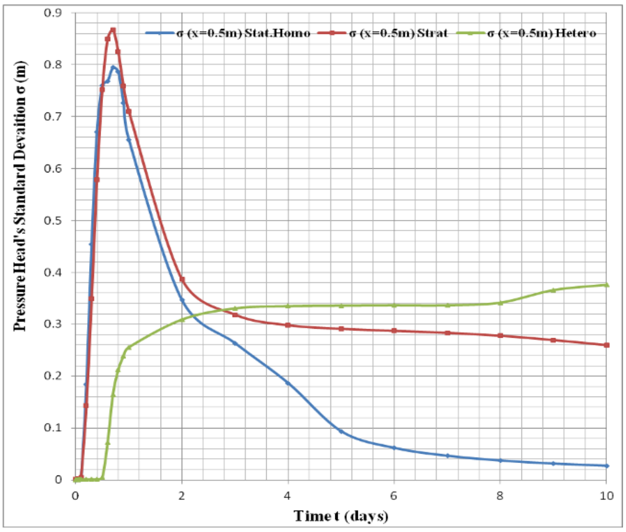

Figure 11- Pressure Head's Standard Deviations for different probabilistic cases (x=0.5m)

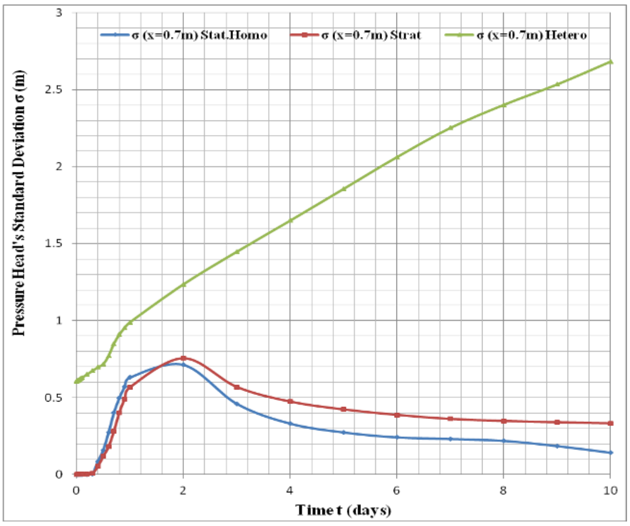

Figure 12- Pressure Head's Standard Deviations for different probabilistic cases (x=0.7m)

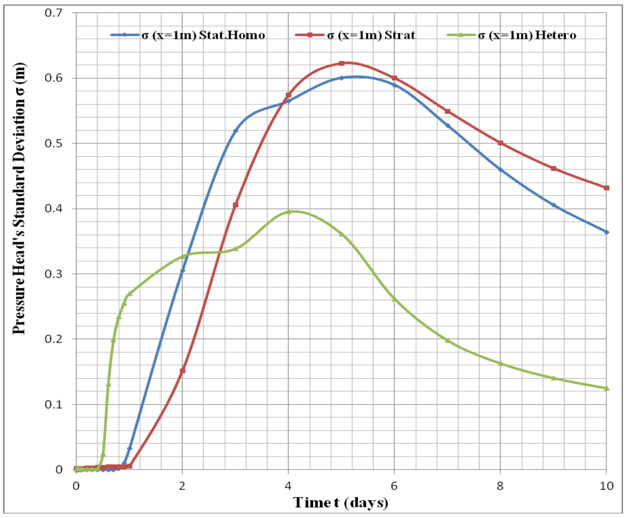

Figure 13- Pressure Head's Standard Deviations for different probabilistic cases (x=1m)

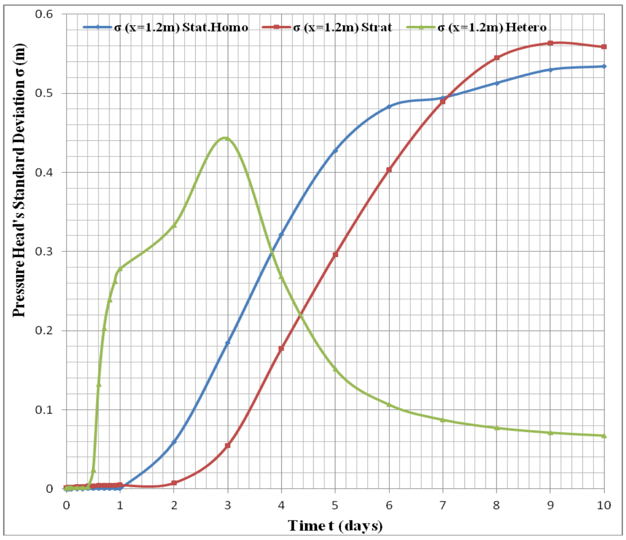

Figure 14- Pressure Head's Standard Deviations for different probabilistic cases (x=1.2m)

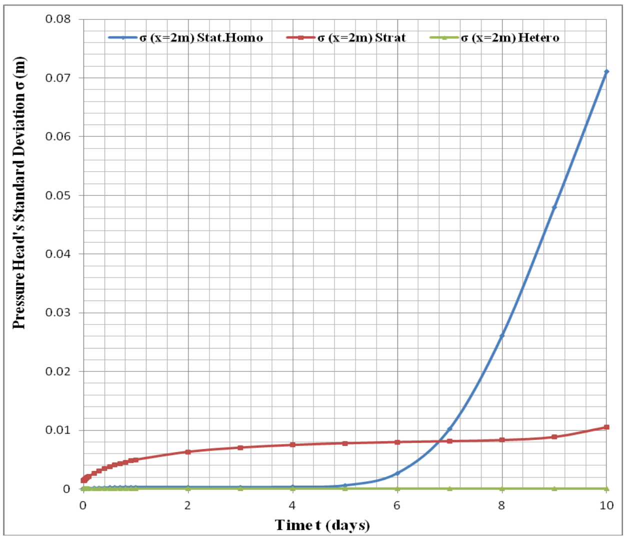

Figure 17- Pressure Head's Standard Deviations for different probabilistic cases (x=2m)

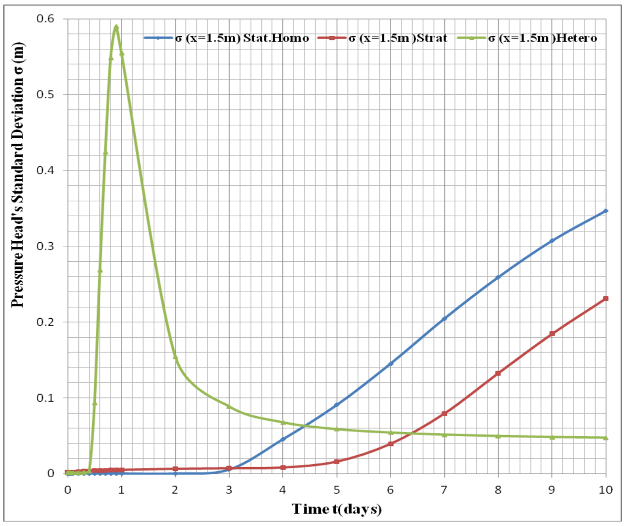

Figure 15- Pressure Head's Standard Deviations for different probabilistic cases (x=1.5m)

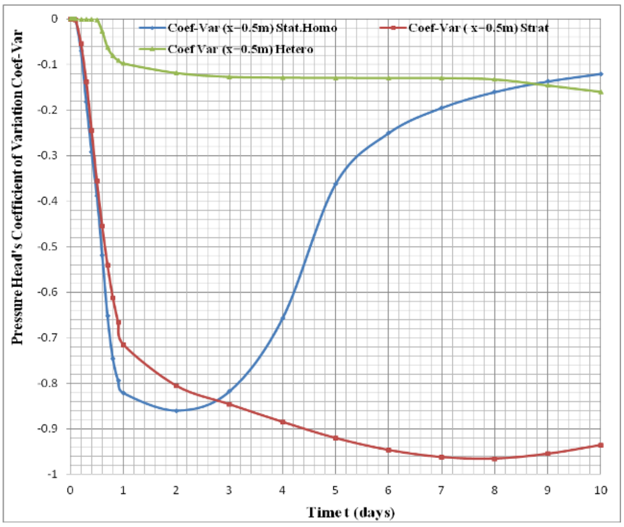

Figure 18- Pressure Head's Coefficients of variation for different probabilistic cases (x=0.5m)

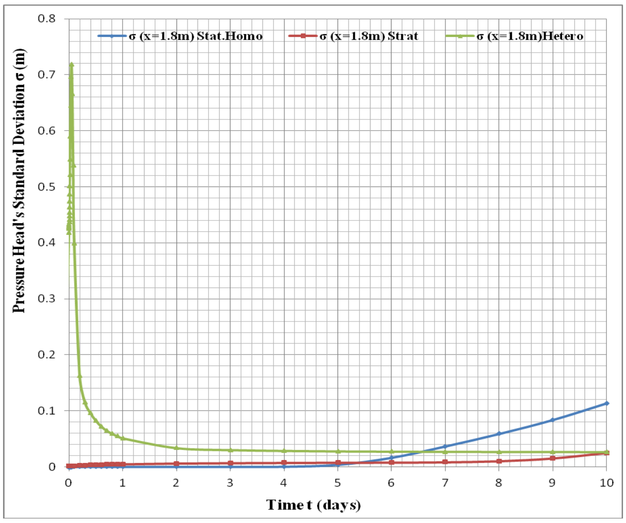

Figure 16- Pressure Head's Standard Deviations for different probabilistic cases (x=1.8m)

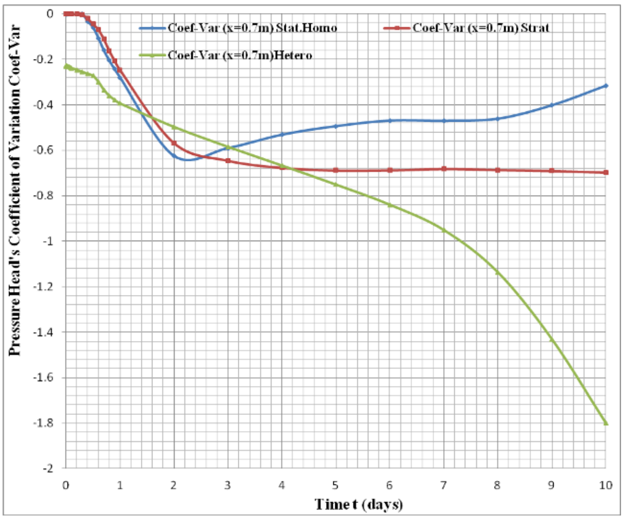

Figure 19- Pressure Head's Coefficients of variation for different probabilistic cases (x=0.7m)

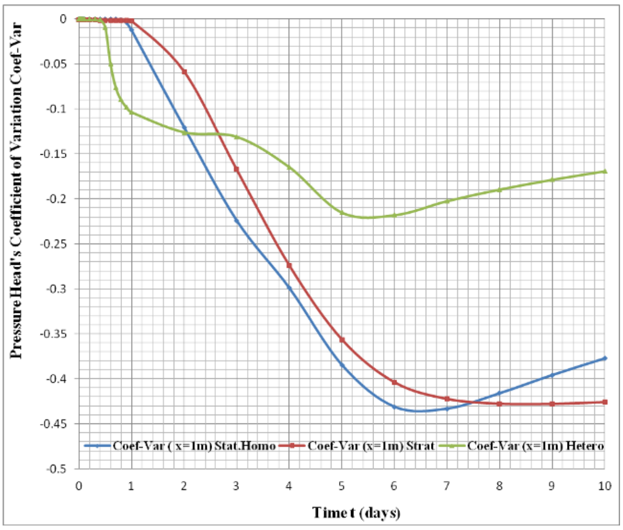

Figure 20- Pressure Head's Coefficients of variation for different probabilistic cases (x=1m)

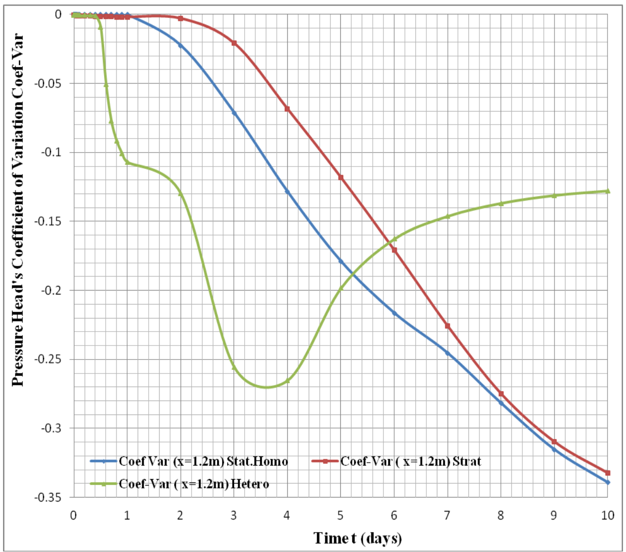

Figure 21- Pressure Head's Coefficients of variation for different probabilistic cases (x=1.2m)

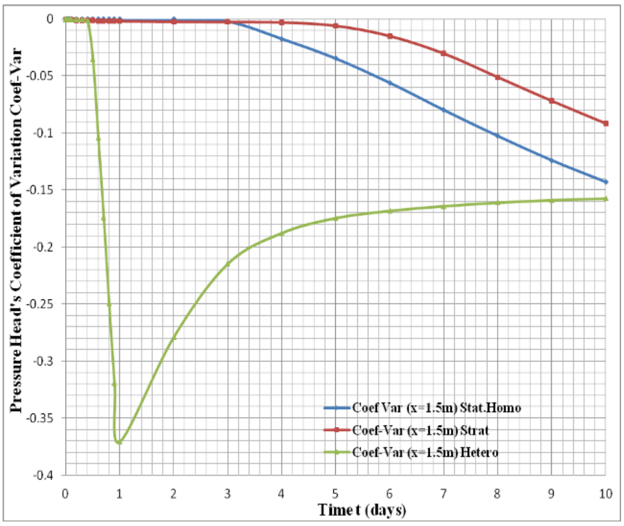

Figure 22- Pressure Head's Coefficients of variation for different probabilistic cases (x=1.5m)

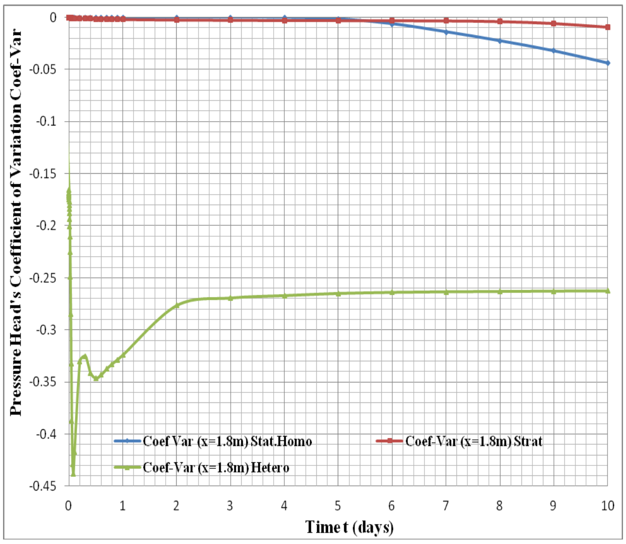

Figure 23- Pressure Head's Coefficients of variation for different probabilistic cases (x=1.8m)

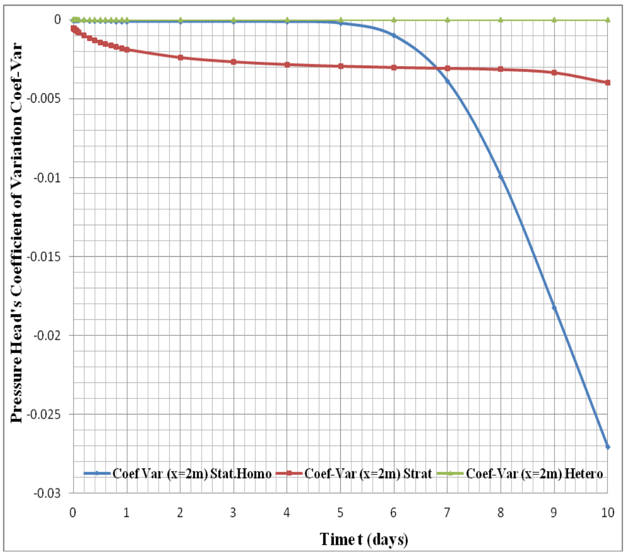

Figure 24- Pressure Head's Coefficients of variation for different probabilistic cases (x=2m)

- Comment: If we can draw a conclusion from the various comparisons above, we can say that it was the last probabilistic calculation (two-dimensional, or heterogeneous) that showed overall more distinction from the first two probabilistic calculations (statistically homogeneous, and one-dimensional or stratified) in terms of soil parameter variability, whose effect and impact was visibly greater and more influential on the problem of flow and transport in porous media. This demonstrates once again the effect of changing the scale of variability of soil parameters on the different results obtained from the various calculations.

9.2 Comparing probabilistic and deterministic calculations

The pressure heads curves obtained through the three probabilistic calculations are compared in Figure 8 with the curves obtained in the deterministic calculation across the various x positions of the soil mass width.

9.2.1 Pressure head: The first two probabilistic calculations (statistically homogeneous, and stratified) gave more or less equal evolutions of the mean values of the pressure heads with those of the deterministic calculation for all positions of the width of the soil mass, as for the third and last calculation (heterogeneous) this one gave respectively evolutions, clearly weaker at position x = 0.5m, x = 0.7m, little different at position x = 1m, and significantly higher at positions x = 1.2m, x = 1.5m, x = 1.8m, x = 2m, than that given by the deterministic calculation.

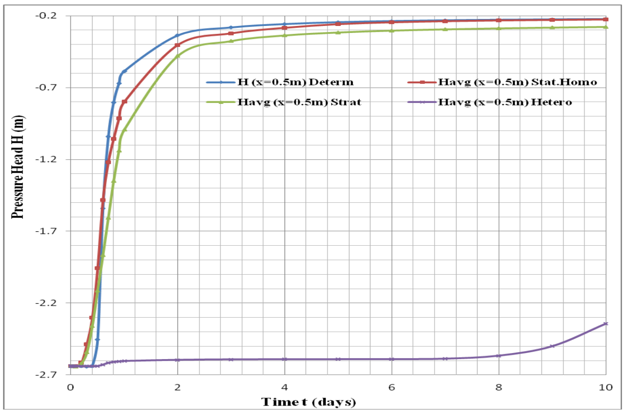

Figure 25- Comparing Average pressure Head for probabilistic and deterministic cases (x=0.5m)

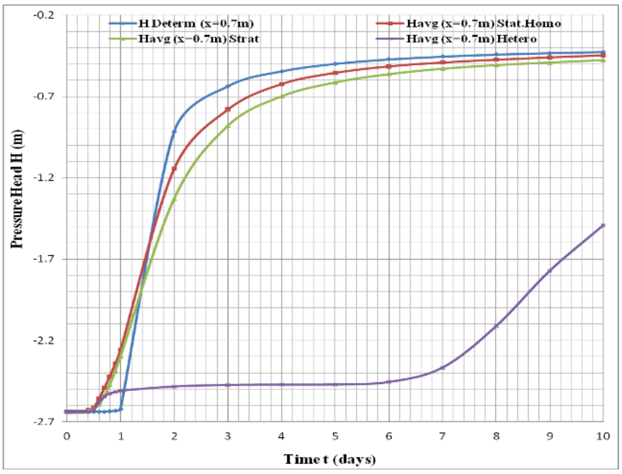

Figure 26- Comparing Average Pressure Head for probabilistic and deterministic cases (x=0.7m)

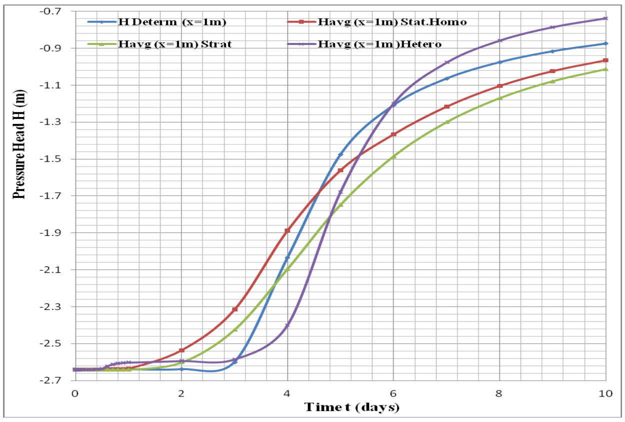

Figure 27- Comparing Average Pressure Head for probabilistic and deterministic cases (x=1m)

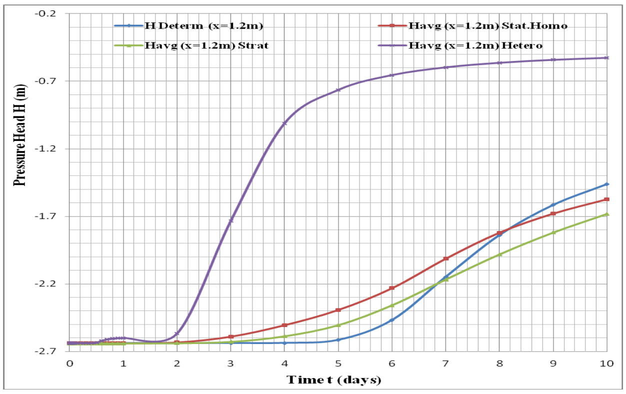

Figure 28- Comparing Average Pressure Head for probabilistic and deterministic cases (x=1.2m)

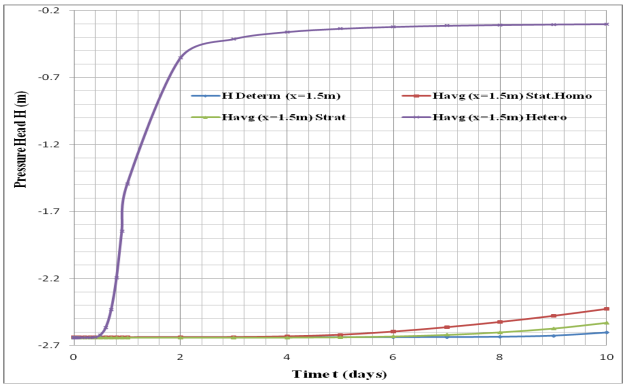

Figure 29- Comparing Average Pressure Head for probabilistic and deterministic cases (x=1.5m)

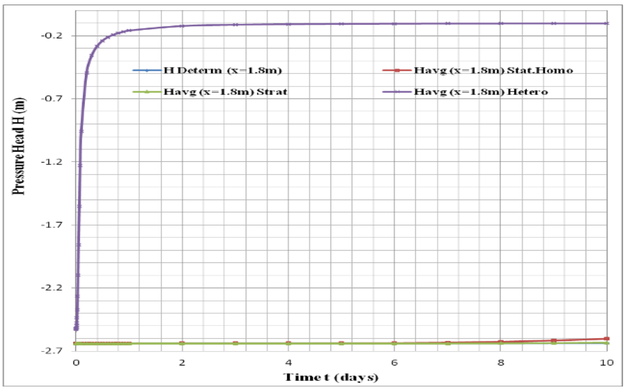

Figure 30. Comparing Average Pressure Head for probabilistic and deterministic cases (x=1.8m)

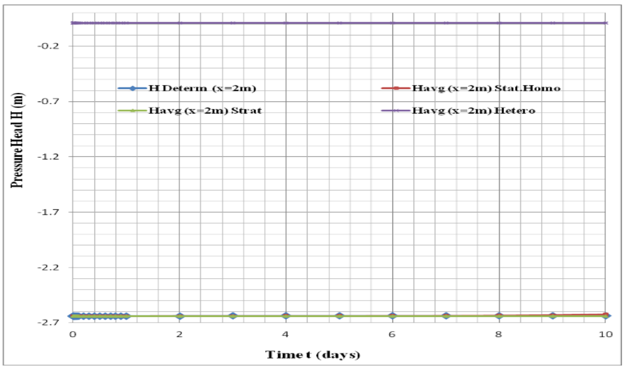

Figure 31. Comparing Average Pressure Head for probabilistic and deterministic cases (x=2m)

- Comment: To sum up with a final comment on the above comparisons, the calculations presented in this article show that there is little difference between the result of a deterministic calculation using an average value for the four (04) random parameters (K_{sat}, θ_{sat}, α, and η) and the average evolution of pressure heads deduced from the first two probabilistic calculations (statistically homogeneous and stratified). Furthermore, the coefficients of variation around the mean curves are low. As for the last probabilistic calculation (heterogeneous), we can say once again that it demonstrated that the effect of soil parameter variability on flow and transport in porous media was visibly the most important and influential on the variability of the results, through the significantly greater differences between the mean load values of the probabilistic (two-dimensional, or heterogeneous) and deterministic calculations than between the latter's average pressure head's values and those of the first two probabilistic calculations (statistically homogeneous, and stratified) at different positions (x) in the width of the soil mass. We also note that the last probabilistic calculation (two-dimensional, or heterogeneous) gave the least similar pressure head's profiles at positions x = 0.5m, x = 0.7m, x = 1m, and, x = 1,2m compared to those of the first two, due to the effect of a change in the scale of variability of soil parameters, resulting in lower moisture levels at these positions, characterized by highly channeled preferential flow paths, i.e. groundwater flows along a minority of paths due to the high variability of soil parameters in the various more or less saturated zones, resulting in lower flow velocities, and thus, a slow accumulation of the pollutant, delaying (delay factor) its mixing with the flow and making its transport further away in these media more complex than in homogeneous media, where the number of available paths is much more limited and highly variable in space and time. These results show that the (theoretical) behaviour of partially saturated porous media is far from the behaviour that can be calculated with average values of soil hydrodynamic parameters.

This issue has also been studied by M. VAUCLIN and G. VACHAUD, who analyzed the influence of hydrodynamic parameters (pressure, water content, hydraulic conductivity) on the flow of infiltration and internal drainage tests carried out on a 1ha plot of bare soil in BAMBEY (Senegal), in order to characterize their variability according to lognormal distribution laws, which tend to become normal as the two tests progress. Based on the assumption of similarity in porous media (verified in this study), these authors also analyzed the results obtained according to the scaling factor theory, which showed, in addition to good agreement with experience, that the mean solution obtained from the mean parameters (deterministic solution) is clearly different from the mean of the solutions (stochastic solution). These differences, as well as the variance, are highly dependent on soil variability, expressed here by the coefficient of variation of the scaling factors. This obviously calls into question the concept of an equivalent porous medium. On the basis of these few results, they concluded that:

- ✓ The use of scaling factor theory for hydrodynamic properties, coupled with classical water transfer equations, has enabled relatively simple modeling of infiltration and drainage at the scale of a one-hectare plot. The good agreement between experimental and numerical values shows the suitability of this type of modeling.
- ✓ The comparison between deterministic and stochastic solutions shows that the deterministic approach is inadequate for modeling in-situ water transfer in non-uniform soils, since the spatial mean of the solutions is different from the mean solution that would be obtained by considering average parameters. This calls into question the concept of an equivalent uniform porous medium, already challenged in hydrogeology for saturated porous media.
- ✓ The approach used here implicitly assumes that spatial variations in soil properties are totally random. Note that the absence of auto-correlation between observations 23 m apart, highlighted here by the semi-variogram, does not necessarily imply independence below this distance. In the case of spatial structure, a conditional simulation approach should be considered.

ANTTI TASKINEN, HANNU SIRVIÖ & MICHAEL BRUEN (2008) analyzed the effects of spatial variation in soil hydraulic conductivity K_{sat} on two-dimensional surface runoff through 2000 synthetic rainfall-runoff events, all generated using actual rainfall events observed on a 12ha agricultural catchment. The soil is assumed to be vertically homogeneous, but K_{sat} could vary according to a lognormal distribution on a 2D horizontal grid. The required parameter values are: (a) the mean of K_{sat}, (b) CV, the coefficient of variation of K_{sat}, and (c) L_xL_y (combination of L_x and L_y due to assumed isotropy) the correlation length of K_{sat}. Both authors extended the analysis of Monte Carlo simulations of runoff to event-based quantitative analysis of variance (ANOVA) to investigate the role of spatial variability in K_{sat}. The aim was to generate a large number of runoff realizations from a specific catchment and apply multivariate statistical analysis methods to study the effects of spatial variability on runoff generation. The steps involved are:

1. A two-dimensional distributed watershed model has already been developed and adapted to the watershed under study.
2. A large number of synthetic realizations of two-dimensional hydraulic conductivity fields with a specified range of means, variations and spatial correlations are generated.
3. The watershed model is run with each of the generated hydraulic conductivity fields to generate a corresponding large set of simulated runoff events. A measured rainfall event is used as input to the simulation.
4. Simple indices are calculated for each simulated event, i.e. peak flow rate and total runoff volume.
5. Multivariate statistical techniques (including ANOVA) are used to study the relationship between the statistical properties of hydraulic conductivity fields and peak flow and runoff volume indices.

They concluded that, the results confirm the relevant value of detailed spatial information on K_{sat}. To this end, the study had shown that, within the catchment, field K_{sat} collection devices with a sampling distance of 50 m or less should be provided if reliable information is required on runoff characteristics, for example for modeling purposes. Samples at 10m intervals on similar fields with high variability may be useful when K_{sat} has a high coefficient of variation. Both Monte Carlo/ANOVA analysis techniques could be applied to any other catchment to determine the spacing of K_{sat} samples from which distributed models can benefit.

The results also suggest that if high variation is combined with a long correlation length, static hydraulic soil properties alone are not sufficient for a good model, but those with dynamic effects on catchment response need to be taken into account. For this purpose, a method with less restrictive assumptions, such as the linear mixed model, provides an analytical tool. ANOVA can estimate relative uncertainty, but cannot accurately identify specific control factors. The detailed results of our calculations (BIR Said, AIT AIDER Hacen, and Ali BOUHERAOUA, 2024) do not allow full comparisons with those of the two case studies cited above, but they do show that the statistical moments (means, standard deviations, and coefficients of variation) of the expected calculation results can vary by as much as a factor of two, depending on the scale of variability of the soil parameters taken into account during their random generation.

10. CONCLUSION

Probabilistic analyses of flow and transport problems in a partially saturated soil mass under different variability conditions, statistically homogeneous, one or two-dimensional show that the variability of the calculation result is significantly lower in the first two analyses than in the last, and that this result is accentuated by the multidimensional aspect of the variability of soil properties. In practical terms, a satisfactory estimate of the average evolution of pressure head seems to be obtained by a probabilistic calculation with two-dimensional variability where the four (04) random parameters (K_{sat}, θ_{sat}, α, and η) are characterized by an average according to a Log-Normal distribution law of the corresponding data. This finding implies that the discrepancies between predictions and measurements of the problem of flow and transport in porous media should not be sought primarily in the influence of the statistical variability of the soil parameters but in the bias (non-representativeness) of the values of these parameters estimated within the framework of geotechnical reconnaissance.

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